Bond characteristics of a Self-Consolidating mix for PC bridge girders

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ABSTRACT: Results from tests used to determine the material and bond characteristics of a proposed SCC mix for bridge girders in the state of Kansas are presented. Eleven full-scale, pre-tensioned SCC flexural specimens were tested to evaluate the transfer and development lengths. These specimens were single-strand specimens that included specimens designed to evaluate the so-called "top-strand" effect. These top-strand specimens, with more than twenty inches of concrete below the strand, were tested to evaluate the current AASHTO requirement of a thirty percent increase in the development length when more than the twelve inches of concrete is cast below the strand. Prior to casting the beams, the pre-stressing strand was pre-qualified using the Large Block Pullout Test procedure. Strand end-slip measurements, used to estimate the transfer lengths, indicated that the proposed SCC mix meets the ACI and AASHTO requirements. In addition, flexural tests on the same specimens, confirmed that the SCC mix also meets the current code requirements for development length. Furthermore, the test results indicated that a thirty percent increase in development length was not necessary to achieve full tensile capacity of the strand in the "top-strand" specimens.

1 INRODUCTION

Self-Consolidating Concrete (SCC) has rapidly become a widely used material in the construction industry. SCC is defined as "a highly workable concrete that can flow through densely reinforced or geometrically complex structural elements under its own weight and adequately fill voids without segregation or excessive bleeding without need for vibration.[1]"

The Interim Guidelines for the use of Self-Consolidating Concrete in PCI Member Plants recommend that "strand bond tests shall be run with new SCC mixes to verify that the bond with SCC is equivalent or better than a conventional concrete of similar design when using similar strand." Furthermore, these guidelines state that "this can be done using a flexural development length test or by direct load testing." Since SCC does not require any external vibration during placement, there has been concern by some design engineers about the ability to achieve adequate bond between the SCC and the pre-stressing strand.

Departments of Transportation, including the Kansas Department of Transportation (KDOT) would like to use SCC in pretensioned bridge members to enhance aesthetics and improve consolidation in congested areas. A drawback with conventional concrete is that in hard-to-vibrate areas, air is trapped at the surface of the form producing "bug" holes (Fig. 1). SCC will help ensure proper consolidation and a smooth finish on these surfaces.

Before allowing the use of SCC in state bridge girders KDOT wanted to investigate the bond and flexural characteristics of an SCC mix proposed by the local precaster. Since SCC is placed without external vibration, KDOT was concerned that the bond between the SCC and strand may not be as strong as that achieved with a conventional concrete mix.

Figure 1 "Bug" holes in bottom flange of IT.

Moreover, at the time of this study, information about the transfer and development length of prestressing steel in SCC and the applicability of the American Concrete Institute (ACI) and American Association of State Highway Transportation Officials (AASHTO) equations to these members, were essentially absent from the literature.

Transfer length is the distance required to transfer the fully effective prestressing force from the strand to the concrete. Development length is the bond length required to anchor the strand as it resists external loads on a member (PCI 1999). As external loads are applied to a flexural member, the member resists the increased moment demand through increased internal tensile and compressive forces. The increased tension in the strand is achieved through anchorage to the surrounding concrete (Khayat et al. 2004).

Current ACI and AASHTO design requirements do not address the use of SCC in prestressing applications. The ACI code expressions for transfer and development lengths are based on tests performed with conventional concrete and are shown below.

Transfer length (L_{tr}):

$$L_{tr} = f_{se} d_b / 3 \quad (1)$$

Development length (L_{dev}):

$$L_{dev} = f_{se} d_b / 3 + (f_{ps} - f_{se}) d_b \quad (2)$$

where d_b = diameter of strand (in.); f_{se} = effective stress is prestressing strand after allowance of prestress losses (ksi); and f_{ps} = stress in prestressing strand at calculated ultimate capacity of section (ksi)

The AASHTO specifications are similar but require an additional 1.6 multiplier to equation 2 for precast, prestressed beams.

Table 1. LBPTs conducted with SCC.

SCC Block with Control Strand		
Specimen	Max load (kips)*	Load at 1st slip (kips)*
# 1	21.8	11.8
# 2	21.4	12.5
# 3	19.7	12.4
# 4	27.5	10.7
# 5	23.2	12.7
# 6	21.4	10.7
Average	22.5	11.8

* 1 Kips = 4.448 KN

Table 2. LBPTs conducted with control mix.

