Sensitivity analysis of a drying model concerning concrete delayed strains in containment vessels by means of a non-intrusive method

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ABSTRACT: Drying impacts the delayed behavior of the containment vessels. Many tests of loss of mass are available in the data base of *Electricité de France* or in the literature and make it possible to evaluate profiles of water content in a test tube. The macroscopic retiming of the drying parameters and the uncertainty level on the latter force to carry out a sensitivity analysis over the lifespan of the vessel. The ranking of the input parameters (sensitivity analysis) was carried out using an expansion of the model response onto the polynomial chaos basis (non intrusive regression method) and computing the Sobol' indices analyticaly from the polynomial chaos coefficients. One is more particularly interested in the delayed vertical and tangential strains. Initially, the parameters of Mensi's law and the water contents actually play the most important role. Then, the parameters of the desorption curve desorption and more particulary the initial water content are the most important parameters.

1 INTRODUCTION

Concrete drying drives the delayed behavior of containment vessels. Many tests of loss of mass are available in the data base of *Electricité de France* or in the literature and make it possible to evaluate profiles of water content in a test-tube or a work. The macroscopic retiming of the drying parameters and the level of uncertainty on the latter force to carry out a sensitivity analysis. It means that we want to quantify the relative importance of each input parameter. In our case, the response is expanded onto the polynomial chaos basis. The coefficients are evaluated from a series of deterministic finite element analysis (regression method). Sobol' indices are computed analytically from the obtained coefficients of the response surface.

2 PRESENTATION OF THE MODEL FOR THE DELAYED STRAIN OF CONCRETE IN CONTAINMENT VESSEL

The containment of French pressurized water reactors is ensured by two concrete vessels. The inner containment is made of reinforced prestressed concrete. Drying impacts the delayed behavior of containment vessels. The constitutive law, implemented in EDF's Finite Element Code, *Code_Aster*[1], is the result of the previous works by L. Granger (Granger 1996) and F. Benboudjema (Benboudjema 2002). The total strain is a function of the temperature T, the hydration degree β, the relative humidity h and the macroscopic stress σ. The conventional strain rate decomposition reads:

$$\begin{aligned} \dot{\varepsilon}\left(T,\beta,h,\sigma\right) = \; & \dot{\varepsilon}_{e}\left(\dot{\sigma}\right)+\dot{\varepsilon}_{th}\left(T\right) \\ & +\dot{\varepsilon}_{as}\left(\dot{\beta}\right)+\dot{\varepsilon}_{ds}\left(h,T\right) \\ & +\dot{\varepsilon}_{bc}\left(\sigma,h\right)+\dot{\varepsilon}_{dc}\left(\sigma,\dot{h}\right) \end{aligned} \tag{1}$$

with ε_e: elastic strain, dependent on σ ; ε_{th}: thermal dilation/contraction ; ε_{as}: autogenous shrinkage, being a function of the hydration degree $\beta \in [0;1]$; ε_{ds}: drying shrinkage, dependent on the drying process controlled by the evolution of h; ε_{bc}: basic creep, naturally being a function of σ, but also, of the hydrous state of the material – basic creep of pre-dried specimen exhibits some dependency with the equilibrated relative humidity–; ε_{dc}: drying creep. The model proposed by Bažant and Chern is assumed (Bažant and Chern 1985). In the sealed specimens (autogenous shrinkage and basic creep), the stress state remains homogeneous. Therefore, the computation is done analytically. Conversely, the drying specimens (loss of weight, drying shrinkage and creep) exhibit a humidity gradient responsible for an heterogeneous stress state. Their analysis requires a numerical simulation performed with *Code_Aster*. The calibration of the parameters is performed on the basis of the previous experimental results and follows the procedure:

[1]This code can be downloaded for free at `http://www.code-aster.org`

1. the drying process is modeled through a non linear thermal analogy,where the water diffusion coefficient D is a function of the water content C, *i.e.* $D(C) = a \cdot \exp(b.C)$. With the back-analysis of the time evolution of the specimen loss of weight, the parameters a and b are retrieved.

2. The autogenous shrinkage is fitted on a simple hyperbolic law: $\varepsilon_{as} = K_{as}\beta$, with $\beta = t/(t + t_{1/2})$.

