Inverse analyses of bending tests for determining the properties of strain hardening cement-based materials

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ABSTRACT: A procedure for determining the mechanical material properties of strain hardening cement-based materials is proposed. It is based on the inverse analysis of bending tests. The experimental results obtained under four-point bending of unnotched specimens allow the determination of the strain hardening curve up to the localization point. A load-crack opening relationship describing the behavior after the localization point is determined on the basis of bending tests at notched specimens. For the numerical simulation of the experiments, a hardening/softening behavior under tension and a non-linear stress-strain curve under compression have been adopted. The optimization required in the inverse analyses is performed by using an evolutionary algorithm which proved be an appropriate as well as efficient numerical tool for the problem to be solved here. The results obtained by inverse analyses of bending tests are compared with those obtained in direct tension tests.

1 INTRODUCTION

Strain hardening materials are characterized by increasing or at least constant stresses after initial cracking. This behavior is accompanied by spatially distributed cracking, i.e. multiple parallel cracks are opening simultaneously. When the first crack starts to exhibit a softening behavior, fracture localization takes place. The further damage is limited to a certain region while outside this region strains are decreasing and cracks are closing. Before fracture localization, the mechanical behavior of the material may be described by a stress-strain relationship, thereafter by a stress-crack opening relationship.

An accepted procedure for determining these material laws are uniaxial tension tests. Such tests are comparably time consuming and expensive. Furthermore, undesirable strain gradients in the ligament cause additional problems as far as the interpretation of the results is concerned. Nonetheless, for the development and optimization of strain hardening materials direct tension tests are inevitable. They allow to directly characterize the uniaxial deformation and fracture behavior which is the standard loading case fundamental studies should be based on.

In practical material testing, the bending test is an alternative way for determining the material parameters of strain hardening materials since such tests are easier to perform and less time consuming. However, the determination of the uniaxial load-deformation relationships is only possible by inverse analyses of the experiments. These analyses include simulations of the fracture processes in the specimen and an iterative fitting of the numerical results to the experimental ones.

For strain softening cement-based materials, several techniques for the inverse analysis of fracture tests have been proposed and used in the past. As an alternative to manual fitting, Roelfstra & Wittmann (1986) presented the Softfit program. It based on the discrete crack approach and uses a gradient descent optimization. Although the program has been successfully used in numerous investigations (Slowik 1995) it reveals the limitations of the applied optimization method. Experience has shown that this method requires a first guess of the softening parameters which is quite close to the final results. This may be attributed to the risk of getting “caught” by local minima in the error function. The Japan Concrete Institute (2001) recommends a method based on a polylinear softening curve. The individual slopes are determined successively by adjusting corresponding sections of the calculated load-displacement curve to the experimental one. Østergaard (2003) proposed and extensively tested a method for the inverse analysis based on a hinge model. Assuming plane sections in a certain distance from the crack path an analytical solution for the softening curve may be derived. This approach appears to be very attractive because of the comparably short computing time and because of the unique solution of the optimization procedure. However, the assumption of plane section is not always justifiable, especially not in laboratory sized wedge splitting specimens. This shortcoming is taken care of by a calibration of the method in order to adopt a suitable value for the distance between the assumed plane sections. For routine material testing, the inverse analysis based on the cracked hinge model is a very effective tool.

Østergaard et al. (2005) modified the hinge model and the corresponding inverse analysis method for the application to strain hardening materials. In the modified model, the inner region of constant moment in a four-point bending beam is assumed to consist of several hinges which are separated by parallel plane sections. After fracture localization, only one of these hinges continues to open while the other ones are closing. The width of the localizing hinge has to be predefined prior to the analysis. It may be adjusted by comparing the results to those of Finite Element simulations. In addition to the advantages of the hinge model already mentioned above, it has to be pointed out that this method allows to determine the complete fracture behavior including hardening and softening by a single inverse analysis which is based on only one type of experiment.

The authors are following a different approach for the inverse analysis of experiments (Villmann B. et al. 2004 & 2006, Slowik et al. 2006). The main objectives of this work are, firstly, to adopt a mechanical model which requires prior to the analysis only inevitable assumptions concerning material behavior and specimen deformation and, secondly, to use a suitable optimization method for this particular problem. As optimization method an evolutionary algorithm including local neighborhood attraction for convergence improvement is proposed. Its advantages are explained in section 2.2. The method has been utilized by the authors for determining different material parameters of cement-based materials (Villmann B. et al. 2006). Softening curves may be determined on the basis of the discrete crack model. Furthermore, inverse analyses of drying experiments allow the determination of the moisture dependent diffusion coefficient. The evolutionary algorithm provides very good fits which could not be achieved by manual fitting. In its current version, the associated software tool allows to optimize models with up to 12 parameters. This corresponds to seven segments of a multilinear hardening curve. The optimization problem described in the present paper requires this comparably high level of variability in the formulation of the material law.

