Creep modelling of ductile fibre reinforced composites

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ABSTRACT: A ductile cement-based composite can be achieved by using short, randomly distributed polymer fibres in a cement-based matrix. This type of composite is known as ECC (Engineered Cement-based Composites). The advantage of this material is that it shows pseudo strain hardening if loaded in tension. The time-dependant behaviour is still an uncertainty for this type of material. Creep, a time-dependant phenomenon of cement-based composites, could cause a structure to fail at a lower than expected load if the load is sustained for a period of time. This is known as creep fracture. The time scale of creep fracture is of vital importance as it could range from a few minutes to hundreds of years. This paper investigates the phenomenon of creep and creep fracture in a computational framework. This is done at a macro level using a homogeneous material model incorporating non-proportional creep for the behaviour of ECC.

Keywords: Cement-Based Composites, Fibre Reinforced Cements, Creep, Computational Modelling

1 INTRODUCTION

Highly ductile fibre reinforced cement-based composites, also known as ECC (Engineered cement-based composites), has become an appealing possibility in the field of construction (Li et al, 2001).

It does not only offer improved durability, but also the ability to absorb orders of magnitude more energy before failing, than normal concrete, or even more than standard fibre reinforced concrete/cements (FRC). Thus it proves to be very useful in high impact structures and buildings in a seismic active region. Other uses with smaller dimensions are also being investigated, e.g. pipes (Aldea et al, 1998) and repair layers (Kabele 1999.)

The prediction of medium to long term behaviour of concrete is an important capability, as the durability of structures is an important aspect that needs to be taken into account in the design process. The time-dependant behaviour is still an uncertainty for ECC.

Creep, a time-dependant phenomenon of cement-based composites, could cause a structure to fail at a lower than expected load if the load is sustained for a period of time. This is known as creep fracture. The time scale of creep fracture is of vital importance, as it could range from a few minutes to hundreds of years. Furthermore, time-dependant loss of the fibre bond can result in a loss of ductility over a period of time. These unknowns have to be addressed before accurate estimations can be made for the reliable design of structures containing ECC materials.

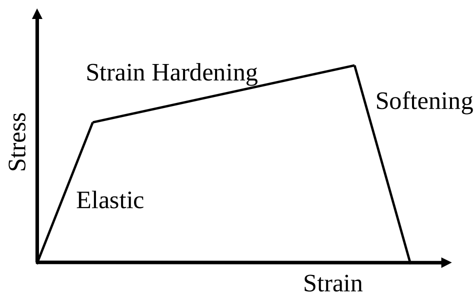

Figure 1. ECC response to uni-axial tensile strain.

Research to a great extent has been done in the modelling of fracture mechanics using finite element methods, for example Bažant and Oh (1983), Rots (1988). This gives the ability to predict the behaviour of a structure before, during and after fracture without doing a destructive test. An assumption is mostly made about the time-dependant behaviour. It is normally neglected and a static analysis is done, thus ignoring an important property of concrete, namely creep and shrinkage.

Creep is a complex phenomenon of a cement-based composite. It plays an even more important role in an ECC material, as normally more than 60% of a typical mix of ECC is cement paste (Li et al, 2001), which is predominantly prone to creep and shrinkage, as shown by Billington et al (2003). This accentuates the importance of a study on the time-dependant behaviour of ECC.

Shrinkage occurs mostly in the early age of concrete. It is acknowledged that shrinkage is a major source of damage and durability impairment to certain classes of concrete application, for instance industrial floors. It will be incorporated in this research project at a later stage for comprehensiveness, but the current focus is on the creep fracture phenomenon

A cement-based composite is, as the name states, a composite of different types of material, thus anything but a homogeneous material. If an accurate prediction of the material is to be made, an analysis of the meso-structure will have to be done, for example (Gitman, 2003). Typically, at a meso-structure level, the aggregates, cement paste, fibres and the interfaces are defined. Due to the complexity of this analysis and the current lack of understanding of the time-dependant behaviour of ECC, a quasi-static analysis is attempted here on the macro-structure level, where the material is analysed as a homogeneous material. The final aim of this research project, however, is to perform multi-level quasi-static analysis, incorporating the meso-level into the macro-level.

