

# Stress-strain model for compressive fracture of RC columns confined with CFRP jackets

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**ABSTRACT:** Catastrophic earthquakes with quite short periods often occur in Turkey. This fact necessitates and paves the way to the preparations of earthquake scenarios of the existing building stock including structures not properly constructed against earthquakes with loss estimations. There are so many buildings in densely populated areas of Turkey and many other earthquake prone countries needing urgent rehabilitation and strengthening. The use of external FRP jackets can provide confinement for RC columns in a fast and efficient way and improve their performance significantly during an earthquake. The noticeable increase in the compressive strength and ductility of concrete is the main reason for the use of FRP jackets in the seismic retrofitting. For that reason, several researchers have proposed various stress-strain models for describing the behavior of concrete confined with FRP jackets. It is worth mentioning that most of the existing models have been developed to improve or modify the analytical result of Richart's tests on concrete cylinders subjected to fluid pressure. However, in these models, confined compressive strength is predicted without a failure criterion of concrete under multi-axial stress state with the exception of the ones adopting Mander model developed for concrete confined with transverse steel reinforcement. As a preliminary study, the stress-strain relations of some of the experimentally tested RC columns confined with CFRP jackets are successfully predicted adopting Köksal model introducing a new failure criterion of concrete under triaxial compression. This model had been previously developed for RC columns confined with conventional reinforcement of steel.

**Keywords:** Compressive fracture, confinement, reinforced concrete column, FRP jackets, stress-strain relation, failure criterion

## 1 INTRODUCTION

The compressive strength of RC columns increases with increasing confining pressure due to the use of several confinement mechanisms such as stirrups, spirals, FRP composite wraps, steel jackets, etc. The axial behavior of confined concrete was primarily researched by Richart et al. (1928) and the following relation was proposed for the ultimate strength of confined concrete based on the test results:

$$f'_{cc} = f_{co} + k_1 f_l \quad (1)$$

where  $f'_{cc}$  is the compressive strength of confined concrete,  $f_{co}$  is the unconfined compressive strength, and  $f_l$  is the effective lateral confining stress. In the recent literature, there are various models for describing the behavior of confined concrete with FRP composite jackets proposed by Saadatmanesh et al. (1994), Karbhari & Gao (1997), Restapol & De Vino (1996), Mirmaran & Shahawy (1998), Samaan et

al. (1998), Spoelstra & Monti (1999), Saafi et al. (1999), Toutanji (1999), Xiao and Wu (2000), Lam & Teng (2002), İlki et al. (2004), and Yan & Pantelides (2006). Most of these models adopt the approach of Richardt et al. (1928) and recommend several different expressions for  $k_1$  in Eq (1). Some researchers (Saadatmanesh et al. 1994, Restapol & De Vino 1996, Spoelstra & Monti 1999) have recommended expressions for the prediction of the compressive strength of FRP confined concrete elements, similar to the relation proposed by Mander et al. (1988) adopting William-Warnke failure surface (1975) for tri-axial compression state of circular RC columns with equal effective lateral confining stresses:

$$\frac{f'_{cc}}{f_{co}} = 2.254 \sqrt{1 + 7.94 \frac{f_l}{f_{co}}} - 2 \frac{f_l}{f_{co}} - 1.254 \quad (2)$$

Based on tests of FRP confined concrete compression members, Yan & Pantelides (2006) proposed two similar equations separately for hardening and softening behavior, adjusting the failure criterion recommended by William & Warnke (1975).

In this paper, the analytical stress-strain model recommended by Köksal (2006), primarily developed for the prediction of the behavior of RC columns, has been employed for the RC columns confined with FRP composite jackets. The stress-strain curves obtained using the existing models (Saadatmanesh et al. 1994, Samaan et al. 1998, Saafi et al. 1999, Mander et al. 1988) and proposed model (Köksal 2006) are compared with large scale experiments carried out by the first author (Turgay 2007) on four RC columns of square cross-section confined with CFRP jackets.

## 2 EXPERIMENTAL WORK

The square RC columns in the test program having a cross-section of 200x200 mm and 1030 mm nominal overall height, have been tested in the structural laboratory of Yıldız Technical University, as a part of the Ph.D. study of the first author about RC columns confined with CFRP jackets (Turgay 2007). A scheme of the test setup and instrumentation is shown in Figure 1.

