Coupling between creep and cracking in tension

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ABSTRACT: An isotropic elastic damage model is extended to taken into account creep failure. It combines a creep model by means of several Kelvin-Voigt chains in serial and the Mazars model, where the equivalent strain threshold includes a part of creep strain. The model has been implemented in an available finite element code. The implementation is straightforward since all the involved equations are defined explicitly. The proposed model is able to reproduce linear visco-elastic strain for low stress levels and also failure caused by tertiary creep.

1 INTRODUCTION

Tensile creep can be an important issue for the behaviour of concrete structures. For instance, at early ages, stresses generated by autogeneous shrinkage and thermal dilation, in massive structures or in restrained elements (reparatio, lifts …) are relaxed by creep in tension (Ostergaard et al. 2001). Moreover, one of the probable reasons behind the collapse of the 2E terminal in the international airport of Roissy (France) is the tertiary creep in tension. Furthermore, in case of accident in nuclear power plants, the internal pressure inside the vessel reaches about 0,5 MPa. This can induce vertical and ortho-radial tensile stresses in internal containment vessels (biaxialy pre-stressed), if creep strain during the service-life of the structure has been underestimated or in particular area (access airlock …). These stresses can be very close to the tensile strength, and cause an initiation of cracking or/and a propagation of existing cracks, and therefore induce an increase of global permeability and consequently the leakage of radioactive elements into the environment.

It is still a difficult task to assess the creep behaviour of concrete, since the associated mechanisms are not well known (Jennings and Xi 1992). Moreover, few experimental results and numerical models are available on tensile creep compared to compressive creep. Especially, tensile creep of aged concrete has been less studied than the one of young concrete.

When compressive stresses level becomes important, we notice that the relation creep strain / stress become non linear (Roll 1964, Li 1994, Mazzotti and Savoia 2003).

In previous studies, this feature can be modelled by several ways:

- From damage theory (Li, 1994), the temporal variable can be introduced explicitly in the evolution law of mechanical damage (in term if rate of damage). One of the shortcomings of this approach is that it is dissociated with linear viscoelastic creep strain;
- According to Bažant and Xiang (1997) this non-linearity is only apparent. The non linear creep does not exist, from their point of view. Non linearity is the result of micro-cracks growing during the time. They proposed a time-dependent generalization of the R-curve model (adaptation of LEFM which considers the process zone as a point), in which the rate of crack growth is a function of the ratio of the stress intensity factor to the R-curve;
- The non linear character can be supposed to be linked to the stress redistribution because of creep, and the non homogeneous damage distribution (Ožbolt and Reinhardt 2001). Indeed, cracking in concrete is heterogeneous in a specimen. It occurs in a localized area. The redistribution takes place between the most damaged areas and the less damaged areas. This hypothesis requires, in the modelling part, to take into account the initial defaults in the material which can corrupt the stress and strain distribution initially homogeneous.
- Linear visco-elastic model can be extended by multiplying the creep compliance by a non linear function of the stress-state (Bažant and Prasannan, 1989). This function is equal to one for low stress level;

- Coupling between creep and cracking can be modeled by combining a visco-elastic and a visco-plastic model (Bertholler et al. 2002). Their model is built on Duvault-Lion approach in which they have integrated a generalized Maxwell model. It allows for reproducing linear visco-elastic creep, creep failure and rate effect on strength;

- Mazzotti and Savoia (2003) proposed to model non linear creep strain by introducing a stress rate reduction factor as a function of damage index in the solidification model of Bažant and Prasannan (1989). Moreover, an effective strain is then defined for creep damage, replacing the equivalent strain for damage evaluation for instantaneous loading case. The effective strain is defined as the sum of instantaneous damaged elastic strain and a fraction of creep strain. Omar et al. (2003) also used a similar approach.

Among these approaches, our model is based on the one proposed by Mazzotti and Savoia (2003). The elastic damage model proposed by Mazars (1984) has been modified in order to get finite fracture energy. The mesh dependency problem is overcome by the use of a characteristic length, related to the size of finite elements. Moreover, no stress factor is introduced. The obtained model allows for retrieving linear creep, non linear creep and partially rate effect on stress-strain curves. Furthermore, since all equations are defined analytically, no local iteratio is needed, rendering the calculations very fast.

