Reinforced and prestressed concrete beams subjected to shear and torsion

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ABSTRACT: Today, the nonlinear finite element method is commonly used by practicing engineers, although design and assessment for shear and torsion in reinforced concrete structures are still made using methods based on sectional forces. By modelling the shear behaviour, using 3D nonlinear FEM, higher load carrying capacity and more favourable load distribution was shown, compared to conventional analysis. A modelling method using four-node curved shell elements with embedded reinforcement was evaluated in this study. Tests of reinforced and prestressed beams loaded in bending, shear and torsion were simulated. The increase in shear capacity, in addition to the reinforcement contribution, was modelled with a relationship for concrete in tension according to the modified compression field theory and compared with the use of a relationship related to the fracture energy of plain concrete. The results show that evaluations of the load-carrying capacity or crack width will be on the safe side, if only the fracture energy is used to define the concrete in tension.

1 INTRODUCTION

For structural design and assessment of reinforced concrete members the non-linear finite element (FE) analysis has become an important tool. However, today, design and assessment for shear and torsion are still made using simplified analytical or empirical design methods based on sectional forces and moments. The current calculation method for reinforced concrete members subjected to combined shear and torsion, in the European Standard EC2 CEN/TC250/SC2 (2004), adds stresses from shear and from torsion linearly without taking into account deformations and compatibility within the member. However, earlier research indicates that there is a redistribution even within concrete members without transverse reinforcement, see Gabrielsson (1999) and Pajari (2004), and that this could be modelled with non-linear FE analyses, see Broo *et al.* (2005) and Broo (2005). Nonlinear FE analyses of concrete members with transverse reinforcement subjected to shear have been studied by several researchers, for example Ayoub & Filippou (1998), Yamamoto & Vecchio (2001), Vecchio & Shim (2004) and Kettil *et al.* (2005). In recent research projects, failures due to shear and torsion was successfully simulated with nonlinear FE analyses, also for members with transverse reinforcement, see Plos (2004). A higher load carrying capacity compared to conventional analysis was shown. Thus, a more favourable load distribution, compared to conventional analysis, was found when the structure was analysed in three dimensions and by including the fracture energy associated with concrete cracking. Modelling R/C and P/C members subjected to shear and torsion needs to be further studied and verified in order to be reliable and practically applicable.

The aim of this study is to work out a modelling method to simulate the shear-induced cracking and the shear failure of R/C and P/C members. The modelling method should be possible to use for analyses of more complicated structures, for example box-girder bridges, subjected to bending, shear, torsion and combinations of these load actions. Engineers using commercial nonlinear FE programs, not especially designed for shear analysis, should be able to use the modelling method in their daily practice. Further aims are to determine the most important parameters that need to be accounted for in the material model or in the material properties used.

The different contributions to the nonlinear response of shear are concisely presented and discussed, as well as the different approaches to shear modelling. The proposed FE approach to R/C and P/C modelling is also shown to fit quite well some available test results concerning concrete beams loaded in bending, torsion and shear.

2 SHEAR BEHAVIOUR

2.1 *The non-linear response in shear*

Both shear forces and torsional moments cause shear stresses that could result in cracks in a concrete member. Cracks due to shear stresses are usually inclined compared to the direction of the reinforcement. To satisfy the new equilibrium after shear cracking, longitudinal reinforcement and transversal reinforcement or friction in the crack is required. After cracking the shear stresses are transmitted by compression in the concrete between the inclined cracks, by tension in the transverse reinforcement crossing the inclined cracks, by tension in the longitudinal reinforcement, by compression in the compressive zone and by shear transfer in the crack. The visual shear cracks are preceded by the formation of micro-cracks. The micro-cracking reduces the stiffness of the member and a redistribution of stresses can occur resulting in strut inclinations smaller than 45°, Hegger *et al.* (2004). Due to the rotation of the struts more transverse reinforcement can be activated. The rotation of the compressive struts can continue until crushing of the concrete between the inclined cracks occurs, Walraven & Stroband (1999). Possible failure modes due to shear cracks are either crushing of the concrete between two shear cracks or sliding along a shear crack. It is well-known that the shear capacity is larger than what can be explained by the reinforcement contribution determined from a truss model. This increase in shear capacity is due to tension stiffening, dowel action, and aggregate interlock, and is also known as the concrete contribution.