For the determination of fracture properties of strain hardening materials a two-step method is proposed. Hardening and softening curves are determined in separate inverse analyses of different types of experiments. For the hardening curve, inverse analyses of four-point-bending tests of unnotched beams are required in order to activate the high strain capacity of these materials. The softening properties are determined on the basis of bending tests of notched beams. Although this two-step procedure appears to be expensive it has some advantages. In addition, it has to be taken into account that in many cases only the hardening behavior is of technical interest for the application of these materials. The major advantage of the two-step procedure is that the width of the fracture process zone in the softening mode is not required as a predefined input parameter for the inverse analysis. This width may even be extracted out of the simulation results. The latter depend on the material parameters and on the specimen geometry only. Furthermore, the mechanical model does not require a plane section assumption. It has to be stated, however, that the fracture process zone width in the notched beams is assumed to be constant along the crack path.

The mechanical model and the optimization method have been used for determining the material parameters of two types of strain hardening cement-based composites (SHCC). For verification of the results, different specimen geometries were included in the investigation.

2 PROPOSED METHOD

2.1 *Numerical Simulation of the experiment*

2.1.1 *Simulation up to the localization point*

The hardening range of strain hardening cement-based materials is characterized by distributed, i.e. multiple cracking. In order to activate it in the experiment, four-point bending tests of unnotched specimens should be performed rather than three-point bending tests or tests of notched specimens. In this way, a wide region of constant moment allows for a spatially distributed damage process. The mechanical model used for the inverse analysis of the experiment is based on non-linear axial members in the inner region of maximum moment and a linear-elastic Finite Element model for the side regions. Taking advantage of symmetry, only half of the beam is considered in the model, see Fig. 1. Cracking outside the inner region of maximum moment is neglected.

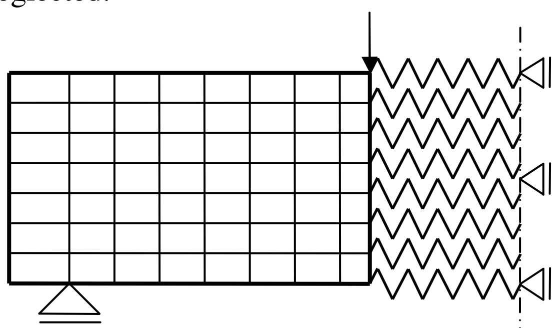

Figure 1. Model for the numerical simulation up to the localization point

The mechanical behavior of the non-linear axial members incorporates a multi-linear stress-strain curve with strain hardening, see Fig. 2. It appeared to be necessary to consider a non-linear stress-strain curve also in the compressive range in order to obtain realistic simulation results. In the analyses presented here, a bi-linear curve was assumed, i.e. a linear-elastic branch and a constant post-peak stress value.

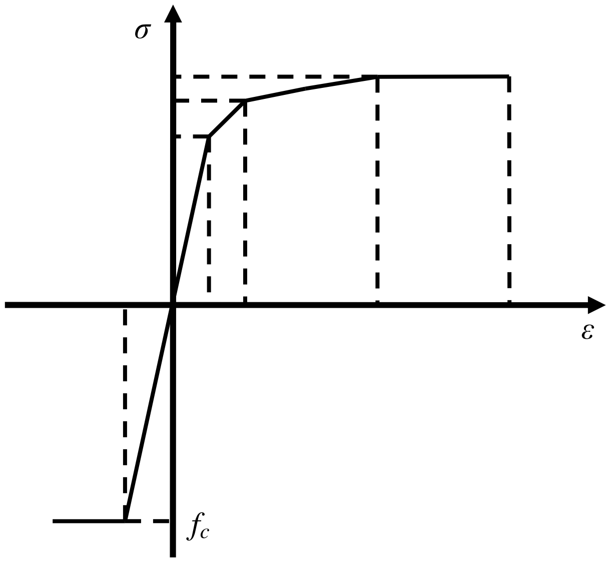

Figure 2. Stress-strain curve assumed for the non-linear axial members

The inverse analysis with the model shown in Fig. 1 and the material law shown in Fig. 2 is performed up to the localization point. After that point, further damage is taking place in a single fracture process zone only and may be described by a discrete crack model as done for softening materials, see Section 2.1.2. A continuing simulation with the model shown in Fig. 1 would not be physically correct. Using an effective stress-strain curve which takes into account localization and unloading is not considered an appropriate way of modeling because of the unknown fracture process zone width, because of the unknown unloading behavior and because of required additional compatibility conditions.