Control Mix with Control Strand		
Specimen	Max load (kip)	Load at 1st slip (kips)
# 1	42.0	28.2
# 2	41.7	27.8
# 3	40.4	27.3
# 4	36.5	24.9
# 5	36.9	24.2
# 6	39.9	25.0
Average	39.5	26.2

2 BACKGROUND

KDOT funded an initial investigation in which Large Block Pullout Tests (LBPTs) were performed at Kansas State University (KSU) using both the standard mix recommended by Logan (1997) and the proposed SCC mix. The results with SCC had both lower first-slip and ultimate load values compared to those values when conventional concrete was used (Tables 1, 2). Both of the LBPTs used strand from the same un-weathered reel and which had exhibited satisfactory bond performance in flexural beam tests.

3 TEST PROGRAM

Based on these early findings it was then determined that full-scale development length girder tests were necessary to further investigate the bond between SCC and the pre-stressing strand. Therefore, KDOT funded an experimental program to evaluate the flexural performance of pre-tensioned concrete members with the proposed SCC mix.

3.1 *Material properties*

3.1.1 *Large block pullout tests*

Prior to casting any flexural test specimens, the prestressing strand that would be used for all test girder specimens was pre-qualified using the LBPTs. Standard LBPT procedures, as stipulated by Logan (1997), were followed while performing these tests. These strand qualification tests were performed with the standard mix proposed by Logan6 and not with SCC. The average first-observed slip was 96.1 KN (21.6 kips) and the average ultimate was 176.1 KN (39.6 kips). The values are both above the minimum values recommended by Logan (1997) of 71.2 KN (16 kips) and 160.1 (36 kips), respectively. Thus, the strand reel was deemed acceptable for use in this study. This reel was then covered to prevent weathering and used for all flexural beams reported herein.

Table 3. SCC and conventional concrete mix design.

Materials	SCC quantity per m^3	Conventional quantity per m^3
Cement (Type III)	338 kg	293 kg
Fine aggregate (MA1 sand)	675 kg	666 kg
Coarse aggregate (CA-6, 1” -#67)	612 kg	656 kg
Air entrainment	148 mL	178 mL
Viscosity modifying agent	0 mL	0 mL
Water	102 L	120 L
W/C ratio	0.30	0.41

3.1.2 *Mix design*

Casting of test specimens was performed at Prestressed Concrete Inc, in Newton, Kansas (PCIN), which is a PCI certified plant that produces bridge members. PCIN developed their proposed SCC mix design with the help of their admixture supplier. The SCC mix used in this study along with the conventional concrete mix that this plant uses is presented in Table 3. It should be noted that, both mixes use a 19.05 mm (¾-inch) maximum aggregate size and have a 0.30 and 0.41 water-to-cementicious materials ratio for the SCC and the conventional concrete mix, respectively. Also note that a different high range water reducer is used for the SCC and conventional concrete mix.

3.1.3 *Fresh concrete evaluation*

During the casting of the specimens, the SCC mix was tested to determine its rheological properties. At the time of casting, there were no existing ASTM standards for testing SCC, but the PCI Interim Guidelines document many test methods to evaluate the plastic properties of SCC for production qualifications. In this study, Inverted Slump Flow (Fig. 2) VSI, J-Ring (Fig. 3) and L-Box (Fig. 4) tests were all performed on the concrete during casting. The Inverted Slump Flow measures the flow separation resistance, stability/settlement resistance, air migration, and relative viscosity. The J-Ring and L-Box are both tests that measure the passing ability and blocking resistance of the SCC mix.

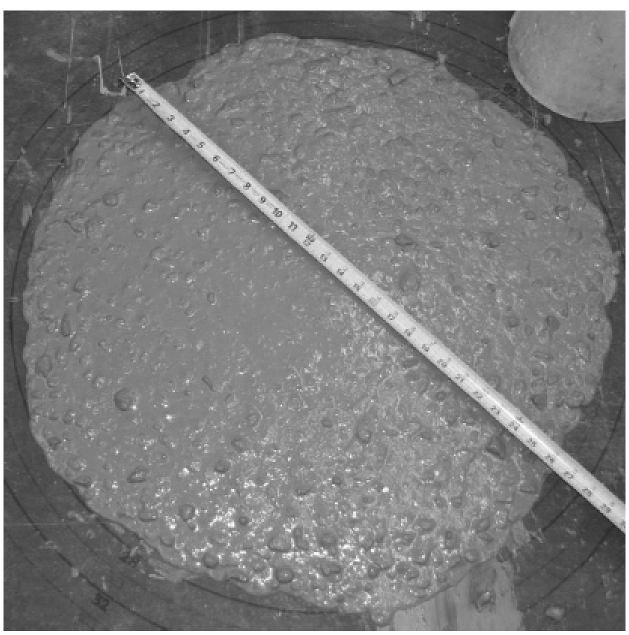

Figure 2 Spread Test for SCC.