Large-scale RC columns have been subjected to monotonic uniaxial compression loading up to the failure. The preparation stages of specimens are consisting of making transverse steel confined square RC columns and hand-apply wrapping the CFRP around these columns. Material properties of CFRP jacket are presented in Table 1.

Table 1. Material properties of CFRP sheets

FRP Composite	Tensile Strength (MPa)	Tensile Modulus (MPa)	Maximum Tensile Strain (mm/mm)	Ply Thickness (mm)
CFRP	3430	230000	0.015	0.165

There is only one type of concrete mix for all test columns. C1, C2, and C3/C5 type columns are tested at the age of 30 days, 60 days and 90 days. The cylindrical compressive strength of the concrete mix of C3/C5 type was 19.36 MPa. All longitudinal bars are 10mm in diameter, and the total number of bars

is illustrated by L4 and L8. The tie spacing is 100mm, and S8 and S12 denote stirrups with 8mm and 12mm in diameter, respectively. The yield strength of reinforcing steel is 422 MPa. In this paper, the test results of four C3 type-columns confined with one layer of CFRP are presented.

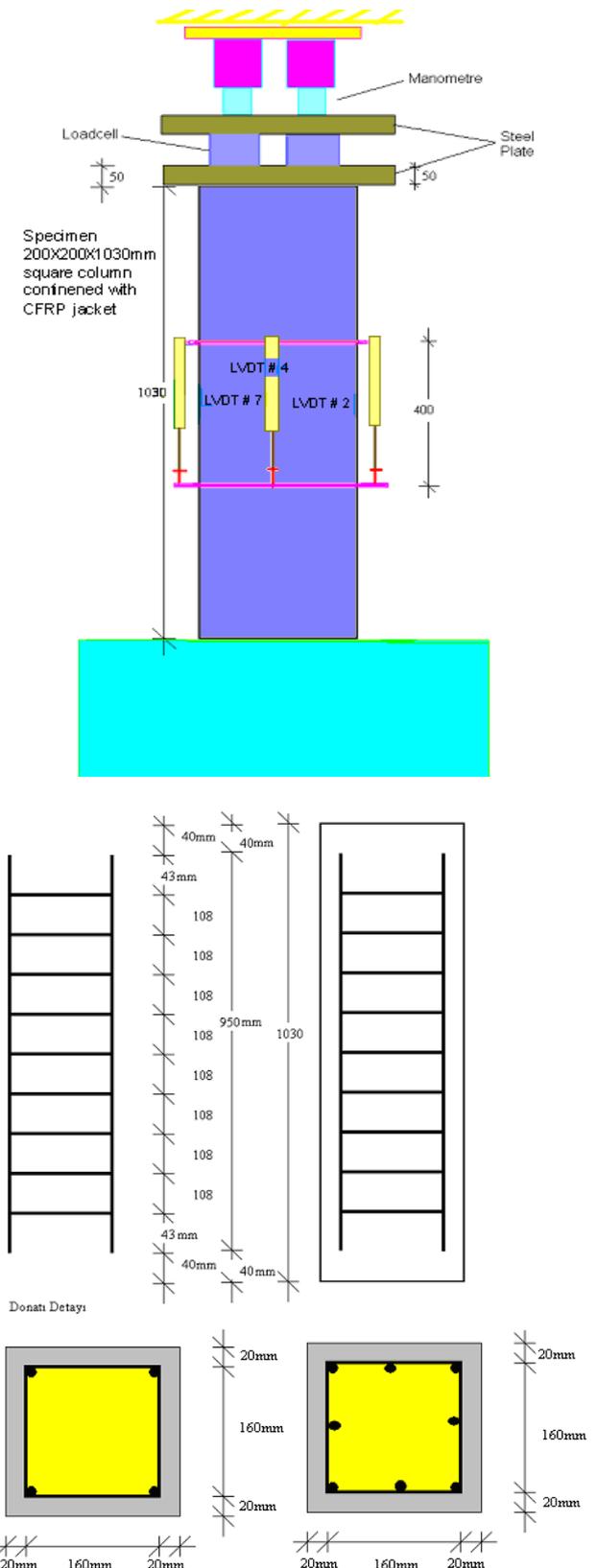


Figure 1. Test setup and the details of test specimens.

