Experimental tests vs. theoretical modeling for FRC in compression

F. Bencardino, L. Rizzuti, G. Spadea
Department of Structural Engineering, University of Calabria, Rende (CS), Italy

ABSTRACT: Several theoretical models and experimental data for the compressive behavior of Fiber Reinforced Concrete (FRC) are available in literature. In this paper the results of experimental compression tests on Steel Fiber Reinforced Concrete (SFRC) specimens carried out according to standard procedures are shown. The complete stress-strain relationship up to failure for each specimen was monitored and plotted. The different post peak curves of the tested SFRC specimens with fiber content of 1%, 1.6%, 3% were compared and the difference in their behavior was highlighted. Aiming to evaluate the reliability of the models available in literature a comparative study between experimental and theoretical stress-strain relationship was developed. For this purpose theoretical models and experimental data published from 1989 to 2006 were collected in a database and critically analyzed. Many of the theoretical models studied agree more with the experimental curves obtained from the same author rather than with the experimental data obtained from other authors. The validity ranges of the examined stress-strain relationship were critically overviewed.

1 INTRODUCTION

The addition of discontinuous fibers plays an important role in the improvement of mechanical properties of concrete: it increases elastic modulus, decreases brittleness, controls crack growth and crack propagation. Debonding and pull out of fibers require more energy absorption, with a substantial increase in toughness and resistance to cyclic and dynamic loads. In particular, the good performance of Steel Fiber Reinforced Concrete (SFRC) suggests the use of such material in many structural applications, with and without traditional internal reinforcement. Therefore the use of SFRC is particularly suitable for structures loaded over the serviceability limit state in bending, shear, impact forces, as it occurs under seismic or cyclic action. However there is still little knowledge on design/analysis of Fiber Reinforced Concrete (FRC) members. The analysis of structural sections requires, as a basic prerequisite, the definition of a suitable stress-strain relationship for each material to relate its behavior to the structural response. Many stress-strain relationships, in tension and in compression, for FRC materials have been proposed in literature by different authors. With reference to the compressive behaviour, experimental data and analytical models, published from 1989 to 2006, were collected in a database in the following sections. The behaviour of a composite material is influenced by the characteristics of each component and by their proportion in the mixture. In particular, when fibers are added in a concrete mix, their type, shape, aspect ratio (L_f/D_f) and volume content (V_f) play an important role. It should be mentioned that variations in specimen geometry, loading versus casting direction, loading rate, maximum aggregate size and fibers characteristics modify the compressive behavior of concrete. Extensive experimental data on standard tests are needed to refine a model to consider the effects of different factors. For this purpose only the experimental data carried out according to standard procedures were collected in the database, which, in addition, has been widened with the results obtained from a set of compressive strength tests on cube and cylinder specimens carried out at the University of Calabria.

2 ANALYTICAL MODELS

The addition of steel fibers in the concrete mix significantly affects the tensile and compressive behavior. Fibers contribute to resist crack growth after first crack and allow larger strains, up to concrete crushing. The considerable increase in toughness and ductility of fibrous composites implies that fibers perform an important confining action inside a loaded structural member. Reliable stress-strain relationships are available for plain concrete (Hognestad 1951, Sargin 1971, CEB-FIP 1993) while for FRC a lot of models have been proposed. For a design at ultimate limit states some guidelines (RILEM TC 162-TDF 2003, CNR-DT 204 2006) proposed the same shape of stress-strain relationship in compression used for ordinary concrete (parabolic-rectangular) with an ultimate compressive strain of 0.0035 and a strain at the peak stress of 0.002. The compressive strength of SFRC should be determined by means of standard tests, either on concrete cylinders or concrete cubes. The design principles are based on the characteristic compressive strength at 28 days. Aiming to calculate the actual flexural strength of tested SFRC elements other authors (Lok & Pei 1998, Lok & Xiao 1999) have proposed a similar stress-strain curve used for plain concrete. In evaluating their experimental data some authors (Soroushian & Lee 1989, Ezeldin & Balaguru 1992, Barros & Figueiras 1999, Mansur et al. 1999, Nataraja et al. 1999), have proposed different analytical equations to reproduce the actual behaviour of FRC material in compression. These analytical models are showed below.