3. The basic creep constitutive law assumes the full uncoupling of the spherical and the deviatoric strains. It requires the knowledge of the strain tensor derived from the experimental test.

4. The drying shrinkage is assumed to be proportional to the loss of water content: $\dot{\varepsilon}_{ds} = -K_{ds}\dot{C}$. The influence of the auto-induced creep due to the stress gradient was found insignificant while calibrating K_{ds}.

5. The drying creep is ultimately modeled once all the other parameters are known. The intrinsic drying creep constitutive equation reads $\dot{\varepsilon}_{dc} = |\dot{h}|\sigma/\eta_{dc}$.

3 NUMERICAL SIMULATION

On the basis of the previously calibrated parameters, some numerical computations (Le Pape, Toppani, and Michel-Ponnelle 2005) are performed on a so-called Representative Structural Volume. This RSV is located approximately at an equal distance from the dome and the raft of hte containment vessel and far enough from the equipment hatch and the tendon buttresses, so that homogeneous strain states may be applied in the directions of the prestressing. ZC1450 is a simplified model; the thickness of the wall is discretized to account for the gradient of humidity. A single element is used in the vertical and the tangential direction due to the total strain homogeneity. The prestressing is introduced by external forces. Iterations on the effective applied prestress are computed over the non linear calculation to account for the loss of prestress induced by the concrete creep. The temperature, humidity and prestress evolutions follow the scenario: 15^{o}C-60%RH inside and outside during building period, 35^{o}C-45%RH inside the containment building when the reactor is in-service. The prestressing is applied graduously following the building stages. Figure 1 presents the mesh used.

4 SENSITIVITY ANALYSIS METHOD

As the numerical model is quite large, a response surface model has to be used to minimize the computational cost of the sensitivity analysis. The expansion

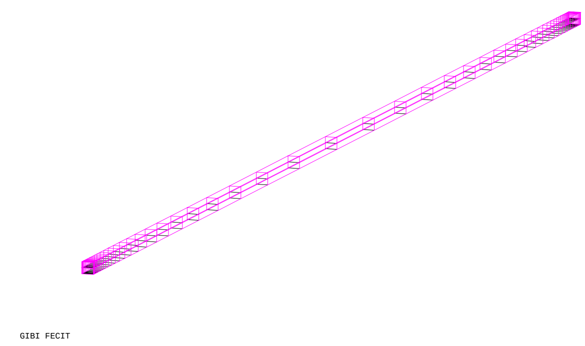

Figure 1: Mesh of the Representative Structural Volume

onto the polynomial chaos (Berveiller 2005; Ghanem and Spanos 1991) is relevant for sensitivity analysis (Sudret 2006a). The non intrusive regression method presented in the sequel only requires deterministic finite element anlyses and an analytical post-processing of the results, which makes it appealing for the non linear coupled problem under consideration.

4.1 Non intrusive regression method

The non intrusive method presented in this communication is based on a least square minimization between the exact solution and the solution approximated using the polynomial chaos (Isukapalli 1999; Berveiller 2005). First the input random variables (gathered in a random vector $\underline{U}$ whose joint PDF is prescribed) are transformed into a standard uniform vector $\underline{\xi}$ (*i.e.* a vector whose components are uniformly distributed over [-1;1]). In our case, we suppose that all variables $\underline{U}$ are uniform. If these M variables are independent, the one-to-one mapping reads :

$$\xi_i = \Phi^{-1}\left(F_i(U_i)\right) = -1 + 2\frac{x - L_{low}}{L_{up} - L_{low}} \tag{2}$$

where Φ is the standard uniform CDF and $\{F_i(U_i), i = 1, \cdots, M\}$ are the marginal CDF of the U_i's wgich expand as in Eq.(2). Suppose now that we want to approximate a response quantity $\underline{S}$ by the truncated series expansion:

$$\underline{S} \approx \underline{\tilde{S}} = \sum_{j=0}^{P-1} \underline{S}_j \Psi_j(\underline{\xi}) \tag{3}$$

where $\{\Psi_j, j = 0, \cdots, P-1\}$ are P multidimensional Legendre polynomials of $\underline{\xi}$ whose degree is less or equal than p. Note that the following relationship holds:

