Mixed-mode pressurized fracture at the dam-foundation joint

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ABSTRACT: When fracture occurs in a concrete dam, the crack mouth is typically exposed to water. Very often this phenomenon occurs at the dam-foundation joint and is driven also by the fluid pressure inside the crack. Since the joint is the weakest point in the structure, this evolutionary process determines the load bearing capacity of the dam. In this paper the cracked joint is analyzed through the cohesive model, which takes into account the coupled degradation of normal and tangential strength. Some numerical results are presented which refer to the benchmark problem proposed in 1999 by the International Commission On Large Dams. During the evolutionary process the horizontal dam crest displacement has been found to be a monotonic increasing function of the external load multiplier. As the fictitious process zone moves from the upstream to the downstream edge a transition occurs in the path of crack formation: the initial phase is dominated by the opening displacement, on the contrary afterwards the shear displacement dominates. Therefore, crack initiation does not depend on dilatancy. On the contary the load carrying capacity depends on dilatancy.

1 INTRODUCTION

When cracking occurs in a concrete dam the crack mouth is typically exposed to water. Very often this phenomenon occurs at the dam-foundation joint and is driven also by the fluid pressure inside the crack. Since the joint is the weakest point in the structure, this evolutionary process determines the load bearing capacity of the dam. In this paper the cracked joint is analyzed through the model proposed by Cocchetti, Maier, and Shen 2002 (shortened CMS), which takes into account the coupled degradation of normal and tangential strength at the dam/foundation interface. The water pressure inside the crack, which reduces fracture energy and increases the driving forces, is analyzed through the model proposed in Reich, Brühwiler, Slowik, and Saouma 1994, Brühwiler and Saouma 1995a and Brühwiler and Saouma 1995b. The crack opening displacement induces two consequences:

- concrete permeability increases,
- water pressure increases.

Each one of these two phenomena drives the other. Some results are presented which refer to the benchmark problem proposed in 1999 by the International Commission On Large Dams (ICOLD 1999). Similar water/fracture interaction phenomena are observed in the analysis of retaining walls and rock slope stability.

2 JOINT MODELS

A joint is a locus of possible displacement discontinuities. The separation phenomenon is analyzed in the plasticity framework since an irreversible process occurs. The displacement vector $\boldsymbol{w}$ is assumed to be the sum of a reversible (superscript e) and an irreversible (superscript p) contribution:

$$\dot{\boldsymbol{w}} = \dot{\boldsymbol{w}}^e + \dot{\boldsymbol{w}}^p \tag{1}$$

$$\dot{\boldsymbol{p}} = \boldsymbol{K_0}\,\dot{\boldsymbol{w}}^e = \boldsymbol{K_0}\,(\dot{\boldsymbol{w}} - \dot{\boldsymbol{w}}^p) \tag{2}$$

where $\boldsymbol{p}$ represents the traction vector across the joint and $\boldsymbol{K_0}$ the stiffness of the joint.

2.1 *Damage initiation phase*

According to the CMS model proposed in Cocchetti, Maier, and Shen 2002 and Bolzon and Cocchetti 2003, damage initiation occurs when the stress path achieves the piecewise linear *yield* or *activation function* shown in Fig. 1, where p_n is the normal traction, χ_0 its ultimate value in pure tension, p_t is the tangential traction, c_0 the cohesion and μ the Coulomb friction angle. The activation function consists of a vector of φ_y whose components or modes correspond to half-planes in the bi-dimensional stress space. The intersection of such half planes is a convex domain that constitutes the region of elastic behaviour of the joint.