3.1.4 *Hardened concrete properties*

The compressive strength and modulus of elasticity of the concrete were measured for future use in analytical computations. Standard ASTM procedures were followed for compressive strength and modulus of elasticity testing. In addition to measuring one-day (release) strengths; compressive strengths were determined just prior to loading the flexural specimens to failure. A set of three 101.6 × 203.2 mm (4 x 8 in.) cylinders were tested for each flexural specimen and the average values were recorded.

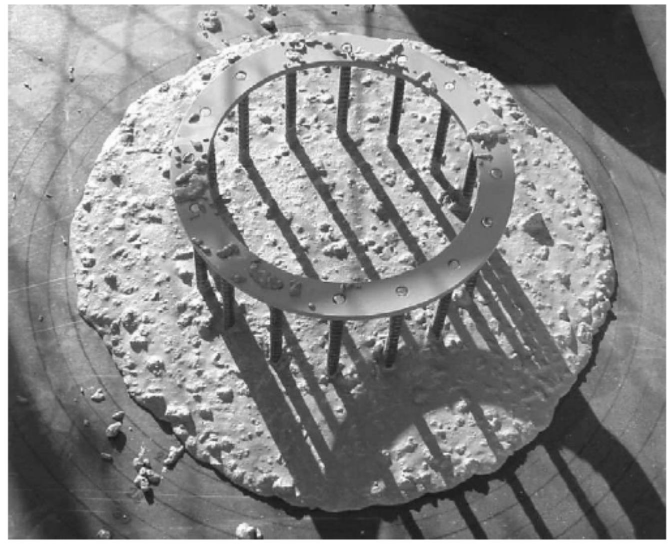

Fig. 3 J-Ring test for SCC.

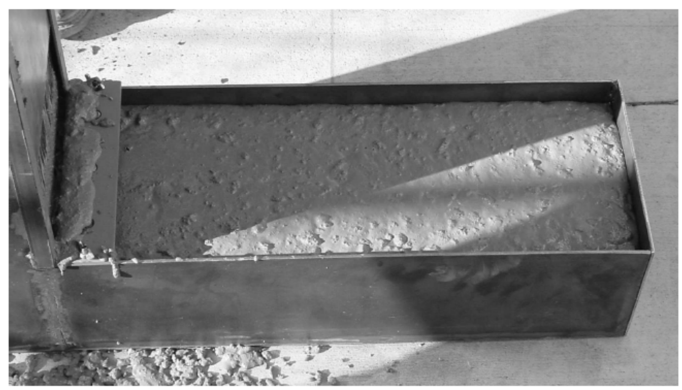

Fig. 4 L-Box test for SCC.

3.2 *Transfer length measurements*

Mast's strand slip theory as presented by Logan (1997) was used to determine the transfer length of the girders experimentally. End-slip values were obtained by measuring the distance that the strand slipped into the beam at the ends. Prior to detensioning, a mark was made on the strand with a saw blade at a distance approximately 25.4 mm from the specimen end. A steel block having a width of exactly 12.7 mm was then held against the concrete at the strand location. The distance between this machined block and the mark on the strand was then measured using a digital caliper having a precision of 0.0254 mm. This value was then used as the baseline for measurements taken after detensioning to determine the amount of end-slip that occurred. Subsequent measurements were taken up to the time of testing of the specimen. The following equations were used to determine the implied transfer length values from the end-slip measurement data.

$$\Delta = {avg\ f_{si} L_{tr}} / {E_{ps}} \tag{3}$$

where Δ = end slip (in.), equal to measured length between steel block and strand minus elastic shortening between the mark on the strand and specimen end; avg f_{si} = average initial strand stress over the transfer length after release of pre-stress (ksi); L_{tr} = transfer length (in.); and E_{ps} = elastic modulus of strand (ksi).

Assume straight line variation in the strand stress from zero at the end of the beam to full pre-stress:

$$\Delta = {0.5 f_{si} L_{tr}} / {E_{ps}} \tag{4}$$

thus

$$L_{tr} = {\Delta E_{ps}} / {(0.5 f_{si})} \tag{5}$$

4 FLEXURAL SPECIMEN TYPES

4.1 *Single-strand development length specimens*

Twelve single-strand development length specimens with different embedment lengths were fabricated and tested in this investigation. However, due to a handling error with one of the specimens only eleven were tested to failure. The single-strand specimens were used to evaluate two different embedment lengths. Two different cross-sections were utilized in order to evaluate the so-called "top strand" effect, having 304.8 mm or more of concrete cast below the reinforcement. ACI requires a 1.3 multiplier on development length for "horizontal reinforcement so placed that more than 304.8 mm (12 in.) of fresh concrete is cast in the member below the development length or splice," (ACI 12.2.4). AASHTO uses a similar 1.3 multiplier for strand development length when using an Alternate Development Length Equation (AASHTO 5.11.4.2-2).