For measurement of axial strains, four linear variable displacement transducers (LVDTs) are mounted on the central 400 mm gage length at each face of a column in a similar way used to eliminate any eccentricity of the applied load as recommended in the study of Shrive et al. (2003). A pre-loading up to the one-fourth of the predicted axial capacity is applied to maintain approximately same LVDT readings so that any eccentricity can be eliminated in the linear elastic stage of the overall behavior.

Although the LVDT placing and readings are provided very close to each other, next to the maximum axial load there can be a significant variation between the minimum and maximum values of shortening (Karakoç et al. 2007). Tests are conducted using 2000-kN compression machine. The load is applied to the specimens at an approximately constant rate until the complete failure of columns and is monitored using a load cell. All measured data were collected by a data logger and stored directly in the computer.

### 3 CONSTITUTIVE MODELS FOR CONFINED CONCRETE

The model proposed by Köksal (2006), is used for the description of the behavior of RC columns confined with CFRP composite jackets. Taking the sign of the compressive stresses as positive because of the dominance of the compressive stresses for the case of RC columns confined with conventional steel reinforcement, and modifying the linear relation of DP criterion, the following failure surface is introduced:

$$f = \sqrt{6} \alpha(\xi)\xi + \rho - \sqrt{2} k(\sigma_1, \sigma_2, \sigma_3) = 0 \quad (3)$$

where  $\rho$  and  $\xi$  are deviatoric and hydrostatic lengths, respectively (Köksal 2006).  $k$  is considered as a function of the lateral confinement pressure and the cylinder compressive strength of concrete,  $f_c'$ :

$$k = k(f_l, f_c') \quad (4)$$

where  $f_l$  is the average of the two principal stresses acting in two orthogonal directions to the cross-section of RC column:

$$f_l = \frac{\sigma_2 + \sigma_3}{2} \quad \text{if } \sigma_1 > \sigma_2 > \sigma_3 \quad (5)$$

and the following form is finally proposed for  $k$  using the test results of Richardt et al. (1928):

$$k = \left( 4.07 \frac{f_l}{f_c'} - 0.89 \left( \frac{f_l}{f_c'} \right)^2 + 0.807 \right) f_c' \quad (6)$$

Material parameter  $\alpha$  in Equation 3 is also given in terms of the hydrostatic length:

$$\alpha = \alpha(\xi) = 0.462 \xi^{-0.2355} \quad \text{if } \frac{\xi}{f_c'} \geq 0.58 \quad (7)$$

Since the ultimate hydrostatic pressure is generally between 20 MPa and 60 MPa during the tests of RC columns,  $\xi/f_c' \geq 0.58$  draws a reasonable lower limit for the confined concrete strength under compression implying that the mean compression stress  $\sigma_m = f_c'/3$ . The geometry of the square and rectangular columns does not allow a uniform distribution of lateral pressure. Therefore,  $k$  in Equation 6 is simply reduced multiplying by 0.85 in order to reflect this fact.

In this paper, the Saenz's equation (1964) is adopted for describing the monotonic stress-strain relationship of confined concrete:

$$\sigma_1 = \frac{\varepsilon_1 E_0}{1 + \left( \frac{E_0}{E_s} - 2 \right) \frac{\varepsilon_1}{\varepsilon_{cc}} + \left( \frac{\varepsilon_1}{\varepsilon_{cc}} \right)^2} \quad (8)$$

where  $\sigma_1$  and  $\varepsilon_1$  are axial compressive stress and strain of concrete, respectively;  $E_0$  is the initial tangent modulus of elasticity in MPa;  $E_s$  is the secant modulus at the point of maximum compressive stress  $f_{cc}'$  which can be determined using Equations 3, and 5-7. The strain  $\varepsilon_{cc}$  corresponding to the maximum compressive stress  $f_{cc}'$  can be found employing the recommended relations in the pioneering work of Richart et al. (1928):

$$\varepsilon_{cc} = \varepsilon_c' \left( 1 + k_2 \frac{f_l}{f_c'} \right) \quad (9)$$

where  $\varepsilon_c'$  is the peak strain at the strength of plain concrete cylinders. Richart et al. (1928) proposed that  $k_2$  could be taken as  $5k_1$ .