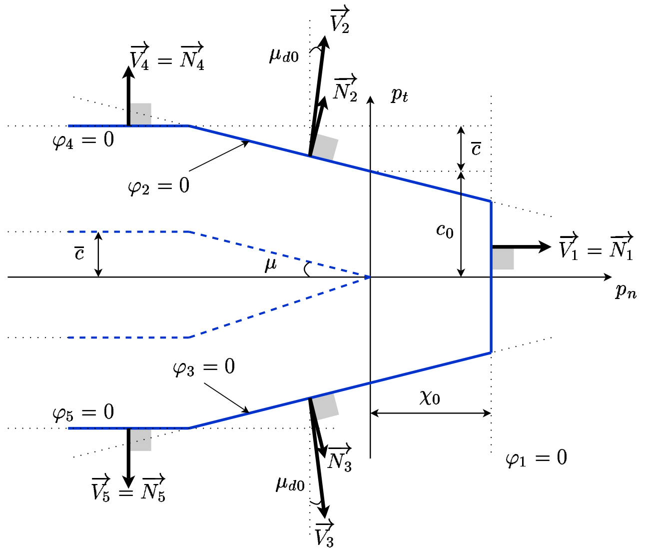

Figure 1. Piecewise linear model.

Each component φ_i depends on cohesive tractions $\boldsymbol{p}$ and static internal variables $\boldsymbol{\chi}$:

$$\varphi_i = \varphi_i(\boldsymbol{p}, \boldsymbol{\chi}) \begin{cases} < 0 & \text{inactive joint} \\ = 0 & \text{active joint} \end{cases}$$

The point where damage initiation occurs is called fictitious crack tip (FCT). During the evolutionary process, it moves from the upstream edge to the downstream edge.

2.2 *Damage evolution phase*

Once the necessary activation condition $\varphi = 0$ is met, irreversible displacements $\dot{\boldsymbol{w}}^p$ can develop along the interface:

$$\dot{\boldsymbol{w}}^p = \frac{\partial Q(\boldsymbol{p}, \boldsymbol{\chi})}{\partial \boldsymbol{p}} \dot{\boldsymbol{\lambda}} \qquad \dot{\boldsymbol{\lambda}} \geq \boldsymbol{0} \tag{3}$$

where the *plastic potential* $Q(\boldsymbol{p}, \boldsymbol{\chi})$ is defined in such a way that the interface fracture work without friction is controlled as explained later. The portion of joint where damage evolves is called fictitious process zone (shortened FPZ).

The main features that differentiates the CMS model from Carol, Prat, and Lopez 1997 and Červenka, Kishen, and Saouma 1998 is that all equations are linearized, hence the nonlinearity of the model is contained only in the complementarity conditions. A first set of five relations, also referred to as *Kuhn-Tucker conditions*, can be written with reference to the plastic multiplier $\dot{\lambda}_y$ associated with the inelastic displacement direction $\boldsymbol{V}_i$ (shown in Fig. 1). Following the notation used by Puntel 2004, we can write:

$$\varphi_y \geq 0 \quad \dot{\lambda}_y \geq 0 \quad \varphi_y \, \dot{\lambda}_y = 0 \tag{4}$$

When the stress path is inside the elastic domain, all components φ_i are positive and therefore all components $\dot{\lambda}_i$ vanish. When the stress path achieves the activation function, a component φ_i vanishes and the corresponding $\dot{\lambda}_i$ becomes positive. A first set of complementarity relations specifies the conditions for the onset of softening along a branch.

Now a second set of complementarity relations has to be introduced. When the traction mode ($\varphi_1 = 0$) is activated, the linear softening law is completely determined by the condition that the energy dissipated is the traditional Mode I fracture energy $\mathcal{G}_F^I$ (Hillerborg, Modeer, and Petersson 1976). The softening branch is bounded; when the displacement discontinuity, along a pure traction mode, reaches the critical values $w_c = 2\,\frac{\mathcal{G}_F^I}{\chi_0}$, the cohesive forces vanish. The condition for the arrest of softening in this case can be written through a sixth complementarity relation.