The first cross-section cast was the standard 203.2 × 304.8 mm (8 × 12 in.) section that was used by Peterman et al. (2000). The nomenclature used for these specimens was Single-Strand Beams (SSB). This section contained a single pre-stressing strand at a depth d_p of 254 mm (Fig. 5). The section chosen was slightly larger than the 165.1 mm wide tested by Logan (1997) in order to provide an increased shear capacity. This was desirable since these specimens did not have any shear reinforcement. Refer to the Appendix for shear capacity calculations and other sample calculations.

Specimens with the second single-strand cross-section used to evaluate the "top-strand" effect, are denoted Top-Strand Beams (TSB). These specimens had a width of 203.2 mm and an overall height of 609.6 mm (Fig. 6). The strand in these specimens was located 558.8 mm from the bottom, and thus greatly exceeded the 304.8 mm height requiring a 1.3 multiplier for development length by AASHTO. At the center portion of these specimens, however, the strand height was only 304.8 mm. At mid-span, a Styrofoam blockout was used to reduce the height from 609.6 mm to 304.8 mm (Fig. 7). These specimens were inverted prior to testing. Note at mid-span, which is the critical section; these specimens had an identical cross-section to the SSB specimens. Therefore, direct comparisons between results are possible.

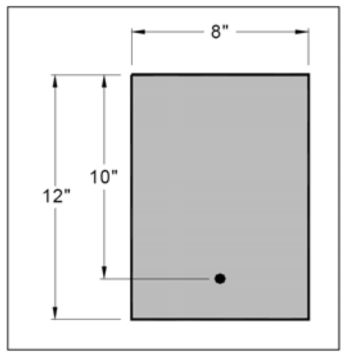

Figure 5. Cross section of bottom strand girders.
Note: 1 inch = 25.4 mm

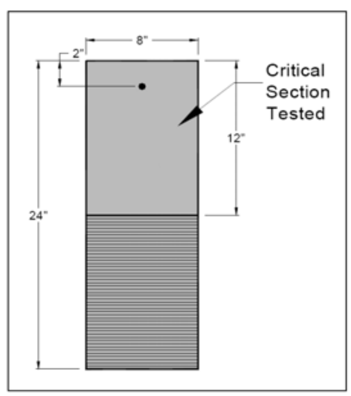

Figure 6. Cross section of top strand girders.
Note: 1 inch = 25.4 mm

Figure 7. Blockouts for top strand beams.

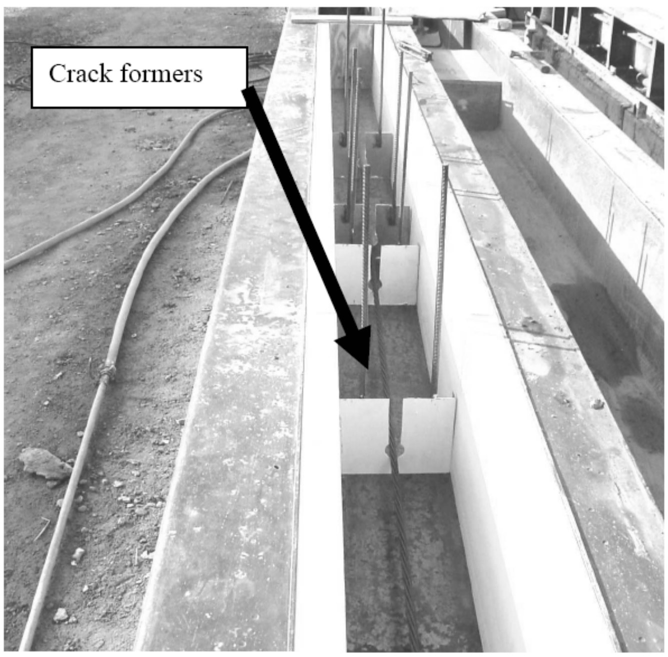

Figure 8. Crack formers.