For comparison purposes, the four stress-strain models of concrete under concentric compression and lateral confinement (Saadatmanesh et al. 1994, Samaan et al. 1998, Saafi et al. 1999, Mander et al. 1988) are evaluated throughout the study. First model is the Mander model proposed for concrete under concentric compression and confined with transverse reinforcement consisting of steel stirrups or spirals. The axial stress-strain curve is plotted according to the Popovics equation (1973):

$$\sigma_1 = \frac{f'_{cc} x r}{r - 1 + x^r} \quad (10)$$

where

$$x = \frac{\varepsilon_1}{\varepsilon_{cc}} \quad (11)$$

$$r = \frac{E_c}{E_c - E_s} \quad (12)$$

and  $\varepsilon_{cc}$  is the strain corresponding to maximum concrete stress  $f'_{cc}$ , calculated by the following equation in this model:

$$\varepsilon_{cc} = \varepsilon'_c \left[ 1 + 5 \left( \frac{f'_{cc}}{f'_{co}} - 1 \right) \right] \quad (13)$$

For the purpose of determining the confined concrete compressive strength  $f_{cc}$ , the five-parameter multi axial failure surface developed by William and Warnke (1975) is adopted as in Equation 2.

Saadatmanesh et al. (1994) presented analytical models for the analysis of concrete columns under monotonic loading and externally confined with FRP composite straps. Stress-strain model for confined concrete proposed by Mander et al. (1988) is adapted for the analysis of circular and rectangular columns confined with fiber composite straps under a slow strain rate and monotonic loading.

Samaan et al. (1998) proposed an analytical model to predict the complete stress-strain response of FRP-confined concrete columns. For the bilinear stress-strain curves are presented for both axial and lateral directions. The bilinear response of FRP confined concrete; the four-parameter relationship of Richard and Abbott (1975) is adopted and calibrated as follows:

$$\sigma_1 = \frac{(E_1 - E_2)\varepsilon_1}{\left[ 1 + \left( \frac{(E_1 - E_2)\varepsilon_1}{f_0} \right)^n \right]^{\frac{1}{n}}} + E_2\varepsilon_1 \quad (14)$$

where,  $E_1$  and  $E_2$  are first and second slopes respectively;  $f_0$  is the reference plastic stress at the intercept of the second slope with stress axis,  $n$  is a curve-shaped parameter that mainly controls the curvature in the transition zone (Fig.3).

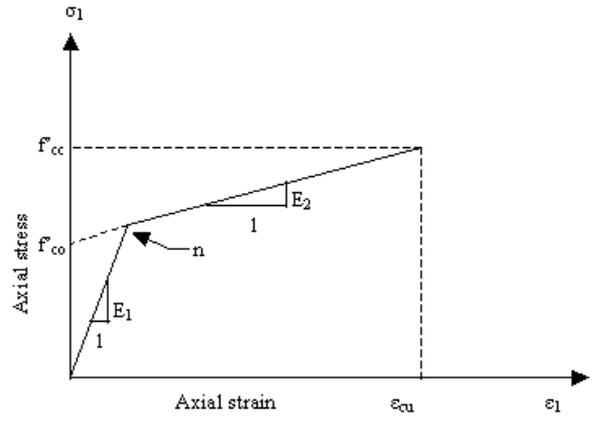


Figure 2. Parameters of bilinear confinement model (Samaan et al. 1998)

Saafi et al. (1999) suggest that the stress-strain response of FRP-confined specimens is bilinear in nature with a small transition zone between the two linear branches.

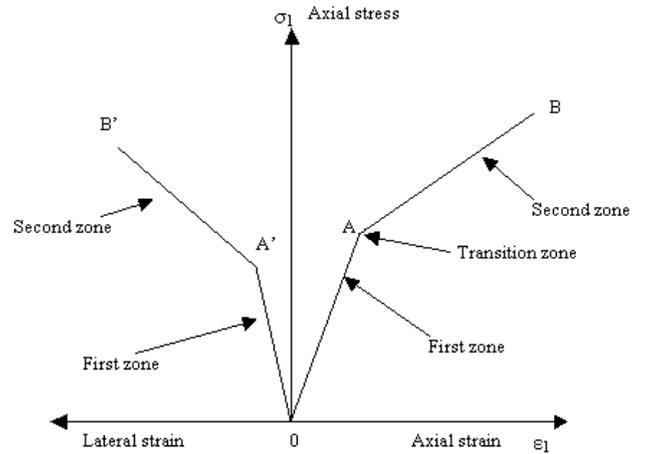


Figure 3. Simplified Stress-strain curves of FRP Grid Confined Concrete (Saafi et al. 1999)

