Influence of cover thickness on the flexural response of fibrous RC beams

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ABSTRACT: An analytical model is proposed in the present paper aimed at describing the response of simply- supported beams loaded in four-point bending. Several cases concerning normal- and high-strength concrete beams, reinforced with longitudinal bars, stirrups and steel hooked fibers are considered, and their load-deflection curves, as well as their moment-curvature curves and the ultimate deflections at the onset of failure in bending or shear, are obtained by means of the proposed, simplified approach. The satisfactory fitting of the test results confirms the potential of the proposed model, that can accurately predict beam behaviour in bending and shear, including the spalling of the cover and the contribution of the fibers.

1 INTRODUCTION

It is well known that the use of short fibers dispersed in fresh concrete in an adequate percentage and geometry ensures, in the hardened state, effective flexural and shear reinforcement in reinforced-concrete beams and deep elements.

For fibrous reinforced concrete (FRC) also the ultimate shear strength increases with increasing flexural reinforcement ratio and with increasing compressive strength, while not increases with the shear span-to-depth ratio.

From the mechanical point of view several studies presented in the literature refer to the calculus of the shear strength of fibrous reinforced-concrete beams made of normal- or high-strength concretes, normal or lightweight.

The analytical expressions given are often of an empirical nature and they account for the increase in the bearing capacity of the beams, owing to the bridging capacity of fibers across the principal cracks.

It is moreover generally assumed that fibers do not significantly influence the mechanisms observed at failure for beams without fibers (e.g. beam and arch actions) and consequently the expressions given separately consider the strength contributions due to the several parameters governing shear failure.

Referring to the determination of the load-deflection curves, several analytical models based on plane section hypothesis are available for beams affected by flexural failure and implemented in well known computer code (e.g. DRAIN-2DX, 1993). Analytical simplified flexural models are also available for fibrous concrete beams with and without steel reinforcements, but very few simple models are able to determine the load-deflection curves when shear failure occurs.

Several models for the accurate prediction of load-deflection curves of beams affected by shear or flexural failure are available based on non linear finite element analyses such as the model proposed by Cervenka (2000) utilizing ATENA code or that proposed by Vecchio (2000) the latter (based on the Modified Compression Field Theory) very successfully utilized in many applications.

Very effective computational truss models are also proposed (e.g. Noghabai, 2000).

Most of these models are very effective but are also quite complex.

They require knowledge of the complete constitutive laws of materials including the most relevant parameters governing phenomena, as concrete size effect and strain softening.

On the basis of these considerations, the focus here is on the development of a simplified model able to determine the load-deflection curves of simply-supported, longitudinally- and transversely- reinforced fibrous beams under flexure and shear.

The model proposed is specific for fibrous reinforced concrete beams with steel fibers (most commonly utilised for structural applications) and is based on knowledge of very few mechanical properties of plain concrete, steel bars and the geometrical properties and volume content of steel fibers.

2 THE CASE STUDY

The case study considers a prismatic reinforced concrete beam with rectangular cross-section, base B and depth h, which is cast with plain or fibrous concrete. The static scheme adopted consists of simply-supported beams under four-point bending tests, in which the beams are subjected to symmetrical vertical loads, V, acting at distance a from the support, as shown in Figure 1.

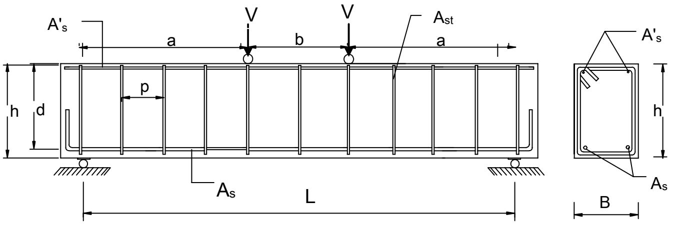

Figure 1. Static scheme of the beam.

The beam is reinforced on the lower side with longitudinal deformed bars having area A_s with a cover thickness c and on the top side with steel bars having area A'_s with a cover thickness c'. The compressed area of longitudinal steel is assumed as being negligible with respect to A_s. Both bars had yielding stress f_y.

In addition, transverse steel stirrups having an area of one leg A_{st} are placed in the beam at pitch p.

The main longitudinal steel bars (bottom bars) are bent at the support with an adequate anchorage length so as to avoid any slippage or premature splitting failure. The beam is reinforced at the bottom with geometrical ratio $\rho = A_s/(B \cdot d)$ with d effective depth. The stirrups had yielding stress f_{yw} and geometrical ratio ρ_{sw} the latter defined as $\rho_{sw} = 2 \cdot A_{st} / (B \cdot p)$.

In the case of fibrous concrete, the fibers utilized have length L_f, equivalent diameter ϕ and volume percentage v_f. In order to take the characteristics of the fibers into account, in the following sections the fiber factor F will be introduced, defined as $F = v_f L_f \beta/\phi$, with β being the shape factor assumed as 1 and 0.5 for deformed and straight fibers respectively.

This assumption for β is in agreement with the results obtained in Banthia and Trottier (1994) from which it appears that the pull-out resistance of straight fibers is less than for hooked or crimped steel fibers (having a similar aspect of ratio) while the values of β are those suggested in Campione et al. (2006).