Where creep fracture has been attempted with a cement-based material, creep has been incorporated as a visco-elastic phenomenon (Van Zijl et al, 2001, Boshoff et al, 2003). This was found to be inadequate to predict the correct time scale of fracture and a calibration factor was introduced, namely the crack mouth opening rate (CMOR). This concept has been used for ECC (Boshoff et al, 2003). It was found that a better description of the time-dependant behaviour is required if a good prediction is to be given for time-dependant behaviour of a structure, especially predicting the time-scale to failure in creep fracture. One particular deficiency is the indirect characterisation of the CMOR parameters. This has motivated the current further study of creep on a macro-level. A physically more appealing approach is followed, by separating irreversible viscous flow associated with micro-cracking from the description of macro-cracking. Thereby, it is attempted to incorporate all rate effects in the creep description. In the process, the determination of the creep fracture modelling parameters is rationalised.

2 CREEP

Creep can be defined as the increase of strain of a structure under a constant load or stress over a period of time. Many theories have seen the light in the past century to describe the mechanisms of creep. The theories have been narrowed down by scholars to a combination of three mechanisms.

Firstly, creep is caused by the migration of the ever present water in the micro-pores in the matrix. This movement is provoked if stress is applied to the material. The second is breaking and re-establishing of bonds on the micro-level. This is believed to be a result of the first mechanism. Lastly, the formation of micro cracks in the matrix is known to cause the non-linearity of creep (Neville, 1970).

A typical strain-time graph of creep is shown in figure 2. The load is kept constant for a period of time. As seen on the graph, the creep strain increases rapidly in the early days after loading and then flattens to an asymptote called the creep limit of the material. Although the total creep deformation continuously increases in time, it is usually considered to stabilize after several years.

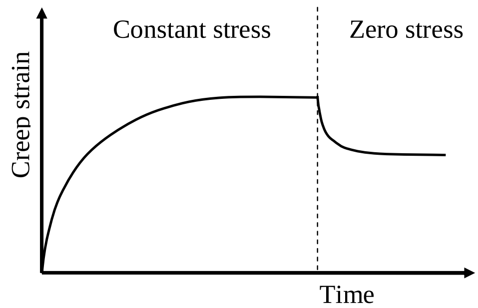

Figure 2. Creep behaviour with time

At unloading, a certain part of the creep strain is recoverable. Typical values of the recoverable creep strain are between 10 and 25% of the total creep strain (Neville 1970). The rest of the creep deformation is thus a permanent deformation.

To distinguish between recoverable creep and permanent creep, Ishai (1962) unloaded creep specimens at different stages and then subtracted the recoverable creep from the permanent creep. This is shown graphically in figure 3. In the context of this paper, the recoverable creep is called primary creep, while the permanent creep will be known as secondary creep.

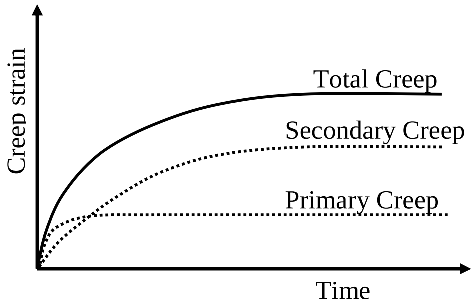

Figure 3. Components of creep strain

The magnitude of stress applied to a structure plays a vital role in the amount of creep strain that occurs. Figure 4 shows the effect of the load, as a ratio to the ultimate strength, has on creep strain.

Up to a certain load ratio, referred to as the non-linear creep load ratio (η) in this paper, the creep strain is proportional to the load. Past this point the creep strain increases exponentially with an increase in load. This non-linearity is believed to be due to micro cracks forming in the matrix and is not recoverable. This is eventually the mechanism that causes creep fracture. Another part of creep is identified and referred to in this paper as tertiary creep.