$$P = \frac{(M+p)!}{M!\,p!} \tag{4}$$

Let us denote by $\{\underline{\xi}^{(k)}, k = 1, \cdots, n\}$ n outcomes of the standard uniform random vector $\underline{\xi}$. For each outcome $\underline{\xi}^{(k)}$, the isoprobabilistic transform yields a vector of input random variables $\underline{U}^{(k)}$ (Eq.(2)). Using a classical finite element code, the response vector $\underline{S}^{(k)}$ can be computed. Let us denote by $\{s^{(k),i}, i = 1, \cdots, N_{ddl}\}$ its components. Using Eq.(3) for the i-th component, one gets:

$$\tilde{s}^i(\underline{\xi}) = \sum_{j=0}^{P-1} s^i_j \Psi_j(\underline{\xi}) \tag{5}$$

where (s^i_j) are coefficients to be computed. The regression method consists in finding for each degree of freedom $i = 1, \cdots, N$ the set of coefficients that minimizes the difference:

$$\Delta s^i = \sum_{k=1}^{n} \left[s^{(k),i} - \tilde{s}^i(\underline{\xi}^{(k)}) \right]^2 \tag{6}$$

These coefficients are solution of the following linear system:

$$\begin{pmatrix} \sum_{k=1}^{n} \Psi_0(\underline{\xi}^{(k)})\Psi_0(\underline{\xi}^{(k)}) & \cdots & \sum_{k=1}^{n} \Psi_0(\underline{\xi}^{(k)})\Psi_{P-1}(\underline{\xi}^{(k)}) \\ \vdots & \ddots & \vdots \\ \sum_{k=1}^{n} \Psi_{P-1}(\underline{\xi}^{(k)})\Psi_0(\underline{\xi}^{(k)}) & \cdots & \sum_{k=1}^{n} \Psi_{P-1}(\underline{\xi}^{(k)})\Psi_{P-1}(\underline{\xi}^{(k)}) \end{pmatrix}$$
$$\begin{pmatrix} s^i_0 \\ \vdots \\ s^i_{P-1} \end{pmatrix} = \begin{pmatrix} \sum_{k=1}^{n} s^{(k),i}\Psi_0(\underline{\xi}^{(k)}) \\ \vdots \\ \sum_{k=1}^{n} s^{(k),i}\Psi_{P-1}(\underline{\xi}^{(k)}) \end{pmatrix} \tag{7}$$

Note that the $P \times P$ matrix on the left hand side may be evaluated once and for all. Moreover it is independent on the mechanical problem under consideration. The crucial point in this approach is to properly select the regression points, *i.e.* the outcomes $\{\underline{\xi}^{(k)}, k = 1, \cdots, n\}$. Note that $n \geq P$ is required so that a solution of (7) exist. (Isukapalli 1999; Berveiller 2005) choose for each input variable the $(p+1)$ roots of the $(p+1)$-th order Legendre polynomial, and then built $(p+1)^M$ vectors of length M using all possible combinations. Then they select n outcomes $\{\underline{\xi}^{(k)}, k = 1, \cdots, n\}$ out of these $(p+1)^M$ possible combinations. (Sudret 2006a) presents a method to obtain the minimum number of points that allows the left matrix to be invertible.

4.2 Computation of the Sobol' indices

Global sensitivity analysis aims at quantifying the uncertainty in the model output due to the uncertainty in the input parameters. More precisely, the so-called *ANOVA techniques* aim at decomposing the variance of the output as a sum of contributions of each input variable, or combinations thereof. Many papers have been devoted to this topic in the last twenty years. A good state-of-the-art of the techniques is available in (Saltelli, Chan, and Scott 2000).