2.1 *Soroushian & Lee (1989)*

The model proposed by Soroushian & Lee (1989) consists of a curvilinear ascending portion followed by a bi-linear descending branch (Eq. 1.a - 1.b).

$$\sigma = -f_{cf}\left(\frac{\varepsilon}{\varepsilon_{pf}}\right)^2 + 2f_{cf}\left(\frac{\varepsilon}{\varepsilon_{pf}}\right) \quad \text{for } \varepsilon \le \varepsilon_{pf} \tag{1.a}$$

$$\sigma = z\left(\varepsilon - \varepsilon_{pf}\right) + f_{cf} \ge f_o \qquad \text{for } \varepsilon > \varepsilon_{pf} \tag{1.b}$$

In this model the stress and the strain at the peak (f_{cf}, ε_{pf}), the residual stress (f_o) and the slope of the descending branch (z) were evaluated empirically as functions of the matrix compressive strength and fiber reinforcement index (I_f=V_f(L_f/D_f)). The compressive strength of fiber concrete (f_{cf}), the strain corresponding to the peak stress (ε_{pf}), are evaluated by adding an additional factor to the matrix strength (f_c), and to the fixed peak strain, ε_{co}=0.0021. This additional factor is obtained by the fiber reinforcement ratio amplified by another constant value. An empirical equation for the different variables of this proposed model, defined using the least square curve fitting the experimental results obtained by the same authors, is shown below:

$$f_{cf} = f_c + 3.6I_f \tag{2}$$

$$f_o = 0.12f_{cf} + 14.8I_f \tag{3}$$

$$z = -343f_c\left(1 - 0.66\sqrt{I_f}\right) \le 0 \tag{4}$$

$$\varepsilon_{pf} = 0.0007I_f + 0.0021 \tag{5}$$

In this model the residual ultimate strain of SFRC was not fixed.

2.2 *Ezeldin & Balaguru (1992)*

Ezeldin & Balaguru (1992) proposed an analytical equation (Eq. 6) to generate the stress-strain curve for normal strength SFRC based on the equation proposed by Carreira & Chu (1985) for uniaxial compression of plain concrete. This equation involves a material parameter β, which is the slope of the inflection point at the descending segment.

$$\frac{\sigma}{f_{cf}} = \frac{\beta\left(\dfrac{\varepsilon}{\varepsilon_{pf}}\right)}{\beta - 1 + \left(\dfrac{\varepsilon}{\varepsilon_{pf}}\right)^{\beta}} \tag{6}$$

In order to quantify the effect of fibers on the compressive behavior of FRC, a least square fitting analysis was performed to establish a connection between the reinforcement index by weight of hooked end fibers (RI=W_f(L_f/D_f)) and the main parameters of the stress-strain curve, namely; the compressive strength (f_{cf}) and the corresponding peak strain (ε_{pf}). Also in this equation f_{cf} and ε_{pf} are calculated by adding an additional factor, linked to fiber property, to the strength and the strain of plain concrete, f_c and ε_{co}, respectively. The following equations were obtained by using the regression analysis performed using experimental data of the same authors.

$$f_{cf} = f_c + 3.51(RI) \tag{7}$$

$$\varepsilon_{pf} = \varepsilon_{co} + 446\times10^{-6}(RI) \tag{8}$$

For hooked end fibers:

$$\beta = 1.093 + 0.7132(RI)^{-0.926} \tag{9}$$

Using the experimental results of Fanella & Naaman (1985) for mortar reinforced with straight fibers, the following equation was proposed:

$$\beta = 1.093 + 7.4818(ri)^{-1.387} \tag{10}$$

Where (ri) is the reinforcement index, by weight, of straight fibers.

Without specific experimental data the authors suggest to use ε_{co}=0.002, according to the International Recommendations (1970).

The equations proposed to evaluate β can be used for reinforcing index values ranging from 0.75 to 2.5 for hooked end fibers and from 2 to 5 for straight fibers. The ultimate strain is not fixed.

