Assessment of the residual fatigue strength in RC beams

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ABSTRACT: In conventional analysis and design procedures of reinforced concrete structures, the ability of concrete to resist tension is neglected. Under cyclic loading, the tension-softening behavior of concrete influences its residual strength and subsequent crack propagation. The stability and the residual strength of a cracked reinforced concrete member under fatigue loading, depends on a number of factors such as, reinforcement ratio, specimen size, grade of concrete, and the fracture properties, and also on the tension-softening behavior of concrete. In the present work, a method is proposed to assess the residual strength of a reinforced concrete member subjected to cyclic loading. The crack extension resistance based approach is used for determining the condition for unstable crack propagation. Three different idealization of tension softening models are considered to study the effect of post-peak response of concrete. The effect of reinforcement is modeled as a closing force counteracting the effect of crack opening produced by the external moment. The effect of reinforcement percentage and specimen size on the failure of reinforced beams is studied. Finally, the residual strength of the beams are computed by including the softening behavior of concrete.

1 INTRODUCTION

Reinforced concrete members subjected to cyclic loading may exhibit both stiffness and strength degradation depending on the maximum amplitude and the number of cycles experienced by the member. Most of the models available currently simulate the cycle dependent stiffness loss that is observed in experiments. The well known Park and Ang (1985) model defines a damage index which is expressed as a linear relation with displacement ratio and absorbed cyclic energy, to describe the hysteretic damage in reinforced concrete members. Garstka et al. (1993) have defined damage indicator in terms of energy ratios, for computing the stiffness loss due to inelastic deformation under earthquake loading. These models are based on elastic-plastic response of RC members, henceforth do not discuss on cracking behavior of concrete. In general, it is accepted that highly reinforced beams that fail by steel yielding are mostly fracture-insensitive. So, structures of these type have not been much investigated from the viewpoint of fracture mechanics. However, there are situations in which fracture plays a role, e.g. failure of normally and lightly reinforced beams (Bazant and Planas 1998). In particular, if crack forms within the tensile zone of RC beams due to fatigue loading, it provokes unstable behavior, which may introduce snap back response in the post peak region. For avoiding such a situation, the criterion to compute the minimum reinforcement for concrete members under flexure, is determined through fracture mechanics approach (Bosco et al. 1990), as the condition for which first concrete cracking and steel yielding are simultaneous. All LEFM models that are currently available in the literature have roots in the model proposed by Carpinteri (1981). In the LEFM based models, to compute the fracture moment, it is assumed that the $K_I = K_{Ic}$, and steel yielding occurs simultaneously. In the present approach, the assumption of steel yielding is not considered. The limiting criterion is assessed in terms of the tip opening displacement, hence the presence of the process zone is incorporated in the formulation through softening laws. Further, the stability of the crack propagation is also determined using crack extension based approach.

2 FRACTURE MECHANICS BASED MODEL OF REINFORCED CONCRETE BEAM

Based on Carpinteri's (1981) seminal work on LEFM models of reinforced concrete beam, the member with a crack of length a subjected to bending is approximated by a beam subjected to the bending moment and to the steel force applied remotely from the crack plane as shown in Figure 1(a), (b) and (c). Next the steel action is decomposed in a standard way into a bending moment and a centric force. Under the action of applied moment M, the steel force is a statically undetermined reaction. Carpinteri assumed that the crack remains closed while the steel is in elastic regime. Therefore, the crack growth takes place only when the steel yields and simultaneously $K_I = K_{Ic}$. Using this condition, the unknown steel force can be computed as described by Carpinteri (1984). With these conditions, the parametric equations of the moment rotation curves could be obtained easily. The limitation of this model in terms of crack closing in elastic regime of the reinforcement was removed by Baluch et al. (1992) and by Bosco and Carpinteri (1992). In the present work, the model proposed by Bosco and Carpinteri (1992) is used. They have modified an earlier model by letting the force of the reinforcement act on the crack faces rather than remotely from the crack plane as shown in Figure 1 (d). Hence, it is no longer necessary to assume that the crack is closed everywhere while the steel is elastic; it is enough to assume that the crack is closed at the point where the reinforcement crosses it. Hence the condition for obtaining the unknown steel force in the elastic regime can be written as,

$$w_s = (w_s)_M - (w_s)_S = 0 \tag{1}$$

where w_s is the crack opening at the level of steel bar; $(w_s)_M$ and $(w_s)_S$ are the crack opening due to bending moment and closure forces exerted by the reinforcement respectively. These are computed using the following relations (Alaee and Karihaloo 2003);