3 PRIMARY CREEP:

As mentioned earlier, the phenomenon of creep has been implemented numerically before as visco-elasticity. This is not ideal because of the full unloading of visco-elastic strain. This does not hold true to the phenomenon of creep, as only part of the creep strain is recoverable. Visco-elasticity is implemented in the proposed model only for the recoverable part of creep.

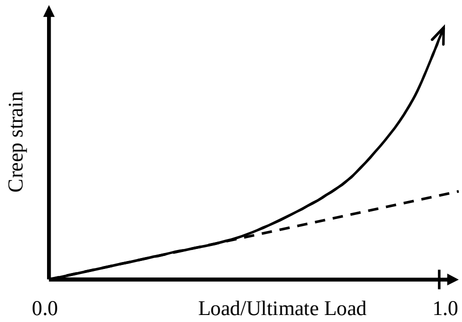

Figure 4. Effect of a load ratio on the creep strain.

Visco-elasticity can be implemented using a rheological model - either that of a Maxwell chain, figure 5, or a Kelvin chain as shown in figure 6. E is the stiffness of the spring and η the viscosity of the damper. A Maxwell chain is ideally suited for relaxation data and a Kelvin chain for creep data.

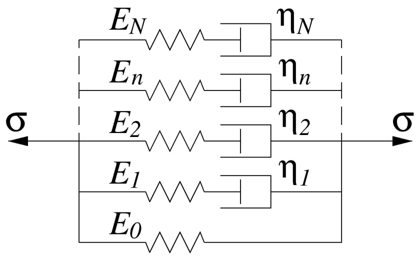

Figure 5. A Maxwell chain

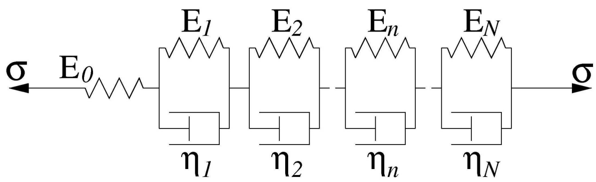

Figure 6. A Kelvin chain.

In the proposed model a Maxwell chain is implemented. Only creep data for ECC is available, thus a relaxation analysis is done with a Kelvin chain in order to be able to fit it to a Maxwell chain.

The typical equations for Maxwell, Equation (1) and Kelvin chains, Equation (2), expressed uniaxially, are:

$$\sigma = \left(E_o + \sum_{i=1}^{n} E_i e^{-\frac{t}{\tau_i}} \right) \varepsilon \qquad (1)$$

$$\varepsilon = \sigma\left(\frac{1}{E_0} + \sum_{i=1}^{n}\frac{1}{E_i}\left(1 - e^{-\frac{t}{\tau_i}}\right)\right) \tag{2}$$

with E_i the stiffness of the specific element, τ_i the relaxation time, expressed as the stiffness divided by the viscosity and n the number of elements.

4 SECONDARY AND TERTIARY CREEP

Both secondary and tertiary creep are not recoverable, thus visco-elasticity is not a suitable implementation. A new approach is needed to model secondary and tertiary creep.

Constitutive isotropic plasticity is chosen to model this part of creep for its features of an activation stress and the plastic deformation which is permanent. It has been shown that creep occurs even at relative small stresses (Neville, 1970). Therefore, it can be deduced that creep does not have an activation stress level. The activation stress level is chosen as zero at the start of loading, but changes due to changes in the load, time and stress history (H).

$$\varepsilon_{Creep} = f(\sigma, t, H) \tag{3}$$

The creep strain after an increment in a numerical analysis is a function of the stress, time and the stress history, see Equation (3). A formulation needs to be devised to be able to compute the amount of creep strain at a specific point in time taking into account the current stress, current time and importantly, the stress history. A simple, but effective method has to be derived to represent the stress history, as it is unpractical to keep record of each stress and time increment and then to compute the new creep strain taking each increment into account.