In order to improve the computational efficiency, the linear-elastic part is pre-analyzed by Finite Elements and replaced by a linear-elastic macro-element. In this way, the number of degrees of freedom to be considered in the inverse analyses is reduced significantly.

In the numerical model, the specimen self-weight is taken into account. Parametric studies have shown, however, that the influence is negligible for practical specimen sizes.

2.1.2 *Simulation after the localization point*

In order to enforce fracture localization and to suppress distributed cracking, bending experiments with notched specimens are undertaken for determining the stress-crack opening behavior of a localized crack. Inverse analyses of such tests are performed by using the same algorithm as for softening materials (Villmann B. et al. 2004). The underlying mechanical model is shown in Fig. 3. Fracture is taking place in the axis of symmetry only where a discrete crack is assumed. The planar part of the specimen exhibits a linear-elastic behavior and is replaced by a pre-analyzed macro-element for the inverse analyses.

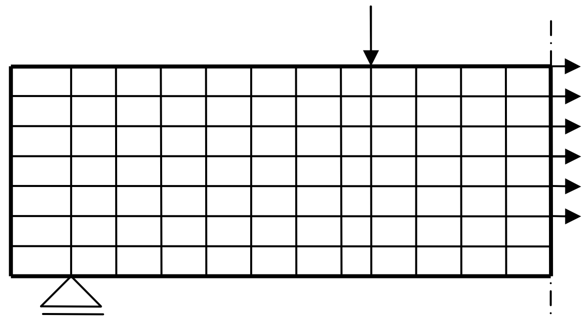

Figure 3. Model for the numerical simulation after the localization point

For characterizing strain hardening materials, it was necessary to increase the number of the curve parameters to be optimized. A multilinear curve with arbitrary shape and up to six segments has been adopted, as stated before. In order to adequately approximate the long descending branch of the load-deflection curves characteristic for strain hardening materials it was necessary to refine the mesh at the end of the ligament. This led to an irregular discretization.

The major shortcoming of the method is the required assumption of a constant width of the fracture process zone along the crack path. If this simplification is accepted, the determination of fracture properties for a localized crack in strain hardening materials is possible. As a result, a stress-crack opening curve is obtained for an unknown constant fracture process zone width. If the strain at the localization point is known, for example as a result of direct tension tests or of inverse analyses according to section 2.1.1, it is possible to extract the fracture process zone width out of the stress-crack opening curve. It has to be taken into account, however, that the fracture process zone width is dependent on specimen size and geometry.

In order to obtain realistic stress distributions in the ligament the material model includes a limitation of the occurring compressive stresses. This is taken care of by predefining a compressive strength as maximum value. In some cases, the limitation of compressive stress along with the characteristic behavior of the strain hardening materials results in situations of local and momentary crack closing. For that reason, the softening law had to be supplemented by a crack closing path, see Fig. 4.

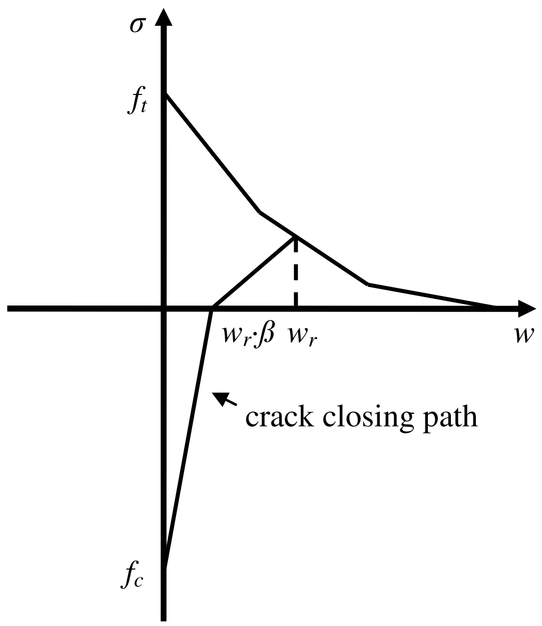

Figure 4. Stress-crack opening curve for the localized crack

In parametric studies, the influence of the self-weight on the results of the inverse analyses was found to be negligible for practical specimen sizes and strain hardening materials. For quasi-brittle softening materials this is not always the case.