2.3 *Barros & Figueiras (1999)*

Based on their experimental results and following the procedure proposed by Mebarkia & Vipulanandan (1992), Barros & Figueiras (1999) proposed the following compression stress-strain relationship.

$$\sigma = f_{cf} \frac{\dfrac{\varepsilon}{\varepsilon_{pf}}}{(1-p-q)+q\left(\dfrac{\varepsilon}{\varepsilon_{pf}}\right)+p\left(\dfrac{\varepsilon}{\varepsilon_{pf}}\right)^{(1-q)/p}} \tag{11}$$

with

$$q = 1 - p - \frac{E_{pf}}{E_c} \tag{12}$$

$$p + q \in \left]0,1\right[, \qquad \frac{1-q}{p} > 0 \tag{13}$$

$$E_{pf} = \frac{f_{cf}}{\varepsilon_{pf}}; \qquad E_c = 21{,}500\sqrt[3]{\frac{f_{cf}}{10}} \tag{14}$$

Where f_{cf} is the average compression strength, experimentally evaluated. The following equations were obtained by the authors by applying the least square method.

For hooked end fibers (L_f=30mm, D_f=0.5mm, L_f/D_f=60):

$$\varepsilon_{pf} = \varepsilon_{co} + 0.0002W_f \tag{15}$$

$$p = 1 - 0.919e^{-0.394W_f} \tag{16}$$

For hooked end fibers (L_f=60mm, D_f=0.8mm, L_f/D_f=75):

$$\varepsilon_{pf} = \varepsilon_{co} + 0.00026W_f \tag{17}$$

$$p = 1 - 0.722e^{-0.144W_f} \tag{18}$$

Where, ε_{co} is the strain at peak for plain concrete. Without specific experimental data the authors suggest to use ε_{co}=0.0022, according to CEB-FIP (1993). W_f is the fiber weight percentage in the mixture.

The values of ε_{pf} and p can be obtained from the equation proposed, for f_{cf} values ranging from 30 to 60 MPa and for concrete reinforced with similar content of fibers used by the authors.

The value of ultimate strain ε_{fu} is not fixed.

2.4 *Nataraja, Dhang, Gupta (1999)*

Nataraja et al. (1999) proposed an analytical equation similar to that of Ezeldin & Balaguru (1992) (Eq. 6) but they used their experimental data by providing other additional factors, related to the fiber reinforcement index, used to determine the strength, the strain corresponding to the peak and the β value.

$$f_{cf} = f_c + 2.1604(RI) \tag{19}$$

$$\varepsilon_{pf} = \varepsilon_{co} + 0.0006(RI) \tag{20}$$

$$\beta = 0.5811 + 1.93(RI)^{-0.7406} \tag{21}$$

According to the authors, without specific experimental data, ε_{co} can assume a value equal to 0.002. The proposed equations can be used for concrete with strength up to 50MPa, reinforced with crimped fibers with a reinforcing index value ranging from 0.9 to 2.7. The value of ultimate strain ε_{fu} is not fixed.

3 EXPERIMENTAL DATA

Experimental data available in literature were analysed but only the data of the experimental analysis carried out following standard procedure were collected in the present database. For each author the main mechanical parameters and the details of the cylindrical tested specimens (D: diameter; H: height; N.: number of specimens) are given in Table 1.

The authors carried out experimental tests on FRC with maximum aggregate size of: 9mm (Ezeldin & Balaguru 1992), 10mm (Dwarakanath & Nagaraj 1991, Wafa & Ashour 1992), 15mm (Barros & Figueiras 1999), 19mm (Mansur et al. 1999), 20mm (Nataraja et al. 1999), 25mm (Jo et al. 2001).

4 EXPERIMENTAL PROGRAM

The experimental investigation was carried out on 24 specimens, cubes and cylinders, considering plain concrete and SFRC with 1%, 1.6% and 3% of fiber content. Compressive strengths were evaluated and, using a suitable experimental set up, stress-strain curves on cylindrical specimens were also recorded to highlight the role of the fibers in the post peak response.

4.1 *Details of materials*

The following materials were used: type I Portland cement, crushed coarse aggregate, spherical quartz, water, condensed silica fume and super plasticizer. Maximum size of the coarse aggregate was 15mm. The steel fibers used in this investigation were hooked end, with a tensile strength of 350-400MPa, a length of 22mm, a diameter of 0.5mm and an aspect ratio of 40.

Table 1. Experimental database.