An assumption is made that superposition can be applied to creep strains. This has been proved valid (Boltzmann, 1876). This implies that the creep strain is the sum of the creep strain caused by each stress and time increment separately. This is explained graphically for two increments in figure 7.

The basic shape of creep strain is assumed to be represented by Equation (4). C_1 defines the total creep that occurs for 1 stress unit, also know as the creep compliance, while C_2 defines the rate the creep occurs. This, however, can not represent the typical shape of creep. Thus, a sum of N terms of

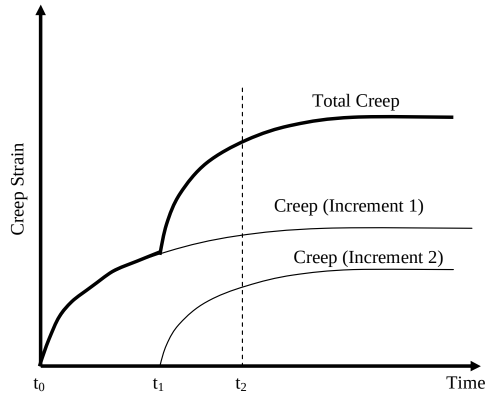

Figure 7. Principal of superposition.

Equation (4) is used, each with different values for C_1 and C_2 as shown in Equation (5).

$$\varepsilon = \sigma.C_1.(1 - e^{-C_2.t}) \tag{4}$$

$$\varepsilon = \sigma.\sum_{N}(C_1(1 - e^{-C_2 t})) \tag{5}$$

For the stress for a specific time increment, t_0 to t_1, and stress increment, σ_0 to σ_1, the equation becomes:

$$\varepsilon_1 = (\sigma_1 - \sigma_0).\sum_{N}(C_1.\varphi.(1 - e^{-C_2(t_1 - t_0)})) \tag{6}$$

This holds true for secondary creep, which is proportional to the stress. If tertiary creep is to be incorporated in this formulation, C_1 has to be increased to include the tertiary creep. For this C_1 is multiplied by a factor, φ, which is introduced to incorporate the non-linearity in this formulation.

$$\varphi = 1 \qquad for \qquad \frac{\sigma_1}{\sigma_u} < \eta \tag{7}$$

$$\varphi = 1 - A + Ae^{B.(\frac{\sigma_1}{\sigma_{ultimate}} - \eta)} \quad for \quad \frac{\sigma_1}{\sigma_u} \geq \eta \tag{8}$$

Where σ_1 is the current stress, σ_u is the ultimate static stress, η the non-linear creep load ratio and A and B two constants defining the shape of the non-linearity. In this implementation A and B are the same for each term in the summation. For simplification of the development of the formulation only one term of the summation is exploited as shown in Equation (9).

$$\varepsilon_1 = (\sigma_1 - \sigma_0).C_1.\varphi.(1 - e^{-C_2(t_1 - t_0)}) \qquad (9)$$

For a next increment, t_1 to t_2, the stress is increased from σ_1 to σ_2. The total creep strain, ε_2, is shown in Equation (10).

$$\varepsilon_2 = (\sigma_1 - \sigma_0).C_1.\varphi_1.(1 - e^{-C_2(t_2 - t_0)})$$
$$+ (\sigma_2 - \sigma_1).C_1.\varphi_2.(1 - e^{-C_2(t_2 - t_1)}) \qquad (10)$$

This simplifies to:

$$\varepsilon_2 = e^{-C_2 t_2}\left[(\sigma_0 - \sigma_1).C_1.\varphi_1.e^{C_2 t_0} + \right.$$
$$\left.(\sigma_1 - \sigma_2).C_1.\varphi_2.e^{C_2 t_1}\right] +$$
$$\left[(\sigma_1 - \sigma_0).C_1.\varphi_1 + (\sigma_2 - \sigma_1).C_1.\varphi_2\right] \qquad (11)$$

