Wet versus dry cement pastes and concretes: A mathematical approach to their strength and fracture properties

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ABSTRACT: The fracture process of a continuous matrix in a porous medium under the combined effect of filtration and external mechanical loads is considered. Taking into account the differences between the failure mechanisms of cement paste under tension and its failure mechanisms under compression, an analytical approach to the relation between water flow and fracture in saturated porous Portland cement pastes is developed. The well known differences in behaviour between the flexural and compressive strengths of wet and dry Portland cement pastes is explained. The extension of the obtained results to the flexural and compressive strength of normal concrete is briefly discussed, including suggestions for further experimental and digital simulation work.

1 INTRODUCTION

Hardened Portland cement paste is a porous and viscoelastic material whose anelastic properties depend strongly of the relative humidity and water redistribution processes in the gel pores. The smaller gel pores (mainly of sub-micrometer dimensions) and the technological pores (from small capillary pores with dimensions of the order of $10\,\mu m$ to big crack-like pores with lengths of the order of $1\,mm$ or more) form a system of communicating cavities or channels, bifurcating and recombining at random. Under appropriate conditions, water flows through these tiny and irregular channels, dominated by viscous friction. Flowing water exerts both normal and shear stresses on the surface of the solid matrix, like an external load. After averaging to smooth the microscopic spatial fluctuations of the fields, this mechanical effect can be described by the so called "filtration pressure", a force per unit volume of the porous medium, acting on the solid matrix and produced by the flow. If there are closed cavities (like trapped air bubbles) that remain outside the flow, they must be considered together with the solid matrix, although they modify the local stress and strain distributions.

Thus, the strain and stresses produced by the filtration process must be combined with the strain and stresses produced by the external loads to describe the mechanical failure of a wet porous medium, like saturated cement paste, mortar or concrete.

It is well known that the flexural strength of a wet cement paste is usually higher than the flexural strength of this same cement paste after it is dried. The opposite happens with the compressive strength (Neville, 1995). Taking into account the differences between the failure mechanisms of cement paste under tension and its failure mechanisms under compression, the analysis of the fracture process in a porous medium is used here as a starting point to develop an analytical approach to the relation between water flow and fracture in saturated porous cement pastes. This approach is then applied to study the relation between the field of filtration pressures and: (a) the compressive strength of cylinders of cement paste, mortar or concrete (used as standard test bodies), (b) the flexural strength of beams of these same materials (that are also used as standard test bodies in four-point bending).

2 MODELLING TOOLS

To model the aforesaid filtration and fracture process in the cement paste we will use three kinds of tools:

(1) An average form of the equations that govern the transport processes in the pore space and the stresses in the solid matrix (Hall and Hoff, 2002; Bear, 1972; Scheidegger, 1963).

(2) Some results for solenoidal and almost gradient fields, taken from classical field theory and differential geometry, and applied to the average field

variables (Ericksen, 1960; Suárez-Ántola, 1983; Suárez-Ántola, 1997).
(3) A certain population of defects included in the matrix and a quantitative expression of the commonly admitted mechanisms of fracture of cement pastes under flexural and compressive loads (Chtchourov, 1980; Wittmann, 1983).

2.1 *Average field equations*

The fields of velocity and stress in the fluid and the stress field in the solid matrix fluctuate randomly along distances of the order of the local pore's diameter d (for a definition of the local pore diameter see Scheidegger, 1963). However it is possible to smooth out these fluctuations by averaging the field variables over regions of characteristic length l, much greater than d and called Darcy's scale. Selecting l, at its turn, much less than a characteristic length L of the whole porous body, we obtain averages that can be considered as true local values. A possible choice for l is the geometric mean $l = \sqrt{d.L}$. In our case we can estimate $d \cong 10^{-5}$ m or less, and $L \cong 10^{-1}$ m or less, so that $l \cong \sqrt{dL}$ between 10^{-4} and $10^{-3} m$ may be a suitable length scale for averaging purposes.