Authors	Standard adopted	Specimens		Steel fibers type	V_f	L_f/D_f	ε_{pf}	ε_{fu}	f_{cf}	f_{fu}
		D, H mm	N.		(%)				(MPa)	(MPa)
Soroushian & Lee	-	150, 300	-	Straight	-	-	0.0033	0.0067	41.2	5.2
					2.00	47	0.0033	0.0116	40.7	23.3
					2.00	83	0.0050	0.0116	43.2	31.2
					2.00	100	0.0050	0.0116	44.6	39.5
Dwarakanath & Nagaraj	-	100, 200	-	Straight	-	-	0.0020	0.0044	24.5	17.6
					1.00	72	0.0021	0.0060	26.0	19.4
					2.00	72	0.0025	0.0070	26.7	19.7
					3.00	72	0.0030	0.0072	31.6	26.5
Ezeldin & Balaguru	ASTM C39	100, 200	36	Hooked end	-	-	0.0022	0.0104	35.9	8.2
					0.38	60	0.0025	0.0177	40.7	7.4
					0.57	60	0.0025	0.0200	40.7	9.6
					0.76	60	0.0031	0.0400	37.9	13.5
Wafa & Ashour	ASTM C39, C31, C192	150, 300	126	Hooked end	-	-	0.0020	0.0027	91.5	80.0
					0.5	75	0.0024	0.0107	94.6	18.3
					1.0	75	0.0018	0.0107	95.6	39.5
					1.5	75	0.0018	0.0107	100.0	57.5
Barros & Figueiras	JSCE SF5	150, 300	32	Hooked end	0.76	75	0.0022	0.0400	38.2	1.8
Mansur et al.	-	100, 200	54	Hooked end	-	-	0.0024	0.0045	103.6	27.0
					0.50	60	0.0026	0.0140	104.7	23.5
					1.00	60	0.0027	0.0140	107.0	39.0
					1.50	60	0.0029	0.0140	103.5	40.0
Nataraja et al.	ASTM C39	150, 300	14	Crimped	-	-	0.0027		43.0	
					0.50	55	0.0031	0.0155	45.8	13.8
					0.75	55	0.0033	0.0167	41.6	17.4
					1.00	55	0.0034	0.0170	47.0	21.2
Jo et al.	JCI SF2	150, 300	75	Hooked end	-	-	0.0021	0.0060	41.8	1.0
					0.50	75	0.0026	0.0050	41.8	5.4
					0.75	75	0.0020	0.0100	37.7	11.6
					1.00	75	0.0020	0.0090	34.3	17.7
					1.50	75	0.0023	0.0100	33.1	25.0
					-	-	0.0021	0.0052	61.4	3.8
					0.50	75	0.0021	0.0050	60.0	5.4
					0.75	75	0.0024	0.0043	60.1	30.0
					1.00	75	0.0023	0.0100	53.8	9.2
					1.50	75	0.0021	0.0100	60.9	3.2
					-	-	0.0020	0.0080	64.6	1.3
					0.50	75	0.0022	0.0080	70.0	1.2
					0.75	75	0.0022	0.0100	71.4	0.0
					1.00	75	0.0022	0.0091	71.0	1.1
					1.50	75	0.0027	0.0075	62.8	3.4

4.2 *Mix design and casting of specimens*

Table 2 shows, the plain concrete (PC) and SFRC (S1%, S1.6% and S3%) mix design for 1m^3 of concrete batch used in the experimental program.

The specimens were mixed, casted and cured in a framework. The concrete was compacted on a vibrating table. All specimens were removed from the moulds within 24hr and cured for 27 more days under water satured sand.

4.3 *Test details*

The compressive strength tests at 28 days were carried out, according to UNI EN 12390-3, using 150mmx150mmx150mm cubes and 150mmx300mm cylinders loaded uniaxially. A Zwick/Roell servo hydraulic closed-loop test machine with a 3000kN capacity was used. Loadings were increased at a rate of 0.05mm/min.

Table 2. Mix design.