After cracking, concrete can transmit tensile stresses due to tension softening and for reinforced concrete also due to tension stiffening. Tension softening is the capability of plain concrete to transfer tensile stresses after crack initiation. In a reinforced concrete member subjected to tensile forces, the concrete in between the cracks carry tensile stresses transferred from the reinforcement through bond, thus contributing to the stiffness of the member. This is known as the tension stiffening. The tension-stiffening effect increases the overall stiffness of the reinforced concrete member in tension compared to that of a bare reinforcing bar. Due to the bond action there are still high transverse tensile stresses in the compressive struts. Cracked concrete subjected to high tensile strains in the direction normal to the compression is softer and weaker than concrete in a standard cylinder test, Vecchio & Collins (1986), Vecchio & Collins (1993) and Belarbi & Hsu (1995).

The complex behaviour of reinforced concrete after shear crack initiation has been explained in several papers, for example ASCE-ACI Committe 445 on Shear and Torsion (1998), Vecchio & Collins (1986), Pang & Hsu (1995), di Prisco & Gambarova (1995), Walraven & Stroband (1999), Zararis (1996), Soltani et al. (2003). Several mechanisms contribute to the non-linear response in shear: bridging stresses of plain concrete (tension softening), interaction between reinforcement and concrete due to bond (tension stiffening), aggregate interlocking, dowel action, and reduction of concrete compressive strength due to lateral cracking. The stress equilibrium can be expressed in average stresses for a region containing several cracks or in local stresses at a crack. The local stresses normal to the crack plane are carried by the reinforcement and by the bridging stresses of plain concrete (tension softening). Along the crack plane, the shear stresses are carried by friction due to aggregate interlocking and dowel action. The stresses will depend on the crack width, the shear slip, the concrete mix-design (strength, grading curve and maximum aggregate size) and of course the reinforcement (type, diameter and spacing), fib (1999).

2.2 *Modelling of the non-linear shear response*

Several analytical models that are capable of predicting the nonlinear response in shear has been presented. for example the modified compression field theory (MCFT), Vecchio & Collins (1986), the distributed stress field model (DSFM), Vecchio (2000), the cracked membrane model (CMM), Kaufmann & Marti (1998), the rotating-angle softened truss model (RA-STM), Pang & Hsu (1995), the fixed-angle softened truss model (FA-STM), Pang & Hsu (1996), and the softened membrane model (SMM), Hsu & Zhu (2002). All these models are based on the smeared approach, i.e. the influence of cracks is smeared over a region and the calculations are made with average stresses and average strains. Stress equilibrium, strain compatibility and constitutive laws are used to predict the shear force for chosen strains. Some models use a rotating crack concept and thus no relationship between shear stress and shear strain is needed. Others are based on a fixed crack concept including a relationship for average shear stresses and average shear strains. Most of the models are also implemented in finite element programs. Soltani et al. (2003) propose a model that calculates local stresses and strains at the crack plane, separating the contribution from tension softening, tension stiffening, aggregate interlock and dowel action, to predict the nonlinear shear response.

If the shear-induced cracking and shear failure is modelled with a nonlinear FE program, not especially designed for shear analysis, parts of the concrete contribution needs to be accounted for by modifying the constitutive relationships used. The required modifications depend on the modelling philosophy, on crack representation (fixed or rotating cracks) and on how the interaction between reinforcement and concrete is modelled.

Modelling the reinforcement and the interaction between reinforcement and concrete can be more or less detailed. When modelling larger structures, i.e. box-girder bridges, the reinforcement can be modelled as embedded in the concrete elements. The embedded reinforcement adds stiffness to the FE model, but the reinforcement has no degree of freedom of its own. Hence, the reinforcement is perfectly bonded to the surrounding concrete and no slip can occur. Embedded reinforcement can be applied to any type of finite element that represents the concrete. In this case the above-mentioned effects of the concrete contribution, must be taken into account in the constitutive relations describing the materials behaviour, e.g. in the concrete tensile response or in the reinforcement response. Different ways of doing this for the tension stiffening effect has been proposed by Lackner & Mang (2003) and Kaufmann & Marti (1998). Relationships for tensile stresses versus crack openings in plain concrete are based on fracture mechanics and related to the fracture energy, G_f; an example is the relation proposed by Hordijk, as described in TNO (2002). In Figure 1 the curve by Hordijk is compared with the expression by Collins & Mitchell (1991) as described below.