3 SIMPLIFIED MODEL

In the following sections the evaluation of the bearing capacity of beams and their load-defection curves will be considered in the case of both shear and flexural failure. In the case of flexural failure, under the hypothesis of perfect bond of steel bars and concrete, the plane section theory will be considered, including also the strength contribution due to the residual tensile strength of fibrous concrete.

No tension stiffening will be considered. Referring to shear failure (cases of shear compression and diagonal tension modes are considered without premature splitting failure) an analytical expression, based on the analysis of beam and arch actions at rupture, will be utilized.

3.1 *Flexural strength of fibrous reinforced concrete beams*

Using translation and rotational equilibrium conditions it is possible to determine the position of the neutral axis x_c, and the flexural moment at cracking, yielding, compressed cover spalling and failure of compressed zone defined as M_c, M_y M_s and M_u and the corresponding curvatures ϕ_c,ϕ_y, ϕ_s and ϕ_u. The above-mentioned stages are analyzed here referring to the strain and stress distribution shown in Figure 2 (I, II, III, IV) and related to the shear force V by the equilibrium condition V=M/a.

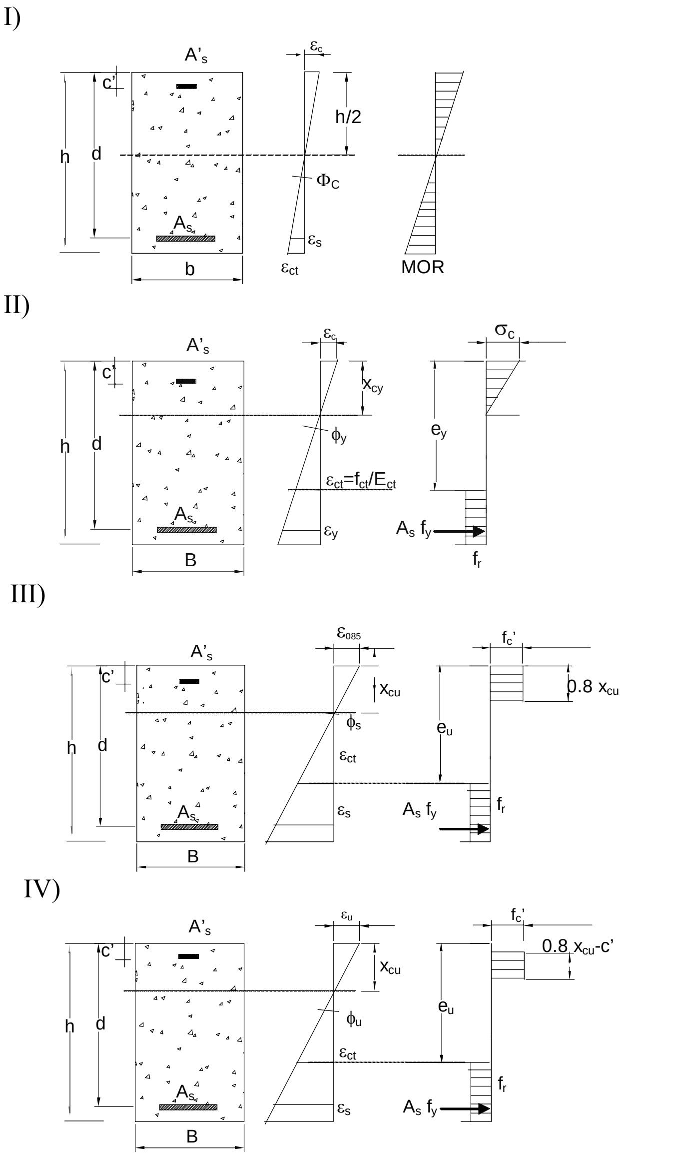

Figure 2. Design assumptions in the analysis of R/C sections.

By adopting the above-mentioned model the simplified moment-curvature diagram shown in Figure 3, (stages I, II, III, IV are also indicated), is obtained. A similar approach was also utilised by Rashid and Mansur (2005) for high-strength concrete beams in flexure.

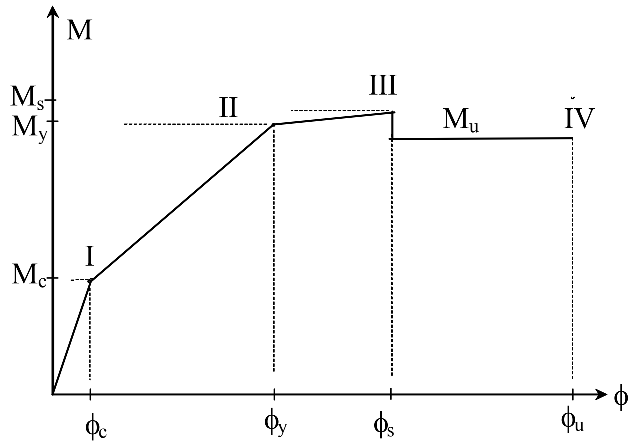

Figure 3. Simplified model for moment-curvature diagram.

