Relationship between compressive strength and modulus of elasticity of High-Strength Concrete

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ABSTRACT: Modulus of elasticity of concrete is frequently expressed in terms of compressive strength. While many empirical equations for predicting modulus of elasticity have been proposed by many investigators, few equations are considered to cover the entire data. The reason is considered to be that the mechanical properties of concrete are highly dependent on the properties and proportions of binders and aggregates. This investigation was carried out as a part of the work of the Research Committee on High-strength Concrete of the Architectural Institute of Japan (AIJ) and National Research and Development Project, called New RC Project, sponsored by the Ministry of Construction. More than 3,000 data, obtained by many investigators using various materials, on the relationship between compressive strengths and modulus of elasticity were collected and analyzed statistically. The compressive strength of investigated concretes ranged from 20 to 160 MPa. As a result, a practical and universal equation is proposed, which takes into consideration types of coarse aggregates and types of mineral admixtures.

1 INTRODUCTION

Modulus of elasticity of concrete is a key factor for estimating the deformation of structural elements, as well as a fundamental factor for determining modular ratio, n, which is used for the design of structural members subjected to flexure. Based on the relationship of modulus of elasticity of concrete that it is proportional to the square root of compressive strength in the range of normal concrete strength, AIJ specifies the following equation to estimate modulus of elasticity of concrete.

$$E = 2.1\times10^{5}\left(\gamma/2.3\right)^{1.5}\left(f_c/200\right)^{1/2} \quad (1)$$

where E = modulus of elasticity (kgf/cm^2)

γ = unit weight of concrete (t/m^3)

f_c = specified design strength of concrete (kgf/cm^2)

Eq. 1 is applied to concrete of a specified design strength of 36 MPa or less which is defined as normal strength concrete, a number of experiments have revealed that Eq. 1 over-estimates the modulus of elasticity as the compressive strength increases (Figure.1). This study aims to derive a practical and universal equation which is applicable to high-strength concretes with compressive strengths greater than 36 MPa, using regress ional analysis of numerous results of experiments published in Japan. The outline of this study was published in 1990.

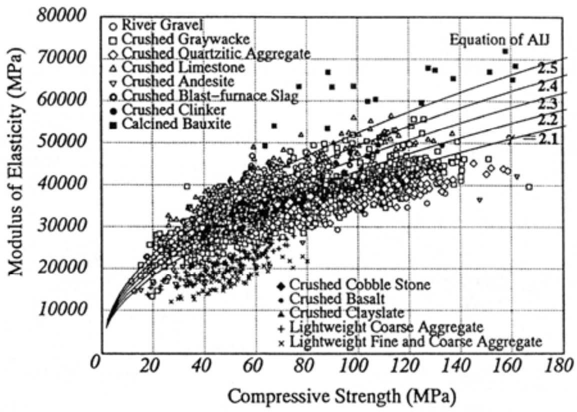

Figure 1.

2 REGRESSIONAL ANALYSIS

Before performing any analysis, it was necessary to create a basic form of the equation for modulus of elasticity. In this study the authors adopted the conventional form, in which modulus of elasticity, E, is expressed as a function of compressive strength, σ_B, and unit weight, γ. Since it is self-evident that the concrete with a compressive strength of 0 MPa has modulus of elasticity of 0 MPa, the basic form of the equation is expressed as Eq. 2.

$$E = a\sigma_B{}^{b}\gamma^{c} \quad (2)$$

The parameters examined are compressive strength, modulus of elasticity, and unit weight of concrete at the time of compression test, as well as types and mechanical properties of materials for producing concrete, mix proportioning, unit weight and air content of fresh concrete, method and temperature of curing, and age.

2.1 *Estimation of the Unit Weight*

Out of the 3000 experimental data collected, only one third included measured unit weight of specimens, γ. In order to express modulus of elasticity as a function of compressive strength and unit weight, the unit weights of hardened concrete had to be estimated when measured unit weights were not available from the data on materials used, mix proportioning, curing conditions, and age.