In this paper, we will consider only basic creep (i.e. no drying) of aged concrete (i.e. no ongoing hydratio) without taking into account the effect of age at loading.

2 MODELLING

2.1 *Basic creep model*

Many mechanisms for basic creep of concrete have been proposed in order to retrieve all the collected experimental evidences. Even no mechanism has been universally accepted yet, several models exist in the literature.

Several models for basic creep are based on rheological elements (springs and dashpots). The most used elements are Kelvin-Voigt and Maxwell chains, which are combined in serial or/and parallel. Here, we used several Kelvin-Voigt chains (see Figure 1). We will show that the use of such elements gives straightforward formula for the computations of strain evolution, in contrary to Maxwell chains for instance, which need the use of an algorithm (such as the so called exponential algorithm proposed by Bažant and Wu, 1974). However, such a model does not allow retrieving irreversible basic creep strains at unloading (which are about 60-70 % of total creep strain in the linear range). Fortunately, if only increasing stresses are concerned, a correct evolution of creep strain is predicted.

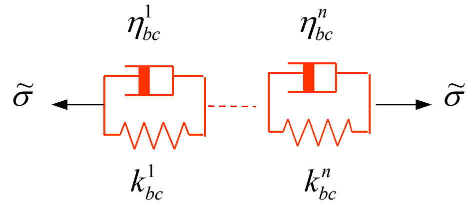

Figure 1. Kelvin-Voigt elements for the prediction of creep strains.

Let us consider a Kelvin-Voigt unit *i*, the basic strain evolution is given by the following relationship:

$$\eta_{bc}^i \dot{\varepsilon}_{bc}^i(t) + k_{bc}^i \varepsilon_{bc}^i(t) = \tilde{\sigma}(t) \tag{1}$$

Where $\tilde{\sigma}(t)$ is the effective stress (See § 2.2), $\varepsilon_{bc}^i(t)$ is the elementary basic creep strain, k_{bc}^i is the stiffness and η_{bc}^i is the viscosity of the Kelvin-Voigt unit *i*.

The total basic creep strain is then obtained from the sum of all the elementary basic creep strain:

$$\varepsilon_{bc}(t) = \sum_i \varepsilon_{bc}^i(t) \tag{2}$$

For variable stresses, the algorithm of the computation of basic creep strain is given in § 2.3.

2.2 *Damage model*

The damage variable is associated to the mechanical degradation process of concrete induced by the development of microcracks. It is defined as the ratio between the area occupied by created micro-cracks, over the whole material area (see figure 6).

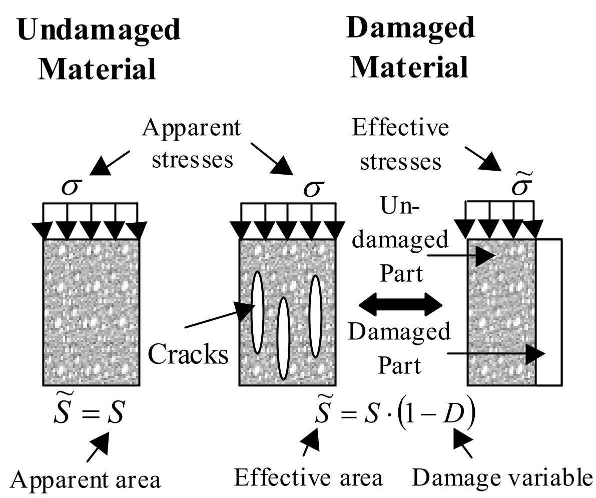

Figure 2. Definition of the damage variable.

The mechanical behaviour of concrete is modelled by an isotropic elastic-damage model coupled with creep. The relationship between apparent stresses $\boldsymbol{\sigma}$, effective stresses $\tilde{\boldsymbol{\sigma}}$, damage D, elastic stiffness tensor $\mathbf{E}_0$, elastic strains $\boldsymbol{\varepsilon}_e$, basic creep strains $\boldsymbol{\varepsilon}_{bc}$, total strains $\boldsymbol{\varepsilon}$:

$$\boldsymbol{\sigma} = (1-D)\cdot\tilde{\boldsymbol{\sigma}} = \mathbf{E}_0 \cdot (1-D)\cdot \boldsymbol{\varepsilon}_e \quad (3)$$