Material		PC	SFRC		
			S1%	S1.6%	S3%
Cement	42.5 R	500	500	500	500
Quartz	0/2 mm	377	377	377	377
	3/6 mm	273	273	273	273
Coarse aggregate	0/5 mm	693	615	567	458
	5/10 mm	290	290	290	290
	10/15 mm	317	317	317	317
Fiber	V_f	0%	1%	1.6%	3%
	(kg)	-	78	126	235
Silica fume	%	6%	6%	6%	6%
	(kg)	30	30	30	30
Super plasticizer	%	1.5%	1.5%	1.5%	1.5%
	(kg)	7.5	7.5	7.5	7.5
Water	w/c	0.35	0.35	0.35	0.35
	(l)	175	175	175	175

For cylindrical specimens three HBM LVDTs with a gage length of 20mm, were mounted at 120-degree intervals along two circumferential ties placed on the specimen at a base length of 100mm. These aluminum ties were able to support the measuring devices and did not confine specimens. Figure 1 shows the position of the measure instrumentation. The data acquisition and signal control were carried out using a HBM Spider 8 control unit.

Figure 1. Experimental set-up.

4.4 *Test Results and discussion*

All test results are summarized in Tables 3 and 4. In these tables the compressive strength of each specimen and for every set of specimens the mean value of the compressive strength, the standard deviation and the coefficient of variation are given. The graphical representations of the stress-strain curves are given in Figure 2. The experimental typical curves of the tested plain concrete and SFRC specimens are compared and showed in Figure 3.

Table 3. Cubic compressive strength.

	R_c	R_m	s	δ
	MPa	MPa	MPa	%
PC_k1	65.5			
PC_k2	57.9	61.5	3.8	6.2
PC_k3	61.1			
S1%_k1	69.8			
S1%_k2	79.0	74.2	4.6	6.2
S1%_k3	73.6			
S1.6%_k1	62.8			
S1.6%_k2	54.3	60.5	5.4	8.9
S1.6%_k3	64.5			
S3%_k1	64.4			
S3%_k2	62.8	62.8	1.6	2.5
S3%_k3	61.2			

Table 4. Cylindrical compressive strength.

	f_c	f_m	s	δ
	MPa	MPa	MPa	%
PC_c1	64.1			
PC_c2	66.3	66.7	2.8	4.2
PC_c3	69.8			
S1%_c1	67.2			
S1%_c2	71.9	69.6	2.4	3.4
S1%_c3	69.8			
S1.6%_c1	61.1			
S1.6%_c2	55.7	58.1	2.7	4.6
S1.6%_c3	57.7			
S3%_c1	59.8			
S3%_c2	54.2	58.5	3.8	6.5
S3%_c3	61.5			