At a given increment the stress and time at the start and end of the increment is known. The parameters of the first term between each square bracket is known at the end of the first increment, while the parameters of second term in each square bracket is known at the end of the second increment, without needing any information about the previous increments. If another increment is added, a third term would arise with the same property, that it is all the parameters of that term is known at the end of that term without needing any information about the previous increments. The equation is simplified to produce Equation (12).

$$\varepsilon_2 = e^{-C_2 t_2}\left[H_1 + (\sigma_1 - \sigma_2).C_1.\varphi_2.e^{C_2 t_1}\right] +$$
$$\left[H_2 + (\sigma_2 - \sigma_1).C_1.\varphi_2\right] \qquad (12)$$

Two new variables are found in Equation (12), namely H_1 and H_2. These are two history variables, which encapsulate the stress history. The update of the history variables after the second increment is as follows:

$$H_{1New} = H_1 + (\sigma_1 - \sigma_2).C_1.\varphi_2.e^{C_2 t_1} \qquad (13)$$

$$H_{2New} = H_2 + (\sigma_2 - \sigma_1).C_1.\varphi_2 \qquad (14)$$

Thus, a formulation is derived where linear and non-linear creep can be modelled in a continuum where the applied stress can change with time. This formulation uses only two variables per summation term to represent the stress history. The complete formulation is shown in Equation (15).

$$\varepsilon_2 = \sum_N \left[e^{-C_2 t_2}\left[H_1 + (\sigma_1 - \sigma_2).C_1.\varphi_2.e^{C_2 t_1}\right] + \right.$$
$$\left.\left[H_2 + (\sigma_2 - \sigma_1).C_1.\varphi_2\right]\right] \qquad (15)$$

This formulation can be used for calculating the yielding criteria for a plastic constitutive implementation for a plane stress element. A Rankine yield surface is used for the compression and tension zones. This is shown in figure 8.

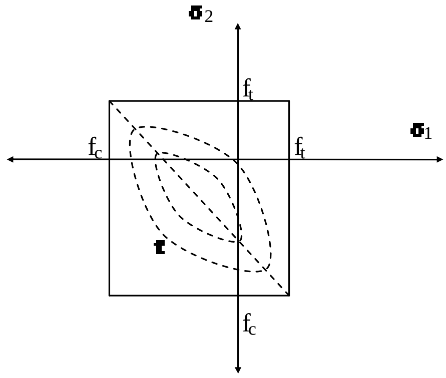

Figure 8. Yield surface used for constitutive plasticity.

To simplify the implementation, the assumption of isotropic creep is made. It is, however, possible to distinguish between compressive and tensile creep. This assumption results in the effect that creep can occur in compression and tension independently.

5 FRACTURE:

This proposed model tries to bring forth a rate effect without incorporating a rate term to adjust the ultimate strength of the material. With the non-linear creep incorporated, the rate effect and the correct time-scale to failure in creep fracture would hopefully stem forth. This can be done if the criterion for fracture is not an ultimate stress, but an ultimate strain. The post peak behaviour is then also determined by the strain. This has the effect that fracture under uni-axial stress always occurs at the same strain. If the material is loaded at a higher rate, there would be less creep and it would have a higher stress than if the material was loaded with a lower rate with more creep. Thus, the higher strain rate would cause a higher ultimate strength in the matrix. This holds true for concrete (Wu and Bažant, 1993, Zhou, 1992) and will be studied experimentally for ECC further along in this project. This is shown in graphically in figure 9.

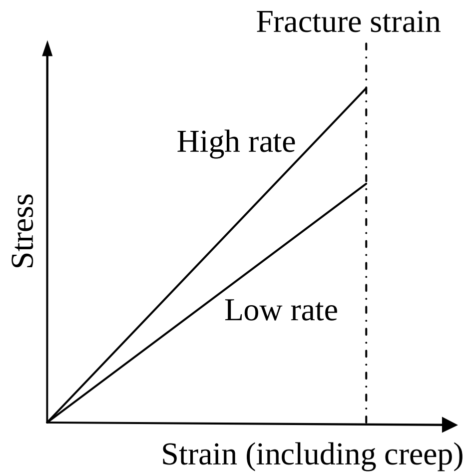

Figure 9. The effect of strain rate on the fracture stress.