$$(w_s)_M = \lambda_{SM} M \tag{2}$$

and

$$(w_s)_S = \lambda_{SS} F_S \tag{3}$$

where λ_{SM} and λ_{SS}, the compliance coefficients due to unit moment M and unit steel force F_S, can be written as,

$$\lambda_{SM} = \frac{2}{BDE} \int_{C_s}^{a} Y_M\left(\frac{x}{a}\right) F_1\left(\frac{x}{a}, \frac{a}{D}\right) dx \tag{4}$$

and

$$\lambda_{SS} = \frac{2}{BE} \int_{C_s}^{a} F_1\left(\frac{x}{a}, \frac{a}{D}\right)^2 dx \tag{5}$$

where C_s is the clear cover to the steel bar. In the above Equations, $Y_M\left(\frac{x}{a}\right)$ and $F_1\left(\frac{x}{a}, \frac{a}{D}\right)$ are the geometry factors expressed as follows:

$$Y_M(\alpha) = \frac{6\alpha^{1/2}(1.99 + 0.83\alpha - 0.31\alpha^2 + 0.14\alpha^3)}{(1-\alpha)^{3/2}(1+3\alpha)} \tag{6}$$

where $\alpha = a/D$ is the relative crack depth;

$$F_1\left(\frac{x}{a}, \alpha\right) = \left\{\frac{3.52(1-\frac{x}{a})}{(1-\alpha)^{3/2}} - \frac{4.35 - 5.28\frac{x}{a}}{(1-\alpha)^{1/2}} + \left[\frac{1.3 - 0.3(\frac{x}{a})^{3/2}}{\sqrt{1-(\frac{x}{a})^2}} + 0.83 - 1.76\frac{x}{a}\right] \left[1 - \left(1 - \frac{x}{a}\right)\alpha\right]\right\} \frac{2}{\sqrt{\pi\alpha}} \tag{7}$$

Once the unknown steel force is computed using the above relation, based on the principle of superposition the stress intensity factor can be expressed as a summation of K_{IM} and K_{IF} as,

$$K_I = K_{IM} - K_{IF} \tag{8}$$

where K_{IM} and K_{IF} are the stress intensity factors produced by the bending moment and steel force, which can be written as follows:

$$K_{IM} = \frac{M}{BD^{3/2}} Y_M(\alpha) \tag{9}$$

and

$$K_{IF} = \frac{F_S}{BD^{1/2}} F_1\left(\frac{C_s}{D}, \frac{a}{D}\right) \tag{10}$$

In the present study, the above described model is used for determining the condition for unstable crack propagation based on the crack extension resistance approach. The tension softening behavior of concrete in the post peak region is also incorporated in the analysis. The detailed description is given in the subsequent section.

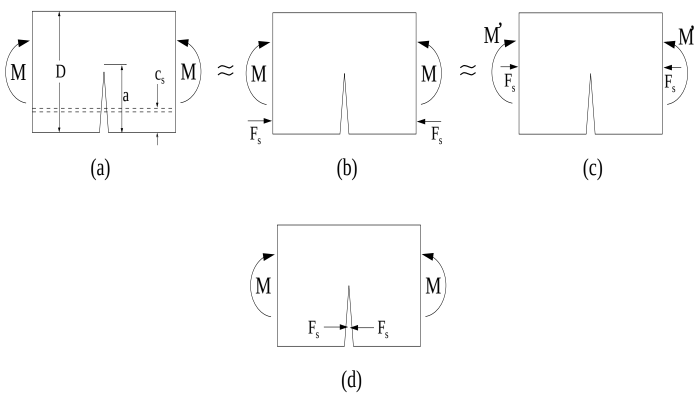

Figure 1: (a), (b), (c), Carpinteri's LEFM approximation for RC beam; (d) Bosco and Carpinteri's Modification