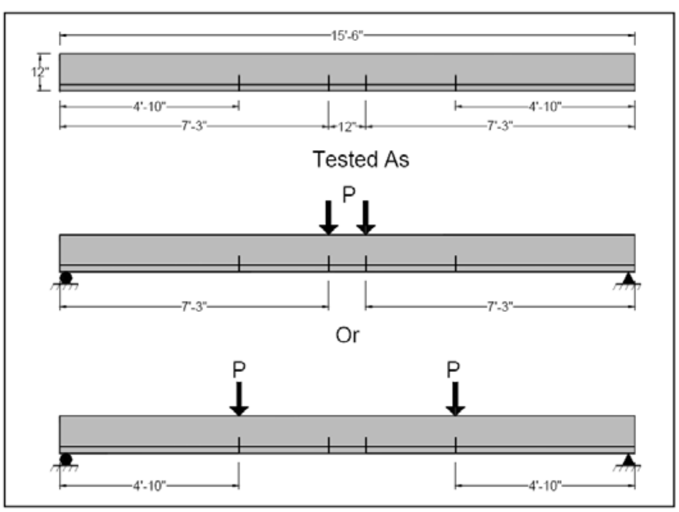

Figure 9. Test setup for 4'-10" embedment length for bottom strand beams.
Note: 1 inch = 25.4 mm

4.2 *Embedment lengths*

At the outset of this experimental program the researchers decided to evaluate two different embedment lengths. Crack formers (Fig. 8) were cast at the embedment length to insure that during loading the first cracks would open at his location. The first set of specimens was tested at an embedment length equal to 100% of the calculated development length (l_{dev}). The second set of specimens was tested at either 80% l_{dev} or 120% l_{dev}, depending on the results obtained from the 100% l_{dev} specimen tests. The second set of specimens was specifically designed to allow for testing at either embedment length as explained below.

If the 100% l_{dev} specimens failed (by flexure) at a moment greater than or equal to the calculated nominal moment capacity, M_n, then the second set of specimens would be tested at an embedment length equal to 80% l_{dev}. However, if the 100% l_{dev} specimens failed (by bond) at a moment less than the calculated nominal moment capacity, M_n, then the second set of specimens would be tested at an embedment length equal to 120% l_{dev}. Since all of the 100% l_{dev} specimens failed by flexure (as later discussed in results section of this manuscript), the second set of specimens were tested at an embedment length equal to 80% l_{dev}.

The different embedment length testing of the second set of specimens was made possible by utilizing four crack formers per beam (Fig. 9). As shown in this figure, the 80% l_{dev} tests required the use of the spreader beam with loading points directly above the outer-most crack former.

4.3 *Loading conditions*

Three types of loading rate conditions were used for evaluating the different embedment lengths. The first loading condition was designated as the SLOW test and was targeted to take about ten hours. During a SLOW test, the specimen was loaded at 444.8 N / min until cracking. Then the loading rate was reduced to 44.48 N / min until failure. This slow loading rate was used in order to accurately measure the amount of strand slip, if any, occurring prior to failure. For the second loading condition, designated as 76.5 % M_n, the specimen was loaded at 444.8 N / min up to 76.5% of nominal capacity of the specimen, and then this load was maintained for twenty-four hours. This load condition was modeled after ACI 20.3.2 for the testing and evaluation of existing structures. If the specimen successfully withstood the load for 24 hours, it was then loaded at 44.48 N / min to failure. The final loading condition, designated as 100% M_n, was similar to the 76.5% M_n procedure, except that load was maintained at 100% M_n for 24 hours. Table 4 shows the loading condition of each specimen tested along with the corresponding development length.

Table 4. Loading conditions for beams tests.

	Beam	Embedment Length*	Loading Condition
Bottom Strand	SSB A	6'-1"	76.5% M_n
	SSB C	6'-1"	SLOW
	SSB D	4'-10"	100% M_n
	SSB E	4'-10"	SLOW
	SSB F	4'-10"	76.5% M_n
Top Strand	TSB A	4'-10"	76.5% M_n
	TSB C	4'-10"	100% M_n
	TSB D	6'-1"	SLOW
	TSB E	6'-1"	76.5% M_n
	TSB F	6'-1"	SLOW

* 1 inch = 25.4 mm.

4.4 *Test setup*

All specimens were tested using a 97.86 KN (22 kips) MTS servo-controlled actuator in the KSU Civil Engineering Department Mechanics of Materials Laboratory. Data was collected for load, mid-span deflection, strand end-slip, and tension face crack opening. End slip readings were monitored by using an LVDT. Figure 10 shows the test frame setup that was used to load all specimens. A spreader beam with rollers was used to apply point loads directly above the crack formers. Roller connections were used to apply the point load at these locations.