For each point in the porous medium we consider an averaging region that has this point as its centre, and then we assign to the point the averaged values of all the field variables, with independence of the true location of the point (in interconnected porous space, in solid matrix, including isolated voids, or in the boundary between them). We thus obtain a vector field of filtration velocities $\vec{v}_f$, a scalar field of averaged pore hydrostatic pressure $\bar{p}$, a scalar field of effective porosity ϕ and a tensor field of averaged matrix stresses $\hat{\sigma}$.

The emergent equations at Darcy scale are:

$$\frac{\partial \phi}{\partial t} = -\nabla \bullet \vec{v}_f \quad (1)$$

$$\vec{v}_f = -\frac{K}{\mu} \nabla \pi_f \quad (2)$$

where $\nabla \bullet ()$ represents the divergence, $\nabla ()$ the gradient, $\bullet$ the scalar product, $\pi_f = \bar{p} + \rho g z$ is the filtration potential (ρ is water density, g is gravitational acceleration, z is the vertical position from a reference horizontal plane, z increasing upwards), μ is water's dynamic viscosity and K is a scalar field that express the local hydraulic conductivity of the porous medium . Equation (2) gives Darcy's law for an isotropic medium. Equation (1) connects the variation of the fraction of pore's volume with water flow (Jaeger, 1969).
If $\hat{\sigma}$ is the tensor of averaged stresses in the matrix, the force acting on an element of surface of area ΔA and unit normal vector $\vec{n}$, at the scale of Darcy is given by the formula:

$$\Delta \vec{F} = \Delta A \, (\hat{\sigma} - \bar{p}\, \phi \, \hat{I}) \bullet \vec{n} \quad (3)$$

Then the complete average stress tensor in the solid matrix verifies the emergent static equation
$$\nabla \bullet (\hat{\sigma} - \phi\, \bar{p}\, \hat{I}) = \vec{X} \quad (4)$$
Here $\hat{I}$ is the unit tensor and $\vec{X}$ is the average external force per unit volume. The average force on the matrix, per unit of volume of a saturated porous medium, due to water flow, is the filtration pressure:
$$\vec{p}_f = -\phi \nabla \pi_f \quad (5)$$
So $\vec{p}_f$ is everywhere orthogonal to a surface of constant π_f. The filtration pressure is related with the average matrix stresses due to the filtration process $\hat{\sigma}_f$ by:
$$\vec{p}_f = -\nabla \bullet \hat{\sigma}_f \quad (6)$$

2.2 *Solenoidal and almost gradient fields*

For an isotropic medium, from equation (2) and (5) it follows that

$$\vec{p}_f = \frac{\phi\, \mu}{K} \vec{v}_f \quad (7)$$

If the phenomena of interest occur in time scales that are much less than the time scale of variation of pore's volume, equation (1) reduces to this one:
$$\nabla \bullet \vec{v}_f = 0 \quad (8)$$

Let us consider a vector line of the field $\vec{v}_f$. If $\vec{t}_f\,(s)$ is the unit tangent vector at the point of intrinsic coordinate s (s being the arc length), given any two points Q and P that belong to the line, we may apply Bjørgum's characterization of solenoidal (divergence-free) fields (Ericksen, 1960; Suárez-Ántola, 1983, Suárez-Ántola, 1997)

$$v_f(Q) = v_f(P)\, e^{-\int_{sP}^{sQ} \nabla \bullet \vec{t}_f(s)\, ds} \quad (9)$$

(Here $v_f = \|\vec{v}_f\|$ is the magnitude of the vector field).