3 EQUATION FOR MODULUS OF ELASTICITY

3.1 *Evaluation of Exponent b of Compressive Strength, σ_B*

As compressive strength increases, Eq. 1 overestimates the modulus of elasticity. It is therefore, considered appropriate to reduce the value of exponent ***b*** of the compressive strength, σ_B, to less than ½ in order to make it compatible to the measured values.

Firstly, range of possible values of exponent **b** in Eq. 2 was investigated evaluating 166 sets of data, each set of which had been obtained from identical materials and curing conditions by the same researcher. Figure. 2 shows the relationship between the ultimate compressive strengths and the estimated exponent **b**. Similarly, Figure. 3 shows the relationship between the exponent **b** and the ranges of compressive strengths in the sets of data. In Figures. 2 and 3, while the estimated values of exponent **b** vary widely, the values show a tendency to decrease from around 0.5 to around 0.3, as the maximum compressive strengths increase and the ranges of compressive strength widens. In other words, whereas modulus of elasticity of normal-strength concrete has been predictable from the compressive strength with exponent **b** of 0.4 to 0.5, the values of 0.3 to 0.4 are more appropriate a general-purpose equation to estimate modulus of elasticity for a wide range of concretes from normal to high-strength. Consequently, the 1/3 is proposed as the value of exponent **b**.

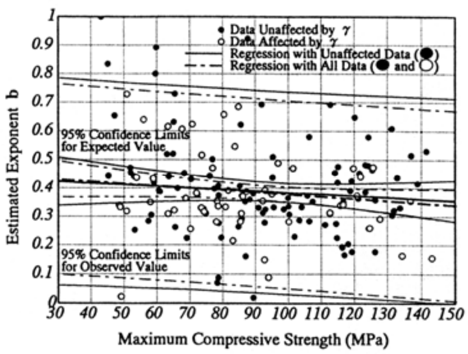

Figure 2.

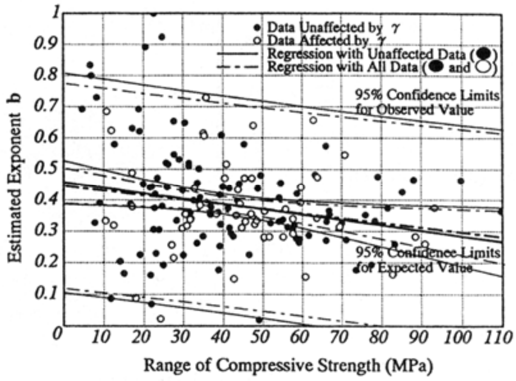

Figure 3.

3.2 *Evaluation of Exponent c of Unit Weight, γ*

Secondly, by fixing exponent **b** at 1/3, as mentioned above, exponent **c** of the unit weight, γ, was investigated. The relationship between the unit weight, γ, and the value obtained by dividing modulus of elasticity by compressive strength to the 1/3 power, $E/\sigma_B^{1/3}$, is shown in Figure. 4 with a regression equation (Eq. 3) that was obtained from data on all aggregates as shown below.

$$E = 1630\sigma_B^{1/3}\gamma^{1.89} \quad (3)$$

From Figure. 4, it can bee seen that Eq. 3 clearly shows the effect of the unit weight on modulus of elasticity, for concretes made with lightweight, normal weight, and heavy weight aggregates (bauxite, for example). The concretes made with normal weight aggregate, however, are scattered over a rather wide range of 6000 to 12000 of $E/\sigma_B^{1/3}$, while they gather in a relatively small unit weight range of 2.3 to 2.5. This suggests differences in the effects of lithological types of aggregates on modulus of elasticity, which will be discussed later in this paper. Whereas 1.5 has been used conventionally as the value for exponent **c**, as indicated in Eq. l, 1.89 was obtained from the regression analysis as the value for exponent **c** that is applicable for a wide range of concretes from normal to high-strength concretes. In consideration of the utility of the equation, a value of 2 is proposed as the value for exponent **c**.