Figure 3 shows a simplified stress-strain response of a concrete specimen confined with FRP grid. A confinement effectiveness coefficient  $k_l$  is proposed using a regression analysis of the experimental data:

$$k_l = 2.2 \left( \frac{f_l}{f'_c} \right)^{-0.16} \quad (15)$$

By substituting Equation 15 into Eq 1, the compressive strength at every point of the second zone is given by

$$f(\varepsilon_l) = f'_c \left( 1 + 2.2 \left( \frac{f_l}{f'_c} \right)^{0.84} \right) \quad (16)$$

where  $f(\varepsilon_l)$  is the axial compressive strength at a radial strain  $\varepsilon_l$ , and a confining pressure  $f_l$ :

$$f_l = \frac{2t_{com}E_{com}\varepsilon_l}{ds} \quad (17)$$

where  $E_{com}$  is the elastic modulus of the composite grid;  $t_{com}$  is composite thickness;  $d$  is the diameter of concrete core; and  $s$  is the spacing of the circular ribs (Saafi et al. 1999). Substituting Equation 17 into Equation 16, the ultimate compressive strength of confined concrete with FRP grids is given as:

$$f'_{cc} = f'_c \left( 1 + 2.22 \left( \frac{2t_{com}}{df'_c} \right)^{0.84} \right) \quad (18)$$

According to Saafi et al. (1999) axial strain reaches maximum:

$$\varepsilon_{cc} = \varepsilon_{co} \left( 1 + (2.6 + 537\varepsilon_{com}) \left( \frac{f'_{cc}}{f'_c} - 1 \right) \right) \quad (19)$$

where  $\varepsilon_{co}$  is the axial strain of unconfined concrete,  $\varepsilon_{com}$  is the ultimate strain of FRP tube.

## 4 RESULTS

In this paper, results relative to C3 test series of four square RC columns confined with CFRP composite jackets are presented. All columns are confined with one layer of CFRP and have fracture surfaces between the mid-height and top surface except C3L4S8. The fracture region was between the bottom and mid-height of C3L4S8. Actually, fracture of cylinders in compression has even traditionally been a difficult problem because of the way that cracks initiate (Landis et al 2003). Sounds started to be heard during approximately half of the ultimate load and were more frequent and louder very close to the ultimate load at the fracture of CFRP jackets. Cone-shaped fracture surface was observed after removing of the CFRP jackets at this region in Figure 4. The failure mode for the confined RC columns was due to the failure of the CFRP jackets and, after this point, the concrete core immediately failed due to the loss of confinement.

As can be seen in Figure 5, the most important observation at the end of the tests is that there is a significant increase up to nearly 100 percent in the deformability of the columns when the diameter of transverse steel changes from 8 mm to 12 mm.