Now, according to equation (2), $\vec{v}_f$ is an almost gradient field. Then each tangent unit vector $\vec{t}_f$ at a point of a filtration line is at the same time a unit normal vector to the surface of constant filtration potential that pass through the same point. In this case we have the additional relation, between $\vec{t}_f\,(P)$ and the mean curvature $H(P)$ of the surface of constant π_f at a given point P of the porous medium:

$$\nabla \bullet \vec{t}_f\,(P) = -2\, H(P) \quad (10)$$

From (9) and (10) it follows that:

$$v_f(Q) = v_f(P)\, e^{2\int_{sP}^{sQ} H(s)\, ds} \quad (11)$$

Then, from (7) and (11) we have:

$$\frac{p_f(Q)}{p_f(P)} = \frac{\phi(Q)\,K(P)}{\phi(P)\,K(Q)}\; e^{2\int_{sP}^{sQ} H(s)\,ds} \qquad (12)$$

To apply this last equation to study the fracture process in the cement paste, we need information about the filtration lines and the distribution of mean curvatures of the surfaces of constant π_f distributed along and orthogonal to the filtration lines. From equations (2) and (8) it follows that the filtration potential verifies the elliptical equation:

$$\nabla \bullet \left(K \nabla \pi_f\right) = 0 \qquad (13)$$

At an impermeable boundary (as the interface between the steel plates of the compression machine and the tested standard cylinder or cube) $\vec{v}_f \bullet \vec{n} = 0$, $\vec{n}$ being the unit normal vector to the interface at the considered point, so

$$\vec{n} \bullet \nabla \pi_f = 0 \qquad (14a)$$

At free water interface (as the free interfaces of the cylinders, cubes or beams used in standard strength tests:

$$\pi_f = constant \qquad (14b)$$

Equation (13) jointly with the boundary conditions (14a) and (14b) for π_f, allows us to determine the surfaces that correspond to constant values of π_f.

2.3 *Population of matrix defects and fracture under flexural and compressive loads*

The characteristic length l of Darcy's scale may be used to classify the population of defects in two broad classes: the microscopic defects whose sizes are much smaller than l and the macroscopic ones, much larger than l. The usually small fraction of defects with the same order of magnitude than l may be called mesoscopic.

At a microscopic level the irregular fracture surface of the cement paste seems to advance through a pull out process that separates the lenticular crystals of hydrated calcium silicate from each other, breaking the van der Waals bonds between these fairly elastic elements.

But from a macroscopic point of view the failure under simple compression begins with the growing of many micro-cracks from small pores, parallel to the external load, and continues with the fusion of the pores in master cracks (Zaitsev, 1980). Thus the compressive strength depends strongly of the cement paste porosity. Also from a macroscopic standpoint, the failure under tension often begins with the growth of a master crack already located in a highly stressed volume (HSV, having tensile stresses between 95 and 100% of the maximum) and often continues with relatively few accompanying small cracks. So, the tensile strength depends weakly of the cement paste porosity, and strongly of the lengths of the big crack-like pores in the HSV (Kendall et al., 1983).

Size and shape of the body, the load distribution applied to it and other systemic properties (Bazant, 2004), have a strong influence on the fracture process and the measured strength values. So, two specific cases of practical interest will be considered first: the test of the compressive strength of standard cylinders and the test of flexural strength of standard beams of square cross section.

3 SOME SIMPLE MODELS OF FLOW AND FRACTURE IN CYLINDERS AND BEAMS OF CEMENT PASTE

3.1 *Filtration effects in uniaxial compression of a porous cylinder*

To study the compressive strength, let us consider a vertical cylinder of saturated porous body, with a population of spherical cavities (Figure 1).

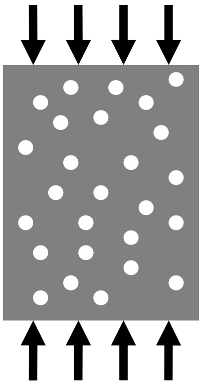

Figure1. Sketch of a compressed porous cylinder with a population of spherical pores