3 DETERMINATION OF FRACTURE STABILITY CRITERION: CRACK EXTENSION RESISTANCE BASED APPROACH

The residual strength assessment of a reinforced concrete member essentially involves the determination of critical condition with respect to fracture failure. In the present study, the condition for unstable crack propagation is found out based on the crack extension resistance based approach. The crack extension resistance based approach originally proposed by Reinhardt and Xu (1998) for plain concrete specimen, in which the crack extension resistance is computed considering the effect of cohesive forces within the process zone. The basic principle of the approach is that the crack extension resistance is composed of two parts. One part is the inherent toughness of the material, which resists the initial propagation of an initial crack under loading, and is denoted as K_{Ic}^{ini}. The cohesive force distributed on the fictitious crack during crack propagation gives another part of the extension resistance. Therefore, it is a function of the cohesive force distribution $f(\sigma)$, tensile strength f_t of the material and the length a of the propagating crack, which can be written as follows,

$$K_R(\Delta a) = K_{Ic}^{ini} + K^c(f_t, f(\sigma), a) \tag{11}$$

The inherent initiation toughness K_{Ic}^{ini} for a standard three-point bending beam can be computed using

$$K_{Ic}^{ini} = K(P_{ini}, a_0) = \frac{3P_{ini}L}{2BD^2}\sqrt{\pi a_0}g_1\left(\frac{a_0}{D}\right) \tag{12}$$

where P_{ini} is the initial cracking load; a_0 is the initial notch length; L, B, D is the span, width and depth of the beam respectively and $g_1(a_0/D)$ is the geometric factor.

Similarly, the general expression of the crack extension resistance due to cohesive force is given by (Reinhardt and Xu 1998),

$$K^c(f_t, f(\sigma), a) = \int_{a_0}^{a} 2\sigma(x)F_1\left(\frac{x}{a}, \frac{a}{D}\right)/\sqrt{\pi a}dx \tag{13}$$

where F_1 is the geometry factor as defined in Equation 7. In the above Equation $\sigma(x)$ is the assumed stress distribution within the fracture process zone. In the present study, the three idealizations for the traction-separation law are considered in order to determine the crack extension resistance and the corresponding critical crack length for which unstable fracture takes place. In available literature, the post-peak softening behavior has been mathematically modeled by different investigators using linear, bilinear, power-law and other relationship depending on the trend followed by experimental results. In this work, we consider the effect of linear, bilinear and power law softening behavior on the fatigue strength of reinforced concrete beams. Amongst these, the simplest approximation is the linear softening relation as proposed by Hillerborg et al. (1976), and stress at any point in the process zone is considered to be a function of the crack opening only. Mathematically, the linear softening relation can be written as (Hillerborg et al. 1976),

$$\sigma = f_t\left(1 - \frac{w}{w_c}\right) \tag{14}$$

where f_t is the tensile strength, w the crack opening displacement and w_c the critical crack opening displacement.

Similarly, the bilinear softening behavior can be mathematically expressed as,

$$\sigma = \begin{cases} f_t - (f_t - \sigma_1)w/w_1 & w \leq w_1 \\ \sigma_1 - \sigma_1(w - w_1)/(w_c - w_1) & w > w_1 \end{cases} \tag{15}$$

where w_1 is the opening displacement when the softening curve changes slope due to bi-linearity and the corresponding stress is σ_1.

The power function suggested by Reinhardt (1984) is given by,

$$\sigma = f_t\left[1 - \left(\frac{w}{w_c}\right)^n\right] \tag{16}$$

where n is an index which is assumed to be 0.248 based on experimental calibrations. After a crack starts from a notch, the size of the fracture process zone grows as the crack advances. The consequence is that the crack resistance K_R to propagation increases. The condition for crack propagation within a member is considered when $K_R(\Delta a)$ equals K_I. K_{IP} is the mode I stress intensity factor under the external loading P, which for a RC beam under three-point bending is given by Equation 8 together with 9 and 10. The important point to be noted here is that, the effect of reinforcement is not considered in terms of resistance, instead it is incorporated while evaluating the stress intensity factor. This is done because, the steel force depends on the applied external moment, therefore not an inherent property of the material.

Further, to determine the instability condition, the well known concept of fracture equilibrium is used (Bazant and Cedolin 1998). If the fracture equilibrium state is unstable, the crack will propagate by itself. Formally, these conditions can be stated as follows:

$$\begin{aligned} K'_R(\Delta a) - K'_{IP} &> 0 \Rightarrow \quad stable \\ K'_R(\Delta a) - K'_{IP}(P) &= 0 \Rightarrow \quad critical \\ K'_R(\Delta a) - K'_{IP}(P) &< 0 \Rightarrow \quad unstable \end{aligned} \tag{17}$$

where the primes in $K's$ indicate the slope of the quantities concerned. From the above conditions it turns out that, when the slope of the resistance curve

Table 1: Details of the RC beam

Depth	150 mm
Width	100 mm
Length	1200 mm
Steel area	113.09 mm^2
Yield stress	544 MPa
E	35.6e3 MPa
f_{ck}	45 MPa
f_t	3.75 MPa
G_F	0.0725 N/mm
w_c (Linear)	0.037 mm
w_c (Bi-linear)	0.094 mm
w_c (Linear)	0.067 mm

is lesser than the slope of the K_I curve, unstable crack propagation takes place. In the present study, a reinforced concrete beam is considered for numerical validation of the above method. The details of the specimen geometry and fracture properties are listed in Table 1. The specimen was originally used by Alaee and Karihaloo (2003) for their study on CARDIFRC.

