Fracture Mechanics of Concrete and Concrete Structures -
Recent Advances in Fracture Mechanics of Concrete - B. H. Oh, et al.(eds)
 ISBN 978-89-5708-180-8

Fracture process zone development and energy dissipation during fracture in concrete wedge-splitting specimens

V. Veselý & P. Frantík
Brno University of Technology, Brno, Czech Republic

S. Seitl
Academy of Sciences of the Czech Republic, Brno, Czech Republic

ABSTRACT: The paper presents a technique for estimation of the size and shape of an inelastic zone evolving around the crack tip during tensile failure in quasi-brittle silicate-based composites. The technique is based on amalgamation of several concepts dealing with failure of structural materials, i.e. multi-parameter linear elastic fracture mechanics, classical non-linear fracture models for concrete, and the plasticity approach. Much attention is paid to the accurate description of the stress field in cracked specimens by means of two-parameter linear elastic fracture mechanics, particularly those used for wedge splitting test. The benefit of the proposed technique is expected to be viewed through the perspective of the determination of fracture characteristics describing the tensile failure of quasi-brittle silicate-based composites. The method is demonstrated using an example of a wedge-splitting fracture test on a compact tension specimen borrowed from literature. Reasonable utilization of the method was therefore a motivation for precise FEM analyses which were performed to enable sufficiently accurate description of the near-crack-tip stress field by two-parameter fracture mechanics.

1 INTRODUCTION

1.1 *Fracture process zone and determination of fracture-mechanical characteristics*

The existence of a fracture process zone (FPZ) accompanying the tensile failure of quasi-brittle materials is a well-known phenomenon. It has been verified experimentally (see e.g. van Mier 1997, Shah et al. 1995) and is widely accepted by the research community in the field of concrete fracture. The FPZ also plays an important role (but only sketchily, philosophically) in some theories developed for capturing the nonlinear character of tensile failure of such materials (Bažant & Oh 1983, Bažant & Kazemi 1990, Hu & Duan 2004, Duan et al. 2003a, 2006). However, the physical properties of the FPZ, i.e. its real size and shape, do not enter the existing procedures of fracture parameters determination, particularly those determining the specific fracture energy (RILEM 1985). This is true in spite of the fact that many researchers put the size and geometry effect (or the boundary effect) on the fracture parameters directly in the context of the mutual relationship of the spatial characteristics of the FPZ and the specimen (i.e. its size, shape, boundary conditions) (Bažant 1996, Duan et al. 2003a,b, 2006, Hu & Wittmann 1992, 2000, Hu & Duan 2004, Karihaloo et al. 2003). Therefore, the research on the incorporation of the FPZ properties into procedures for the determination of fracture parameters of quasi-brittle materials seems to be promising.

In this paper, a proposal of such a technique is sketched which is aimed at eliminating or minimizing the above-mentioned effects. The method is based on the relation of the work of fracture, i.e. the energy dissipated in the fracture mechanisms in the vicinity of the macroscopic crack tip, to the volume of the zone in which the failure processes take place. The paper proposes the construction of the FPZ based on a combination of several approaches: *i)* multi-parameter linear elastic fracture mechanics via the stress field approximation in the cracked body, *ii)* equivalent elastic crack models via the estimation of the location of the crack tip during the fracture process, *iii)* the plasticity theory via the estimation of the zone of the current onset of material failure, and *iv)* cohesive crack models via the introduction of the cohesive law into the procedure of the FPZ range estimation. Special attention is paid to the point *i)* in the paper, i.e. to the accurate description of the near crack-tip stress field in the test specimen, particularly in the wedge-splitting test configuration.

In the case of quasi-brittle materials the FPZ size substantially exceeds a range in which the stress state in a cracked body can be described accurately enough only by means of classical linear fracture mechanics. Therefore the constrain-based fracture mechanics considering several initial terms of Williams' series (Williams 1957) approximating the stress field in the cracked body is used to describe the more distant surroundings of the crack tip. The procedure of the estimation of the FPZ is illustrated on examples of the results of wedge-splitting fracture tests taken from literature (Xu et al. 2007).