2.2 *Optimization*

Optimization in the inverse analyses presented here is a nontrivial problem. For the adequate approximation of the stress-strain curves a comparably large number of parameters is required. For that reason, a manual optimization is not only time consuming but nearly impossible and expected to be not objective. Another problem is that large variations in the hardening function cause only comparably small differences of the numerical stress-strain curve (Slowik et al. 2006). Hence, only quite exact approximations provide reliable results of the inverse analysis, i.e. sufficiently accurate hardening functions. That means the modification of the hardening parameters requires an automatic routine. Furthermore, the n-dimensional error function (n is the number of model parameters to be optimized) of the approximation is quite jagged and there is a risk of getting caught by non-attractive local minima.

A general alternative for such optimization problems are Evolutionary Algorithms (EAs) (Schwefel 1981). EAs are biologically motivated iterative stochastic optimization methods, the roots of which point to biological genetics as described by Mendel's laws of heredity and Darwin's evolution model by natural selection. They have several advantages when compared to classical optimization methods:

- The variation of the model (parameters) to be optimized is separated from its evaluation. Therefore, the underlying physical model can be easily modified without any changes in the optimization algorithm. Furthermore, there are no restrictions concerning type and mathematical formulation of the model.
- In general, the optimization is possible for an arbitrary number of parameters.
- Because of the stochastic nature of the algorithm the risk of getting caught by local minima is reduced.
- EAs show a comparably good convergence behavior. It may be further improved by using a special local search procedure named neighborhood attraction (Villmann T. 2001).
- The required computing time is moderate.
- After a sufficient number of generations (see below), very good approximations may be obtained.

The basic principle of the EAs is the following: The model to be optimized is characterized by a vector of model parameters called individual. For a given individual a so-called fitness may be calculated, inverse to the error describing the deviation of the model prediction from the experimental data. The task is to find an individual which minimizes the error. Initially, a generation of a given number of individuals is randomly chosen and the fitness of each individual is calculated. In each time step, a new generation (offspring) of new individuals is created by certain genetic operations. Such operations may be (Bäck 1996):

- random variations of vector element values (mutations) or
- randomly cutting two strings and gluing them together in a crossed way (combination).

After the genetic operations, all offspring individuals are evaluated, i.e. their fitness is determined. Then, a fitness based selection procedure follows in order to create a new generation. This selection may comprise either the “best”, i.e. fittest, individuals of the offspring only or of both the offspring and the adult generation. In the investigations presented here, a combination of these two selection methods is used (Bäck 1996, Villmann T. 2002). The fittest individual of the actual generation is the currently found solution of the EA. For a more detailed explanation see Bäck (1996), Villmann T. (2001) and Villmann T. et al. (2004).

3 APPLICATION OF THE METHOD

3.1 *Material composition and uniaxial tension tests*

For testing the proposed method inverse analyses of different experiments were carried out. The development of this method was motivated by the advances in the field of strain hardening cement-based composites (SHCC) with polymeric fiber reinforcement. These materials were formerly referred to as Engineered Cement-based Composites (ECC) (Li 1998). Their very high strain capacity is achieved mainly by the addition of high modulus polymeric fibers of short length, in most of the cases polyvinyl alcohol (PVA) fibers.

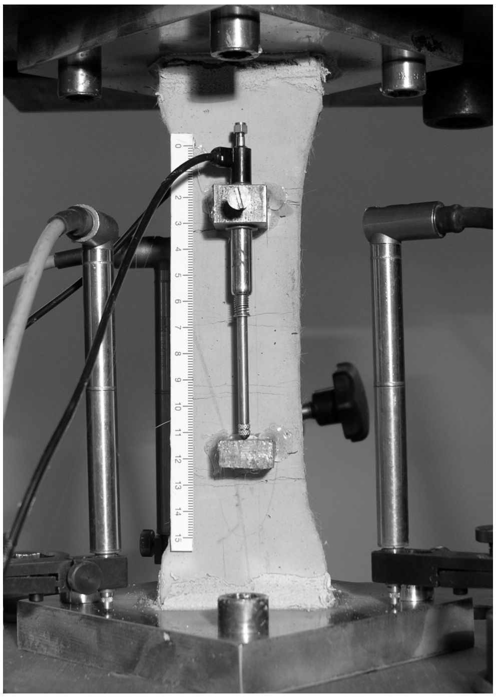

Figure 5. Direct tension test of a strain hardening cement-based composite

The materials used in the investigation presented here were a result of an optimization process aimed to a high tensile strain capacity. Their composition is characterized by fine aggregates and moderate water to binder ratios. The uniaxial tensile strength is comparably low in order to provide a smooth transition from initial cracking to the hardening range. Table 1 shows the material composition of the two materials included in this investigation. Both materials were reinforced with 8 mm long PVA fibers (Kuraray REC15). The fiber content was 2.2 % by volume.