The effect of the strain rate on the strain hardening part of ductile fracture is still unknown. To the knowledge of the authors, no research has been done on the effect of strain rate on the fibre strength and bond. The assumption is made here that the rate of loading has no effect on the strength or the ductility of the fibres. The proposed fracture curve of ECC in tension at different rates can be seen in figure 10. Note that even if the rate decreases and the matrix strength is reduced, the strength of the fibres stays the same. This is can easily be altered if evidence is produced of rate-dependent ultimate strength of ECC.

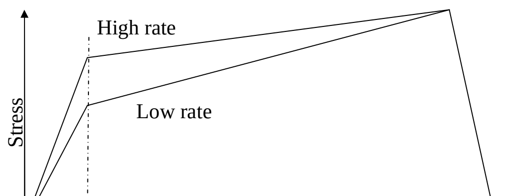

Figure 10. The post fracture behaviour of ECC.

Fracture is implemented using an isotropic damage formulation. The basic equation for this in one dimension is:

$$\sigma = (1-\omega).E.\varepsilon \qquad (16)$$

and for a plane stress application:

$$\overline{\sigma} = (1-\omega).\overline{\overline{D}}.E.\overline{\varepsilon} \qquad (17)$$

with ω the damage factor ranging from 0 (no damage) to 1 (complete damage). Secant unloading for this part of the constitutive law is implied, while the creep formulation accounts for time-dependent unloading to the irrecoverable viscous strain.

The effect of the damage model limit function can be seen in figure 11. It is implemented in terms of principal stress. The vector in the figure represents the current stress state. The damage variable is adjusted to reduce the stress to the intersection of the limit surface, as shown in the graph. A Rankine limiting surface is chosen for compression as well as tension for simplicity. The biaxial strength enhancement in compression will be considered in future work, for instance by considering a Hill-type limit function in compression (Van Zijl et al. 2001)

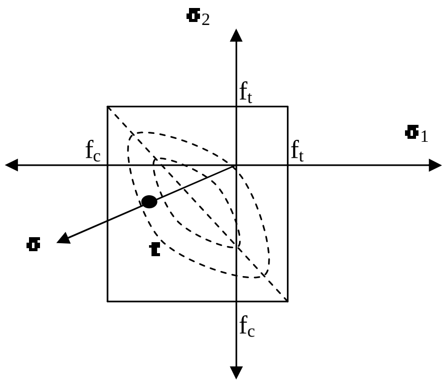

Figure 11. Reduction of stress vector due to damage.

6 CONCLUSIONS

A model has been proposed for the computational simulation of time-dependent behaviour of cement-based material, with special attention for ductile fibre reinforced composites. The model incorporates reversible and irreversible creep separately. Most importantly, it captures micro cracking through a plasticity-based formulation of tertiary creep. Macro cracking is captured by computational damage. For viable application in structural analysis, the macroscopic level has been chosen for the formulation.

The model has been designed to capture the main features of time-dependent behaviour of cement-based composites, in order to accurately simulate the time scale of fracture. This is essential for life-cycle analysis of structures fabricated of these materials. Consideration of the various components of creep separately has led to a simple formulation of non-proportional creep, which does not require complicated and computationally expensive accounting of the loading history. It is believed that the non-proportional creep will encapsulate the rate-effect in the fracture process zone of these materials.

To validate this model and produce data for characterisation of the model parameters, creep tests on ECC are planned. Parallel to the creep tests, creep fracture tests will be executed to experimentally determine the time to failure and compare it to the values predicted by finite element analyses incorporating the constitutive material model proposed in this paper. Creep fracture at different load levels will be determined to characterise the time to failure.

Furthermore, the meso-level will be modelled to study mechanisms of the features captured by the current, proposed continuum model. On this level the time-dependent straining and slip of fibres from the matrix will be studied both computationally and experimentally, to verify/modify the current rate independence in the continuum model.

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