Consider a scalar response quantity s^k (*i.e.* a component of $\underline{S}$ in Eq.(3)). The Sobol' decomposition of its variance D reads (Sobol' 1993):

$$D \equiv \text{Var}\left[s^k\right] = \sum_{i=1}^{n} D_i + \sum_{1 \leq i < j \leq n} D_{ij} + \cdots + D_{12\ldots n} \tag{8}$$

where each term $D_{i_1 i_2 \ldots i_s}$ represents the part of the variance associated with the combination of variables $\{i_1, i_2, \ldots, i_s\}$. The Sobol' indices are nothing but the normalized version of these partial variances:

$$\delta_{i_1 i_2 \ldots i_s} = D_{i_1 i_2 \ldots i_s}/D \tag{9}$$

They sum up to 1 and thus represent the fraction of the response variance that may be attributed to the combination of input variables $\{i_1, i_2, \ldots, i_s\}$. Monte Carlo estimates of these so-called partial variances are usually used, leading to an unaffordable computational cost when the reponse is the result of a finite element analysis.

Alternatively, (Sudret 2006a; Sudret 2006b) shows that the decomposition of the total variance is straightforward once the model response has been expanded onto the polynomial chaos basis.

First remember that each multivariate polynomial in Eq.(3) is completely defined by a list of M non-negative integers $\{\alpha_1, \cdots, \alpha_M\}$ as follows:

$$\Psi_j(\underline{\xi}) \equiv \Psi_\alpha(\underline{\xi}) = \prod_{i=1}^{M} P_{\alpha_i}(\xi_i) \quad , \quad \alpha_i \geq 0 \tag{10}$$

where $P_q(.)$ is the q-th Legendre polynomial. Let us denote by $\mathcal{I}_{i_1, \ldots, i_s}$ the set of $\underline{\alpha}$ multi-indices such that only the indices $(i_1, \ldots, i_s)$ are non zero:

$$\mathcal{I}_{i_1, \ldots, i_s} = \left\{ \underline{\alpha} : \begin{array}{lclll} \alpha_k & > & 0 & \forall k = 1, \ldots, n, & k \in (i_1, \ldots, i_s) \\ \alpha_j & = & 0 & \forall k = 1, \ldots, n, & k \notin (i_1, \ldots, i_s) \end{array} \right\}$$

(11)

Note that $\mathcal{I}_i$ corresponds to the polynomials depending only on parameter x_i. Using this notation, the $P-1$ terms in Eq.(3) corresponding to the polynomials $\{\Psi_j,\, j=1,\,\ldots,\,P-1\}$ may now be gathered according the parameters they depend on:

$$
\begin{aligned}
s^k = s_0^k &+ \sum_{i=1}^{n} \sum_{\underline{\alpha}\in\mathcal{I}_i} s_{\underline{\alpha}}^k\, \Psi_{\underline{\alpha}}(x_i) \\
&+ \sum_{1\le i_1<i_2\le n} \ \sum_{\underline{\alpha}\in\mathcal{I}_{i_1,i_2}} s_{\underline{\alpha}}^k\, \Psi_{\underline{\alpha}}(x_{i_1},x_{i_2}) + \ldots \\
&+ \sum_{1\le i_1<\cdots<i_s\le n} \ \sum_{\underline{\alpha}\in\mathcal{I}_{i_1,\ldots,i_s}} s_{\underline{\alpha}}^k\, \Psi_{\underline{\alpha}}(x_{i_1},\ldots,x_{i_s}) \\
&+ \cdots + \sum_{\underline{\alpha}\in\mathcal{I}_{1,2\ldots,n}} s_{\underline{\alpha}}^k\, \Psi_{\underline{\alpha}}(x_1,\,\ldots,\,x_n)
\end{aligned} \quad (12)
$$

Thus the Sobol-PC sensivity indices of the k-th component of the response vector:

$$\delta_{i_1\ldots i_s}^k = \sum_{\underline{\alpha}\in\mathcal{I}_{i_1,\ldots,i_s}} (s_{\underline{\alpha}}^k)^2\, \mathrm{E}\left[\Psi_{\underline{\alpha}}^2\right] / \mathrm{Var}\left[s^k\right] \quad (13)$$

where the total variance of s^k is easily obtained from the PC coefficients:

$$\mathrm{Var}\left[s^k\right] = \sum_{j=1}^{P-1} (s_j^k)^2\, \mathrm{E}\left[\Psi_j^2\right] \quad (14)$$

In the sequel, the first order Sobol-PC sensitivity indices are computed.

5 SENSITIVITY ANALYSIS OF THE DRYING MODEL ON THE DELAYED STRAIN OF CONCRETE IN CONTAINMENT VESSEL

The macroscopic retiming of the drying parameters and the level of uncertainty on the latter force to carry out a sensitivity analysis. It means that we want to quantify the relative importance of each input parameter.