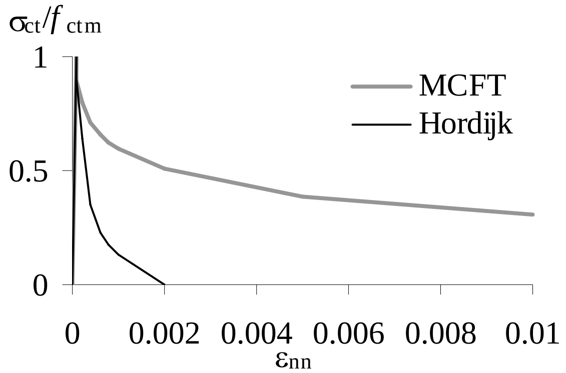


Figure 1. Different tension-softening relations used, for the four-point bending beam. For the curve by Hordijk the fracture energy is smeared over a length of 107 mm (the crack band with, *h*), which corresponds to the mean crack spacing obtained in the test.

For reinforced concrete members subjected to shear also the contribution from dowel action and friction due to aggregate interlock can be accounted for in the constitutive relations. Such relationships that link average tensile stresses to average tensile strains for orthogonally reinforced cracked concrete have been established trough shear panel tests, Vecchio & Collins (1986) and Pang & Hsu (1995). The relationship

$$\sigma_{c1} = \frac{f_{ctm}}{1+\sqrt{500\cdot\varepsilon_1}} \tag{1}$$

is the one used in the modified compression field theory (MCFT), Collins & Mitchell (1991). Here σ_{c1} is the mean principal tensile stress, f_{ctm}, the mean tensile concrete strength and ε_1 is the mean principal tensile strain. This relationship has later been modified by Bentz (2005).

The relationship should be limited so that no concrete tensile stress is transmitted after the reinforcement has started to yield. This is a problem when modifying the relationship for the concrete tensile response in a FE program since there is no obvious link between the steel strain in the reinforcement direction and the concrete strain in the principal stress direction. Hence, the cracked concrete can transfer tensile stresses in the principal stress direction even after the reinforcement in any direction yields.

The relationships by Collins & Mitchell (1991), Pang & Hsu (1995) and Bentz (2005) were established for analysis of orthogonally reinforced concrete specimens subjected to shear. However, more general applicability for members with deviating reinforcement or specimens subjected to, for instance, bending or tension is not shown and is likely doubtful. In Broo *et al.* (2006) some of the shear panels tested by Vecchio & Collins (1986) and Pang & Hsu (1995) were analysed with the FE program Diana, TNO (2002), and the use of the tension softening curve by Hordijk and a curve according to MCFT, Equation 1, were compared. The results showed that – by merely considering plain-concrete fracture energy - the capacity was underestimated and the average strains, i.e. the crack width, were overestimated. On the other hand, if the concrete contribution was modelled with the expression from MCFT, the capacity was overestimated and the average strains underestimated for most specimens, except for the panels tested by Vecchio & Collins (1986). It should be mentioned that results from these panel tests are included in the test results, used to calibrate the expression in the MCFT. This means that the concrete contribution to shear capacity can be accounted for by modifying the constitutive relationship used for concrete in tension. However, caution is recommended in order not to overestimate the capacity. If no modification of the tension-softening curve is done, the shear capacity will at least not be overestimated. Moreover, it was found that it is important to include the reduction of compression strength due to lateral cracking if the failure mode is crushing of the concrete between the shear cracks. In the analyses presented here, the results obtained by using Hordijk's tension-softening curve and MCFT's curve are compared for a reinforced concrete beam subjected to bending and shear and a prestressed box-beam subjected to bending, shear and torsion.