Table 1 Composition of the tested materials

	material I	material II
Cement [kg]	1.00	1.00
Water [l]	1.10	1.20
Fly Ash [kg]	2.30	2.30
Silica Flour [kg]	1.00	1.00
Silica Sand [kg]	0.40	0.00
Plasticizer [kg]	0.040	0.040
Stabilizer [kg]	0.015	0.015
Fiber Content [Vol.-%]	2.20	2.20

After casting, the specimens were left in their metal moulds and covered for three days. After that, they were stored at 65 % relative air humidity and 20°C. The compressive strength after 14 days amounted to 24.8 N/mm² for material I and to 18.8 N/mm² for material II.

With both materials, direct tension tests have been carried out. Fig. 5 shows the experimental setup. Dog bone shaped specimens with a cross-section of 40 mm by 40 mm were tested under displacement control. The inner part of the specimens with constant cross-section had a length of 100 mm which was also the base length for the displacement measurement.

Fig. 6 shows measured stress-strain curves. The latter are characterized by nearly constant stresses up to the localization point. For material I, the strain at the localization point amounted to about 2%; for material II about 2.5% were reached.

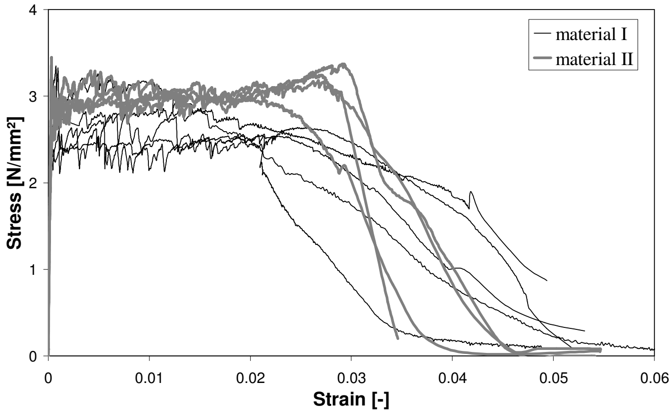

Figure 6. Stress-strain curves obtained in direct tension tests

3.2 *Determination of hardening curves by inverse analyses of bending tests with unnotched beams*

As stated before, the hardening behavior may be determined by inverse analyses of four-point bending tests with unnotched beams. For a verification of the proposed method, different specimen geometries were included in the investigations. Fig. 7 schematically shows the specimen geometry; the corresponding dimensions are given in Table 2.

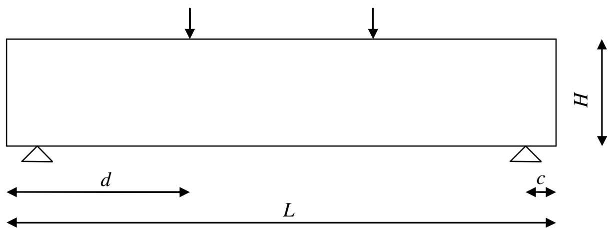

Figure 7. Geometry of the tested beams (dimensions in Table 2)

Table 2. Specimen dimensions

	B 50	B 75	B 100	B 125	B 150
midspan [mm]	50	75	100	125	150
H [mm]	40	40	40	40	80
L [mm]	180	205	230	255	400
c [mm]	15	15	15	15	25
d [mm]	65	65	65	65	125
thickness [mm]	40	40	40	40	80

For material I, some of the load-deflection curves obtained with the beam geometries B 75 and B 150, respectively, are shown in Fig. 8. Whereas approximately the same ultimate load levels were measured for the individual samples having the same geometry, the deflections at the localization point show a comparably large scatter. The best fitting numerical results obtained by optimization are also shown in Fig. 8. It can be seen that the evolutionary algorithm yields excellent approximations of the experimental results.