To develop the simplified model the following stress-strain curve, proposed by La Mendola and Papia (2002), under monotonic loading is assumed:

$$\frac{\sigma}{f_{cf}} = \frac{A \cdot \frac{\varepsilon}{\varepsilon_{0f}} + (D-1) \cdot \left(\frac{\varepsilon}{\varepsilon_{0f}}\right)^2}{1 + (A-2) \cdot \frac{\varepsilon}{\varepsilon_{0f}} + D \cdot \left(\frac{\varepsilon}{\varepsilon_{0f}}\right)^2} \tag{1}$$

in which A= E_c /E_0, with E_0 the secant modulus at peak stress defined as $E_0=f_{cf}/\varepsilon_{of}$, being f_{cf} and ε_{0f} the maximum compressive strength and corresponding strain respectively, and E_c the initial modulus of elasticity in compression assumed as in Razvi and Saatciouglu (1999) to be variable with the compressive strength.

The D parameter governs the slope of the descending branch and was the chosen variable with F, according to the expression:

$$D = 0.3136 + 0.175 \cdot F \tag{2}$$

The equation (2) was calibrated on the basis of experimental data available in the literature for fibrous concrete compressed members with hooked steel fibers (data from Fanella and Naaman 1983).

The maximum compressive strength f_{cf} and corresponding strain ε_{0f} of normal strength FRC with steel fibers, proposed by Nataraja et al.(1999), can be assumed as:

$$f_{cf} = f_c' + 6.913 \cdot F \tag{3}$$

$$\varepsilon_{0f} = \varepsilon_0 + 0.00192 \cdot F \tag{4}$$

being ε_0 the strain at peak stress assumed as in Razvi and Saatciouglu (1999) to be variable with the compressive strength.

It is possible to obtain the value of strain ε_{085} in a closed form by using Eq. (1), assuming that the σ value is equal to 0.85 f_{cf} for $\varepsilon = \varepsilon_{085}$ resulting in the following second order equation:

$$\begin{aligned}&\varepsilon_{0.85}{}^2 \cdot (1 - 0.15 \cdot D) - \varepsilon_{0.85} \cdot (1.7 + 0.15 \cdot A) \cdot \varepsilon_{of} + \\ &+ 0.85 \cdot \varepsilon_{of}{}^2 = 0\end{aligned} \tag{5}$$

This value will be assumed in the following sections for the calculus of M_s and ϕ_s.

Referring to Case I of Figure 2 for the calculus of the cracking moment M_c (if the contribution due to the longitudinal bars is neglected) it is possible to adopt the following relationship:

$$\frac{M_c}{B \cdot h^2} = \frac{MOR}{6} \tag{6}$$

MOR being the modulus of rupture in flexure of FRC well defined in ACI 544 (1988).

The corresponding curvature is:

$$\phi_c = \frac{2 \cdot MOR}{h \cdot E_c} \tag{7}$$

in which E_c can be evaluated with the expression $E_c=4200 \cdot (f'_c)^{0,5}$ (MPa) proposed by ACI 318-02 (2002).

Referring to the calculus of the yielding moment M_y reference is made to Case II (Fig. 2) in which the yielding strain ($\varepsilon_y = f_y/E_s$) is attained in the longitudinal bars in tension and the residual strength f_r of fibrous concrete in tension is reached (this post-cracking tensile strength of FRC is better defined in the following sections). For concrete in compression linear elastic behavior is supposed to be due to the lower deformation reached. To obtain the neutral axis position x_{cy} the stress distribution of Figure 2 (stage II), is assumed. From the translational equilibrium we obtain the following equation:

$$\begin{aligned}&\frac{1}{2} \cdot \frac{E_{cr} \cdot f_y}{E_s} \cdot \frac{x_{cy}}{d - x_c} \cdot B \cdot x_{cy} - f_y \cdot A_s + \\ &- f_r \cdot B \cdot (d - e_y) = 0\end{aligned} \tag{8}$$

e_y being the distance between the more compressed fiber of the transverse cross-section and the fiber with the maximum tensile strength in concrete, while E_{cr} is a reduced value of initial modulus of elasticity assumed 0.5 E_c as in Rashid and Mansur (2005) to include an average amount of cracking and tension stiffening effects. Its values can be obtained by considering the plane section hypothesis resulting in:

$$e_y = \frac{\frac{f_t}{E_{ct}} \cdot (d - x_{cy}) + \varepsilon_y \cdot x_{cy}}{\varepsilon_y} \tag{9}$$

E_{ct} being the elastic modulus of concrete in tension assuming half of that in compression.