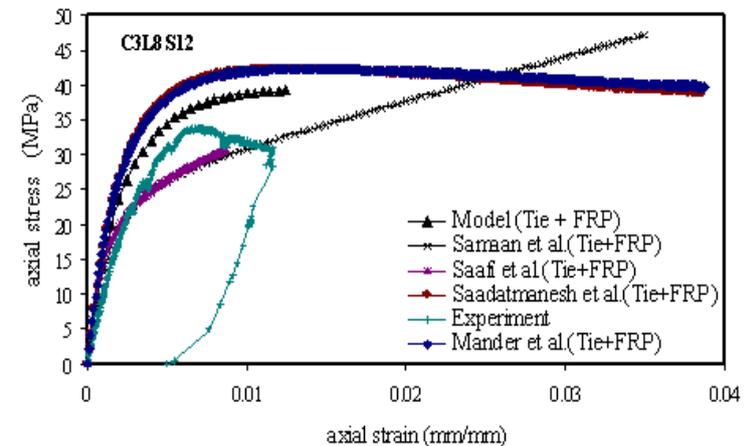
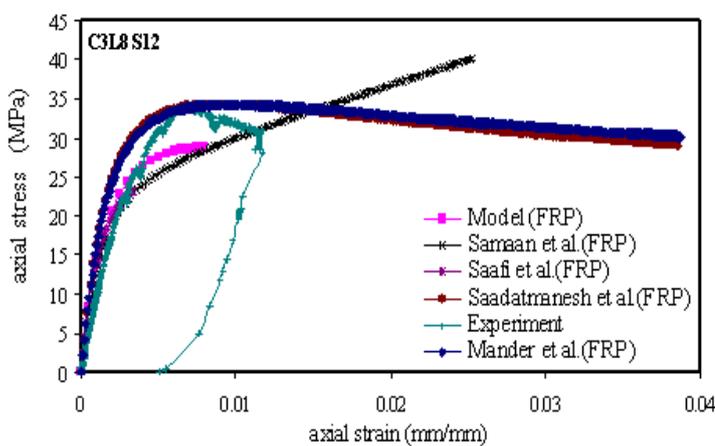
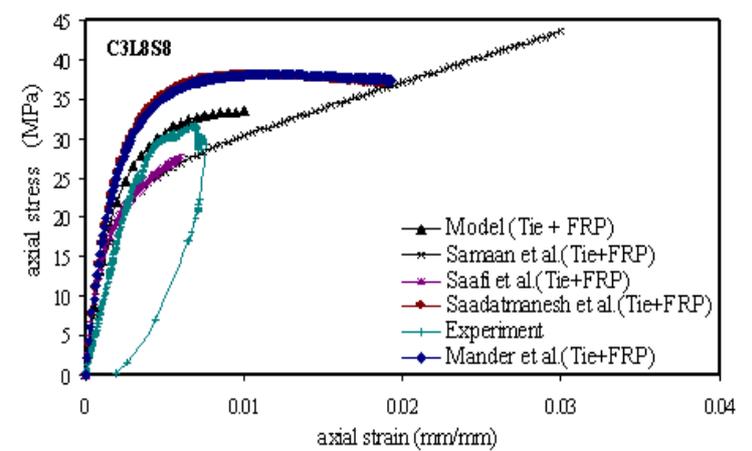
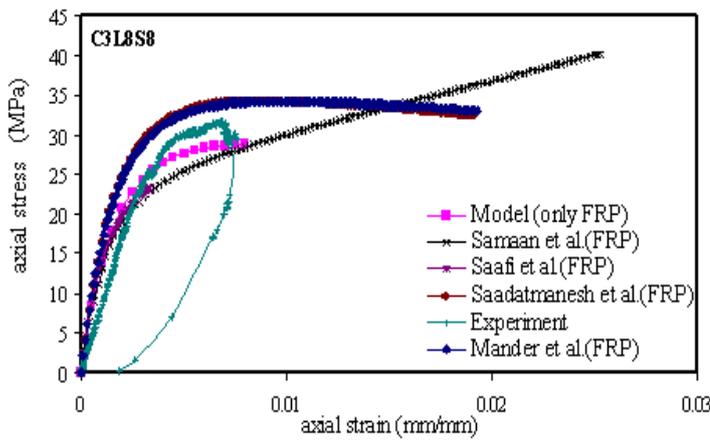
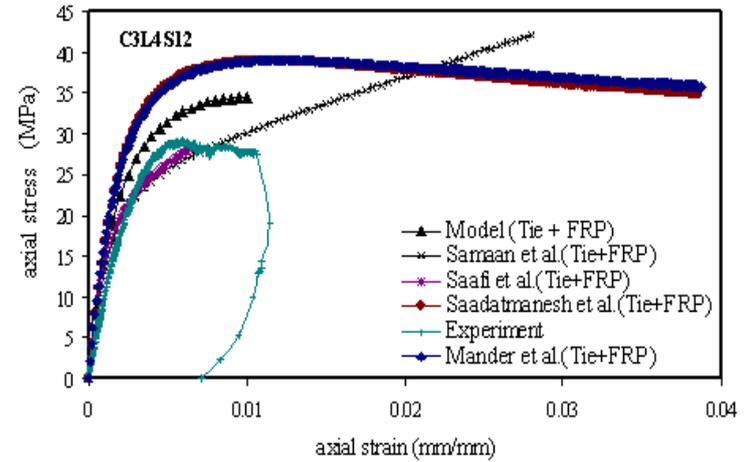
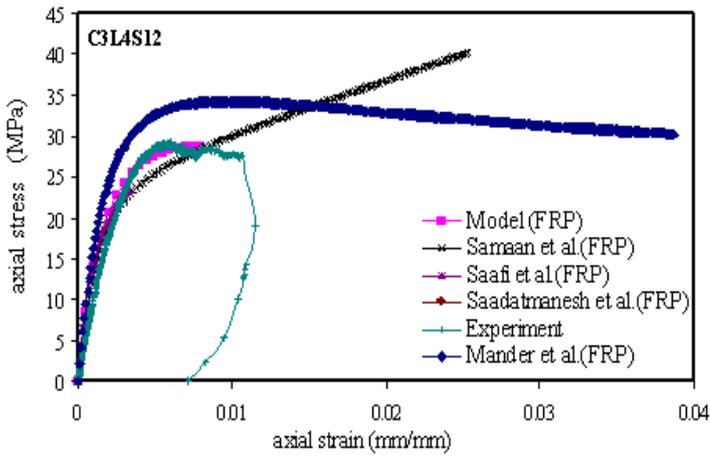
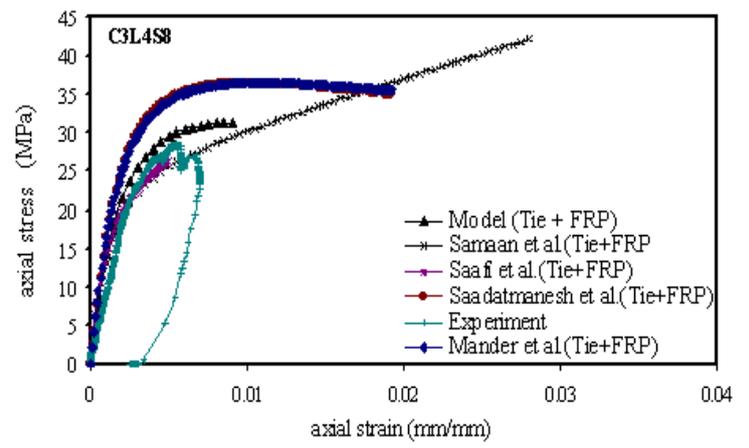
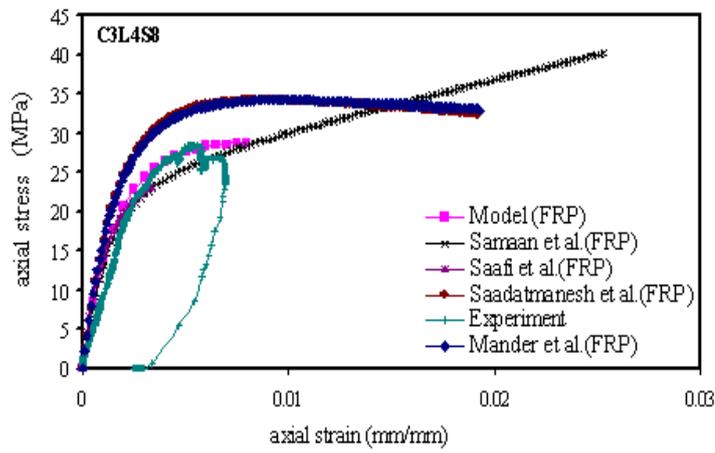
Figure 4. Failure mode for C3L8S8 and C3L4S12