3 MODELLING TECHNIQUES

To investigate the general applicability of the modelling method worked out for shear panels in Broo *et* *al.* (2006) for members with non-orthogonal reinforcement and subjected to mixed loading, two beam tests were modelled. In all analyses, the concrete was modelled with four-node curved shell elements. For the specimens modelled here, plane-stress elements or even beam elements would have been more appropriate, but the aim was to work out a modelling method that can be used also for more complicated structures, for which curved shell elements are more suitable. Full interaction was assumed between the reinforcement and the concrete, which was modelled with a constitutive model based on nonlinear fracture mechanics. A rotating crack model based on total strain, see TNO (2002), was used in all analyses. In the analyses of the beams, concrete compressive stresses localise into one element, whose size does not correspond to the size of the specimens used to calibrate the compression relationship by Thorenfeldt as described in TNO (2002). Hence, if the relationship by Thorenfeldt was used the model could not predict the response. This disadvantage was overcome by modelling the concrete in compression with an elastic-ideal plastic relationship. The reduction of the hardening in compression due to lateral cracking was modelled according Vecchio and Collins, as described in TNO (2002). For the tension softening, two approaches to account for the concrete contribution from tension stiffening, dowel action and friction due to aggregate interlock were compared. The curve by Hordijk, see TNO (2002), merely based on plain-concrete fracture energy is compared with the expression in Equation 1, taken from the MCFT, Collins & Mitchell (1991), which attempts to take also the concrete contribution into account, see Figure 1. The concrete material properties for the beams analysed are presented in Table 1. The concrete tensile strength, f_{ct}, the concrete modulus of elasticity, E_c, and the fracture energy G_f were calculated according to CEB (1993), from the mean cylinder compressive strength, f_{cm}, reported from the tests. For the curve by Hordijk the fracture energy is smeared over a length, h, the crack band with that corresponds to the mean crack spacing obtained in the test.

Table 1. Material properties for concrete used in the analyses.

Test	f_{cm}	f_{ctm}	E_c	G_f	h
	MPa	MPa	GPa	Nm/m^2	m
NSC 3	27.3	2.16	30.05	88.9	0.107
Beam 5	24.9	1.97	29.14	47.3	0.050

The constitutive behaviour of the reinforcement and the prestressing steel was modelled by the von Mises yield criterion, with an associated flow law and isotropic hardening. In Table 2 the material properties of the reinforcement used for the beam analyses are presented. No hardening parameters were presented for the reinforcement used in the box-beam test; the values presented in Table 2 are mean values taken from several other test reports using the same kind of reinforcement, from the same laboratory and the same time period.

Table 2. Material properties for reinforcement and prestressing strands used in the analyses.

Test	Dim and Qual.	f_y	f_u	ε_{sy}	ε_{s2}	ε_{su}	E_s
		MPa	MPa	‰	‰	‰	GPa
NSC 3	ϕ 8 K500 ST	574	670	3.1	29.3	99	199.8
	ϕ 20 K500 ST	468	600	2.4	21.5	132	195.6
Beam 5	½" St 150/170	1840	-	-	-	-	-
	ϕ 8 Ks40s	456	600*	2.09	-	150*	218
	ϕ 16 Ks60	710	900*	3.05	-	110*	233

* Values taken as mean value of test values from several other reports using same kind of reinforcement from the same time period.

4 ANALYSES OF P/C BOX-BEAM

4.1 *FE model*

A prestressed box-beam (Beam 5) provided also with ordinary longitudinal and transverse reinforcement tested by Karlsson & Elfgren (1976) was analysed. The box-beam was subjected to bending, shear and torsion and the final failure was due to large opening of a shear and torsion crack in the loaded web. Figure 2 shows the dimensions and support conditions of the simulated box-beam.