D is linked to the equivalent tensile strain $\hat{\varepsilon}$ (Mazzotti and Savoia, 2003):

$$\hat{\varepsilon} = \sqrt{\langle \boldsymbol{\varepsilon}_e \rangle_+ : \langle \boldsymbol{\varepsilon}_e \rangle_+ + \beta \langle \boldsymbol{\varepsilon}_{bc} \rangle_+ : \langle \boldsymbol{\varepsilon}_{bc} \rangle_+} \quad (4)$$

where $\langle \boldsymbol{\varepsilon}_{xx} \rangle_+$ corresponds to the positive part of the strains and β is a material parameter which can be identified from non linear basic creep data. Therefore, damage will be affected also by basic creep strain. This will allow for retrieving non linear basic creep evolution for high stress levels. By taking $\beta = 0$, the initial formula proposed by Mazars is retrieved.

The damage variable represents the effect of progressive microcracking, due to external mechanical loads, in term of degradation of the current Young's modulus of the material. The evolution of the damage variable in tension D_t and in compression D_c is an exponential form:

$$D_x = 1 - \frac{\kappa_0}{\hat{\varepsilon}}\left[(1 + A_x)\exp(-B_x\hat{\varepsilon}) - A_x \exp(-2B_x\hat{\varepsilon})\right] \quad (5)$$

where A_x and B_x ($x = t$ for tension and $x = c$ for compression) are constant material parameters which controls the softening branch in the stress-strain curve; κ_0 is the tensile strain threshold.

The damage variable is related to the compressive and tensile ones by the following relationship:

$$D = (1-\alpha_t)D_c + \alpha_t D_t \quad (6)$$

where α_t is related to the tensile and compressive strain created by principal tensile and compressive stresses, ε_{T_ii} and ε_{C_ii}, respectively (Mazars 1984):

$$\alpha_t = \sum_i \frac{\varepsilon_{T\_ii}(\varepsilon_{T\_ii} + \varepsilon_{C\_ii})\cdot\left[H_i(\varepsilon_{T\_ii} + \varepsilon_{C\_ii})\right]}{\hat{\varepsilon}^2} \quad (7)$$

where H is the Heaviside function.

The damage criterion is given by (Mazars 1984):

$$f = \hat{\varepsilon} - \kappa_0 \quad (8)$$

Strain softening induces inherent mesh dependency and produces failure without energy dissipation (Pijaudier-Cabot et al. 1987). In order to avoid such shortcomings, a characteristic length l_c is introduced. This length is related to the mesh size (Rots 1988, Cervera and Chiumenti 2006) in order to dissipate the same amount of energy after mesh refinement, when strains localize in one row of finite elements.

For the adopted model, the dissipated energy density g_{fx} at failure in compression ($x = c$) and tension ($x = t$) is reads:

$$g_{fx} = \frac{f_x(1 + A_x/2)}{B_x} \quad (9)$$

where f_c and f_t are the strength in compression and tension, respectively.

It is related to the fracture energy G_{fx} and the characteristic length l_c:

$$g_{fx} = \frac{G_{fx}}{l_c} \quad (10)$$

2.3 *Numerical algorithm*

The effective stresses are linearized, for each time step:

$$\tilde{\boldsymbol{\sigma}}(t) = \tilde{\boldsymbol{\sigma}}_n + \Delta\tilde{\boldsymbol{\sigma}}_{n+1}\frac{(t - t_n)}{\Delta t_n} \quad (11)$$

with $t \in [t_n, t_{n+1}]$, $\Delta t_n = t_{n+1} - t_n$, $\Delta\tilde{\boldsymbol{\sigma}}_{n+1} = \tilde{\boldsymbol{\sigma}}_{n+1} - \tilde{\boldsymbol{\sigma}}_n$ and where t_n is the time at time-step number n; $\tilde{\boldsymbol{\sigma}}_n$ is the effective stress vector at time step number n.