The proposed method of obtaining the instability condition is applied for the RC beam under consideration, for three different values of external moment as $M = 2E6, 3.5E6$ and $6E6Nmm$ as shown in Figure 2. In this particular case, only linear softening is assumed. It is seen that, for the lowest value of M, the slope of the resistance curve remains higher than the slope of K_{IP} curve throughout the assumed crack length regime; hence the crack propagation is stable for this case. For $M = 3.5e6$ N-mm, the resistance curve intersects the stress intensity factor curve at a point $\alpha = 0.418$, and K'_R is lesser than K'_{IP} for α greater than 0.418. Hence, according to the stability condition stated above, the crack propagation remains stable upto relative crack depth of 0.418, becomes critical at that particular value of α, and the unstable region follows in case of further crack propagation. For the M value of $6e6N - mm$, the resistance is always lesser than the stress intensity factor throughout the crack propagation region resulting in an unstable fracture phenomenon. It can be concluded that as the applied moment value increases the α value corresponding to fracture instability decreases. The study is further extended to determine the influence of different softening approximations as described by Equation 14, 15 and 16, on the computation of instability limits. Figure 3 shows the K_R, K_{IP} curves obtained considering the three softening laws under the external moment of $3.5e5N - mm$. Since, the external load remains constant, and the softening does not take part in K_{IP} computation, the K_{IP} curves are the same for all the three cases. Only the resistance curve will depend on the softening approximations. The comparative study reveals that the linear

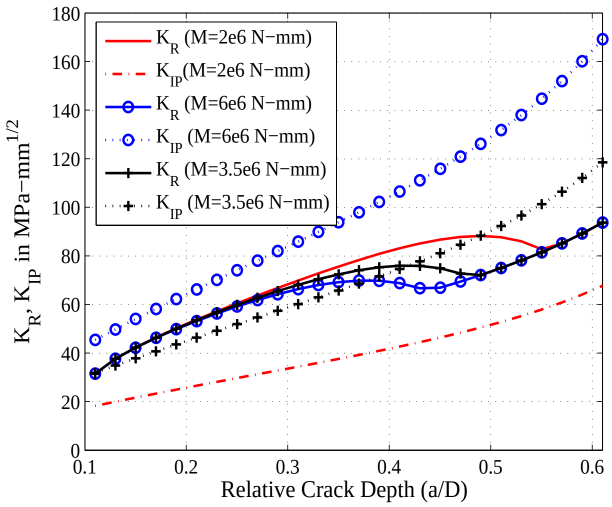

Figure 2: K_R, K_{IP} for Linear Softening

softening predicts highest value of $\alpha_C = 0.418$ corresponding to unstable condition, followed by bilinear ($\alpha_C = 0.375$) and power-law ($\alpha_C = 0.195$). Therefore, one has to correlate the experimental data of unstable crack propagation with the numerical predictions and conclude about the ideal softening approximation, which would result into realistic prediction on stability condition. Once the critical value of α is determined, the residual strength of the RC beam has to be computed as a function of increasing crack length in the stable region. Before entering into the discussion of fatigue behavior, it is to be noted here, that a parametric study is performed on the steel percentage and the size of the specimen, to find out the influence of these factors on fracture stability issue. Figure 4 represents the K_{IP} and K_R curves for three different percentages of steel and considering linear softening. The resistance will be same for all the three cases and the SIF due to applied loading also does not vary with p unless steel yielding occurs. Therefore, the curves coincide with each other and intersect at a specific point. Hence, it can be concluded, that the reinforcement percentage does not have any perceivable effect on the stability criterion. The critical value of α remains unchanged for increasing as well as decreasing value of steel percentage. Whereas, a strong effect of specimen size is observed on the instability condition as predicted in Figure 5. Here, three different values of specimen depth are considered ($D = 75, 150$ and $300mm$). It is observed that as the specimen size decreases, the zone corresponding to stable crack propagation almost vanishes as observed for $D = 75mm$; where the resistance is always lesser than the SIF value.