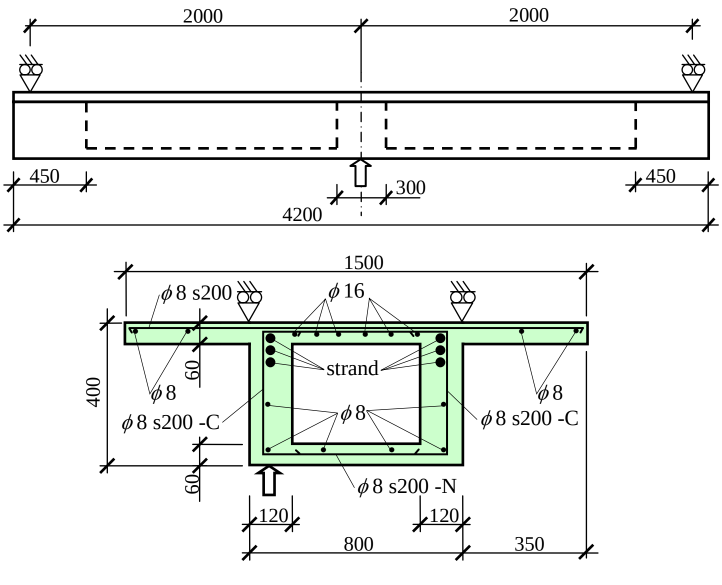


Figure 2. Test set-up, geometry and reinforcement of the prestressed box-beam, Beam 5, Karlsson & Elfgren (1976).

Due to symmetry only half the beam was modelled, as shown in Figure 3, using curved shell elements and material properties as described above. The box-beam was reinforced as shown in Figure 2. The prestressing strands and the longitudinal reinforcement with dimension 8 mm were modelled as embedded bars, TNO (2002). All other reinforcements were modelled as embedded grids, TNO (2002).

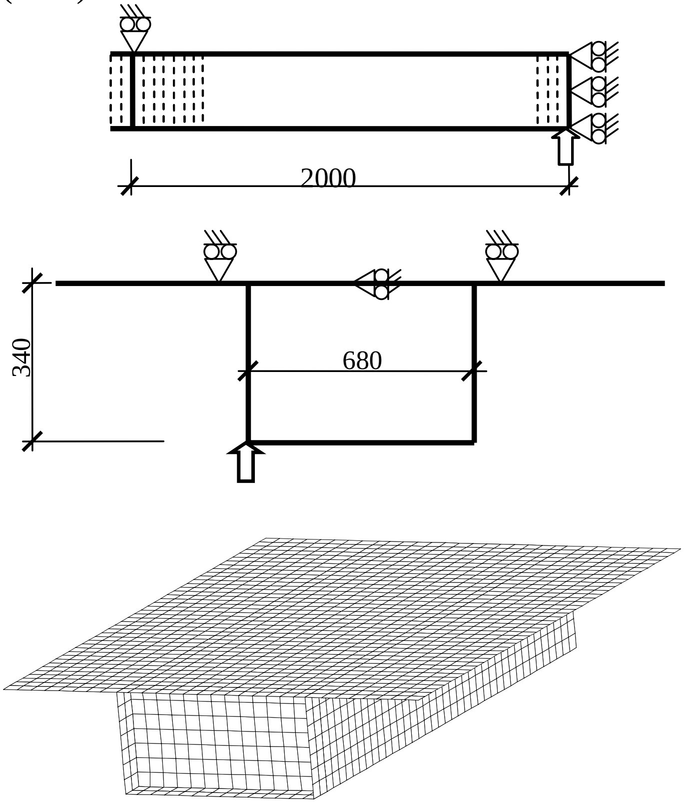


Figure 3. Restraints and finite-element mesh used to model the box-beam tested by Karlsson & Elfgren (1976). Dashed lines mark the cross-sections that are stiffened and tied to keep the cross-section in plane.

The support conditions of the model are shown in Figure 3. In the test the box-beam was supported on roller bearings with a load distributing support plate. In the analyses, the nodes in the centre of the supports were fixed in the vertical direction. The nodes on each side of this node were forced to have the same vertical displacement but in opposite direction, thus enabling a rotation and simulating a free support with a distribution length equal to the support plate in the test.

Stiffeners at the support and at the mid-span where the load was applied were taken into account in two ways. All shell elements for the box wall and flanges in the area of the stiffeners were given a thickness twice the thickness of the elements outside these parts. The density of the concrete was also modified to maintain the correct self-weight of the box-beam. Furthermore, all nodes in each cross-section of the stiffened areas were tied so that each cross-section remained plane.