1.2 *Wedge-splitting test geometry*

A convenient alternative to usual bending or tensile tests for determination of the fracture parameters of cementitious composites presents the wedge splitting test (WST) proposed by Linsbauer & Tschegg (1986) and later developed by Brühwiler & Wittmann (1990). The WST is an adaptation of the standard compact tension (CT, see e.g. ASTM 2000) test which eliminates the disadvantages stemming from the usually insufficient toughness of the fittings between the CT specimen and the testing machine (cumulation of elastic energy resulting in worse stability of the test). The WST is extensively used for various experimental studies and recently an increase in usage of the testing method has been registered (e.g. Kim & Kim 1999, Löfgren et al. 2005, Østergaard et al. 2002, Xu et al. 2007)

Determination of the fracture-mechanical properties of materials from records of fracture tests is conditioned by a proper fracture-mechanical description of the test in question. In the case of common testing geometries relevant information can be found summarized in the classical works from the field of fracture mechanics (e.g. Anderson 2004, for concrete e.g. Bažant & Planas 1998, Karihaloo 1995, Shah et al. 1995), handbooks (Tada et al. 2000, Murakami 1987) or other works (Knésl & Bednář 1998). Fracture parameters for the WST are not so widely reported. Stress intensity factors for particular variants of the WST (cube-shaped specimens) can be found e.g. in Guinea et al. (1996), RILEM (1991). However, they are utilizable within the classical (single-parameter) fracture mechanics approach only, which is inaccurate (insufficient) for application in the case of quasi-brittle fracture. Expressions of the terms of Williams' series approximating the stress field in the cracked body up to the order of 5 for particular dimensions and boundary conditions of cube-shaped WST specimens were introduced in Karihaloo et al. (2003). These results are valid only for a limited range of variants of boundary conditions (configurations of load and supports on the bottom side of the specimen).

This paper brings a partial refinement of those results, as boundary conditions of the WST test are considered in a more realistic way in the presented calculations. However, the refining is performed in the framework of two-parameter fracture mechanics only. The problem is solved numerically by means of the finite element method; the K-factor and the T-stress for a particular case of cube-shaped WST specimens are determined.

2 THEORETICAL BACKGROUND

2.1 *Multi-parameter linear elastic fracture mechanics*

Asymptotic stress and displacement fields in a cracked body (i.e. the fields for the very vicinity of the crack tip) can be described by only one single parameter – usually the stress intensity factor K – within the scope of classical linear elastic fracture mechanics (LEFM). The failure of an ideally brittle material starts at a single point, the crack tip, for which the asymptotic description holds true. In the case of quasi-brittle materials, however, the FPZ arises and evolves around the crack/notch tip; the size of this zone cannot be neglected in relation to the dimensions of the cracked body (including the crack length). The FPZ size substantially exceeds the range in which the stress state can be described accurately enough only by means of classical linear fracture mechanics, i.e. based only on the K-factor.

2.1.1 *Near-crack-tip stress field approximation*

Williams' solution for an elastic homogeneous 2D body with a crack (Williams 1957, Knésl & Bednář 1998) provides an approximation of stress and deformation fields by means of its expansion into a power series. For a stress tensor it holds:

$$\sigma_{ij} = \sum_{n=1}^{\infty} \left(A_n \frac{n}{2} \right) r^{\frac{n}{2}-1} f_{ij}(n,\theta) \tag{1}$$

where r, θ = polar coordinates, coefficients A_n = known constants and f_{ij} = known functions. A closer look at the three stress components reveals the following form:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \sum_{n=1}^{\infty} \left(A_n \frac{n}{2} \right) r^{\frac{n}{2}-1} \cdot \\ \cdot \begin{Bmatrix} \left[2 + (-1)^n + \frac{n}{2} \right] \cos\left(\frac{n}{2} - 1 \right)\theta - \left(\frac{n}{2} - 1 \right) \cos\left(\frac{n}{2} - 3 \right)\theta \\ \left[2 + (-1)^n - \frac{n}{2} \right] \cos\left(\frac{n}{2} - 1 \right)\theta + \left(\frac{n}{2} - 1 \right) \cos\left(\frac{n}{2} - 3 \right)\theta \\ -\left[(-1)^n - \frac{n}{2} \right] \sin\left(\frac{n}{2} - 1 \right)\theta + \left(\frac{n}{2} - 1 \right) \sin\left(\frac{n}{2} - 3 \right)\theta \end{Bmatrix} \tag{2}$$

The first term of the series is singular with regard to the distance r from the crack tip and the constant A_1 at this term corresponds to the stress intensity factor K. The second term is constant with respect to the value of r and is referred to as T-stress. The rest of the terms take finite values for arbitrary r.