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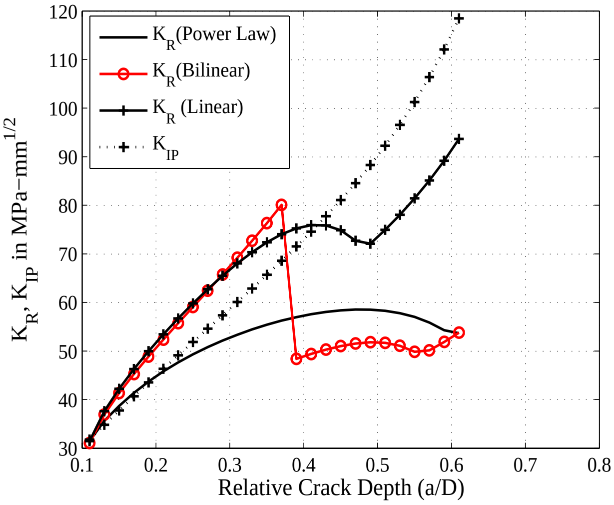

Figure 3: K_R, K_{IP} for Linear, bilinear and Power-law Softening

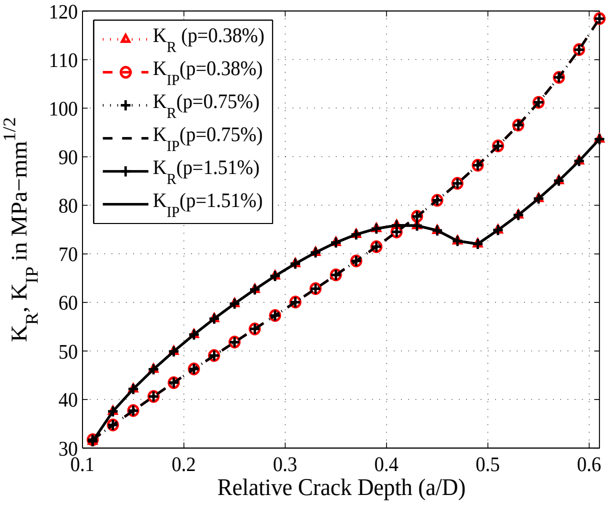

Figure 4: K_R, K_{IP} for three different percentage reinforcement

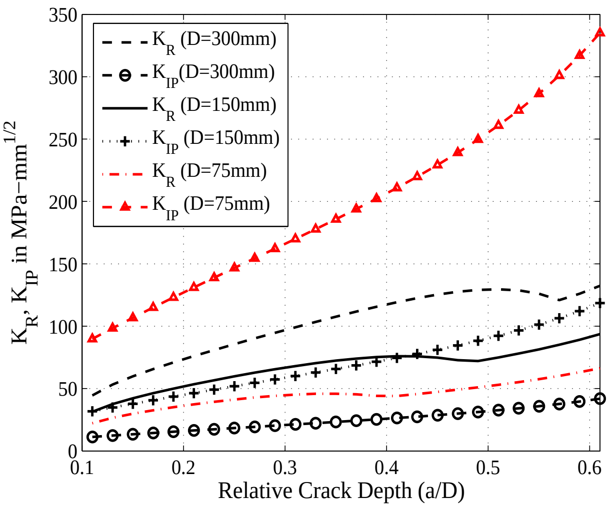

Figure 5: K_R, K_{IP} for three different depth of specimen

4 FATIGUE CRACK PROPAGATION IN REINFORCED CONCRETE

In the present study, to analyze the fatigue behavior of reinforced concrete, the LEFM based fatigue law as proposed by Slowik et al. (1996) is used, with suitable modifications to incorporate the effect of reinforcement. The fatigue law proposed by Slowik et al. (1996) for describing the complex phenomenon of crack propagation in concrete is given by:

$$\frac{da}{dN} = C\frac{{K_{Imax}}^m \Delta {K_I}^n}{(K_{IC} - K_{Isup})^p} + F(a, \Delta\sigma) \qquad (18)$$