In the box-beam test the load was applied in steps of 40 kN up to 320 kN. Thereafter, the load was increased by controlling the mid-deflection in steps of 1-2.5 mm. In the analyses, the load was applied as a prescribed deformation of the loading node, i.e. the bottom corner node in the symmetry section. The box-beam analysis had to be performed in two phases. In the first phase the loading node was not supported; here the prestressing force (110 kN) was released and the self-weight was applied. In the second phase, the loading node was supported vertically at the location obtained in the first phase. Thereafter, the loading was applied by increasing the vertical displacement of the loading node. An implicit solving method was used. Iteration was made with constant deformation increments of 0.1 mm. For each increment equilibrium was found using the BFGS secant iteration method, TNO (2002). The analysis was continued if the specified force, energy or displacement convergence criterion was fulfilled, according to default values, see TNO (2002). If the convergence criterion was not fulfilled within twenty iterations, the analysis was continued anyhow. Afterwards the convergence criteria ratios were checked.

4.2 *Results*

The applied load versus vertical displacements from the analyses and the test are compared in Figure 4. The results show, as expected, that if only the fracture energy of plain concrete was taken into account, the capacity was underestimated and the vertical deflections were overestimated. However, when the concrete contribution was modelled with the expression from MCFT, the capacity was still underestimated but the fitting of the results was satisfactory for the vertical deflections.

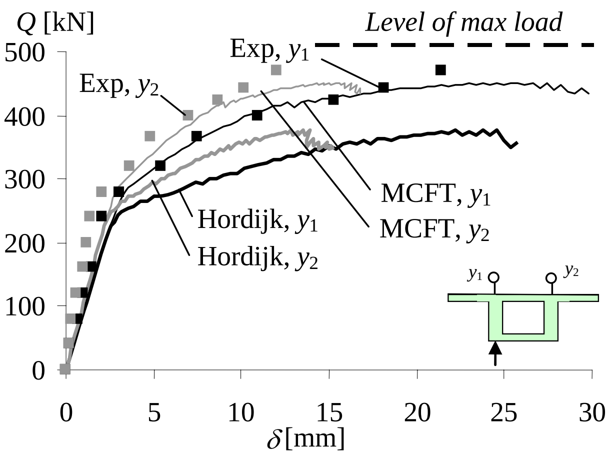
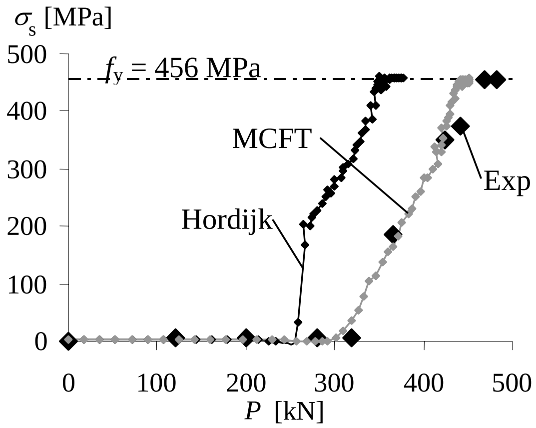


Figure 4. Comparison of results from test and analyses of a prestressed concrete box-beam subjected to bending, shear and torsion; applied load versus mid-span displacement.

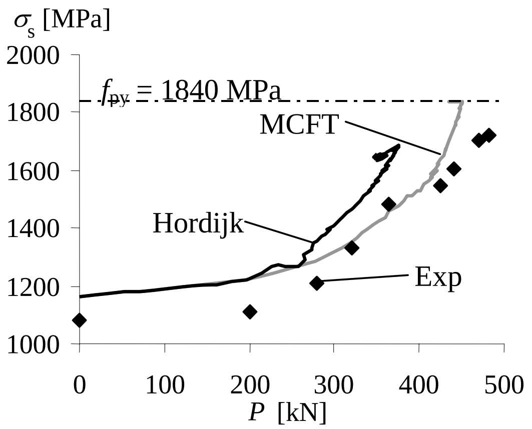
In the test, the first crack, going in transverse direction across the top flange, occurred at a load of 240 kN due to bending. This crack propagated downwards in the most loaded web at a load of 280 kN. At a load of 320 kN the first shear and torsion crack appeared near the support. The final failure, at a load of 510 kN, was due to large opening of a shear and torsion crack in the loaded web. The angle of the cracks in the most loaded web varied between

45 and 60 degrees, while they remained vertical in the other web. The crack propagation and the crack pattern from both analyses agree well with those observed in the test.