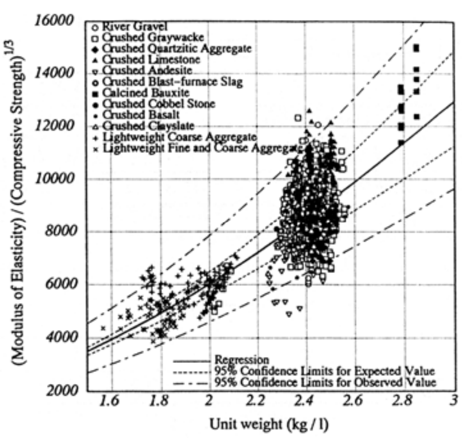

Figure 4.

3.3 *Evaluation of Coefficient a*

Thirdly, after fixing exponent **b** and exponent **c** at 1/3 and 2, respectively, the value for coefficient ***a*** was investigated. The relationship between modulus of elasticity, *E*, and the product of compressive strength to the 1/3 power and unit weight to the second power, $\sigma_B^{1/3}\gamma^2$, is shown in Figure.5, together with regression equation (Eq. 4) obtained from the data on all aggregates as shown below.

$$E = 1486\sigma_B^{1/3}\gamma^2 \quad (4)$$

The coefficient of determination is as high as 0.769, and the 95% confidence interval of modulus of elasticity is within the range of ±8000 MPa, as shown in Figure.5. The relationship between modulus of elasticity and $\sigma_B^{1/3}\gamma^2$ can therefore be virtually expressed by Eq. 4.

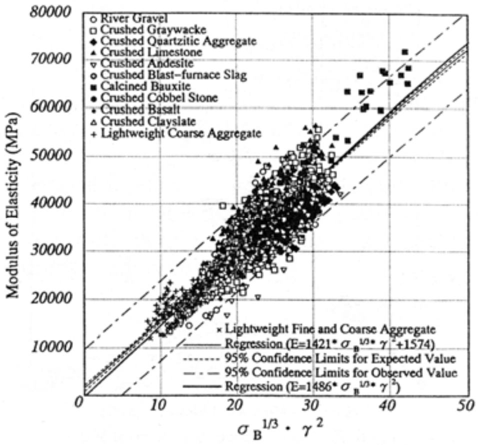

Figure 5.

3.4 *Evaluation of Correction Factor k*

In the conventional equation for modulus of elasticity, Eq. 1, the only difference in the type of coarse aggregate taken into account is the difference in the specific gravity, the effect of which is represented by the unit weight of concrete, γ. However, use of a wide variety of crushed stone has revealed that the difference in unit weight is not the only factor to account for the differences in moduli of elasticity of concretes of the same compressive strength. Lithological type should also be considered as a parameter of coarse aggregate. Besides, it has also been pointed out by many researchers that modulus of elasticity cannot be expected to increase with an increase in compressive strength, when the concrete contains a mineral admixture for high-strength, such as silica fume. This suggests the need to include the type of admixtures as another factor affecting modulus of elasticity. Thus, the type of coarse aggregate, as well as type and amount of mineral admixtures should be considered on the investigation of the values of correction factor, ***k***.

$$E = k \cdot 1486\sigma_B^{1/3}\gamma^{1/2} = k_1 k_2 \cdot 1486\sigma_B^{1/3}\gamma^2 \quad (5)$$

Where $k = k_1 \cdot k_2$

k_1 = correction factor corresponding to coarse aggregates

k_2 = correction factor corresponding to mineral admixtures.