The values of x_{cy} and e_y are obtained by introducing Eq. (9) in Eq. (8) and producing the second degree equation for position of the neutral axis:

$$\left[\frac{1}{2}\cdot\frac{E_c\cdot f_y}{E_s}\cdot B + f_r\cdot B\cdot\left(\frac{\varepsilon_{ct}-\varepsilon_y}{\varepsilon_y}\right)\right]\cdot x_{cy}^2 + \left(f_r\cdot B\cdot c + f_y\cdot A_s\right)\cdot x_{cy} + \left(f_r\cdot B\cdot d^2\cdot\frac{\varepsilon_{ct}-\varepsilon_y}{\varepsilon_y} - f_y\cdot d\cdot A_s\right) = 0 \tag{10}$$

The internal arm value can be expressed by means of:

$$z = d - \frac{x_c}{3} \tag{11}$$

From the rotational equilibrium it is possible to obtain the yielding moment M_y in the following form:

$$\frac{M_y}{B\cdot d^2} = \rho\cdot f_y\cdot\left(1-\frac{0.33\cdot x_{cy}}{d}\right) + f_r\cdot\left(\frac{h}{d}-\frac{e_y}{d}\right)\cdot\left(\frac{2}{3}\cdot\frac{x_{cy}}{d}-\frac{h-e_y}{2\cdot d}\right) \tag{12}$$

The curvature corresponding to step M_y (first yielding or Stage II in Fig. 2) results in:

$$\phi_y = \frac{f_y}{E_s}\cdot\frac{1}{d-x_{cy}} \tag{13}$$

When referring to the calculus of M_s, corresponding to the cover spalling process (Stage III in Figure 2) and of M_u, at the ultimate state (Stage IV in Figure 2), it was assumed that longitudinal bars are yielded and maximum compressive strength of concrete is reached.

With reference to the symbols in Figure 2 (stage III) and considering the translational equilibrium we obtain:

$$x_{cu} = \frac{d}{0.8}\cdot\frac{\rho\cdot f_y + f_r\cdot\frac{h}{d}}{f_c^{'} + f_r\cdot\frac{f_t/E_{ct}+\varepsilon_{085}}{0.80\cdot\varepsilon_{085}}} \tag{14}$$

The moment corresponding to the cover spalling process from the rotational equilibrium results in:

$$\frac{M_s}{B\cdot d^2} = \rho\cdot f_y\cdot\left(1-\frac{0.4\cdot x_{cu}}{d}\right) + f_r\cdot\left(\frac{h}{d}-\frac{e_u}{d}\right)\cdot\left(\frac{h}{d}-\frac{h-e_u}{2\cdot d}-\frac{0.4\cdot x_{cu}}{d}\right) \tag{15}$$

e_u being the distance between the more compressed fiber and the fiber at which the maximum tensile strength in concrete is reached.

$$e_u = x_{cu}\cdot\frac{\frac{f_t}{E_{ct}}+\varepsilon_{085}}{\varepsilon_{085}} \tag{16}$$

Therefore the curvature at spalling state is:

$$\phi_s = \frac{\varepsilon_{0.85}}{x_{cu}} \tag{17}$$

From the rotational equilibrium the ultimate moment results in:

$$\frac{M_u}{B\cdot d^2} = \rho\cdot f_y\cdot\left(1-\frac{0.4\cdot x_{cu}}{d}-\frac{c^{'}}{2\cdot d}\right) + f_r\cdot\left(\frac{h}{d}-\frac{e_u}{d}\right)\cdot\left(\frac{h}{d}-\frac{h-e_u}{2\cdot d}-\frac{0.4\cdot x_{cu}}{d}+\frac{c^{'}}{2\cdot d}\right) \tag{18}$$

Therefore the curvature at ultimate state is:

$$\phi_u = \frac{\varepsilon_{su}}{x_{cu}} \tag{19}$$

ε_{su} being the maximum strain of bar in tension assumed 0.01.

Moreover, the arm of the internal forces is j_0 d in which j_0 can be expressed by the equilibrium across the compressive centroid in the following form:

$$j_0 = \frac{\rho\cdot f_y\cdot\left(1-0.4\cdot\frac{x_{cu}}{d}\right)+f_r\cdot\left(\frac{h}{d}-\frac{e_u}{d}\right)\cdot\left(\frac{h+e_u}{2\cdot d}-0.4\cdot\frac{x_{cu}}{d}\right)}{\rho\cdot f_y + f_r\cdot\left(\frac{h}{d}-\frac{e_u}{d}\right)} \tag{20}$$

The previous equations highlight the influence of the main parameters governing the flexural response of the transverse cross-section including the effect of cover thickness also. It is interesting to observe that the cover thickness of the compressed bars plays a role on the ductility of the cross-section also. But, because low values of cover thickness are generally utilized, no significant changes in the overall flexural behavior of the cross-section occurs, while its contribution can be significant in the presence of higher cover thickness that can be required for durability reasons.

In the previous equations, f_t and f_r indicate the maximum and the residual strength of fibrous concrete. For the maximum tensile strength, the same value found in the literature plain concrete was assumed, while for the residual strength the proposal by Marti et al. (1999) was adopted:

$$f_r = 0.375\cdot F\cdot\left(f_c^{'}\right)^{0.66} \quad \text{(in MPa)} \tag{21}$$

To validate the proposed model a comparison with data given in Swamy and Al-Ta'an (1981) is shown. The experimental research refers to a third point bending test on fibrous reinforced concrete beams of 2250 mm length between the two lateral supports. The beams had a rectangular cross-section with B = 130 mm, h = 203 mm, c = 18 mm, the area in tension constituted by two deformed bars of 12 mm diameter, and the area in compression constituted by two 10 mm deformed bars. Plain concrete of f'_c = 30 MPa and steel bars with yielding stress f_y = 460 MPa. The beams were also reinforced with transverse stirrups of 6 mm diameter and pitch p = 125 mm. Hooked steel fibers were utilised with 50 mm length and 0.5 mm diameter at volume percentage of 0, 0.5 and 1%.