Similarly, when two shear modes ($\varphi_4 = 0$) or ($\varphi_5 = 0$) are activated, the linear softening law is completely determined by the condition that the energy dissipated is the Mode II fracture energy $\mathcal{G}_F^{IIa}$ under high normal confinement and no dilatancy proposed by Carol, Bažant, and Prat 1992 in the context of the microplane model. The determination of pure Mode II fracture energy $\mathcal{G}_F^{II}$ would require a pure shear test, without normal confinement, which is extremely difficult to perform. That is the reason why $\mathcal{G}_F^{IIa}$ is preferred as a material property. The softening branch is bounded; when interface fracture work without contribution from friction, along a pure shear mode, reaches the critical value $\mathcal{G}_F^{IIa}$, the cohesive tractions vanish and the interaction forces are due to friction alone. The condition for the arrest of softening in this case can be written through a seventh complementarity relation. When the cohesive-frictional modes ($\varphi_2 = 0$ or $\varphi_3 = 0$) are activated, the critical condition is related to both displacement discontinuity components as shown in Cocchetti, Maier, and Shen 2002. Along this separation mode, when the condition for the arrest of softening is reached, the residual tangential stress is assumed as constant (see term $\bar{c}$ in Fig. 1).

The last complementarity relation of the model regards the dilatant behaviour associated with λ_2 and λ_3 (see μ_{d0} in Fig. 1). It appears reasonable to assume that there is a limit to the dilatancy of a joint. Therefore, plastic multiplier λ_8 is activated in order to store the total of λ_2 and λ_3 exceeding the parameter w_{dil}. Along this separation mode, when the condition for the arrest of softening is reached, the residual tangential stress is assumed to be dependent on Coulombian friction (see the dashed line, i.e., $\mu\, p_n$, of Fig. 1).

It should be remarked that the model takes into account a bilinear relationship between tensile strength and crack opening and between cohesion and crack opening. The coordinates of knee point are (χ_1, w_1)

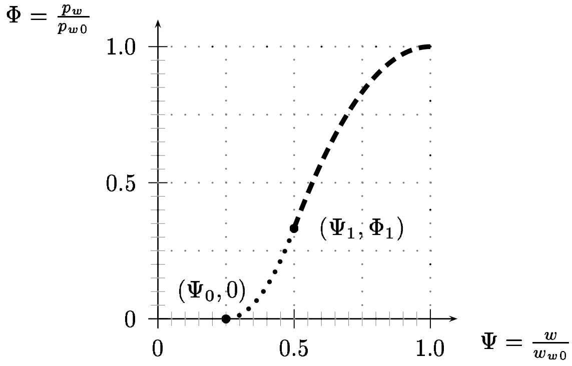

Figure 2. Hydrostatic pressure transition ($\kappa = 4, \Psi_0 = 0.25, \Psi_1 = 0.5$).

while the slopes of the branches are h_{n0} and h_{n1} for the former and (c_1, w_{c_1}) and h_{t0} and h_{t1} for the latter.

3 MODELING WATER INSIDE THE CRACKS

3.1 Damage inside the cracks

As a consequence of additional damage occurring inside the FPZ due to the presence of water, it is assumed that fracture energy $\mathcal{G}_F$ reduces as pressure p_{w0} increases. The apparent value of $\mathcal{G}_F$ is assumed to be expressed by the following relationship (Reich, Brühwiler, Slowik, and Saouma 1994):

$$\hat{\mathcal{G}}_F = \mathcal{G}_F \left[1 - 2\,\frac{p_{w0}}{\chi_0} + \left(\frac{p_{w0}}{\chi_0}\right)^2 \right] = \mathcal{G}_F\, S \qquad (5)$$

The ratio $\frac{p_{w0}}{\chi_0}$ is identified as damage number. If $\frac{p_{w0}}{\chi_0} = 0$, i.e., $S = 1$, the material is considered undamaged and therefore, the softening law is derived from the traditional fracture energy measured in dry conditions. If $\frac{p_{w0}}{\chi_0} = 1$, i.e., $S = 0$, the material is considered fully damaged and fracture energy vanishes. The stress-opening law is now assumed in such a way that the openings are scaled through the factor S:

$$\hat{w} = S\, w \qquad (6)$$

3.2 Pressure distribution

The pressure distribution is assumed to be described by two polynomial functions. Defining $\Psi = \frac{w}{w_{w0}}$ and $\Phi = \frac{p_w}{p_{w0}}$, we can write:

$$\Phi = f_1(\Psi) = A_1 + B_1\Psi + C_1\Psi^2 + D_1\Psi^3 \quad \Psi \leq \Psi_1 \ (7)$$

$$\Phi = f_2(\Psi) = A_2 + B_2\Psi + C_2\Psi^2 + D_2\Psi^3 \quad \Psi \geq \Psi_1 \ (8)$$