The radii of these cavities are at least of the same order that the Darcy's scale for the porous medium, so that they don't belong to the interconnected pore space used to construct the averaged fields. Many little cracks begin at the surfaces of the cavities, and go into the matrix, in all directions. If we apply compressive external loads to the flat faces, a radial filtration pressure field will be produced, directed toward the curved face which is both free from stress and a free-water boundary of the porous body. The vertical component of filtration field, due to gravity, may be neglected in the modelling of fracture in compressive strength tests. Then, the nominal stress field is the superposition of a vertical compressive stress (due to external load) and a horizontal plane tensile stress field (due to filtration) with both radial $\sigma_{f,r}$ and tangential $\sigma_{f,\theta}$ principal components. The compressive stress σ_z by itself produces a tensile stress in the matrix surrounding the upper and lower parts of the spherical cavities. If this stress is big enough, a crack located there will grow and a fracture process will begin. The radial and tangential tensile fields will produce an increment of the tensile stress already induced in the matrix by the compressive external load. As a consequence, due to filtration, the compressive strength of the wet porous body will be less than its compressive strength when it is dry. The symmetry of the problem simplifies the derivation of formulae for the filtration stresses $\sigma_{f,r}$ and $\sigma_{f,\theta}$, once the field of filtration pressures $p_f(t,r)$ is known. Let r_0 be the radius and h the height of the standard cylinder. Applying a volume balance to a coaxial cylinder of radius r less than r_0, and height h, ($2.\pi.\mathrm{r.h.v_f} = -\pi.r^2.h.\frac{\partial\phi}{\partial t}$) we obtain the field of radial filtration velocities

$$v_f = -\frac{r}{2}.\frac{\partial\phi}{\partial t} \tag{15}$$

From (15) and (7), it follows that

$$p_f(t,r) = f(t).r \tag{16a}$$

$$f(t) = -\frac{\mu.\phi}{2.K}.\frac{\partial\phi}{\partial t} \tag{16b}$$

As the time scale of the filtration process is at least an order of magnitude greater than the duration of the strength test, during the period of load application $f(t) \approx f(0)$. But from (5) it follows that: $\frac{\partial\pi_f}{\partial r} = \frac{\mu}{2.K}.\frac{\partial\phi}{\partial t}.r$ Integrating between 0 and r_0 and taking into account that $\pi_f(t,r_0) \approx \pi_f(0,r_0) = \pi_0$ is the atmospheric pressure (neglecting gravity) and is thus negligible relative to $\pi_f(t,0) \approx \pi_f(0,0) \approx |\sigma_z|$, we obtain:

$$|\sigma_z| \approx -\frac{\mu.r_0^{\,2}}{4.K}.\frac{\partial\phi}{\partial t} \tag{17}$$

Then, from (16b) and (17),

$$f(t) \approx f(0) \approx \frac{2.\phi}{r_0^{\,2}}.|\sigma_z| \tag{18}$$

Now, the problem of finding the filtration stresses when the filtration pressure verifies (16) and the solid matrix is homogeneous, isotropic and linear elastic is mathematically the same as the well known problem of finding the stresses in a linear elastic rotating disc (Nadeau, 1964). Adapting the solution of this classical problem to the filtration case we obtain (being ν Poisson's ratio):

$$\sigma_{f,r} = \frac{(3+\nu).\phi}{4}.|\sigma_z|.\left[1-\left(\frac{r}{r_0}\right)^2\right] \tag{19a}$$

$$\sigma_{f,\theta} = \frac{(3+\nu).\phi}{4}.|\sigma_z|.\left[1-\left(\frac{1+3\nu}{3+\nu}\right)\left(\frac{r}{r_0}\right)^2\right] \tag{19b}$$

Figure 2 shows a spherical cavity located at a distance r from the axis of the cylinder.