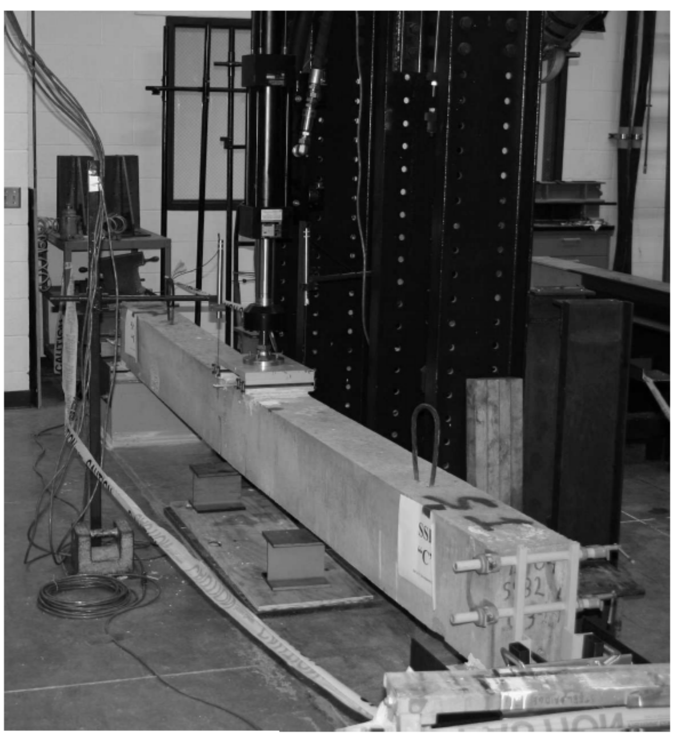

Figure 10 Beam setup.

5 RESULTS

5.1 *Transfer length*

As described earlier, end-slip measurements were used to estimate the transfer length of each girder. In these calculations, f_{si} was assumed to be 1352.75 MPa (196.2 ksi) for all single strand specimens. For all bottom strand beams, none had a longer implied transfer length (21-day) than assumed by the ACI code. The average implied transfer length was 533.4 mm for the SSB specimens and 812.8 mm. for the TSB specimens. Figure 11 presents the range and average implied transfer lengths for all the specimens tested, along with the ACI code assumptions.

5.2 *Flexural test results*

Flexural failure by strand rupture was the failure mode of all specimens tested in this study. In each case, the experimental moment exceeded the calculated nominal moment capacities by 10%- 20% (Table 5). In Table 5 the spread column refers to the test referred to in Figure 2. Furthermore, the maximum end-slip recorded for all specimens during testing was less than 0.01 in.

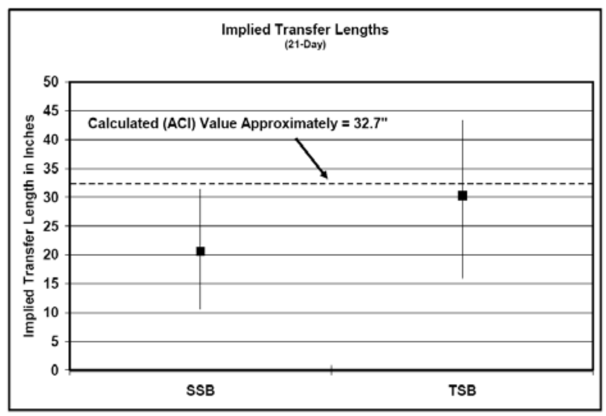

Figure 11. Transfer length results.
Note: 1 inch = 25.4 mm.

Table 5. Results of specimens tested.

	Beam	Spread (in)*	% M_n Achieved	Strand Rupture	Strand slip >0.01"
SSB	A	21	110.9	Yes	No
	C	21	115.4	Yes	No
	D	22	117.2	Yes	No
	E	22	113.3	Yes	No
	F	22	115.7	Yes	No
TSB	A	28	116.2	Yes	No
	B	28	116.4	Yes	No
	C	28	115.3	Yes	No
	D	28	112.6	Yes	No
	E	28	116.6	Yes	No
	F	28	114.0	Yes	No

* 1 inch = 25.4 mm.

6 CONCLUSIONS

Transfer lengths estimated from 21-day strand end-slip measurements were in general accordance with the values assumed by the AASHTO and ACI specifications. The average implied transfer lengths for the top-strand beams were approximately 50% greater than those for the corresponding bottom-strand beams.

Flexural tests indicated that the current ACI (and thus also the AASHTO) equations for strand development length were conservative for the SCC mix and specimen geometry used in this study. Moreover, all of the load tests conducted on specimens with an embedment length equal to eighty percent of the ACI development length, including those with more than 304.8 mm of concrete below the strand, failed in flexure by strand rupture.

REFERENCES

AASHTO. 2004. *LRFD bridge design specifications, 3rd edition*. Washington, DC. American Association of State Highway and Transportation Officials.

ACI Committee 318. 2002. *Building code requirements for structural concrete (ACI 318-02) and commentary (ACI 318R-02)*. Farmington Hills, MI. American Concrete Institute.