In Figure 5, the load versus steel stresses for one strand and one stirrup, from the test and the analyses, are compared. The steel stresses increases first when the box-beam starts to crack. In the analysis with the tension softening modelled according MCFT the steel stresses increase is slower which corresponds better with the steel stresses measured in the test.



(a)



(b)

Figure 5. *Comparison of results from test and analyses of a prestressed concrete box-beam subjected to bending, shear and torsion; applied load versus steel stresses in; a) a stirrup in the loaded web and 400 mm from the load b) the top prestressing strand in the loaded web and the most loaded section.*

5 ANALYSES OF P/C BEAM

5.1 *FE model*

A R/C beam loaded in four-point bending, NSC3, tested by Magnusson (1998) was modelled with curved shell elements. The beam was subjected to bending and shear and failed in bending due to yielding of the longitudinal reinforcement and crushing of the concrete in the compressive zone in the mid-span part of the beam. Figure 6 shows the geometry and support conditions of the simulated bending beam.

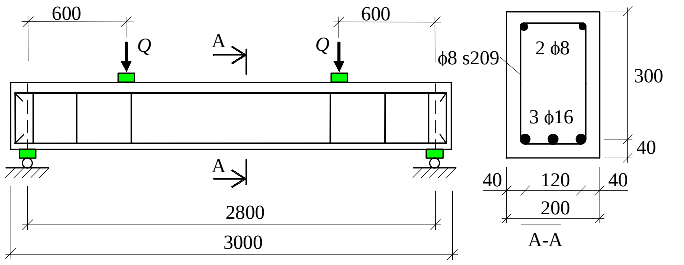


Figure 6. Test set-up, geometry and reinforcement of the R/C beam loaded in four-point bending, NSC3, Magnusson (1998).

Due to symmetry only half the beam was modelled, as shown in Figure 7, using curved shell elements and material properties as described in Section 3. The beam was reinforced as shown in Figure 6. The longitudinal reinforcement and the stirrups between the support and the load were modelled as embedded bars, TNO (2002). The stirrups in the middle part of the beam were modelled as an embedded grid, TNO (2002).

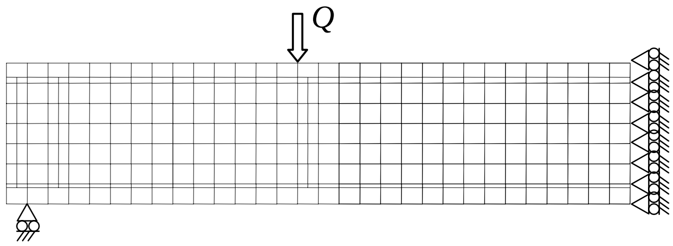


Figure 7. FE mesh used to simulate the R/C beam loaded in four-point bending.

In the test the beam was supported on roller bearings with a load distributing steel plate. In the analyses, the node in the centre of the support was fixed in the vertical direction. The nodes on each side of this node were forced to have the same vertical displacement but in opposite direction, thus enabling a rotation and simulating a free support with a distribution length equal as the support plate in the test.

The loading of the bending beam was controlled by displacement both in the test and in the analysis. In the analysis the loading was applied by increasing the vertical displacement of the loading node in steps of 0.1 mm. In the test the load was distributed by a loading plate. In the analyses, this was simulated by locking the nodes on each side of the loading node to the loading node, in such a way that their vertical displacement remained in a plane.