## 5 CONCLUDING REMARKS

Generally, analytical models developed for concrete confined with FRP composite jackets, are based on the results of small-scale experiments instead of testing full-scale columns. For these large-scale specimens the failure mechanisms can be very different compared to the theoretical models (Köksal 2006). Therefore, the validity of the comparison of the proposed models can be best achieved for the experiments of large-scale RC columns.

The models of Mander and Saadatmanesh overestimate the ultimate strength of RC columns especially for the case in which the confinement effects of tie and CFRP are both considered. It can be noted that the predictions of Saadatmanesh and Mander are so close as indicated by the almost same common curve in Figs. 5 and 6. The analytical models of Saafi and Samaan predict the shape of the curve fairly well although Samaan model over-predicts the ultimate strength.

Köksal model predicts not only the ultimate strength of the columns closely but also the trends of the stress-strain plots. Its predictions are somewhat higher for the second case, accounting for the confinement effects of both CFRP jackets and transverse steel, as indicated in Figure 6. The interaction between the confinement mechanisms of transverse steel and FRP composites is a future and important issue to be considered for the load and deformability capacity of RC columns.



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Figure 5. Plots of the analytical models for test columns with the confinement effects of only CFRP.

Figure 6. Plots of the analytical models for test columns with the confinement effects of both tie and CFRP.

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