Then, basic creep strains are calculated by solving the differential equation (1), using Eq. 11:

$$\Delta\varepsilon_{bc}^{n+1} = \varepsilon_{bc}^{n+1} - \varepsilon_{bc}^{n} = a_{bc}\varepsilon_{bc}^{n} + b_{bc}\cdot\tilde{\sigma}_n + b_{bc}\cdot\tilde{\sigma}_{n+1} \quad (12)$$

where $\boldsymbol{\varepsilon}_{bc}^n$ is the basic creep strains vector at time-step number n; a_{bc}, b_{bc} and c_{bc} depend only upon material parameters, t and Δt:

$$\begin{cases} a_{bc} = e^{-\frac{\Delta t}{\tau}} - 1 \\ b_{bc} = \sum_i \frac{1}{k_{bc}^i}\left[\frac{\tau_{bc}^i}{\Delta t}\left(1 - e^{-\frac{\Delta t}{\tau_{bc}^i}}\right) - e^{-\frac{\Delta t}{\tau_{bc}^i}}\right] \\ c_{bc} = \sum_i \frac{1}{k_{bc}^i}\left[1 - \frac{\tau_{bc}^i}{\Delta t}\left(1 - e^{-\frac{\Delta t}{\tau_{bc}^i}}\right)\right] \end{cases} \quad (13)$$

where the characteristic time is defined by:

$$\tau_{bc}^i = \frac{\eta_{bc}^i}{k_{bc}^i} \quad (14)$$

Therefore, we only need to know the stresses and the creep strains at time step number n to calculate the creep strains at time-step number n+1 (the storage of the stress history is not needed). In the case of linear visco-elastic strains, Eq. 12 and 13 are strictly equivalent to Boltzmann superposition principle (Benboudjema 2002).

Eq. 12 is extended in 3D, by mean of a creep Poisson ratio, which has been taken equal to the elastic one in the numerical simulations:

$$\Delta\boldsymbol{\varepsilon}_{bc}^{n+1} = a_{bc}\boldsymbol{\varepsilon}_{bc}^{n} + \mathbf{B}_{bc}\cdot\tilde{\boldsymbol{\sigma}}_{n} + \mathbf{C}_{bc}\cdot\tilde{\boldsymbol{\sigma}}_{n+1} \qquad (15)$$

The effective stresses increment $\Delta\tilde{\boldsymbol{\sigma}}_{n+1}$ at the end of the time step number n is updated by the following relationship:

$$\Delta\tilde{\boldsymbol{\sigma}}_{n+1} = \mathbf{E}\cdot\Delta\boldsymbol{\varepsilon}_{e}^{n+1} = \mathbf{E}\cdot\left(\Delta\boldsymbol{\varepsilon}^{n+1} - \Delta\boldsymbol{\varepsilon}_{bc}^{n+1}\right) \qquad (16)$$

Finally, if one makes use of the Eq. (14), the effective stresses vector increment $\Delta\tilde{\boldsymbol{\sigma}}_{n+1}$ reads:

$$\Delta\tilde{\boldsymbol{\sigma}}_{n+1} = \mathbf{E}_{bc}\cdot\left(\Delta\boldsymbol{\varepsilon}^{n+1} - a_{bc}\boldsymbol{\varepsilon}_{bc}^{n} - \left(\mathbf{B}_{bc} + \mathbf{C}_{bc}\right)\cdot\tilde{\boldsymbol{\sigma}}_{n}\right) \qquad (17)$$

With

$$\mathbf{E}_{bc} = \left(\mathbf{1} + \mathbf{E}\cdot C_{bc}\right)^{-1}\cdot\mathbf{E} \qquad (18)$$

Then, the elastic strain, the equivalent tensile strain, the damage variable and finally the apparent stresses can be calculated (see § 2.2).

The algorithm is summarized in Figure 3.