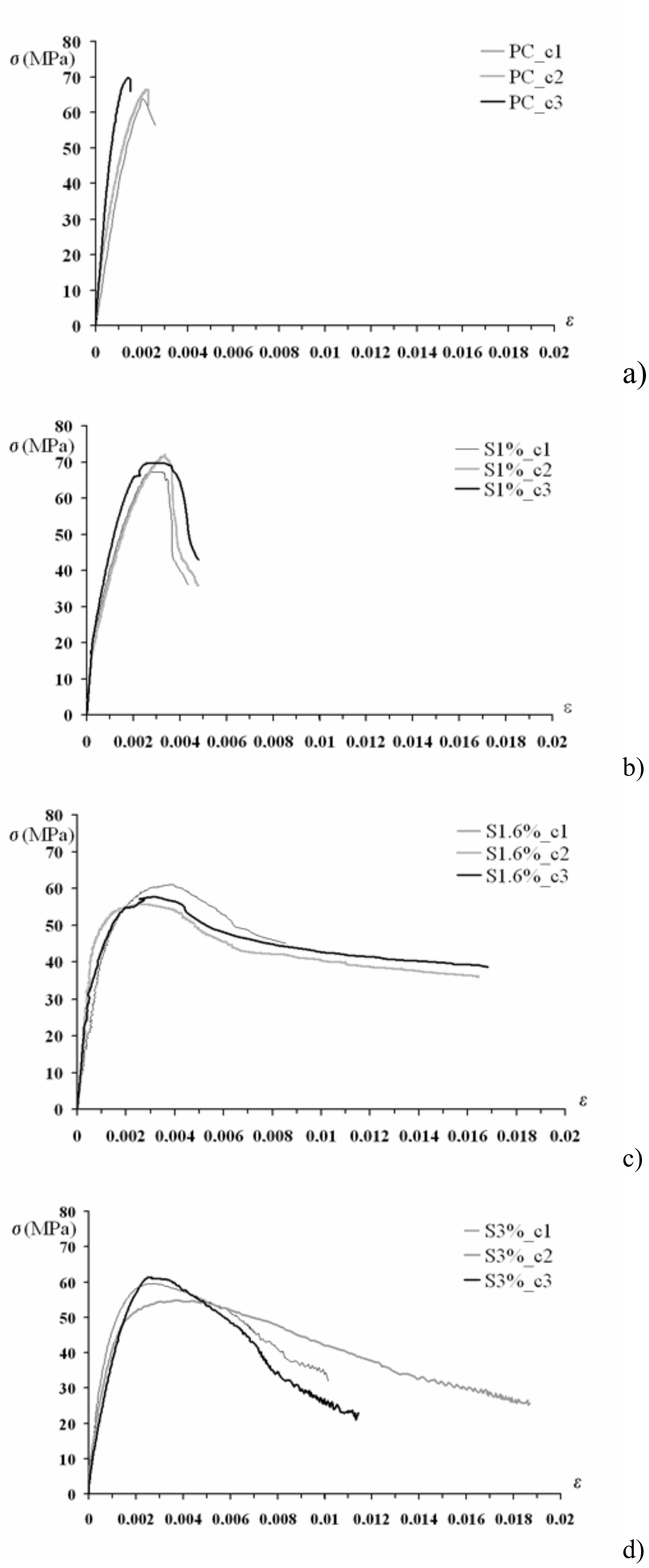


Figure 2. Stress-strain curves of cylindrical specimens: a) plain concrete, SFRC b) V_f=1%, c) V_f=1.6%, d) V_f=3%.

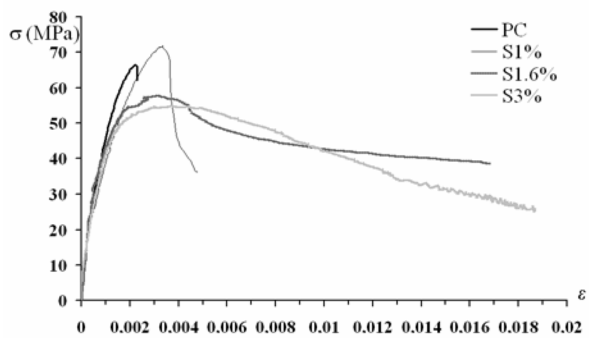


Figure 3. Typical stress-strain curves without and with fibers.

Tests results show that the addition of fibers, compared to plain concrete, slightly affects the compressive strength value but more of it influences the post-peak response (Fig. 3). Concrete reinforced with medium content of steel fibers (V_f=1%) shows a slight improvement in the descending or softening branch compared to plain concrete while concrete reinforced with higher content of fibers (V_f=1.6%, 3%) shows a more extended softening branch. The presence of fiber modified the failure mode of the concrete cylinder from a brittle to a less brittle failure mode. The post-test aspects of the specimens showed, in the case of plain concrete, either a single shear plane or a cone-type failure. By contrast, SFRC specimens showed a large number of longitudinal cracks near the failure zone, which were oriented in the direction parallel or sub-parallel to the external compressive stresses.

5 EXPERIMENTAL TESTS V/S THEORETICAL MODELING

5.1 *Theoretical models: critical review*

With reference to fixed mechanical parameters, according to the studied models by increasing fiber content, the FRC compressive strength and the strain at the peak stress increase.

Example:

Plain concrete: f_c=30MPa, ε_{co}=0.002.

Steel fibers: L_f=30mm, D_f=0.5mm, L_f/D_f=60.

– *Soroushian & Lee (1989)*

V_f=1%, f_{cf}=32.2MPa (+7.3%), ε_{pf}=0.0025 (+25%)

V_f=2%, f_{cf}=34.3MPa (+14.3%), ε_{pf}=0.0029 (+45%)

V_f=3%, f_{cf}=36.5MPa (+21.7%), ε_{pf}=0.0034 (+70%)

– *Ezeldin & Balaguru (1992)*

V_f=1%, f_{cf}=36.3MPa (+21%), ε_{pf}=0.0028 (+40%)

V_f=2%, f_{cf}=42.6MPa (+42%), ε_{pf}=0.0036 (+80%)

V_f=3%, f_{cf}=48.9MPa (+63%), ε_{pf}=0.0044 (+120%)

– *Nataraja et al. (1999)*

V_f=1%, f_{cf}=34.1MPa (+13.7%), ε_{pf}=0.0032 (+60%)