Figure 4 shows the experimental and the analytical moment-curvature diagrams for cases of 0 and 0.5 % of fibers.

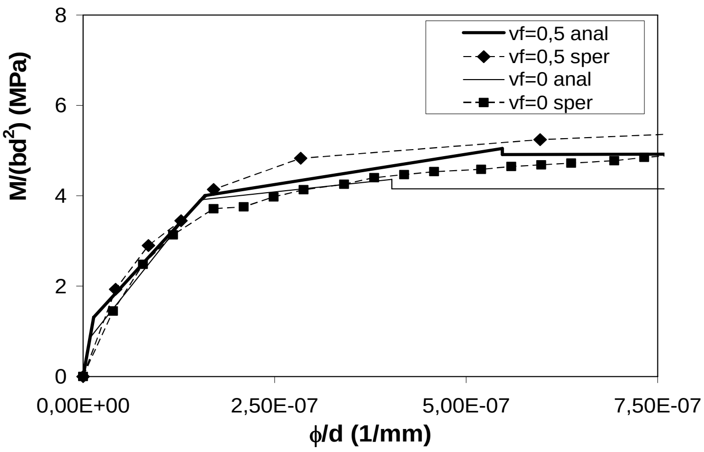

Figure 4. Experimental and analytical moment-curvature curves of beams with v_f =0 and 0.5% (data from Swamy and Al Ta'an, 1981).

The comparison shows the ability of the simplified model to predict the experimental response including yielding of main bars and crushing of concrete. The model is also able to include the spalling of compressed cover occurring at high curvature.

3.2. *Shear strength of fibrous reinforced concrete beams*

Bazant and Kim (1984) propose a mechanical model to calculate the flexural capacity in shear of reinforced concrete beams considering the sum of the strength contributions due to the beam and arch actions. These contributions are identified, as shown in Fig. 5, by imposing conditions of equilibrium of the beam enclosed between the support and the loaded section (shear span a). With reference to the symbols shown in Fig. 5, the bending moment M and the shear force V at the generic cross-section can be related to the axial force T in the longitudinal bar and to the internal arm jd by means of:

$$M = V \cdot x = T \cdot jd \qquad (22)$$

Moreover, between the shear force V and the bending moment M there is the following, well-known relationship:

$$V = \frac{dM}{dx} = jd \cdot \frac{dT}{dx} + T \cdot \frac{d(jd)}{dx} \qquad (23)$$

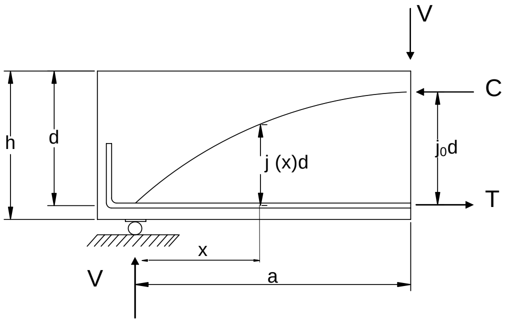

Figure 5. Beam and arch actions.

obtaining by means of Eq. (22) in Eq. (23) the two fundamental strength contributions well known in the literature as beam effect (jd constant) and arch effect (jd variable).

According to this model, well justified in Bazant and Kim (1984) and here utilized also for fibrous concrete beams, the following simplified hypotheses are assumed: - beam and arch effects are considered separately; - in the evaluation of arch effect it is assumed that the tension force in longitudinal bar remains constant across the shear span;- in the evaluation of beam effect the tension force is supposed to be variable across the shear span. In this was the expression given by Campione et al. (2006) was obtained including in the beam and arch effect the effect of fibers and also considering the contribution made by the fibers through the main cracks. For the beam effect it was assumed that the bond stress q_b is proportional to the tensile strength of the concrete, which in turn is proportional to the square root of the cylindrical compressive strength f'_c . This choice is valid for plain concrete and assumed in ACI 318-02 (2002) and, as suggested in the original work of Bazant and Kim (1984), was also supported experimentally by Harajili et al. (1995) for fibrous concrete. Moreover the expression given by Eq.(20) for the arm of internal forces was assumed also taking into account the presence of fibers by means of the residual tensile strength of the composite. The strength contribution V_2 to the arch action was evaluated, with reference to the mechanism shown in Figure 5, by relating the shear force to the variation in j, T assumed to be constant. In the case of fibrous concrete Campione et al. (2006) to estimate the tensile force in main bars was also included the strength contribution due to the residual strength of composite resulting in a fictitious geometrical ratio of main steel defined as:

$$\rho_t = \rho + \rho_f = \frac{f_r}{\eta \cdot f_y} \cdot \frac{h}{d}\left(1 - \frac{e}{h}\right) \qquad (24)$$

in which $\eta = \sigma_s / f_y$, with η (assumed 0.3) a share of the yielding stress assumed in accordance with experimental data available. Finally by considering the sum of the strength contributions due to the beam and arch actions and due to the fibers across the principal cracks we obtain:

$$v_u = \xi \left\{ \begin{array}{l} j_0 \cdot \left[1.3 \cdot \rho^{1/2} \cdot \sqrt{f_c'} + 0.3 \cdot f_y \cdot \varepsilon \cdot \rho_t \cdot \left(\frac{d}{a} \right)^{1.8} \right] \\ + 0.27 \cdot F \cdot \sqrt{f_c'} \end{array} \right\}$$