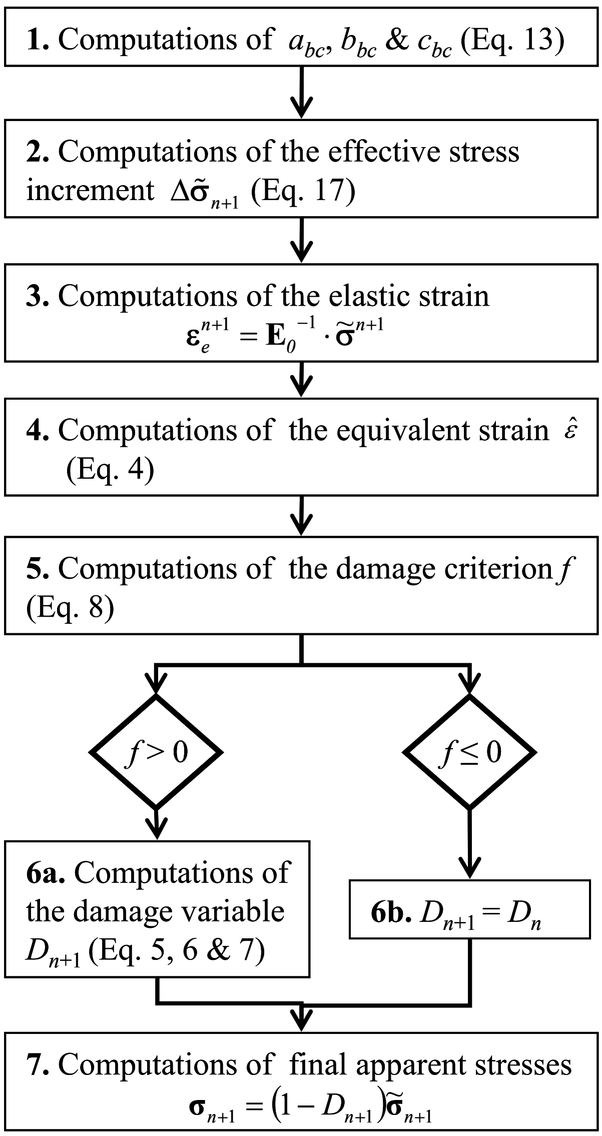

Figure 3. Local algorithm.

Therefore, the numerical implantation is straightforward. Indeed, all involved equations are defined analytically and can be calculated without any local iteratio.

3 NUMERICAL SIMULATIONS

The model presented in § 2 has been implemented in cast3m finite element code. Some numerical simulations are performed in order to validate the model in the case of uniform compressive stresses, as in bending.

Experimental results of Roll (1964, quoted by Mazzotti and Savoia, 2001) have been used. The specimens have been loaded at an age of 28 days in a normal ambiance (20°C and 60 % relative humidity). The compressive strength at 28 days is equal to 42 MPa.

3.1 *Uniaxial behavior in compression and tension under quasi-static loading*

Typical stress-strain curves in tension and compression have been used (see Fig. 4 and Fig. 5.

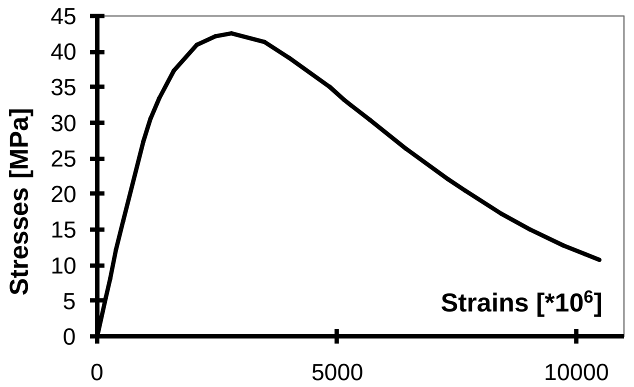

Figure 4. Stress – strain relationship in compression ($\beta = 0$, Eq. 4).

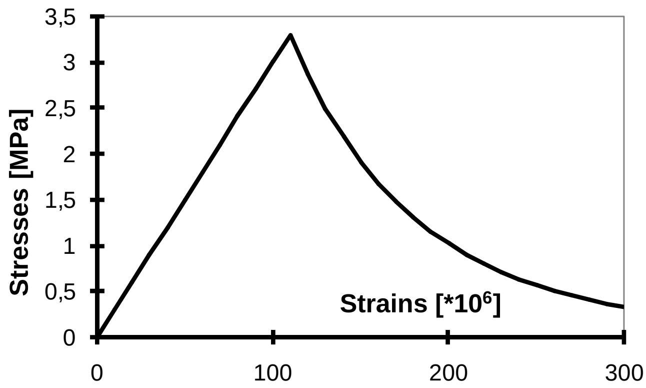

Figure 5. Stress – strain relationship in tension ($\beta = 0$, Eq. 4).

Mechanical parameters are given in Tab. 1.