In the bending beam analyses, large compressive strains localised in one element, which was also subjected to large lateral tensile strains due to a flexural shear crack. This flexural shear crack was also observed in the test, but there it did not go into the compressive zone. Consequently, reducing the compressive strength due to lateral strains resulted in an unreasonable response. Therefore, for these analyses, the compressive strength was not reduced. In these analyses, the loading was applied by increasing the vertical displacement of the loading node. An implicit solving method was used. Iterations were made with constant deformation increments of 0.1 mm. For each increment, equilibrium was found using the BFGS secant iteration method, TNO (2002). The analysis was continued if the specified energy convergence criteria was fulfilled, according to default value, see TNO (2002). However, if the convergence criterion was not fulfilled within twenty iterations, the analysis was continued. Afterwards the convergence criteria ratios were checked.

5.2 *Results*

The applied load versus vertical displacements from the analyses and the test are compared in Figure 8. These results show that, if tension softening is modelled according to MCFT, the behaviour was too stiff and the capacity was overestimated. Hence, the cracked concrete transfers tensile stresses even if the longitudinal reinforcement yields.

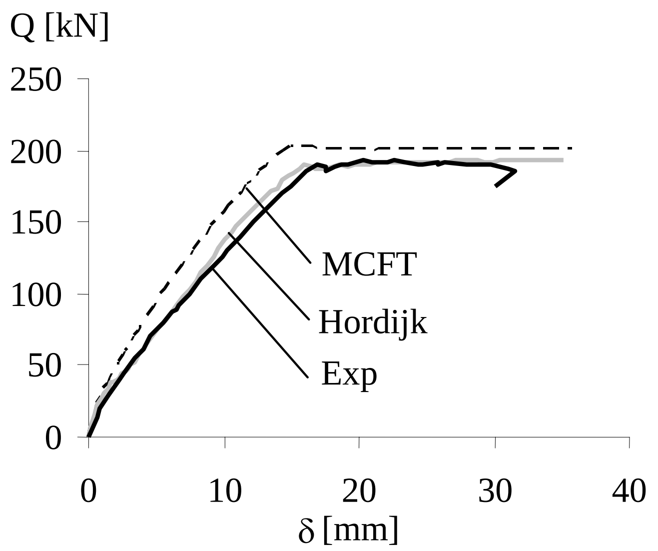


Figure 8. Comparison of results from test and analyses of the R/C beam loaded in four-point bending; applied load versus mid-span displacement.

With only the fracture energy of plain concrete taken into account, the capacity is very well estimated and the behaviour is just a little bit to stiff. The conclusion is that if a tension softening curve including the concrete contribution to shear capacity is used, it needs to be modified, so that no tensile stresses are transferred when the reinforcement yields. Otherwise the capacity will be overestimated for the parts of a member, which are subjected to tension or bending.

6 CONCLUSIONS

It is well-known that the shear capacity is larger than what can be explained by the reinforcement contribution determined from a truss model. This increase in shear capacity is due to tension stiffening, dowel action, and friction due to aggregate interlock, and is also known as the concrete contribution. If the shear-induced cracking and shear failure is modelled nonlinear with a FE program, not especially designed for shear analysis, parts of the concrete contribution needs to be accounted for by modifying the constitutive relationships used.

In the analyses presented here the use of the tension-softening curve by Hordijk and a curve according to MCFT are compared for a prestressed box-beam subjected to bending, shear and torsion, and for a reinforced concrete beam subjected to bending and shear.

The commercial FE program Diana was used to model a test of a box-beam that failed in shear. It was shown that four-node curved shell elements with embedded reinforcement can describe the nonlinear shear response also for P/C members loaded in bending, shear and torsion. The results show that – by merely considering plain-concrete fracture energy – the capacity is underestimated and the vertical deflections are overestimated. However, when the concrete contribution was modelled with the expression from MCFT, the capacity was still underestimated but the fitting of the results was satisfactory for the vertical deflections.

By modelling a test of a R/C beam loaded in four-point bending that failed in bending it was found that, when the tension softening was modelled according to MCFT, the behaviour was to stiff and the capacity was overestimated. Hence, the cracked concrete transferred tensile stresses even when the longitudinal reinforcements yield. This implies that an analysis of a concrete member subjected to shear, torsion, and bending will be on the safe side when evaluating the load-carrying capacity or crack width, if only the fracture energy is used to define the unloading branch of the concrete in tension.

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