5.1 Presentation of the random variables

In this sensitivity analysis, we take into account 7 random variables (gathered in Table 1). The humidity in the intersapce between the two concrete vessels is called outside humidity. The desorption curve (*cf.* Fig.2) relates the water content to the humidity. It is usually difficult to get data about this curve. Thus it is important to take this parameter into account in the sensitivity analysis. The empirical curve on Fig.2 is not that of the concrete of the vessel under consideration. It is difficult to model this parameter by a random variable. We make the choice to enclose this curve by two lines, which are defined with two random variables: the initial humidity C_0 and the absorption abscissa ABS_H.

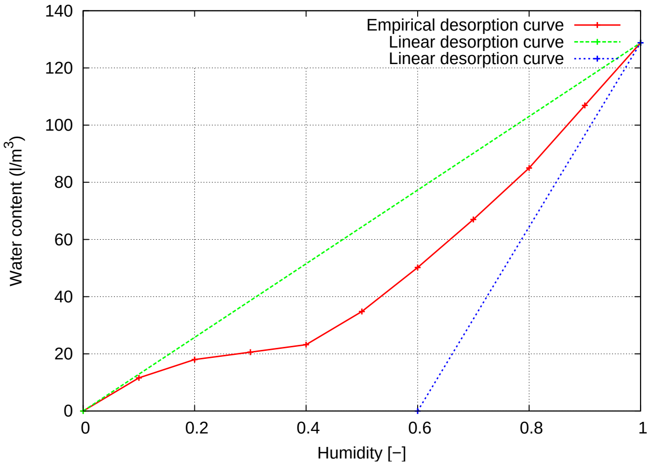

Figure 2: Desorption curve

The first step in the sensitivity analysis is to expand the evolution of delayed strain onto the polynomial chaos basis. As all input random variables are uniform, the polynomial chaos is built from Legendre polynomials. (Sudret 2006a) shows that an order 2 gives the best ratio accuracy/efficiency for computing Sobol' indices. In our case, which involves 7 random variables, we have 36 coefficients to compute. $n=56$ finite element analyses are enough to obtain the whole expansion of delayed strain onto the polynomial chaos by using the non intrusive regression method presented in section 4.1.

5.2 Results of the sensitivity analysis

Figures 3 and 4 show the evolution of the Sobol' indices for each input parameter versus time on the tangential and vertical strains. One can first of all notice that for $t\in[0;5]$ years, the Sobol' indices vary a lot. This corresponds to the date when the installation of prestressing is modelled. The parameters Q, C_{INT} and C_{EXT} have a relatively low index both for the tangential and the vertical strains. On the other hand, the other parameters play a considerable role. As it can be seen on each figure, the reduction of the importance of C_0, a and b (all three directky influencing drying) coincide with the increase in importance of ABS_H. This could mean that when drying reaches a certain level, it is the desorption curve which plays the most important role. This means that the role of drying process decreases in time and that its variability has no influnece on the amplitude of the delayed strains after a certain time. This implies that the very

Table 1: Presentation of input random variables

Parameter	notation	Type of distribution	Lower limit	Upper limit
Initial humidity	C_0	Uniform	116.4 l/m^3	135.8 l/m^3
Outside humidity	C_{EXT}	Uniform	65.61 l/m^3	80.19 l/m^3
Inside humidity	C_{INT}	Uniform	49.12 l/m^3	54.28 l/m^3
Thermal activation energy	Q	Uniform	4465 K	4935 K
Parameter a of the drying process	a	Uniform	$1.05 \cdot 10^{-13} m^2/s$	$1.95 \cdot 10^{-13} m^2/s$
Parameter b of the drying process	b	Uniform	0.06118 m^3/l	0.06762 m^3/l
Desorption abscissa	ABS_H	Uniform	0	0.6

fine modeling of the kinetics of drying is not necessary when one is only interested in the delayed strains in the long run.

The figure 5 presents the temporal evolution of the coefficients of variation of the tangential and vertical deformations. One notices that these coefficients of variation are relatively low (lower than 4 %). That means that the importance of drying on the differed deformations is relatively low.