These functions are plotted in a non dimensional space in Fig. 2 (f_1: dotted, f_2: dashed). The slope at $(\Psi_0, 0)$ and $(1, 1)$ is equal to zero; the slope at (Ψ_1, Φ_1) is continuous. Value Ψ_0 corresponds to crack

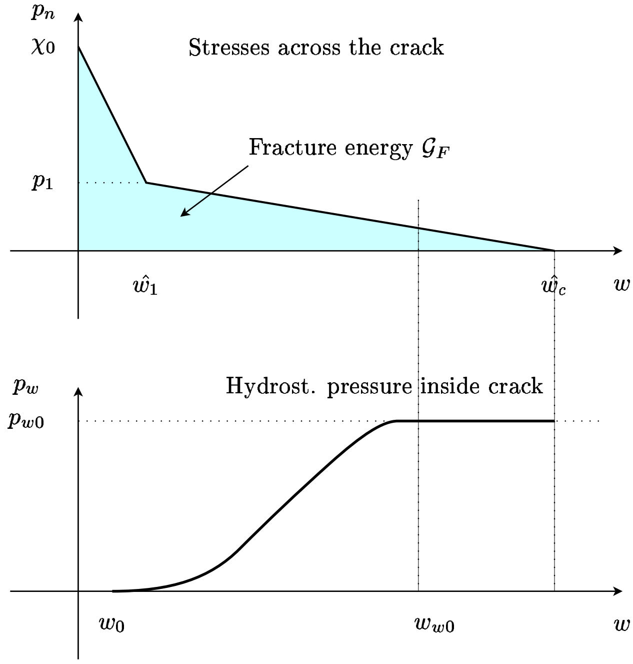

Figure 3. Water pressure distribution inside the crack. The aspect ratios of Petersson's softening law are $\frac{\hat{w}_1}{\hat{w}_c} = \frac{2}{9}$ and $\frac{p_1}{\chi_0} = \frac{1}{3}$.

opening w below which $p_{w0} = 0$, while Ψ_1 corresponds to the knee point w_1. Value Ψ_0 is defined as:

$$\Psi_0 = \Psi_1 - \frac{2}{\kappa}\,\Psi_1 \qquad (9)$$

where $\kappa \geq 2$ is a constant.

The transition point between f_1 and f_2 is defined by the coordinate Ψ_1, see Eq. 9, and Φ_1:

$$\Phi_1 = \frac{2\,\Psi_1}{2\Psi_1 + \kappa\,(1 - \Psi_1)} \qquad (10)$$

The value w_{w0} shown in Fig. 3 is assumed to be:

$$w_{w0} = \hat{w}_1 + \frac{2}{\xi}\,(\hat{w}_c - \hat{w}_1) \qquad (11)$$

4 EXAMPLE OF APPLICATION

4.1 Numerical model

The numerical simulations are performed in the framework of the finite element code ABAQUS 2005 by means of the so called "user subroutines".