3.4.1 *Evaluation of Correction Factor k_1 for Coarse Aggregate*

Figure 6. shows the relationship between the values estimated by Eq. 4 and the measured values of modulus of elasticity of concretes without admixtures. According to Figure. 6, most of the measured values/the calculated values, i.e. values of $\boldsymbol{k_1}$ in Eq. 5, fall in the range of 0.9 to 1.2, indicating that each lithological type of coarse aggregate tends to have an inherent $\boldsymbol{k_1}$. The correction factor $\boldsymbol{k_1}$ for each coarse aggregate is presented in Table 1. According to Table 1, the effects of coarse aggregate on modulus of elasticity are classified into three groups. The first group, which requires no correction factor, includes river gravel, crushed greywacke, etc. ;the second group, which requires correction factors of greater than 1, includes crushed limestone and calcined bauxite; and the third group, which requires correction factors smaller than 1, includes crushed quartzitic aggregate, crushed andesite, crushed cobble stone, crushed basalt, and crushed clayslate. Consequently, the value for each type of coarse aggregate is proposed as shown in Table 2, in consideration of the utility of the equation.

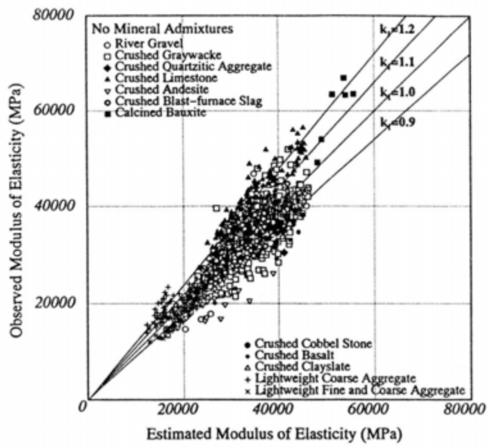

Figure 6.

Table 1.

Aggregate	k_1
River Gravel	1.005
Crushed Graywacke	1.002
Crushed Quartzitic Aggregate	0.931
Crushed Limestone	1.207
Crushed Andesite	0.902
Crushed Basalt	0.922
Crushed Clayslate	0.928
Crushed Cobbel Stone	0.955
Blast−furnace Slag	0.987
Calcined Bauxite	1.163
Lightweight Coarse Aggregate	1.035
Lightweight Fine and Coarse Aggregate	0.989

Table 2.

k_1	Lithological Type of Coarse Aggregate
1.20	Crushed Limestone, Calcined Bauxite
0.95	Crushed Quartzitic Aggregate, Crushed Andesite, Crushed Basalt, Crushed Clayslate, Crushed Cobbel Stone
1.00	Coarse Aggregate Other Than the Above

3.4.2 *Evaluation of Correction Factor k_2 for Admixtures*

Table 3 presents the averages of correction factor $\boldsymbol{k_2}$ obtained for each lithological type of coarse aggregate as well as for each type and amount of admixtures. When fly ash is used as an admixture the value of correction factor $\boldsymbol{k_2}$ is greater than 1, but when strength-enhancing admixtures, such as silica fume, ground granulated blast furnace slag, or fly ash fume (ultra fine powder produced by condensation of fly ash) are used, the correction factor $\boldsymbol{k_2}$ is smaller than 1. The proposed values of correction factor $\boldsymbol{k_2}$ for admixtures are shown in Table 4, in consideration of the utility of the equation.

Table 3.

Coarse Aggregate	Silica Fume			Glanulated Blast–furnace Slag		Fly Ash Fume	Fly Ash
	<10%	10–20%	20–30%	<30%	30%<		
River Gravel	1.045	0.995	0.818	1.047	1.118	–	1.110
Crushed Graywacke	0.961	0.949	0.923	0.949	0.942	0.927	–
Crushed Quartzitic Aggregate	0.957	0.956	–	0.942	0.961	–	–
Crushed Limestone	0.968	0.913	–	–	–	–	–
Crushed Andesite	–	1.072	0.959	–	–	–	–
Crushed Basalt	–	–	–	–	–	–	1.087
Calcined Bauxite	–	0.942	–	–	–	–	–
Lightweight Coarse Aggregate	1.026	–	–	–	–	–	–
Lightweight Fine and Coarse Aggregate	1.143	–	–	–	–	–	–

Table 4.

k_2	Type of Addition
0.95	Silica Fume , Ground Glanulated Blast–furnace Slag , Fly Ash Fume
1.10	Fly Ash
1.00	Addition Other Than the Above