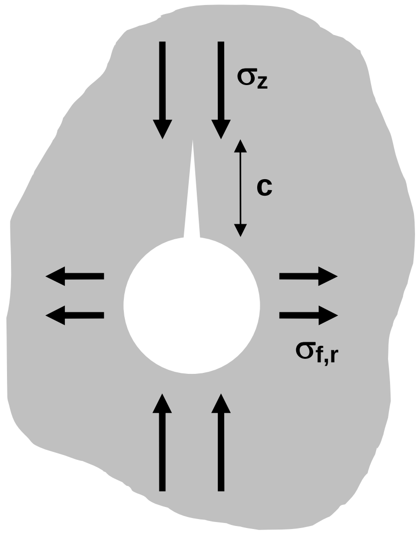

Figure 2 Sketch of a spherical cavity with a vertical crack of length c

A vertical crack extends from the upper surface of the cavity, with its plane orthogonal to the radial direction (tangential crack). Let us suppose that the onset of crack propagation occurs when the nominal tensile stress field at the crack location reaches a threshold value σ_{th}. If there is no filtration stress, the only nominal tensile stress is due to the compressive load P_c and its value is $|\sigma_z| = \frac{|P_c|}{\pi.r^2{}_0}$. At the critical state, the threshold load is $\pi.r_0^{\,2}.\sigma_{th} = |P_{c,th}|$. If there are filtration stresses, $\sigma_{f,\theta}$ has no effect in this case, but $\sigma_{f,r}$ produces a nominal tensile stress $3.\sigma_{f,r}$ at the location of the crack, so that now the total nominal tensile stress is $|\sigma_z| + 3.\sigma_{f,r}$. Taking into account equation (19a), the critical stress would be reached when

$$|\sigma_z|.\left[1+\frac{3.(3+\nu)}{4}.\phi.\left(1-\left(\frac{r}{r_0}\right)^2\right)\right] = \sigma_{th}$$

The new threshold load $|\tilde{P}_{c,th}|$ will be related with the old one by the relation:

$$\left|\tilde{P}_{c,th}\right|\cdot\left[1+\frac{3.(3+\nu)}{4}.\phi.\left(1-\left(\frac{r}{r_0}\right)^2\right)\right]=\left|P_{c,th}\right| \qquad (20a)$$

If the plane of the vertical crack is radial, $\sigma_{f,r}$ has no effect and $\sigma_{f,\theta}$ is active now. In this case we obtain from (19b):

$$\left|\tilde{P}_{c,th}\right|\cdot\left[1+\frac{3.(3+\nu)}{4}.\phi.\left(1-\left(\frac{1+3.\nu}{3+\nu}\right)\left(\frac{r}{r_0}\right)^2\right)\right]=\left|P_{c,th}\right| \qquad (20b)$$

3.2 *Filtration effects in bending of a porous beam in a flexural strength test*

Now, consider a wide master crack in a saturated porous beam with a continuous matrix, such that the crack reaches the lower boundary of the body. If the beam is in the air and is flexed as suggested in fig. 3, water will tend to flow downwards.

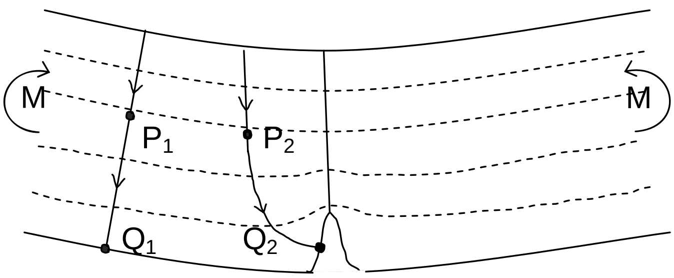

Figure 3. Sketch of a flexed porous beam with some filtration lines (represented by continuous curves) and several surfaces of constant filtration potential (represented by dotted curves).