For an approximate description of stress and deformation fields in the vicinity of the crack tip it is possible to consider only the first term (in the case of classical fracture mechanics) or the first two terms (in the case of two-parameter fracture mechanics) of the series and neglect the terms for $n > 2$, as they converge to zero for $r \to 0$. However, if the more distant surroundings of the crack tip need to be described, which is the case with quasi-brittle materials, higher order terms of Williams' series must be taken into account. This approach can be referred to as an application of multi-parameter LEFM.

2.1.2 *Determination of the stress field*

In the fracture mechanics approach, the interest is focused on the singularity point (crack tip) where stress becomes (mathematically, but not physically) infinite. Near such singularities usual polynomial-based finite element approximations perform badly. Therefore, singular elements should be used around the singularity point. Along the singular element edges the derivatives $\partial u/\partial x$ (strain) vary as $1/r$ where r is the distance from the corner node at which the singularity develops.

In the linear elastic problem, the stress intensity factor K and the T-stress values can be computed by means of the finite element method using various procedures (for a review see e.g. Ayatollahi 1998, Xiao & Karihaloo 2007). In this contribution both the direct technique (Yang & Ravi-Chandar 1999) and quarter-point crack-tip elements (Tan & Wang 2003) are used. Generally, the direct methods need extreme mesh refinement close to the crack tip in comparison with the method employing quarter-point elements. For the direct method the estimation of the fracture parameters is derived directly from the singular stress description, see Equation 2.

2.2 *Classical non-linear fracture models for concrete*

The characteristic features of quasi-brittle fracture were briefly noted in the introduction. The fundamental characteristic of the tensile failure of quasi-brittle materials is the existence of the FPZ at the macroscopic crack tip. This phenomenon is the reason for the non-linear fracture behaviour of the materials in question and introduces the main topic of the paper.

2.2.1 *Equivalent elastic crack models*

The simplest non-linear models within the mechanics of continuum capturing the "real" fracture behaviour of concrete are known as equivalent elastic crack models. They simulate the cohesive fracture of quasi-brittle materials by replacing the real body, which has a crack of a certain initial length and a FPZ ahead of it, with a brittle body with an effective crack longer than the initial one, and then forcing both bodies to exhibit the same structural behaviour. The essential advantage of these models is the LEFM apparatus preserved for the analyses within these models.

A representative of this group utilized in the presented research is the effective crack model (Nallathambi and Karihaloo 1986).

2.2.2 *Cohesive crack models*

More accurate models considering the mutual cohesive effect of crack faces in the vicinity of the crack tip, which is typical of quasi-brittle materials, are referred to as cohesive crack models (Hillerborg et al. 1976, Bažant & Oh 1983). According to this approach a crack and the FPZ evolving in front of the crack tip in a quasi-brittle body are modelled by an extension Δa of the original crack of length a on a distinct section of which the crack faces are clamped by cohesive forces.

A technique for reconstruction of the FPZ size and shape introduced hereinafter employs tools of the fictitious crack model by Hillerborg et al. (1976) which is a representative of cohesive crack models for quasi-brittle materials.

2.3 *Determination of crack-tip plastic zone – plasticity theory*

The size and shape of the plastic zone in elastic-plastic materials influence the fracture behaviour substantially (see e.g. Knésl & Bednář 1998, Knésl et al. 2000). In the procedure of the plastic zone contour calculation an equivalent (comparative) near-crack-tip stress field amplitude, determined e.g. from principal stresses by using of von Mises' or Tresca's failure criterion, is compared with a particular critical value (Anderson 2004), which is the value of yield stress or yield stress in shear, respectively. Equation 2 can be rearranged in these cases into a closed form that explicitly returns the radius r for a particular angle θ. In the case of the stress field description using even more terms of Williams' expansion than two it is convenient to calculate the values of r for a selected sequence of θ numerically, e.g. by Newton's method.