3.5 *Practical Equation for Modulus of Elasticity*

Eq. 5 was derived as an equation for modulus of elasticity. Meanwhile, conventional equations such as Eq.1 have been convenient in such a way that standard moduli of elasticity can be obtained simply by substituting standard values of compressive strength and unit weight in the equation. In this study, Eq. 6 is proposed as the equation to be used for modulus of elasticity calculations. The equation is based on 60 MPa, a typical compressive strength of high-strength concrete, and uses a unit weight of 2.4, which leads to the compressive strength of 60 MPa.

$$E = k_1 k_2 \cdot 3.35 \times 10^4 (\gamma/2.4)^2 (\sigma_B/60)^{1/3} \qquad (6)$$

4 COMPARISON OF EQUATIONS

Figures 7-10 show the accuracy of estimation by Eq. 1 and Eq. 6 as well as by ACI 363R and CEB-FIP equations, which are presented in Table 5.

As pointed out by a number of researchers, the equation by the Architectural Institute of Japan (AIJ)(Figure. 7) tends to overestimate moduli of elasticity in the range of compressive strength over 40 MPa, except in the cases where crushed limestone or calcined bauxite is used as the coarse aggregate. The residuals also tend to increase in relation to the compressive strength.

The equation by ACI 363R (Figure. 8) slightly underestimates moduli of elasticity when crushed limestone or calcined bauxite is used as the coarse aggregate, regardless of the compressive strength. In the case of other aggregates, the equation tends to overestimate the moduli, though marginally, as compressive strength increases.

The CEB-FIP equation (Figure. 9) leads to clear differences in residuals depending on the lithological type of coarse aggregate. When lightweight aggregate is used, the equation overestimates the moduli, and the value of the residuals tends to decrease as the specific gravity of coarse aggregate increases from crushed quartzitic aggregate to crashed graywacke, crushed limestone, and calcined bauxite.

The residuals by the new equation (Figure. 10), as a whole, fall in the range of ±5000MPa, regardless of the compressive strength levels, although a portion of data display residuals near ±10000MPa. The new equation is therefore assumed to be capable of estimating moduli of elasticity for a wide range of concretes from normal to high-strength.

Table 5.

ACI 363R Equation	$E = (40000 \cdot \sigma_B^{0.5} + 1.0 \cdot 10^6) \cdot (\gamma/2.346)^{1.5}$	(psi)	State–of–the–Art Report on High–Strength Concrete ACI JOURNAL, July–August 1984
CEB–FIP Equation	$E = \alpha \cdot 21500 \cdot (\sigma_{28}/10)^{1/3}$ α = 1.2 : Basalt, Dense Limestone Aggregates 1.0 : Quartzitic Aggregates 0.9 : Limestone Aggregates 0.7 : Sandstone Aggregates	(MPa)	CEB–FIP MODEL CODE 1990 Final Draft Contribution a la 28e Session Pleniere du CEB Vienne – Septembre 1991

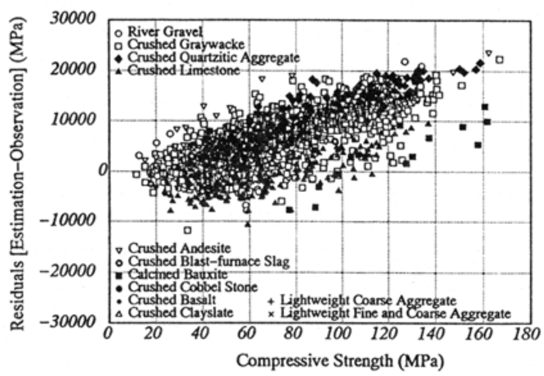

Figure 7.

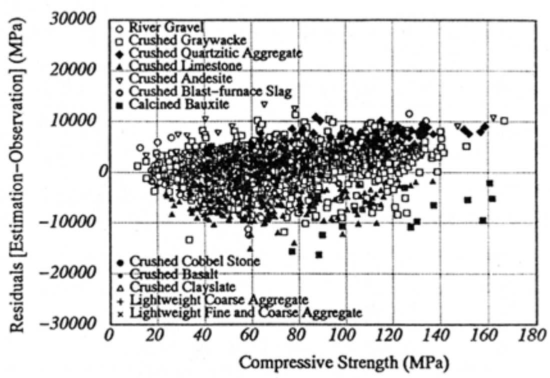

Figure 8.