Table 1. Values of the mechanical parameters used in the simulations.

a_c	b_c	κ_0	β	E [GPa]
12.5	920	1.1×10^{-4}	0.24	30
a_t	b_t			
-0.52	9.21×10^{3}			

3.2 *Creep behavior in compression*

3.2.1 *Identification of material parameters*

Basic creep parameters have been identified from experimental results for a stress to strength ratio equal to 0.2 (still in the linear visco-elasticity domain). Three Kelvin-Voigt chains are sufficient to cover the whole time range (13 to 210 days). The Levenberg-Marquardt algorithm has been used (Press 1994).

The identified parameters are given in Tab. 2.

Table 2. Values of the identified creep parameters.

k_1^{bc} [GPa]	k_2^{bc} [GPa]	k_3^{bc} [GPa]
40.2	24.5	53.4
τ_1^{bc} [days]	τ_2^{bc} [days]	τ_3^{bc} [days]
300	43	1

Next, delayed strains (defined as total strain minus initial strain) evolutions are simulated for different stress to strength ratios, without taking into account creep/cracking interaction (β = 0, Eq. 4). Results are plotted in Fig. 6.

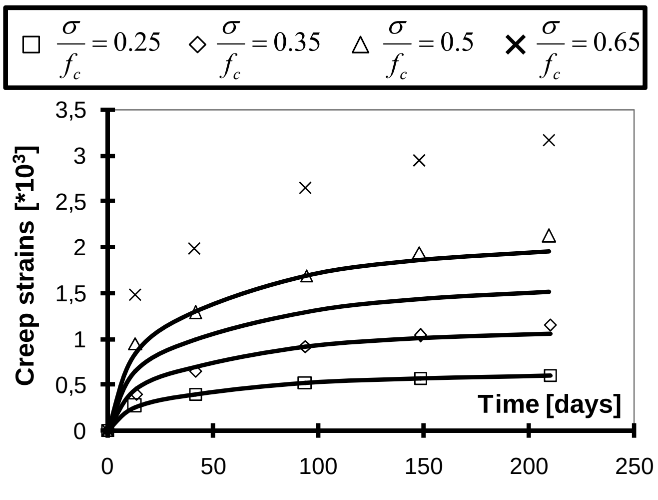

Figure 6. Delayed evolutions for different stress to strength ratios without taking into account creep/cracking interaction (β = 0, Eq. 4) – (experimental results from Roll, 1964).

We observed that delayed strains are underestimated, since creep/cracking interaction has not been taken into account. Experimental results stray from linear visco-elasticity for a stress to strength greater than 0.35. This has been previously simulated by Mazzotti and Savoia (2003).

Parameter β (Eq. 4), which renders creep/cracking interaction phenomenon, is thus identified in order to retrieve experimental results. Using again Levenberg-Marquardt algorithm (Press 1994), the best obtained value is β = 0.24. This value is slightly lower than the ones proposed by Mazzotti and Savoia (2003). This is probably due to the fact that we do not use a stress rate reduction factor.

A good agreement is obtained between experimental and numerical results (see Fig. 7). Such a good agreement is obtained by Mazzotti and Savoia (2003). However, our calculations highlight that:

- A stress rate reduction factor is not necessary to retrieve non linear delayed strains;
- The used algorithm is fast and easy to implement, since no local iteratios are needed;
- Modeling creep by Kelvin-Voigt elements allows for retrieving non linear creep strains. This is not consistent with previous calculations performed by Omar et al. (2003), which showed that only Maxwell elements are suitable. Nevertheless, one should be aware that the use of Kelvin-Voigt elements does not allow for retrieving irreversible creep strains at unloading. Therefore, the proposed model is rather relevant for constant and increasing stresses. (Torrenti and al. 2007)