V_f=2%, f_{cf}=38.3MPa (+27.7%), ε_{pf}=0.0043 (+115%)

V_f=3%, f_{cf}=42.4MPa (+41.3%), ε_{pf}=0.0055 (+175%)

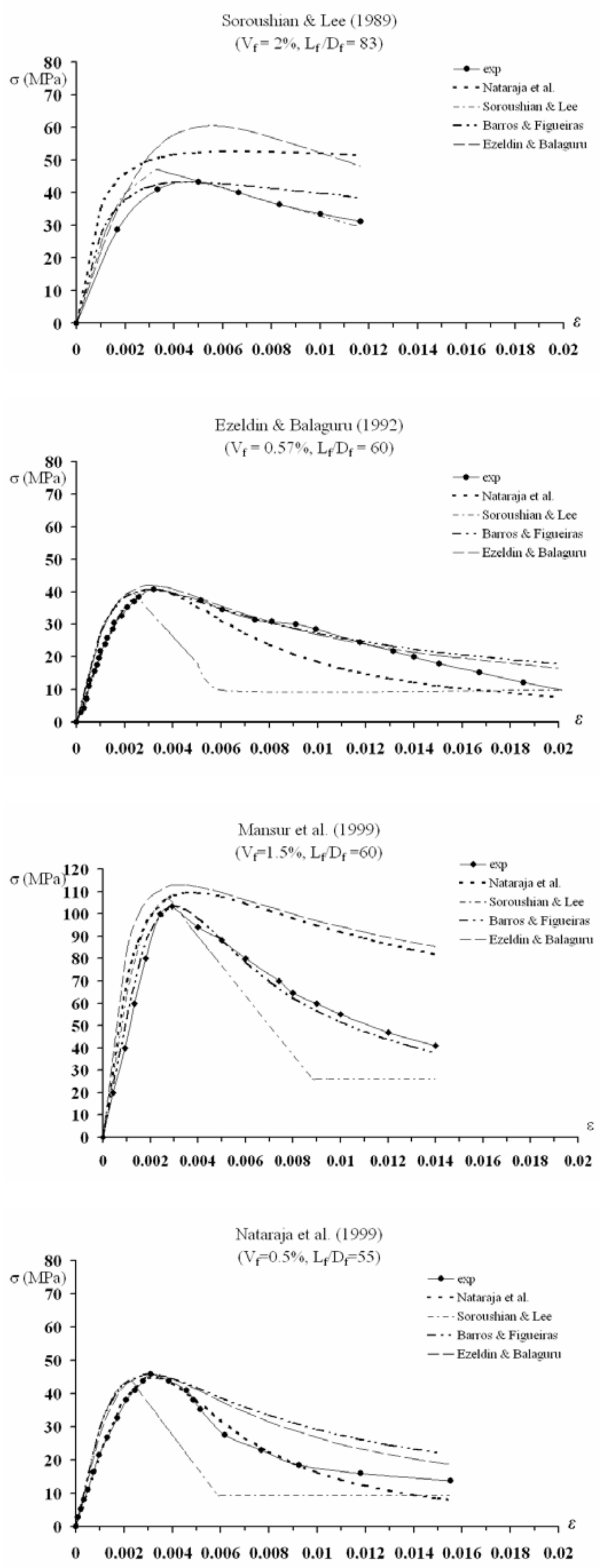
The experimental data collected show that the FRC compressive strength does not increase significantly compared to the strength of plain concrete.

5.2 *Cross comparison*

A cross comparison between the experimental curve and those obtained by using the theoretical models analysed was carried out. This comparison highlights that every model agrees well with their respective experimental data (Fig. 4) and less well with the experimental data obtained by other authors, because each proposed equation was obtained by using a regression analysis to interpolate their own experimental data. The behavior of a concrete reinforced with a medium content of fibers (V_f=1-2%) is better performed by the model proposed by Soroushian & Lee (1989). According to this model in the case of low content of fibers added into the concrete matrix (V_f<1%), the post-peak response is underestimated. With reference to concrete reinforced with medium content of fiber (V_f>1%) the models proposed by Ezeldin & Balaguru (1992) and by Nataraja et al. (1999), predict a stiffened ascending branch and overestimate the descending branch. These authors worked out an evaluation for the experimental data of SFRC with low content of fibers. The descending branch of these stress-strain relationships is affected by the β value. As the value of β decreases the experimental softening effect is not modelled. The stress-strain relationship proposed by Barros & Figueiras (1999) agrees with the experimental data of Ezeldin & Balaguru (1992) and of Mansur et al. (1999) because the authors also used these experimental data to define their model. In this proposed model the average compression strength of FRC was used while the other models calculated the compressive strength of SFRC and the strain corresponding to the peak stress adding an empirical factor to the values of matrix stress and strain.