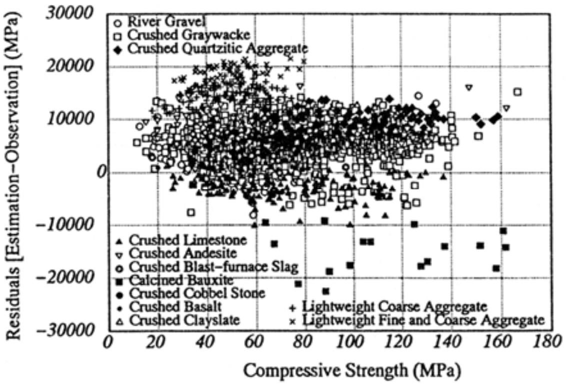

Figure 9.

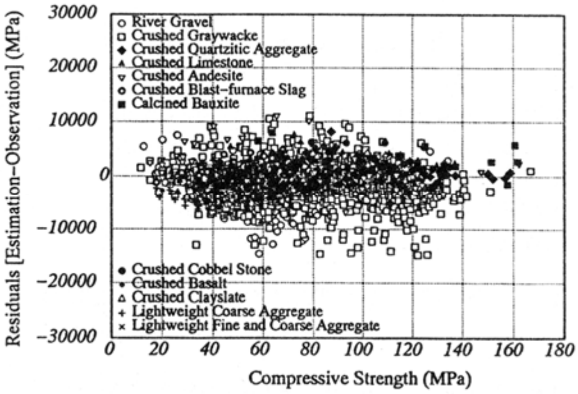

Figure 10.

5 EVALUATION OF 95 % CONFIDENCE INTERVALS

The accuracy of the equation being enhanced by incorporating the correction factors, 95% confidence intervals should be indicated, because the reliability of the estimated values is required in structural design and is used when determining materials and mix proportioning so as to ensure safety.

Excluding the ease of using fly ash as an admixture, only five values of the product of the correction factors, $\boldsymbol{k_1}$ and $\boldsymbol{k_2}$, are possible, i.e. 1.2, 1.14, 1.0, 0.95, and 0.9025.

A regression analysis of Eq. 2 was conducted for the combinations of a coarse aggregate and an admixture corresponding to each of the five values of $\boldsymbol{k_1 \cdot k_2}$, to obtain 95% confidence intervals of both estimated and measured moduli of elasticity. The results are shown in Figures.11-15. The curves indicating the upper and lower limits of 95% confidence of the expected values for all $\boldsymbol{k_1 \cdot k_2}$ are within the range of approximately ±5% of the estimated values, regardless of compressive strength and unit weight. The curves indicating those for the observed values are also within the range of approximately ±20% of the estimated values. Consequently, the 95% confidence limits of the new equation (Eq. 6) are expressed in a simple form as Eq. 7, and the 95% confidence limits of measured modulus of elasticity can be expressed as Eq. 8.

$$E_{e95} = (1 \pm 0.05)E \tag{7}$$

$$E_{o95} = (1 \pm 0.2)E \tag{8}$$

where E_{e95} = 95% confidence limits of expected modulus of elasticity.
E_{o95} = 95% confidence limits of observed modulus of elasticity.

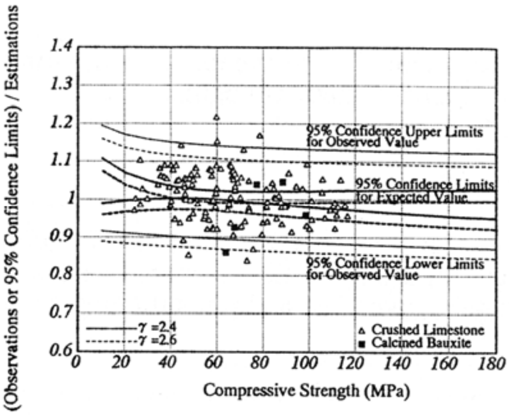

Figure 11.