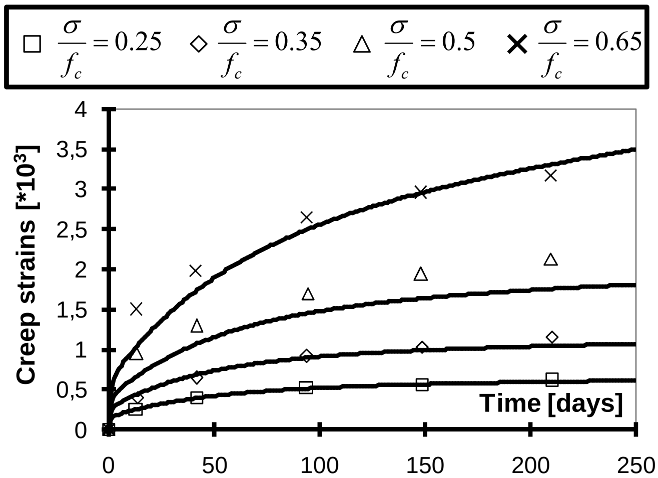

Figure 7. Delayed strains evolutions for different stress to strength ratios (β = 0.24, Eq. 4) – (experimental results from Roll, 1964).

3.2.2 *Advanced analysis*

Creep strains are also predicted for higher stress to strength ratios (see Fig. 8). The evolution of damage with respect to time is also plotted in Fig. 9.

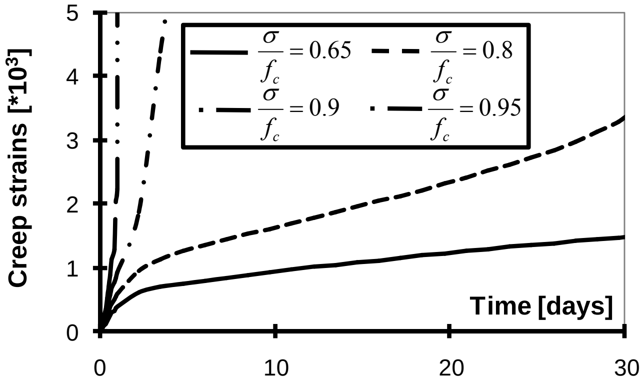

Figure 8. Delayed strains evolutions for different stress to strength ratios ($\beta = 0.24$, Eq. 4).

The model is therefore able to reproduce failure due to creep for very high stress to strength ratios within a finite time, which is in accordance to experimental results (e.g. Li, 1994).

However, due to the use of an elastic damage model, increasing of apparent Poisson ratio cannot be reproduced, when cracking occurs: Poisson ratio remains constant with respect to time and equal to the elastic one. This is one of the shortcomings of the proposed approach.

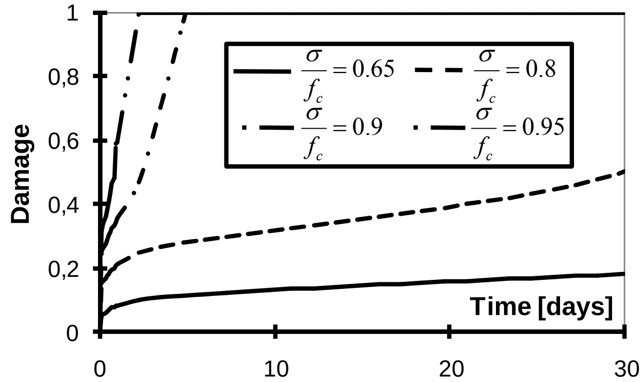

Figure 9. Damage evolutions with respect to time for different stress to strength ratios ($\beta = 0.24$, Eq. 4).

Delayed strain is defined as total strain minus initial strain. In fact, since damage evolves with respect to time, effective stresses increase. Therefore, elastic strain is also contributing to time dependent deformation. For a stress to strength ratio equal to 0.8, both evolutions of "real" creep (Eq. 15) and elastic strain are given in Fig. 10.

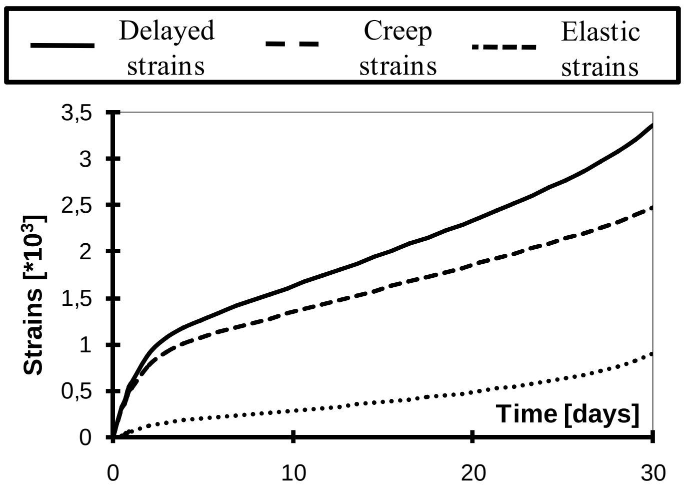

Figure 10. Contribution of "real" creep and elastic strains to delayed strains ($\beta = 0.24$, Eq. 4).