When water is filtering, a congruence of regular surfaces of constant π_f fills the porous body. The flow velocity and the vector of filtration pressure are at every point orthogonal to the surface of constant filtration potential that passes through this point. At each boundary with free water conditions, a surface of constant π_f closely follows the boundary. The water inside the wide crack can be considered as free water. Then a surface of constant filtration potential will follow closely the lateral faces of the crack, and the other surfaces of constant π_f will be disposed as is shown qualitatively in the figure. The tip of the crack is in a tensile nominal stress field due to the external loads. However the filtration lines will arrive orthogonally to the crack surface, so that the magnitude of the tensile components will be diminished by the effect of the filtration forces (that tend to close the crack). So, while the presence of the crack lowers the local elastic energy, the filtration pressure around the crack raises this local elastic energy and modifies the global energy balance that appears in the energy theory of fracture. Indeed, from equation (3) it follows that the tensile components of the matrix stress tensor will be larger in locations where the product $\phi\bar{p}$ is higher, if we assume static conditions. If we follow a given filtration line, the average pore pressure $\bar{p}$ will be a decreasing function of the arc length, so that if ϕ doesn't increase too much and the other conditions remain constant, the tensile components of the matrix stress tensor will decrease along the filtration line. Beginning with equation (4), it is possible to derive an approximate formula for the elastic energy release in the solid matrix, due to the presence of the crack, including the effect of the filtration process. If the master crack crosses completely the beam of width a, and if c is its length, then the elastic energy released is

$$-\pi.c^2.a.\left(\frac{\sigma^2{}_x}{2.E}-\frac{\phi.\bar{p}.\sigma_x}{E}\right) \qquad (21)$$

Here σ_x is a horizontal nominal tensile stress in the matrix, $\bar{p}$ is a nominal pressure in the pore space, and E is Young's modulus. If G_s is the average energy absorption in the fracture process zone (FPZ) per unit area of crack advance, the increase in the beam's energy due to the presence of the crack is $2.G_s.a.c$ (22). The sum of these two energies will have a maximum when the stress in the matrix verifies the threshold condition:

$$\sigma^2{}_x-2.\phi.\bar{p}.\sigma_x-\frac{2.G_s.E}{\pi.c}=0 \qquad (22)$$

Both σ_x and $\bar{p}$ are proportional to the bending moment M produced by the loads during the strength test:

$$\sigma_x=k_\sigma(z).M \qquad (23a)$$

$$\bar{p}=k_p(t,z).M \qquad (23b)$$

As the master crack length may be several mm, while the height a of the beam is not less than $50mm$, we may suppose that $k_\sigma\approx\frac{a}{2.J}$ (J is the moment of area of the cross section). The dependence of k_p on the system's parameters remains to be studied. Nevertheless, for the same reasons that in the compressive strength test, we can suppose that $k_p(t,z) \cong k_p(0,z)$

From (22) and (23) we obtain:

$$\tilde{M}_{th} \approx \frac{M_{th}}{\sqrt{1 - 2.\phi.\dfrac{k_p}{k_\sigma}}} \tag{24}$$

By definition: $M_{th} = \dfrac{\sigma_{th}}{k_\sigma}$ and $\sigma_{th} \approx \sqrt{\dfrac{2.G_s.E}{\pi.c}}$ is a well known stress threshold of fracture mechanics. As a consequence the flexural strength of the porous beam will be higher when it is wet and saturated, relative to its value when the body is dry. This difference should increase when the average porosity (of the pore space in which water flows) increases. If as a coarse approximation we put $k_\sigma \approx k_p$, from (24) it follows that the quotient of threshold bending moments is only a function of the porosity of the pore space engaged in filtration.