Interim guidelines for the use of self-consolidating concrete in precast/ prestressed concrete institute member plants, first edition. 2003. Chicago, IL.

Khayat, K.H., Assaad, J., & Daczko, J. 2004. Comparison of field-0riented test methods to assess dynamic stability of self-consolidating concrete. *ACI Materials Journal*101(2):168-176.

Logan, D.R. 1997. Acceptance criteria for bond quality of strand for pretensioned prestressed concrete applications. *PCI JOURNAL*42(2):52-90.

PCI. 1999. *PCI design handbook, fifth edition*. Chicago, IL Precast/Prestressed Concrete Institute.

Peterman, R.J., Ramirez, J.A. & Olek, J. 2000. Influence of flexure-shear cracking on strand development length in prestressed concrete members. *PCI JOURNAL*45(5):76-94.

NOTATION

AASHTO = American Association of State Highway Transportation Officials;
ACI = American Concrete Institute;
CC = Conventional concrete;
db = Diameter of prestressing strand;
dp = Depth of prestressing strand;
Ec = 28-day modulus of elasticity of con crete;
Eci = Release modulus of elasticity of concrete;
Eps = Modulus of elasticity of prestressing strand;
fpj = Stress in strand after tensioning;
fps = Stress in prestressing strand at failure;
fse = Effective prestress, after all time – dependent deformations;
fsi = Prestress after transfer before time-dependent losses;
FT = Fully tensioned;
KDOT = Kansas Department of Transportation;
LBPT = Large-block pullout test;
IT = Inverted-T;
Ldev = Development length;
Le = Embedment length;
Ltr = Transfer length;
Mn = Nominal moment;
PCI = Prestressed/Precast Concrete Institute;
SCC = Self-consolidating concrete;
SSB = Single-strand beam;
TB = T-beam;
TSB = Top-strand beam;
UT = Untensioned;
VWSG = Vibrating wire strain gage;
□ = Measured end slip;
□ = Strain; and
□ = Stress.

APPENDIX

Prestress Losses and Nominal Calculations for SSB specimen

Assume that

$$\varepsilon_c = 0.003 \qquad f_c^{'} = 8,000\,\text{psi} \qquad f_{pu} = 270\,\text{ksi}$$

See Figure 4 for rectangular prestressed concrete beam with following properties:

$$\text{b} = 8\,\text{in} \quad \text{h} = 12\,\text{in} \quad \text{d}_\text{p} = 10\,\text{in}$$

$$\text{A} = 96\,\text{in}^2 \;\; \text{I} = 1152\,\text{in}^4 \;\; \text{y}_\text{b} = 6\,\text{in}$$

$$\text{e} = 4\,\text{in} \quad \beta_1 = 0.85 - [0.05(f_c^{'} - 4,000)/1,000] = 0.65$$

$$\text{E}_\text{ps} = 28,500\,\text{ksi} \;\; \text{A}_\text{ps} = 0.153\,\text{in}^2 \;\; \text{f}_\text{pj} = 0.75*(270) = 202.5\,\text{ksi}$$

$$\text{P} = 202.5*0.153 = 30.983\,\text{kips} \qquad \text{E}_\text{ci} = 3,600\,\text{ksi}$$

$$\text{E}_\text{c} = 5,000\,\text{ksi} \quad \text{Self Weight} = 93.33\,\text{lb/ft}$$

$$\text{L}_\text{b} = 13.17\,\text{ft} \quad \text{M}_\text{sw} = \frac{93.33*(13.17)^2}{8}*12 = 24,376\,\text{lb-in}$$

$$\text{V/S} = 2.33 \qquad \text{RH} = 65\%$$

Loss Calculations (Based on PCI Handbook 5th Edition)

$$\text{ES} = \text{K}_\text{es}*\text{E}_\text{ps}*\text{f}_\text{cir}/\text{E}_\text{ci}$$

$$\text{f}_\text{cir} = \text{K}_\text{cir}\left[\frac{\text{P}}{\text{A}} + \frac{\text{P*e}^2}{\text{I}}\right] - \frac{\text{M}_\text{sw}*\text{e}}{\text{I}}$$

$$= 0.9\left[\frac{30,983}{96} + \frac{30,983*4^2}{1152}\right] - \frac{24,276*4}{1152}$$

$$594 \;\; \text{psi}$$

with $\text{K}_\text{cir} = 0.9$ for pretensioned members

$\text{K}_\text{es} = 1.0$ for pretensioned members

$$\text{ES} = 1.0*28,500*0.594/3600 = 4.70 \;\; \text{ksi}$$

Creep (CR)

$$\text{CR} = \text{K}_\text{cr} \times (\text{E}_\text{p}/\text{E}_\text{c}) \times (\text{f}_\text{cir} - \text{f}_\text{cds})$$

with

$$\text{CR} = 2.0 \times (28,500/5000) \times 0.594 = 6.77\,\text{ksi}$$

Shrinkage (SH)

$$SH = (8.2x10^{-6}) \times K_{SH} \times E_p \times (1 - 0.06 \times (V/S)) \times (100 - RH)$$

with

$$SH = (8.2x10^{-6}) \times 1.0 \times 28,500 \times (1 - 0.06 \times 2.33) \times (100 - 65) = 7.04\,\text{ksi}$$