where C is a parameter which gives a measure of crack growth per load cycle, K_{Isup} is the maximum stress intensity factor ever reached by the structure in its past loading history, K_{IC} the fracture toughness, K_{Imax} is the maximum stress intensity factor in a cycle, N is the number of load cycles, a is the crack length, ΔK is the stress intensity factor range, and *m, n, p*, are constants. These constant co-efficients are determined by Slowik et al. through an optimization process using the experimental data and are 2.0, 1.1, 0.7 respectively. In an earlier work, the above law is modified by the authors (Sain and Chandra Kishen 2004), to incorporate the effect of frequency of the cyclic loading, and the effect of overloads, the details of which are not repeated here.
In case of reinforced concrete beams, the stress intensity factor is considered to be a combined effect of applied loading and the tensile reinforcement as mentioned in Equation 8. The presence of reinforcement introduces a negative SIF (resistance to crack opening), which essentially reduces the crack propagation rate. When the steel is in the elastic regime, the unknown reaction force in steel is computed using the method as described in earlier section. The crack propagation procedure is applied for the considered specimen, for three different values of reinforcement percentage ($p = 0.36, 0.75, 1.51\%$). Figure 6 shows the $a - N$ curve obtained using the proposed method, for three different steel area under constant amplitude fatigue load with maximum moment of $3.5E6Nmm$ and a minimum value of zero. It is observed that the rate of crack propagation remains same for all the three p values, if the steel does not yield. The steel yielding occurs only for $p = 0.36\%$, at $\alpha = 0.5$, under the given loading condition. Hence, after $\alpha = 0.5$, the crack propagation rate differs from the other two case as shown in the Figure.

5 RESIDUAL STRENGTH ASSESSMENT

The tensile strength and toughness of concrete are usually disregarded in the strength assessment of reinforced concrete member. In the present study, the post-peak behavior is considered in terms of the

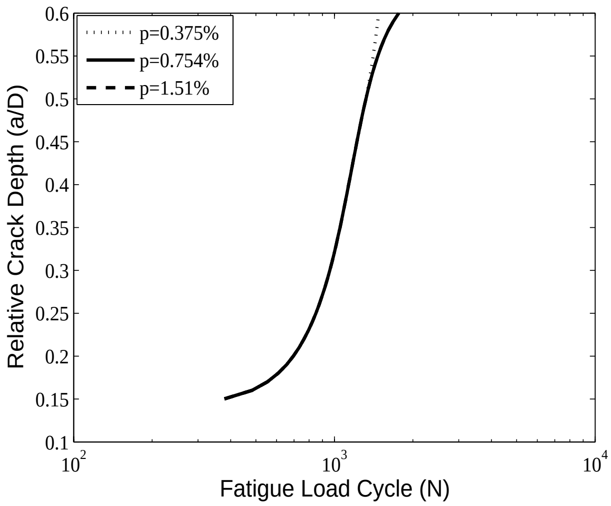

Figure 6: Fatigue crack propagation in RC beam

tension-softening law, as described earlier, for computing the residual capacity of a cracked RC beam. The available methods for determining the fracture moment, either assume the limiting condition as yielding of reinforcement or assume the length of the process zone. These assumptions are relaxed in the foregoing analysis. The criterion used for computing the ultimate moment capacity is the crack tip opening displacement, w at the tip of each incremental crack length reaching the critical crack tip opening displacement, w_c, which is a material parameter. Hence, the reinforcement does not necessarily reach yielding corresponding to all the crack length values. The following assumptions are made in the analysis regarding the stress-strain distribution along the cracked section:

1. Strain varies linearly across the depth of beam during bending.

2. The crack opening profile is linear.

3. The softening behavior is known in terms of cohesive force-crack opening law. Alternatively, an average strain ϵ_t on the continuum scale may be defined as representative of the opening displacement of the microcracks within an effective softening zone width h_s. In this way, an effective stress-strain constitutive relationship can be adopted in the spirit of the nonlocal continuum concept (Bazant and Oh 1983). The crack opening displacement in the discrete crack model and the post-peak strain in the continuum model are related by $w = h_s \epsilon_t$. In the present study, h_s is taken as $0.5D$, where D is the beam depth.

By fixing the limiting tip opening displacement, corresponding equivalent strain is calculated following assumption (3). The ultimate tensile strain corresponding to $w = w_c$ is denoted as ϵ_{tu}, and the strain corresponding to elastic limit (in other words ϵ for $w = 0$ or $\sigma = f_t$) is represented as ϵ_{tp}. The fracture process zone of length l_p is assumed to form in front of the crack tip. It comprises of the zone starting from the crack tip where $(w = w_c)$ or equivalently $\epsilon_t = \epsilon_{tu}$ and extending until $w = 0$ or $\epsilon_t = \epsilon_{tp}$. To compute the moment carrying capacity for the assumed strain distribution, an incremental procedure as proposed by Raghuprasad et al. (2005), is adopted. The method is based on the fundamental equilibrium equation for the progressive failure of concrete beams. The uncracked ligament portion $(d - \alpha d - l_p)$ as shown in Figure 7 is divided into a number of segments (say $10,000$); each having a segment of depth $\delta x = [(1-\alpha)d - l_p]/10,000$. To calculate the neutral axis depth factor k, a trial and error procedure is adopted. Knowing k, by the linearity assumption, l_p can be computed as,

$$l_p = \left(1 - \frac{\epsilon_{tp}}{\epsilon_{tu}}\right)(1 - k - \alpha)d \tag{19}$$