To quantify the concentration of filtration pressures due to the crack, let us follow a filtration line in the flow direction. If the line ends in a flat portion of the lower boundary of the beam, there is not a significant variation in p_f along the line, when we pass from a point P_1 in the bulk of the medium to a point Q_1 in the interface (fig. 3), because the integral $\int_{s_{P1}}^{s_{Q1}} H(s)\,ds$ that appears in formula (12) will be nearly zero. If the filtration line ends in the bottom of a cavity, then when we pass from a point P_2 in the bulk of the porous medium to a point Q_2 in the interface, p_f can increase significantly, because in a neighbourhood of the interface the line will cross the surfaces of constant π_f in points where the mean curvature is positive and non-negligible (fig. 3). When the depth of the cavity increases and the mean curvature at Q_2 increases also, the integral $\int_{s_{P2}}^{s_{Q2}} H(s)\,ds$ will increase too. The extrapolation of the results of the analysis of several idealized models with plane symmetry (for example, an infinite groove with half-elliptical cross section in the surface of a half-space filled with a porous medium, homogeneous and isotropic; see Harr, 1991) suggest that $p_f(Q_2)$ must be an increasing function of the product of a measure of the depth of the groove (or notch) by the mean curvature at the bottom. For lengthen cavities like cracks, $p_f(Q_2)$ may be one or two orders of magnitude greater than $p_f(P_2)$.

4 MORTARS AND PLAIN CONCRETES

Up to now, we studied the onset of crack propagation under the combined effects of external loads and filtration pressure, in two idealized situations: a spherical cavity with a vertical crack in a compressed cylinder of hardened cement paste, and a vertical master crack in a beam of cement paste in four points bending.

In order to extend the analysis of the effects of the filtration pressure to the cases of compressive and tensile failures of normal mortars and concretes, several things must be considered. Amongst them:

(1)The transition ring, that appears at the interface between the paste and the aggregates (Maso, 1980).

(2) The difference in stiffness between the cement paste and the fine and coarse aggregates.

(3)The gradual strain softening and the crack band in the FPZ (Bazant and Oh, 1983).

(4)The stages of crack growth prior to fracture: initiation, slow stable crack growth, crack arrest, a true threshold condition, and unstable crack propagation up to definitive rupture.

Let us consider the typical case of a mortar or concrete with relatively non-porous aggregates, stiffer and stronger than the cement paste that surrounds them.

At the time of mixing, a film of water appears between the surface of the non-porous aggregate and the bulk of the water-cement mix. Due to this circumstance a transition ring of cement paste is formed, usually weaker and more porous than the bulk of the paste.

If the aggregates are stiffer than the bulk of the cement paste, this usually produces stress concentrations in the aggregates and in the adjacent transition rings. In a nominal one dimensional stress field, like the one found in a standard compression test of a cylinder of mortar or plain concrete (neglecting the complications introduced by the restrictive action of the plates of the testing machine), or in a small enough region of a beam in the standard four-point bending test), a non-uniform three dimensional state of stress is produced in the composite material due to the abovementioned difference between the elastic moduli of the paste and the aggregates.

If the aggregates are stronger than the hardened cement paste, a growing crack will run around the aggregate travelling in the transition ring and eventually leaving it behind if the crack growth continues. Then, the crack path in normal concretes will be fairly tortuous, and the aggregate particles will act as crack arresters. In high strength concretes things are usually different (Wittmann, 2002).

In both, mortars and normal concretes, the fracture process begins with the growing of one or more cracks suitably located and oriented in relation with the stress field. Satellite micro- cracks emerge from the growing tortuous crack. A band-like fracture process zone (associated with the strain softening already mentioned) is produced. As a consequence of the crack arresting processes and the complex pattern of cracks that is produced, hardened mortars and concretes are usually less brittle than hardened cement pastes. Concretes show two critical loads in standard compressive strength tests. The first critical load corresponds to crack growth initiation. In concrete, above the first critical load, Poisson's modulus begins to increase monotonically towards 0.5. Between the first and the second critical loads there is stable crack growth. At the second critical load the cubic dilatation of the stressed cylinder reaches its maximum value.

Above the second critical load there is unstable crack propagation to definitive rupture.

However, the effect of the filtration field in the fracture processes of mortars and concretes, at least up to the first critical load, should be in its main traits fairly similar to the effect already discussed for cement pastes, not only for the compression test, but also (and mainly) for the bending test.

Consider first a vertical cylinder of concrete, porous and wet saturated, under uniaxial compression. Due to the assumed differences in stiffness, the bulk of cement paste is less stressed than the transition rings, and these are less stressed than the cores of the aggregate particles. These variations are mild in relation with sand particles, but very significant in the case of coarse aggregates.