(25)

where to take the size effect into account the ξ coefficient given by Bazant and Kim (1984) was adopted:

$$\xi = \frac{1}{\sqrt{1 + \frac{d}{25 \cdot d_a}}} \qquad (26)$$

where d_a is the maximum aggregate size of the concrete. Eq. (25) was calibrated on the basis of experimental data described in the literature as in Campione et al. (2006).

Figure 6 shows the comparison between the analytical expression here proposed (which has a similar structure to the ACI equation for shear strength) and the expression proposed by CNR-DT 204 (2006) in which the equivalent residual strength is evaluated by Eq. (21). The comparison in terms of shear strength versus a/d ratio referring to beams with and without fibers shows good agreement between the two proposed equations although the origin of the two equations differs.

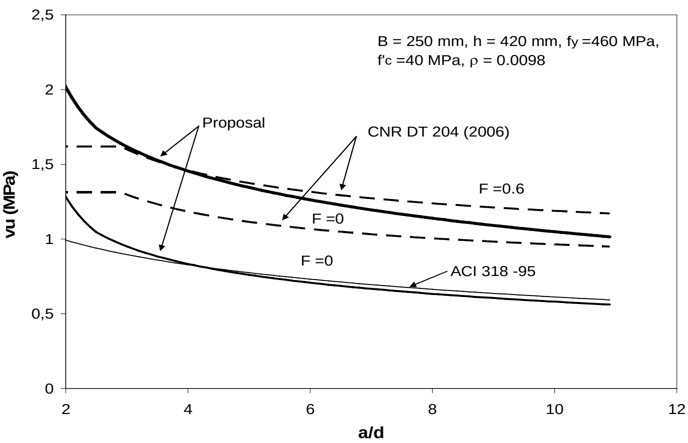

Figure 6 Comparison between analytical expressions.

Several studies show that it is possible to obtain the shear strength of reinforced concrete beams with stirrups by adding the contributions due to beam and arch actions with the strength contribution due to the stirrups bridging the principal crack and not considering the interaction between the single mechanisms.

Moreover, the inclination of the principal cracks is assumed to be 45°. Russo and Puleri (1997) have shown that the stirrups do not always yield at beam rupture and the effective stress can be estimated if the contributions of beam action to the sum of beam and arch action are known. Therefore, they introduce an effectiveness coefficient able to determine the share of yielding stress in the stirrups at beam rupture.

It is possible to include the effect of fibers in the effectiveness function of stirrups originally proposed by Russo and Puleri (1997) and determine the shear strength contribution due to stirrups, expressed as:

$$v_{st} = \Phi_f \cdot \rho_{sw} \cdot f_{yw} \qquad (27)$$

where the yielding stress in the stirrups is reduced by Φ_f defined as in Campione et al. (2006). This function which has to be cut at maximum value of one reflects the influence of the beam action (including the effect of fibers also) on the whole strength contribution of the beam and is expressed:

$$\Phi_f = \frac{2.17 \cdot j_0 \cdot \rho^{1/2} \cdot \sqrt{f_c'}}{j_0 \cdot \left[1.3 \cdot \rho^{1/2} \cdot \sqrt{f_c'} + 0.3 \cdot \varepsilon \cdot f_y \cdot \rho_t \cdot \left(\frac{d}{a} \right)^{1.8} \right] + 0.27 \cdot F \cdot \sqrt{f_c'}} \qquad (28)$$

Finally, using Eq. (27) and Eq. (28) we obtain the expression of the shear strength in the presence of stirrups:

$$v_u = \xi \cdot \left\{ j_0 \cdot \left[1.3 \cdot \rho^{1/2} \cdot \sqrt{f_c'} + 0.3 \cdot \varepsilon \cdot f_y \cdot \rho_t \cdot \left(\frac{d}{a} \right)^{1.8} \right] + 0.27 \cdot F \cdot \sqrt{f_c'} \right\} + \qquad (29)$$
$$+ \Phi_f \cdot \rho_{sw} \cdot f_{yw}$$

Campione et al. (2006) also support the proposed expression with available experimental data.

4 LOAD-DEFLECTION CURVES UNDER FLEXURE AND SHEAR

It is possible to determine the load-deflection curves of the simply supported beam knowing the moment curvature-diagrams of the loaded sections, as determined in the previous section, by using Mohr's analogy which considers a fictitious beam simply supported and loaded by the curvature diagrams as shown to provide the simplified load-deflection curves shown in Figure 7.