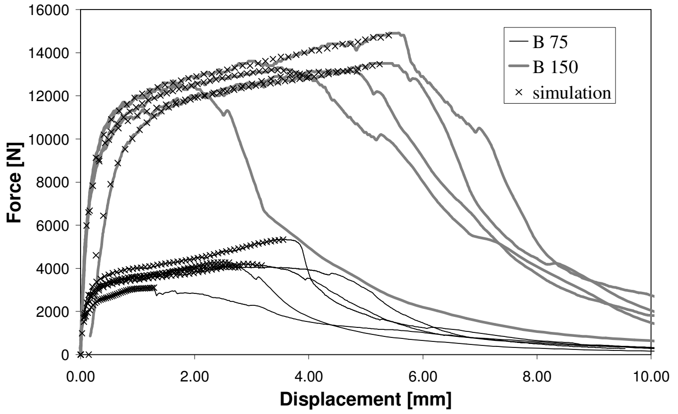

Figure 8. Comparison of measured and simulated load-deflection curves for material I

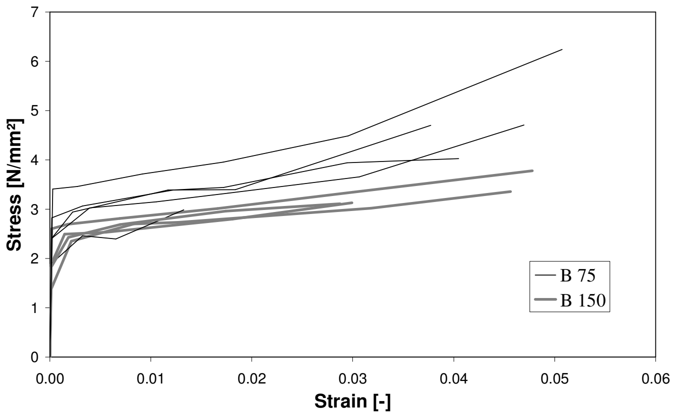

Figure 9. Obtained hardening curves for material I

The obtained hardening curves for the samples used for Fig. 8 are displayed in Fig. 9. As stated before, these curves are physically correct only up to the localization point. For that reason, they were truncated at the point of maximum stress although the numerical simulations were continued slightly beyond this point. Usually, the point of local maximum stress, resulting in localization, is reached before the point of ultimate external load. In the case of bending, fracture localization does not necessarily lead to an immediate decrease of the external load.

The curves shown in Fig. 9 form a band with a clear trend. For the different geometries roughly the same hardening curves were determined although there is a tendency of higher stresses in the smaller beams. The ultimate hardening strain values show a comparably large variation. Considering the differences between the curves in Fig. 8 it may be expected that the corresponding hardening curves differ as well. However, all the hardening curves show about the same stress level in the hardening range.

In Fig. 10, the variation of the ultimate hardening strain for different geometries is shown. No clear trend may be identified. That means, neglecting the cracks outside of the inner region of the beams with constant moment does not result in significantly size dependent results. For the saw-cut specimens lower hardening strain values were reached in comparison to all the other specimens which had molded surfaces. The random fiber orientation at saw-cut surfaces leads to lower ultimate strains. During the experiments, it was observed that the cracks at saw-cut surfaces were not as equally distributed as those at molded surfaces.

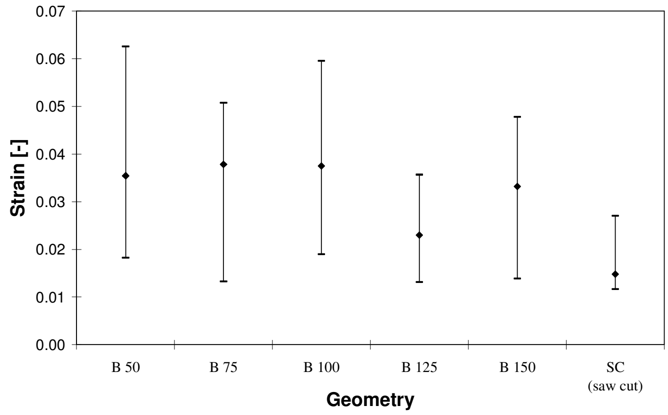

Figure 10. Ultimate hardening strain values obtained in inverse analyses for material I and different geometries

Fig. 11 shows the load-deflection curves for material II. The corresponding results of the inverse analyses are presented in Fig. 12. As in the case of material I, for different geometries roughly the same hardening curves were obtained by the inverse analyses.