6 CONCLUSIONS

- The experimental curves obtained from this investigation show that the ultimate strain of SFRC reaches higher values than 0.0035, usually adopted in current guidelines.
- The addition of fibers does not affect significantly the compressive strength of concrete; with the increase of fiber content a more extended softening branch is observed.
- SFRC specimens with fiber content of 1.6% and 3% show, at 0.01 strain, a residual stress of about 70% and 50% of the peak stress, respectively, and both reach an ultimate strain three times greater than the ultimate strain fixed by current guidelines.
- The analytical equations proposed by many authors are not rational but generally empirical; therefore they agree more with their own experimental data rather than the experimental data of other authors. For this reason each model cannot be assumed as universally valid.



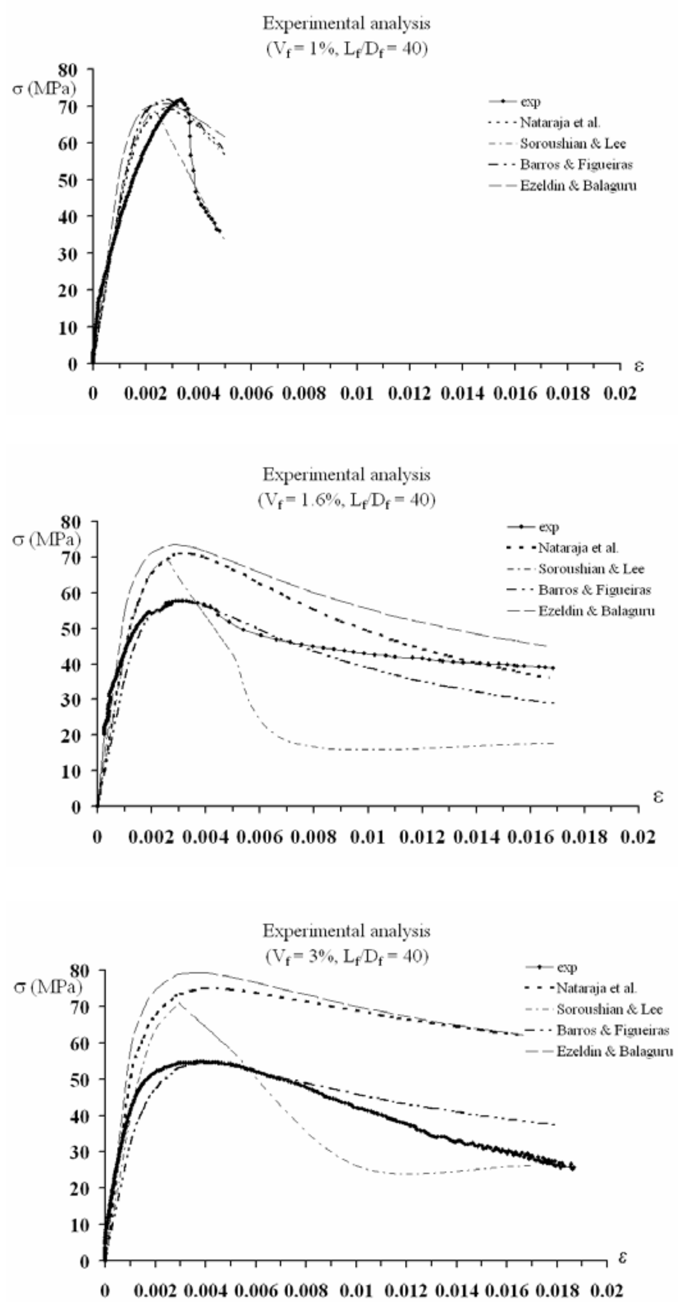


Figure 4. Comparisons: experimental v/s theoretical curves.

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