Relaxation (RE)

$$RE_L = [K_{RE} - J*(SH + CR + ES)]*C$$

with

$$RE_L = [5.0 - 0.04*(4.7 + 6.77 + 7.04)]*1.0 = 4.26$$

$$RE_i = f_{st}\left\{\left[\log 24t - \log 24t_1\right]/24\right\}*\left[f_{st}/f_{py} - .55\right]$$

$$= 202,500*\left[\log 18\right]/45*\left[202,500/243,000 - .55\right] = 1.60\,\text{ksi}$$

Total Losses

$$f_{si} = f_{pj} - ES - RE_i$$

$$= 202.5 - 4.7 - 1.6 \approx 196\,\text{ksi}$$

$$f_{se} = f_{pj} - ES - CR - SH - RE_L$$

$$= 202.5 - 4.7 - 6.77 - 7.04 - 4.26 \approx 180\,\text{ksi}$$

Calculated Transfer Length (using equation 1)

$$L_{tr} = f_{se}d_b/3$$

$$196*.5/3 = 32.67\,\text{inches}$$

Experimental Implied Transfer Length (Sample Calculation)

$$\Delta_{end-slip} = .060 - \left(\frac{PL}{AE}\right)_{elastic\,shortening} = .060 - \left(\frac{30.98*1}{.153*28,500}\right) = .055\,\text{inch}$$

$$\text{L}_\text{tr} = \frac{\Delta E_{ps}}{.5\,f_{si}} = \frac{.055*28,500}{.5*196} = 16\,\text{inch}$$

Calculated Development Length (using Equation 2)

$$L_{dev} = f_{se}d_b/3 + (f_{ps} - f_{se})d_b$$

$$= 180*.5/3 + (268.2 - 180)*.5 = 74\,\text{inch}$$

Nominal Capacity using Strain Compatibility

$$P_e = f_{se}*A_{ps} = 180*0.153 = 27.54\,\text{kips}$$

$$\varepsilon_1 = f_{se}/E_p = 180/28,500 = 0.00623$$

$$\varepsilon_2 = \frac{1}{E_c}\left[\frac{P_e}{A} + \frac{P_e*e^2}{I}\right] = \frac{1}{5000}\left[\frac{27.54}{96} + \frac{27.54*4^2}{1152}\right] = .000134$$

Assume $f_{ps} = 268.2\,\text{ksi}$

$$a = \frac{A_{ps}*f_{ps}}{0.85*f_c^{'}*b} = \frac{0.153*268.2}{0.85*8*8} = 0.754$$

$$c = a/\beta_1 = 0.754/0.65 = 1.16$$

$$\varepsilon_3 = \left(\frac{d_p - c}{c}\right)*\varepsilon_c = \left(\frac{10 - 1.16}{1.16}\right)*0.003 = 0.0229$$

$$\varepsilon_{ps} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 0.00623 + 0.000134 + 0.0229 = 0.0294$$

From Curve in Handbook

$$f_{ps} = 270 - \frac{0.04}{\varepsilon_{ps} - 0.007} = 268.2\,ksi\,(Equals\ assumed\ value)$$

$$M_n = A_{ps}*f_{ps}*\left(d_p - \frac{a}{2}\right) = 0.153*268.2*\left(10 - \frac{.754}{1}\right) = 394.9\,\text{kip-in}$$

32.9 kip-ft

Shear Capacity

$$Test\ Span \qquad L_{test} = 12.83\ ft$$

$$Shear\ Span \qquad a_{test} = 5.92\ ft$$

$$M_D + M_L = 32.9\,kip - ft$$

$$\frac{(0.0933)*(12.83)^2}{8} + \frac{P_F}{2}(5.92) = 32.9$$

$$P_F = 10.5\,kips$$

$$V_{\max} = 0.0933*(6) + \frac{10.5}{2} = 5.8\,kips$$

$$V_c = 2*\sqrt{f_c^{'}}*b*d_p = 2*\sqrt{8,000}*8*10 = 14.3\,kips$$

$$V_c > V_{\max} \quad (Good)$$