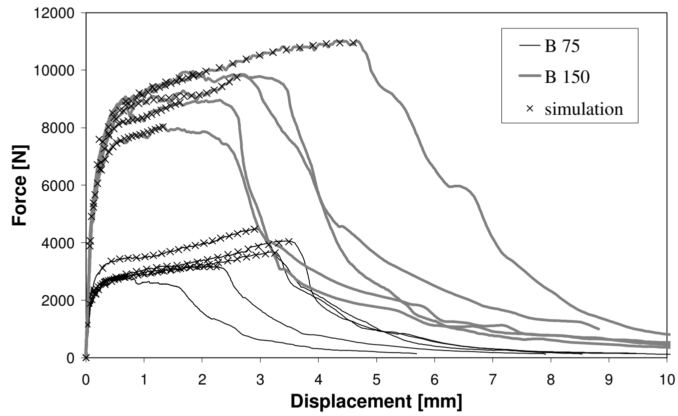

Figure 11. Comparison of measured and simulated load-deflection curves for material II

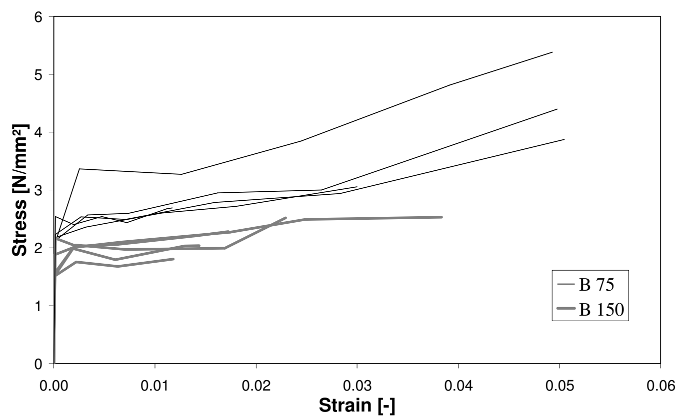

Figure 12. Obtained hardening curves for material II

Although in general the variation of these results is regarded as being high, it may be noticed that the scatter of the hardening curves obtained for the smaller beams (B 75) is lower.

3.3 *Determination of softening curves by inverse analyses of bending tests with notched beams*

The discrete crack approach is used for simulating the experiments. Thereby, a constant fracture process zone width is assumed. All strains and dislocations within the fracture process zone are considered to be included in the discrete crack opening value.

Two different specimen geometries were included in the investigation, B 75 with a notch length notch 10 mm and B 150 with a notch length of 20 mm.

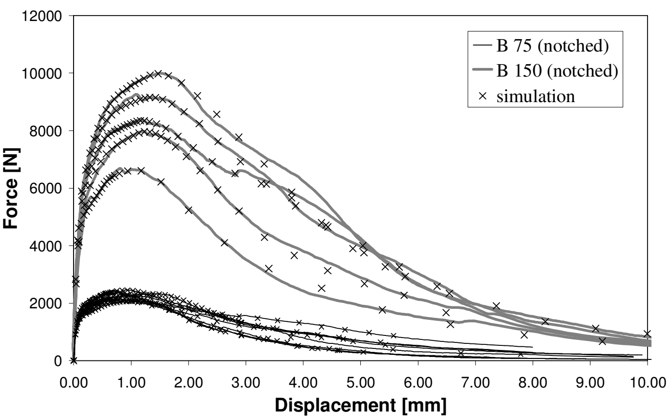

Figure 13. Measured and simulated load-deflection curves for material I (notched beams)

Fig. 13 shows the experimental and numerical results. The scatter of the maximum load appears to be larger than in the case of unnotched beams. This may be attributed to the spatially delimited fracture process zone controlling the global loading capacity of these inhomogeneous specimens.

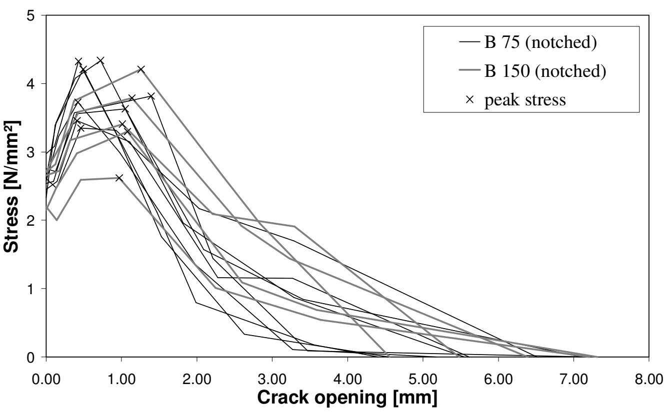

Figure 14. Obtained stress-crack opening curves for material I

The results of the inverse analyses are presented in Fig. 14. As in the case of the hardening curves, see section 3.2, the stress-crack opening curves form a band and for the different specimen geometries the inverse analysis yields nearly the same results. However, the crack opening at maximum stress is clearly higher for the larger specimens. This may be explained by the size dependence of the fracture process zone.

The maximum stress values in the stress-crack opening curves should correspond to the stress at the localization point in the hardening curves. This is confirmed by the obtained results of the inverse analyses.