3.3 *Creep behavior in bending*

The same parameters as in §3.1 and §3.2 are used. It is also assumed that creep compliance is equal in compression and in tension, which is accordance with experimental evidences for mature concretes (Brooks and Neville, 1977).

Numerical simulations are performed on 3 points bending beams (b = 10 cm, D = 10 cm, L = 35 cm, l = 30 cm, a_0 = 1.5 cm). The geometry is reported on Figure 11.

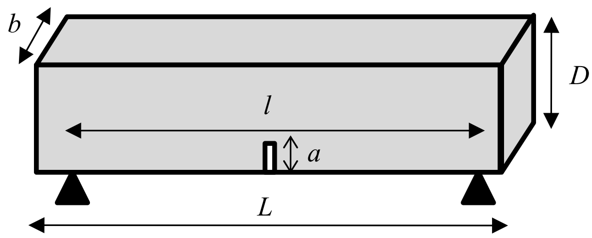

Figure 11. Geometry of the 3 points bending beam.

Numerical simulations are performed in stress plane conditions. The evolution of delayed deflection is plotted in Fig. 12 for different load to peak load ratios. It highlights that the proposed model is also suitable for the computations of non linear creep in bending (and therefore in tension). However, more simulations (and especially comparisons with experimental data) have to been carried out to validate the model.

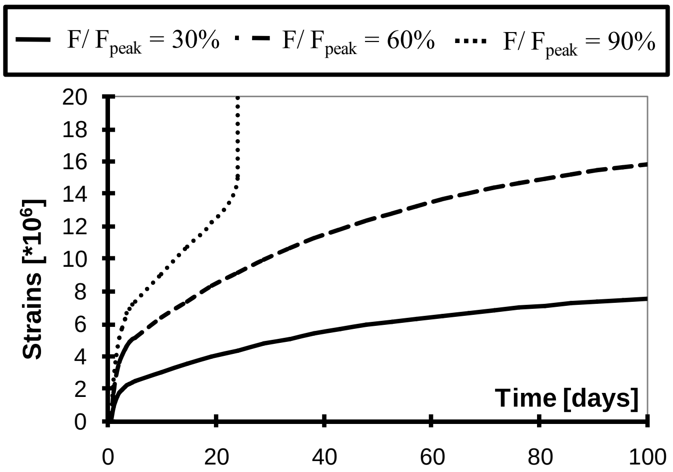

Figure 12. Delayed deflection with respect to time for different load to peak load ratios.

4 CONCLUSIONS

An available elastic damage model has been modified in order to take into account linear and non linear creep. It has been achieved by solving analytically linear creep laws (based on Kelvin-Voigt elements) during a time step by assuming a linear evolution of effective stresses. Therefore, the storage of stress history is not needed.

Coupling between cracking and creep is modeled by:

- Creep of un-cracked material is assumed to remain visco-elastic. Therefore, creep strains are governed by effective stresses;
- The Mazars criterion, expressed in term of strain, is modified by incorporating only a part of creep strain in the definition of tensile equivalent strain.

Therefore, no local algorithm is needed to compute damage and stresses. The model is therefore easy to implement in a finite element code and the computations are very fast (conditioned by the finite element code efficiency).

Numerical simulations have been performed in compression and bending. They show that:

- The proposed model is able to reproduce non linear creep as failure of concrete for high stress levels;
- Delayed strains are due not only to creep strains, but also to elastic strains since effective stresses are increasing during a creep test;

However, the model is not adapted if strong decreasing effective stresses are expected. Moreover, the increase of effective Poisson ratio before failure cannot be predicted accurately. Furthermore, concrete behavior becomes anisotropic after cracking. Therefore, an anisotropic extension has to be performed. More numerical simulations (especially a comparison with experimental data in bending) have to be performed.

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