The fields of filtration velocities and filtration pressures are locally modified by the non-uniformities in the stresses (that produce variations in the dimensions of the interconnected pore space), by the stress-independent variations in porosity (higher in the transition rings than in the bulk of the matrix), and by the blocking effect of the impervious aggregate particles over the water flow in the pores. Nevertheless, on the average, they point outwards from the cylinder axis, as in the case of the cement paste, and tend to enhance the growth of cracks, mainly in vertical direction. Thus, the first critical load in compression must be smaller for a wet cylinder in comparison with a dry one (all the other things remaining equal). We could expect that for a splitting mode of crack growth, the difference in the first critical load should increase with porosity in a way similar to the cement paste threshold load studied above. Between the first and the second critical load, the changing and expanding pattern of cracks and micro-cracks produces a progressive modification of the filtration field and the corresponding filtration pressures.

The filtration field and filtration pressures in bended beams of mortars and concretes are locally modified by the heterogeneities as were described for a cylinder under compression. However, the lower portion of the beam, where the fracture process begins with the growing of a master crack, is under tensile nominal stress. On the average this enhances the filtration process and increases the filtration pressures that tend to close the crack for the same reasons already considered for the cement paste beam. Nevertheless, the arresting effect of the aggregates and the transition rings increases the number and tortuosity of the pattern of satellite micro-cracks connected with the master crack. In spite of the blocking effect of the impervious aggregates, this could enhance the local filtration process even more. As a consequence, for a wet saturated beam of mortar or concrete, the critical value of the bending moment that corresponds to the onset of a master crack growth should be greater than the critical value for a dry beam, all the other things remaining equal.

CONCLUSIONS

Now we have a solution to the problem posed in the introduction: to explain the differences in the behaviour of the flexural and the compressive strengths of a body of cement paste when it is tested wet and saturated or when it is tested dry.

In the case of compressed cylinders, given a crack with a tangential plane and a threshold stress, from (20a) it follows that the most dangerous cracks from the standpoint of the onset of crack propagation are the ones located on the axis of the cylinder. The tangential cracks located near the surface of the cylinder are almost not affected by filtration stresses. Given a crack with a radial plane, from (20b) it follows that again the most dangerous cracks are located on the axis. However, now the cracks located near the surface of the cylinder are affected by filtration stresses, and if they size is big enough, a fracture onset may happen in one of these radial cracks. In the case of bended beams, the model should be completed with a formula for k_p. In principle this could be done from a water balance as was done for the cylinder, but the arguments are subtler and lengthier and will be developed elsewhere.

Albeit the analytical formulae obtained in this work are crude approximations to very complex situations, they can be improved by means of more realistic analytical models of the onset of crack propagation. The energetic approach used here could be extended to cope with the essentially heterogeneous nature of fracture processes in cement pastes, mortars and concretes. The obtained results may be compared with true experimental results, or with the results of digital simulations done with realistic nonlinear models and suitable computer codes.

Even in the case of cylinders of cement paste, compressed without filtration pressure effects, it is well known that the pattern of crack growth that leads to fracture may sometimes correspond to what seems to be a shear mode. In fact, if the pores are randomly distributed, the pore cracks are randomly oriented and the pores are near enough, both digital simulations and experiments suggest that the probability of having a pattern of crack propagation that ultimately leads to rupture in an apparent shear mode instead of a vertical splitting mode, increases with the pore density (Zaitsev, 1980). From the qualitative discussion of the fracture processes of mortars and normal concretes under filtration pressure fields, given in part 4, it follows that the analytical approach could be only a guide to further experimental and numerical studies in the subject of the present paper. Only a thorough numerical approach can afford suitable tools and realistic results for an in depth study of the fracture processes under the effects of filtration pressure fields in concrete, especially in problems related with structures.

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