4.2 Benchmark problem

As an example of application, the benchmark problem proposed in 1999 by the International Commission On Large Dams (ICOLD 1999) was analyzed. The gravity dam shown in Fig. 4 was discretized through 57313 triangular elements and the foundation through 11020.The joint was discretized through 1000 quadrilateral elements (0.01m thick and 0.06m wide), the

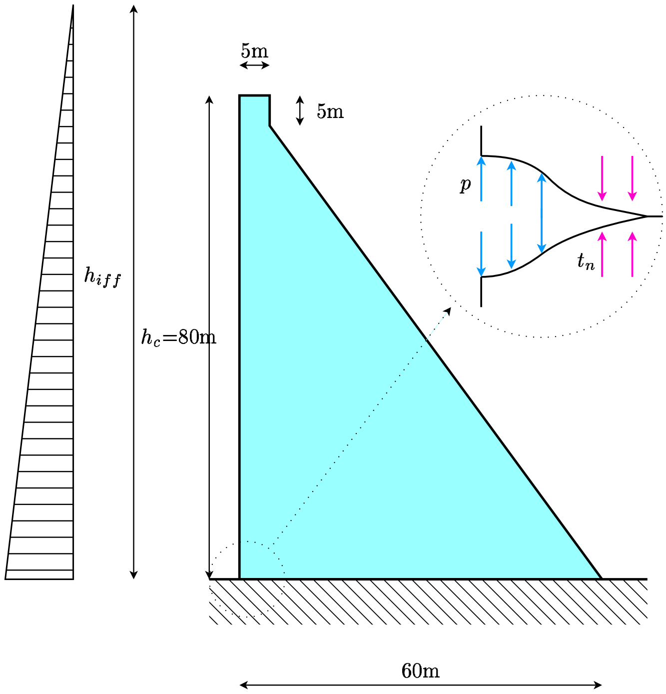

Figure 4. Gravity dam proposed as benchmark by ICOLD (1999).

boundary through 115 infinite elements. The following lists show the material properties assumed.

- Dam and foundation Young modulus: 2.4e10Pa
- Dam and foundation Poisson ratio: 0.15
- c_1: 2.33e6Pa
- $\bar{c}$: 1.0Pa
- c_0: 6.0e6Pa
- μ: 0.577
- μ_{d0}: 0.1
- χ_0: 2.0e6Pa
- $\mathcal{G}_F^{IIa}$: 514N/m
- $\mathcal{G}_F^{I}$: 147N/m
- p_{w0}: water pressure
- w_{dil}: 2.0e-3m
- χ_1: 0.66e6Pa
- c_1: 2.33e6Pa
- w_1: 1.5e-4m
- w_1: 6.75e-4m

4.3 Numerical results

The dam is analyzed under the following conditions:

- self weight application,
- reservoir filling,
- imminent failure flood.

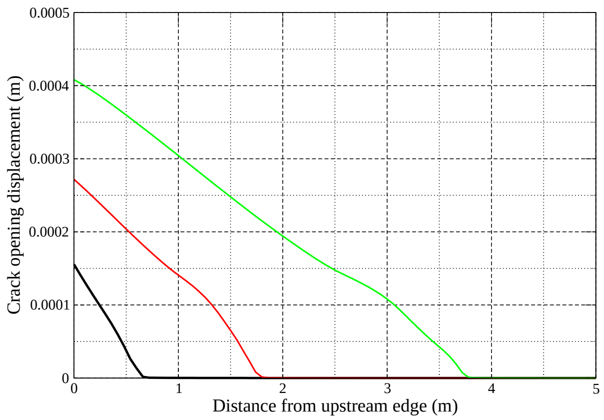

Figure 5. Crack opening displacement vs. distance from upstream edge.

Since the joint is the weakest part in the structure, the remaining material behaves in a linear elastic way.

In order to deal with a ratio $\frac{p_{w0}}{\chi_0}$ belonging to the range tested by Reich, Brühwiler, Slowik, and Saouma 1994, an appropriate value of tensile strength χ_0 is chosen. After the application of the self weight, the structure behaves linearly up to 87.5% of hydrostatic water pressure corresponding to the height of the dam crest ($h_c = 80$m). Above this level, starting from the upstream right angle, where the elastic stress field is singular, a fictitious process zone begins to grow along the joint. As the load proportionality factor grows from 0.875 to 1 the crack mouth opening displacement reaches the value w_{w0} and the water pressure penetrates into the crack and becomes an additional driving force for crack propagation. Nevertheless, when the water level reaches the dam crest, the crack turns out to be still stable in load control. In the last load step the water level is fictitiously raised up to the level that leads to the collapse of the dam. This level is often termed as the level of *imminent failure flood* h_{iff}. The load-carrying capacity and the safety of the dam against failure are evaluated in terms of the maximum overtopping coefficient $\gamma_{iff} = \frac{h_{iff}}{h_c}$. After each load increment, the fluid pressure acting on the crack faces is updated according to the new values of displacement discontinuity. All the states reached during the evolutionary quasi-static analysis are stable in load-control.