Hence, the resistance provided by the softening zone, (assuming linear softening behavior) can be expressed as,

$$T_s = \frac{1}{2} B l_p f_t \tag{20}$$

Next in the uncracked ligament portion, the stresses are calculated at each segment for the compressive strains $\epsilon_{c1}, \epsilon_{c2} \ldots \epsilon_{cm}$ and for the tensile strains $\epsilon_{t1}, \epsilon_{t2} \ldots \epsilon_{tn}$; where $m =$ number of segments in compression zone, and $n =$ number of segments in tension zone. Then the compressive forces $f_{c1}, f_{c2}, \ldots f_{cm}$ and the tensile forces $f_{t1}, f_{t2}, \ldots f_{tm}$ are calculated incrementally. The stress and strain in the tensile reinforcement can be computed as;

$$\epsilon_{st} = \epsilon_{tu} \frac{1-k}{1-\alpha-k} \tag{21}$$

and the stress, can be written as,

$$\sigma_{st} = E_s \epsilon_{st} \quad \leq f_Y \tag{22}$$

where, E_s is the elastic modulus of steel and f_Y is the yield stress. Steel is assumed to behave in an elastic perfectly plastic manner. Hence, the resistance provided by the reinforcement is expressed as,

$$T_{st} = \sigma_{st} A_{st} \tag{23}$$

where, A_{st} is the area of reinforcement. The depth of the neutral axis is calculated such that the total compressive force $(C = f_{c1} + f_{c2} + \ldots + f_{cm})$ equals the total tensile force $(T = T_s + f_{t1} + f_{t2} + \ldots + f_{tm} + T_{st})$. Once the neutral axis depth factor is known the moment carrying capacity can be computed for that

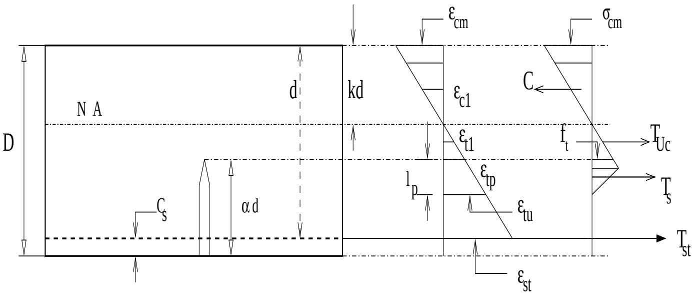

Figure 7: Stress-Strain distribution of the RC beam

equilibrium configuration. The moment of resistance can be computed as,

$$M_R = M_{soft} + M_{UT} + M_{st} \tag{24}$$

where, M_{soft} is the moment of resistance provided by the softening zone and equals,

$$M_{soft} = T_s[(1 - \alpha - k/3)D - 2/3l_p] \tag{25}$$

and M_{UT} is the moment of resistance provided by the uncracked tension concrete, which is given by,

$$M_{UT} = T_{Ut}[(1 - \alpha - k/3)D - xx/3 - l_p] \tag{26}$$

where T_{Ut} is the tensile resistance provided by the uncracked concrete and xx is the length of the corresponding uncracked portion. Finally, the moment of resistance due to the reinforcement M_{st} is computed as,

$$M_{st} = T_{st}\left(1 - \frac{k}{3}\right)d \tag{27}$$

The procedure is repeated for different crack lengths $a_1, a_2 ... a_n$ as long as equilibrium is satisfied.
The proposed method is applied to determine the moment carrying capacity for the RC beam specimen as considered above, as a function of increasing crack length. Figure 8 shows the normalized moment value obtained for the considered specimen as a function of increasing crack length. It is observed that, before steel yielding, the moment carrying capacity increases along with increase in crack length, and the value starts decreasing once the steel undergoes plastic deformation, which is reasonable in case of reinforced specimen. The normalization is done with respect to $(K_{Ic}BD^{3/2})$.
Figure 9 shows the normalized moment value for the specimen, computed using the above method as well as assuming the LEFM criteria, which considers the failure condition to be $(K_I = K_{Ic})$ together with the assumption of steel yielding. In the second case the ultimate moment can be expressed as follows;

$$M_F = \frac{K_{Ic}BD^{3/2}}{Y_M(\alpha)} + \frac{F_P D}{Y_M(\alpha)} \left[F_1\left(\frac{x}{a}, \alpha\right) + Y_M(\alpha)\left(\frac{1}{2} - \frac{C_s}{D}\right)\right] \tag{28}$$