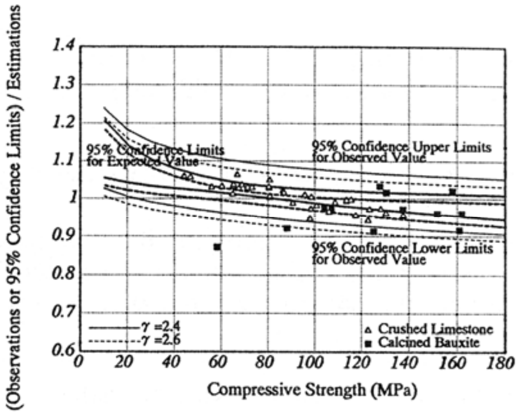

Figure12.

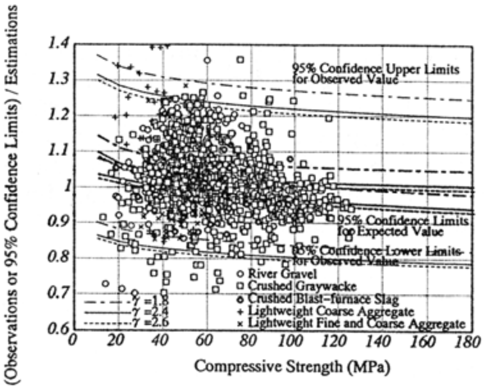

Figure 13.

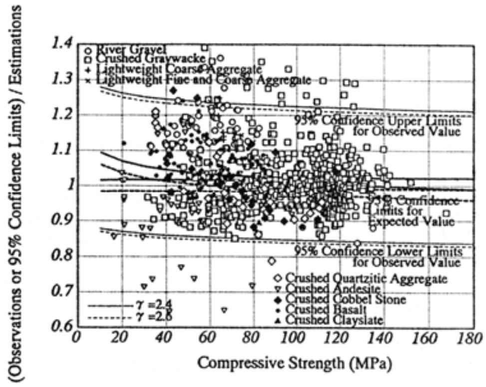

Figure 14.

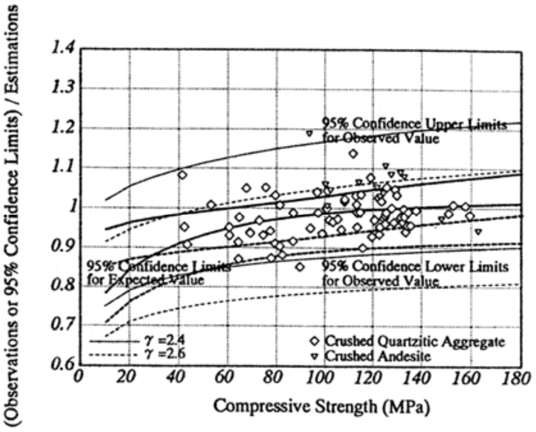

Figure 15.

6 CONCLUSIONS

Multiple regression analyses were conducted using a great deal of data published in Japan regarding the relationship between compressive strength and modulus of elasticity of concrete, by assuming compressive strength and unit weight as explanatory variables and modulus of elasticity as the target variable. As a result, a new equation (Eq. 6) is proposed as a practical and universal equation for modulus of elasticity. It is applicable to a wide range of concretes from normal to high-strength. The 95% confidence limits of the new equation were also examined, and Eq. 7 and 8 were proposed as the equations to indicate the 95% confidence limits for the expected and observed values, respectively.

REFERENCE

Tomosawa, F., Noguchi, T. and Onoyama, K. “Investigation of Fundamental Mechanical Properties of High-strength Concrete”, Summaries of Technical Papers of Annual Meeting of Architectural Institute of Japan, pp.497-498, October 1990.