Figure 5 and 6 show the crack opening and sliding distribution near the crack, Fig. 7 the displacement paths. Figure 8 and 9 show the related normalized (with respect to χ_0) normal and tangential stress distribution.

Finally, Fig. 10, 11 and 12 show the overtopping coefficient as a function of the horizontal crest displacement, crack mouth opening and sliding displace-

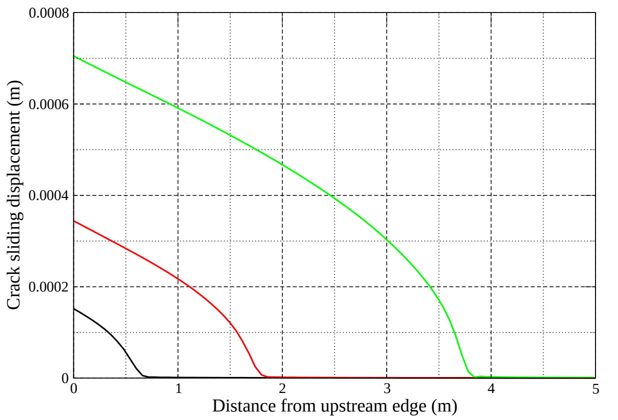

Figure 6. Crack sliding displacement vs. distance from upstream edge.

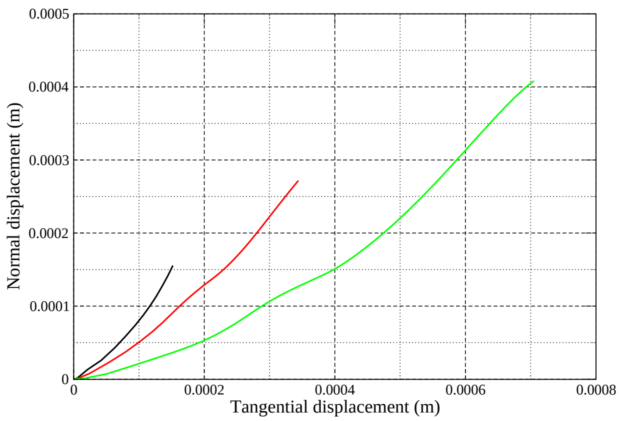

Figure 7. Crack mouth displacement path.

ment, respectively.

5 CONCLUSIONS

The main contribution of this research is to assess the influence of water penetration inside a dam/foundation joint. For the material properties and boundary conditions analyzed the following conclusions can be drawn:

- During the evolutionary process the horizontal dam crest displacement has been found to be a monotonic increasing function of the external load multiplier.

- As the fictitious process zone moves from the upstream to the downstream edge a transition occurs in the path of crack formation: the initial phase is dominated by the opening displacement, on the contrary afterwards the shear displacement dominates.

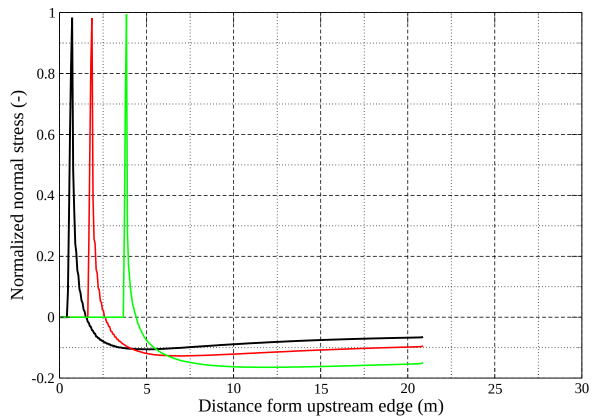

Figure 8. Normal stress vs. distance from upstream edge.