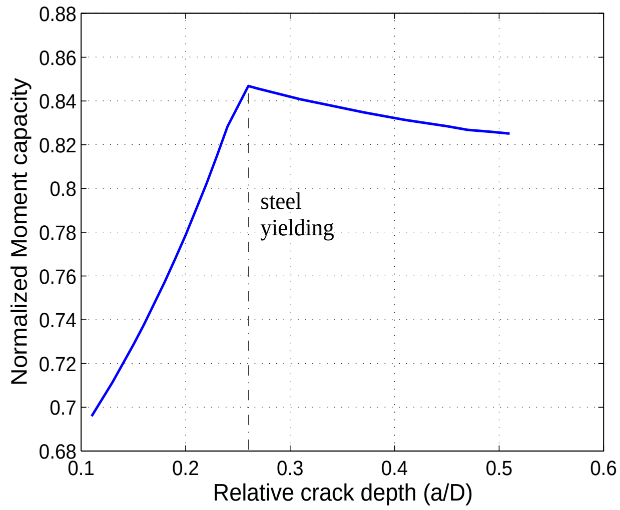

Figure 8: Normalized moment capacity computed through present method

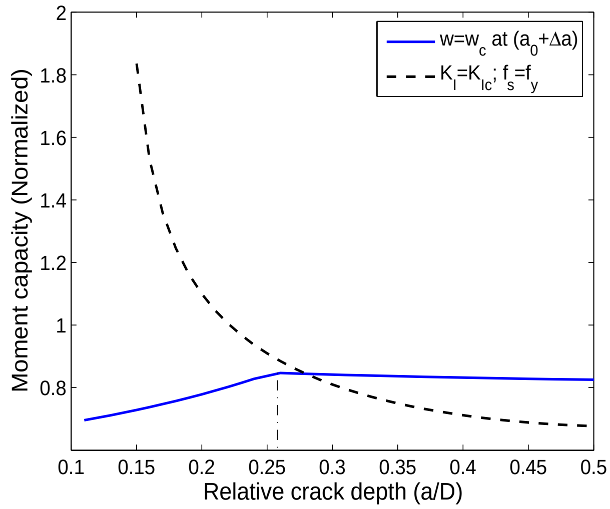

Figure 9: Normalized moment capacity (Comparison between proposed method and LEFM theory)

It is seen from the results, that before steel yielding, the second condition predicts higher value than the first case. Whereas, after steel yielding it is lower than the present method. The moment value computed using Equation 24 becomes greater than those computed using Equation 28. Hence, it can be concluded that the assumption of steel yielding for throughout the crack propagation regime, overestimates the capacity of the member in the initial crack propagation stage. However, a situation may arise, when the member fails due to crack propagation even though the steel may not have yielded. Therefore, the present method relaxes the assumption of steel yielding, instead it considers the formation of full process zone at the crack tip. The failure is to be governed by the crack tip opening displacement as mentioned earlier, which is more reasonable in case of residual strength assessment of cracked member.

6 CONCLUSIONS

In the first part of the present study, a method is proposed to determine the condition for unstable crack propagation in a RC beam, considering the post-peak softening response of the concrete. The crack extension resistance based approach is followed to determine the critical value of relative crack depth. Three standard approximations namely, linear, bilinear and power-law are used for describing the softening zone behavior. A parametric study is performed over the reinforcement percentage and the depth of the beam, to find out the influence of each on the stability criterion. It is observed that the percentage reinforcement does not effect the stability phenomenon, unless the steel yields, whereas the depth of the specimen has a strong influence on fracture instability. As the depth increases, the critical crack length corresponding to instability decreases. In the second part of the analysis, the residual strength of a cracked RC beam is assessed by considering the critical tip opening displacement as the governing parameter. The numerical example shows, that the prediction through proposed method is reasonable in the sense, it relaxes the assumption of steel yielding for each and every crack length, which is commonly followed in LEFM based analysis. It is observed that the capacity of the member increases along with crack length when the reinforcement is within the elastic regime, whereas after steel yielding the value reduces with further propagation of crack.

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