6 CONCLUSION

This paper presents a sensitivity analysis of concrete drying in the delayed behavior of containment vessels. Only the parameters influencing drying are described by random variables. The analysis of sensitivity was carried out using the non intrusive method of regression, which makes it possible to have a stochastic response surface of the delayed strains. The Sobol' indices, which allow to rank the input variables according their weight in the response variance, are calculated in an analytical way from the coefficents of the response surface rank. The input random parameters are:

- Initial humidity;
- Outside humidity;
- Inside humidity;
- Thermal activation energy;
- Two parameters of Mensi's law;
- The curve of desorption.

The response variables are the tangential and vertical delayed strain computed over the time range [0,60 years]. As far as the tangential and vertical delayed strains are concern, it appears that the parameter which has the most importance is the curve of desorption, which utilizes two random variables, in particular the initial humidity. However we have little information on this curve, so it would be intersting to experimentally determine this curve for the concrete mix of the containment vessel. The two parameters of Mensi's law are rather important until approximately 40 years. Then their importances decrease. The initial water content C_0 also plays a part in drying when one considers this parameter coupled with the outside humidity C_{EXT}. The inside humidity and the thermal activation energy almost do not have importance over the lifespan of the vessel. It should be also noted that the coefficients of variation of the delayed strains are relatively low (less than 4 %). That means that the importance of drying on the delayed strains is relatively low. This does not deteriorate in anything the results on the Sobol' indices which do not depend on the variance of the response quantity considered.

Finally, the analysis shows that it is necessary to accurately determine the desorption curve and the parameters of Mensi's law to get accurate prediction of the delayed strains.