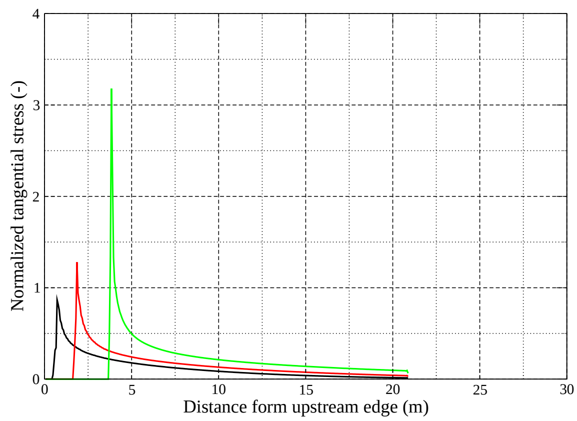

Figure 9. Tangential stress vs. distance from upstream edge.

- The crack initiation does not depends on dilatancy. On the contary the load carrying capacity depends on dilatancy.

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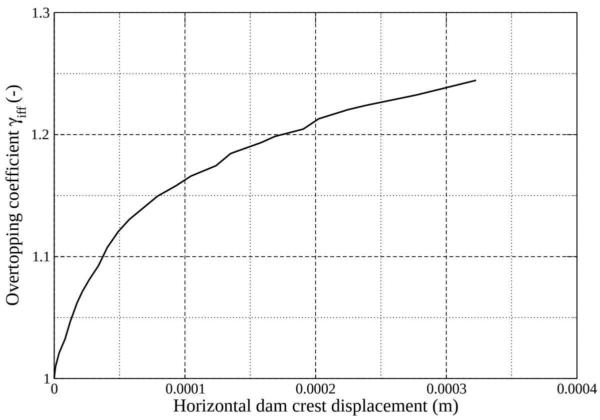

Figure 10. Overtopping coefficient γ_{iff} vs. horizontal crest displacement.

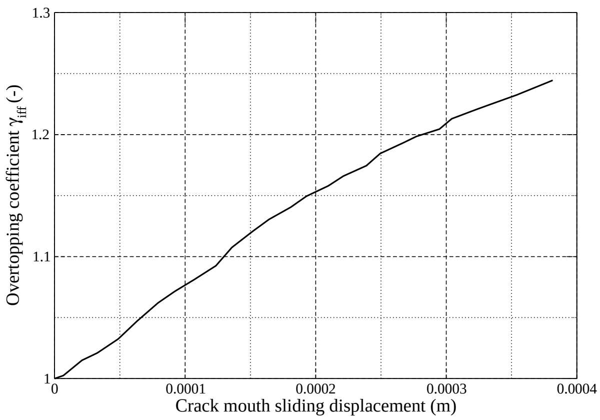

Figure 12. Overtopping coefficient γ_{iff} vs. crack mouth sliding displacement.

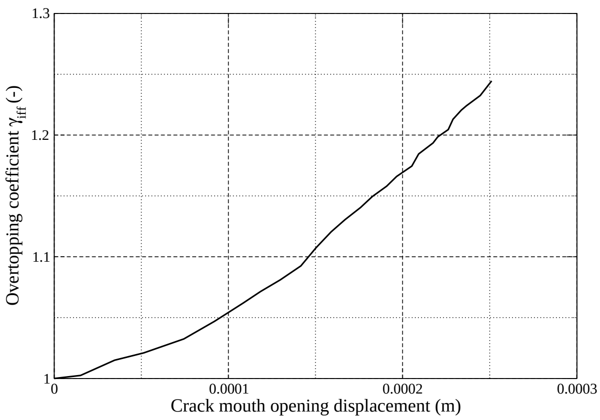

Figure 11. Overtopping coefficient γ_{iff} vs. crack mouth opening displacement.

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