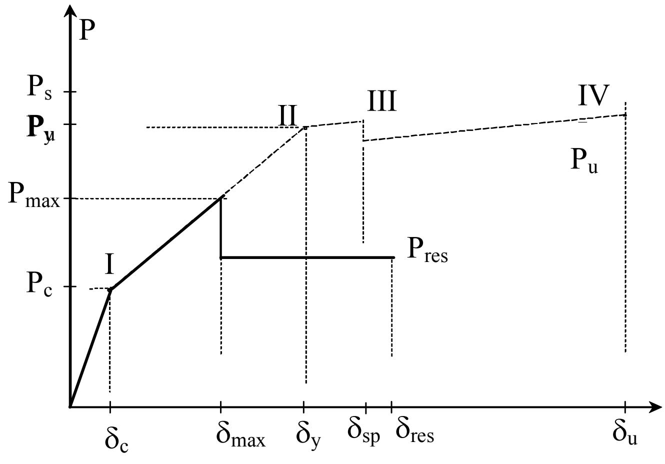

Figure 7. Simplified model for load-deflection curves.

It emerges from the curve that if overstrength in shear is attained and flexural failure occurs, the response will be that of the simply supported beam in flexure which can be approximated by a four linear diagram (first cracking, yielding, cover spalling and ultimate load). However, if the failure is in shear the diagram can be characterized by a bilinear behaviour constituted by two linear branches up to the maximum load corresponding to shear failure and by a second characterized by residual strength essentially governed by the stirrups and fibers. Consequently the load-deflection curve is that shown in Figure 7 where it must be remembered that instead of V ($V=v_u$ bd) the whole load P = 2V appears for the different values of V. According to Mohr's analogy the deflection of the beam is the moment in the middle section of the fictitious beam. In particular these deflections are expressed in the cases of cracking, yielding, cover spalling and ultimate moments as:

At first cracking:

$$\delta_c = \phi_c \cdot \left[\frac{a^2}{3} + \frac{b^2}{8} + \frac{ab}{2} \right] \tag{30}$$

At first yielding:

$$\delta_y = \phi_c \cdot \frac{a}{6} \cdot (a + x_f) + \phi_y \cdot \left(\frac{a^2}{3} - \frac{a \cdot x_f}{6} + \frac{a \cdot b}{2} + \frac{b^2}{8} - \frac{x_f^2}{6} \right) \tag{31}$$

with $x_f = (M_c/M_y) \cdot a$.

At the moment corresponding to the cover spalling:

$$\delta_s = \frac{\phi_c}{2} \cdot \left[a \cdot (x_f - x_y) + x_y \cdot \frac{x_y - y_f}{3} \right] + \frac{\phi_y}{6} \cdot \left[a \cdot (a + x_y) - x_f \cdot (x_y + x_f) \right] + \phi_s \cdot \left(\frac{a^2}{3} - \frac{a \cdot x_y}{6} + \frac{a \cdot b}{2} + \frac{b^2}{8} - \frac{x_y^2}{6} \right) \tag{32}$$

with $x_y = (M_y/M_s) \cdot a$. At the ultimate moment the deflection is expressed by the Eq. (32), with $x_y = (M_y/M_u) \cdot a$.

Should shear failure occur before the yielding of the longitudinal reinforcement, beam capacity would be limited to $M_{max} = (v_u \cdot b \cdot d) \cdot a$.

In this case the corresponding deflection δ_{max} can be obtained as shown in Figure 7 (if we consider for simplicity that up to this stage the response is essentially flexural) as the intersection between the constant value line P_{max} ($2 \cdot v_u \cdot b \cdot d$) and the slope line connecting the first cracking and the yielding stage ultimately state resulting in:

$$\delta_{max} = \delta_c + (v_u \cdot B \cdot d \cdot a - P_c) \cdot \frac{\delta_y - \delta_c}{M_y - M_c} \tag{33}$$

$$P_c \le P_{max} \le P_y$$

Should shear failure occur past the attainment of the yielding moment, the maximum deflection would be as follows:

$$\delta_{max} = \delta_y + (v_u \cdot B \cdot d \cdot a - P_y) \cdot \frac{\delta_u - \delta_y}{M_u - M_y} \tag{34}$$

Moreover, if it is supposed (as also observed experimentally in Campione et al. 2003) that after shear failure occurs the main contribution to the residual strength is due to the effects of stirrups and fibres (beams and arch effects are not included), it is possible to obtain the residual strength $P_{res} = 2\, v_{res} \cdot b \cdot d$, v_{res} being expressed by:

$$v_{ures} = \xi \cdot f_r \cdot \left(\frac{h}{d} - \frac{e_u}{d} \right) + \rho_{sw} \cdot f_y \text{ (in MPa)} \tag{35}$$

To validate the proposed model a comparison with data given in Swamy and Al-Ta'an (1981) mentioned in the previous section and also data from Mansur and Rashid (2005) is shown. The experimental research of Rashid and Mansur (2005) refer to a four point bending test on reinforced concrete beams of 3400 mm length between the two lateral supports. Beams had rectangular cross-section with B = 250 mm, h = 400 mm, c = 20 mm, the area in tension constituted by four deformed bars of 25 mm diameter, and area in compression constituted by two 13 mm deformed bars. Plain concrete were f'_c = 42.8 MPa and steel bars were of yielding stress f_y = 460 MPa. Beams were also reinforced with transverse stirrups of 10 mm diameter and pitch p = 200 mm. Both researches consider the flexural failure. Figures 8 and 9 show the experimental and analytical load-deflection curves for cases of 0 and 0.5 % of fibers.