3.4 *Discussion of the results*

On the basis of both the crack opening value at maximum stress and the ultimate strain in the hardening curve it is possible to determine the width of the fracture process zone for the particular geometry of the notched beams. For B 75 this width was determined to 20.2 mm and for B 150 to 32.9 mm.

In the pre-peak region of the stress-crack opening curves, see Fig. 14, hardening takes place in the fracture process zone. This phenomenon could also be visually observed during the experiments. Parallel cracks were formed in a zone with a width in same the order of magnitude like the values given above.

The results of the inverse analyses may be used for assembling a complete stress-strain curve for the strain hardening material including both hardening and softening. Up to the localization point, the obtained hardening curve, see Section 3.2, is used. Starting with the average ultimate hardening strain, a softening branch is added to the curve. For that, the softening curve needs to be extracted from the stress-crack opening curve. This is done by subdividing the crack opening displacement obtained by inverse analysis into a portion resulting from the strain in the hardening fracture process zone and into a portion coming from the opening of the final crack, see Fig. 15. Here, a remaining strain of 75% of the maximum has been assumed. This is in good accordance with experimental observations during uniaxial unloading and reloading.

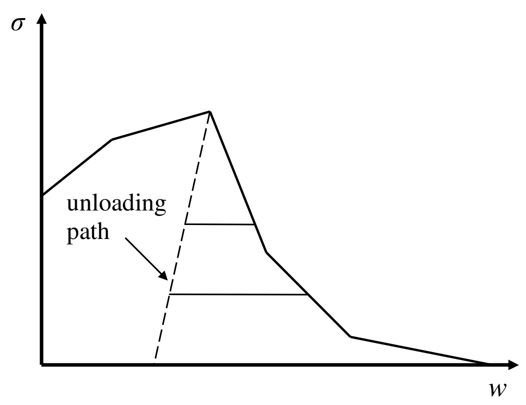

Figure 15. Extraction of the softening curve from the load-crack opening curve obtained by inverse analysis of bending experiments with notched beams

If the discrete crack opening in the softening crack is known, it may be transformed into an equivalent strain value for a given gage length in the direct tension test. Then, the uniaxial stress-strain curve may be completed by adding the calculated softening strain values to the unloading branch starting in the localization point. Fig. 16 shows such curves calculated for several specimens in comparison to results of direct tension tests.

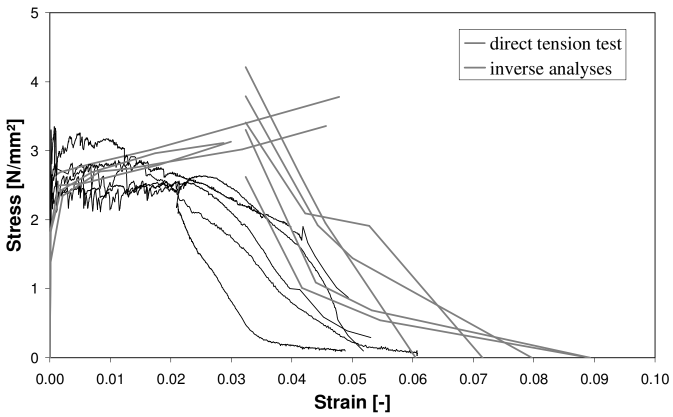

Figure 16. Results obtained by inverse analyses in comparison to those measured in direct tension tests (material I)

Whereas the hardening stress levels found in direct tension tests and by inverse analyses, respectively, are in good accordance, there are significant deviations in the ultimate hardening strain values. Furthermore, only the curves obtained by inverse analysis exhibit increasing stresses in the hardening range. These differences may be at least partially attributed to a different material behavior under uniaxial tension and in bending, respectively. In bending tests, lateral stresses as well as the inclined fiber pull-out direction may result in a larger strain capacity of the fiber reinforced material. In addition, strain gradients occurring in direct tension tests might lead to a reduction of the average ultimate strain. Up to a strain of about 2%, however, the results of the inverse analyses are considered to resemble the uniaxial deformation behavior in a satisfactory manner for technical applications.

4 CONCLUSIONS

The method for determining fracture properties of strain hardening cement-based materials by inverse analyses of bending tests yields physically sound results. For different geometries, roughly the same material parameters were obtained. The optimization by using an evolutionary algorithm allows excellent approximations of the experimental results. For strain values above 2%, significant deviations between the results of direct tension tests and those obtained by inverse analyses of bending tests were found. These differences may partially be explained by specific effects occurring in the different experiments and will be subject of further investigations. Furthermore, the model assumptions concerning the compressive failure need to be refined.

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