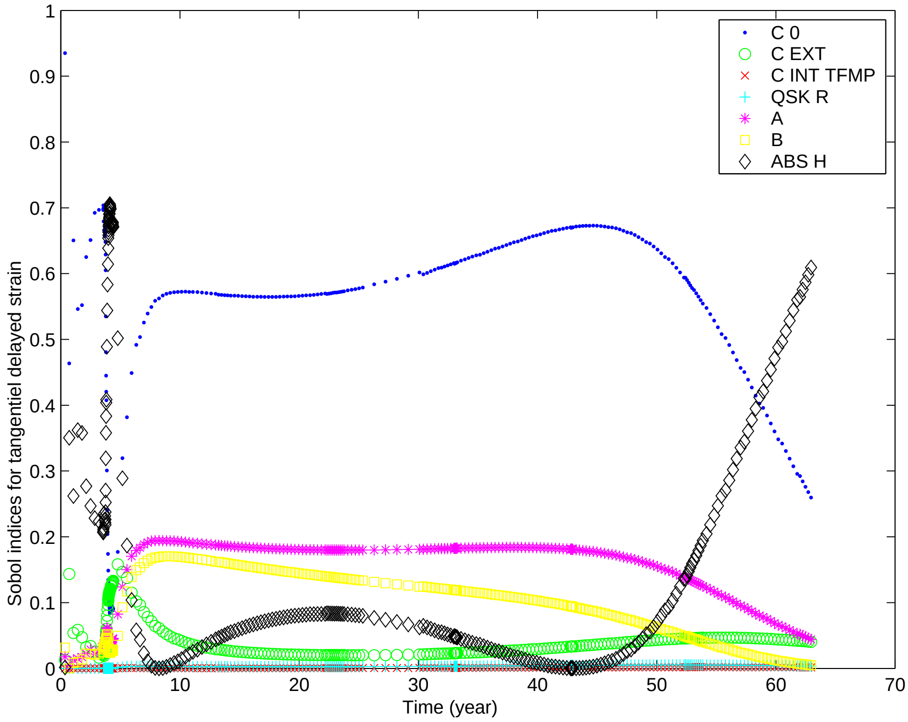

Figure 3: Evolution of Sobol’ indices for tangential strains

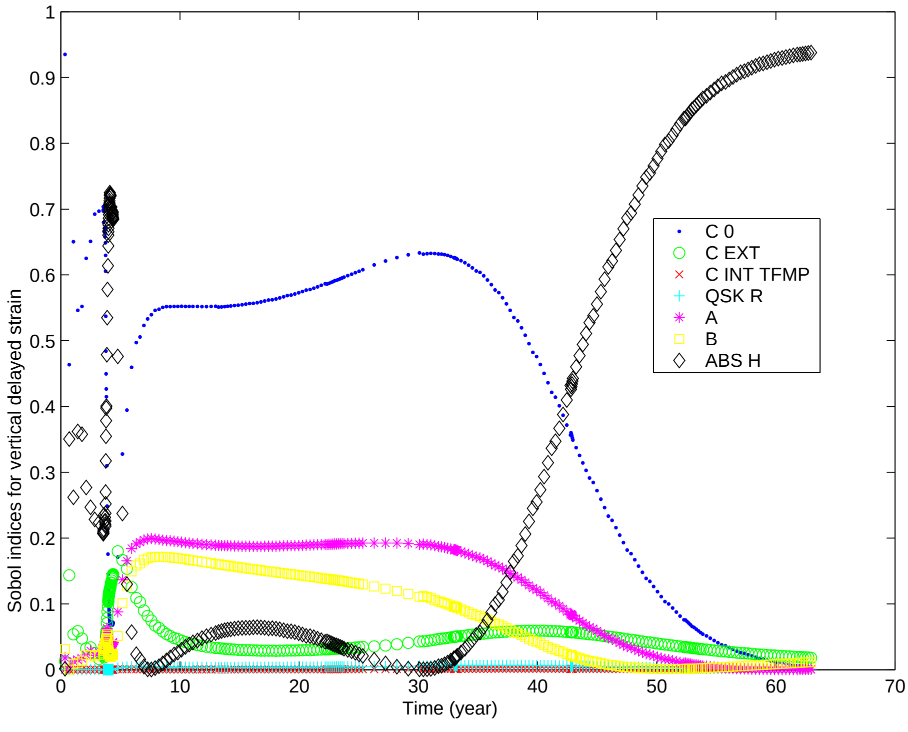

Figure 4: Evolution of Sobol’ indices for vertical strains

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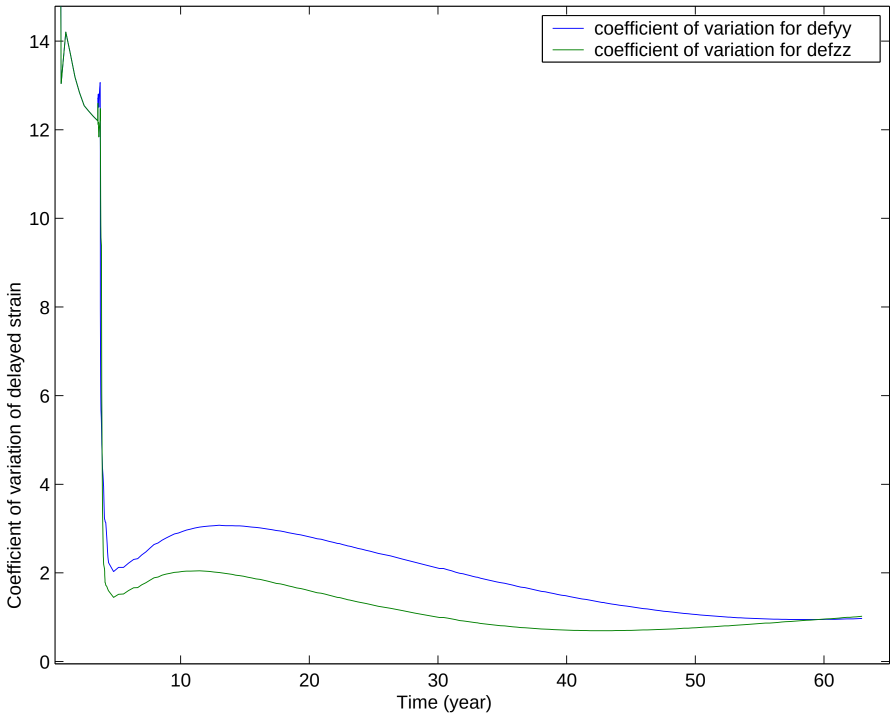

Figure 5: Evolution of coefficient of variation of tangentials and vertival strains

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