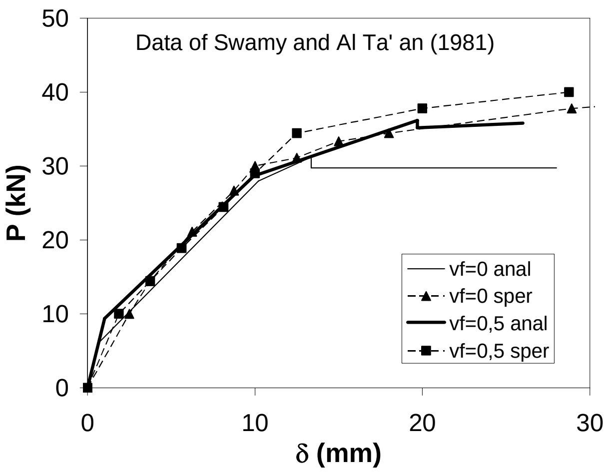

Figure 8. Experimental and analytical load –deflection curves of beams with v_f =0 and 0.5% (data from Swamy and Al Ta'an, 1981).

The comparison shows the ability of the simplified model to predict the experimental response including yielding of main bars and crushing of concrete. The model is also able to include the spalling of compressed cover occurring at high curvature levels when fibers are added or high strength concrete is utilized.

The results from Campione et al. (2003) refer to four-point bending tests on medium size beams of 2500 mm length and a/d = 2.8 in the presence of longitudinal bars, stirrups and fibers. The stirrup diameter was 6 mm, pitch 200 mm and with yielding stress of 510 MPa. In the case of fibers F = 0, 0.6 and 1.2 were assumed (Fig. 10). In this case shear and flexural failures are obtained depending on fiber amount.

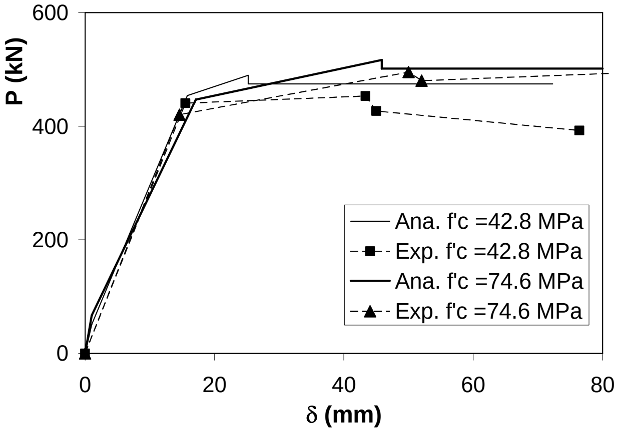

Figure 9. Experimental and analytical load –deflection curves of beams (data from Rashid and Mansur 2005).

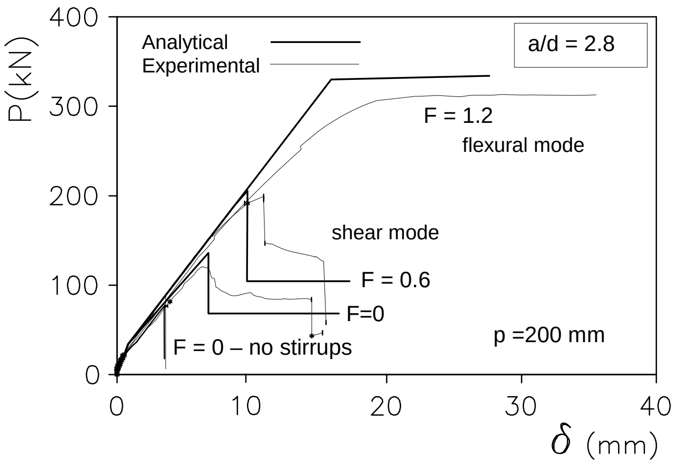

Figure 10. Experimental and analytical load – deflection curves of beams with data from Campione et al. (2003).

In all cases examined the model shows a good ability to describe the overall flexural behavior of the beams including flexural and shear modes of failure and cover spalling processes.

5 CONCLUSIONS

In this paper an analytical model is proposed and discussed, its primary objective being the description of the load-deflection curve in simply-supported R/C beams, containing stirrups and steel fiber, and subjected to bending and shear.

More specifically, the model: a) accurately predicts the bearing capacity in bending and shear; b) takes care of the spalling process ensuing from the compressive stresses acting in the cover of any given section; and c) makes it possible to evaluate the residual capacity in shear, after shear failure, taking into account stirrups and fiber contributions.

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