3 METHOD PROPOSAL

As was noted above, the proposed technique for determination of "true" fracture characteristics should relate the energy dissipated within the various failure mechanisms in the FPZ to its volume in order to eliminate or considerably decrease the effects of the size, shape and boundary conditions of the specimen on the parameters evaluated from the records of fracture tests performed on laboratory specimens.

3.1 *FPZ extent estimation*

The proposed concept of the construction of the size and shape of the FPZ is based on an amalgamation of several approaches which are listed in the following subsections. These approaches are used within the processing of fracture test records; typically load–displacement diagrams (*P–d, P–CMOD*).

3.1.1 *Estimation of effective crack tip*

The effective crack length in the loaded body for each loading step (i.e. each point of the *P– CMOD* diagram) is determined based on the change of secant compliance of the body between the initial and the current stage of the fracture process by using the effective crack model (Nallathambi & Karihaloo 1986, Karihaloo 1995), see Section 2.2.1.

3.1.2 *Estimation of the crack-tip stress field*

For the approximation of the stress field within the specimen with the current crack length (corresponding to current stage of fracture according to its location on the *P– CMOD* diagram) the multi-parameter fracture mechanics (see Section 2.1) is utilized. The stress state in a body with an effective crack is approximated through Williams' power series, whereas the number of terms of the series has to be chosen with respect to the mutual relation between the expected FPZ size/shape and the size/shape of the body (with respect to the distance of the FPZ to the free boundaries of the body).

3.1.3 *Estimation of "plastic" zone at the effective crack tip*

The extent of the zone where the until-now elastic material starts to fail is determined by comparing the proper characteristic of the stress state around the crack tip (some sort of equivalent stress σ_{eq}, for cementitious composites e.g. the Rankine, Drucker-Prager or another suitable failure criterion can be employed) to tensile strength f_t of the material.

3.1.4 *Estimation of crack opening profile*

The crack opening displacement values at the propagating crack faces are calculated from appropriate LEFM formulas, e.g. those from Tada et al. (2000), Murakami et al. (1987).

3.1.5 *Estimation of cohesive zone extent*

In agreement with the cohesive crack approach, the FPZ is supposed to extend from the zone of the current failure around the current crack tip, where the selected stress state characteristic (equivalent stress σ_{eq}) exceeds the tensile strength f_t, up to a point at the crack faces where the value of crack opening displacement w reaches its critical value w_c (i.e. the value of cohesive stress drops to zero).

This method of FPZ definition for the current crack tip is based on the assumption that the energy dissipation in the failure processes occurs at those points in the body where the equivalent stress σ_{eq} appropriate to the prior stages of the fracture has exceeded the tensile strength f_t (failure mechanisms started to proceed there), and simultaneously the value of cohesive stress $\sigma(w)$ corresponding to the prior crack tip plastic zone is positive.

A detailed description of the construction of the FPZ evolving during fracture in quasi-brittle materials can be found in Veselý et al. (2009), Veselý & Frantík (in prep.). In this paper, only examples of the reconstructed FPZs are shown evolving at the crack tip during the fracture process in WST specimens.

3.2 *Work-of-fracture specification by the stress free crack area and the FPZ volume*

The size of the FPZ is significant in the case of quasi-brittle materials; moreover it is a spatial issue. Methods for the determination of fracture parameters based on classical non-linear models for concrete fracture ignore the 3D character of this zone. For that reason, the specimen size, shape and boundary conditions have an observable influence on fracture-mechanical parameters determined in such a way. According to the best knowledge of the authors, there exists no consistent research relating quasi-brittle fracture behaviour to the explicit expression of the FPZ size and shape. However, extensive numerical experiments (Veselý et al. 2007, Řoutil et al. 2008) clearly demonstrated the need to link the fracture behaviour of quasi-brittle materials to FPZ parameters.

The authors attempted to separate the energy amounts consumed via the two basic different energy dissipation mechanisms (the Griffith-Irwin one and the Dugdale-Barenblatt one) during quasi-brittle fracture and to specify them using the spatial characteristics of the propagating crack and the FPZ evolving at its tip (Veselý et al. 2009). The portion of energy consumed in order to create new crack surfaces is specified by the area of the projection of the surfaces to the crack plane resulting in a critical energy release rate referred to as fracture energy G_f, the value of which is assumed to be a material property. The other part of the entirely dissipated energy is consumed in the failure mechanism taking place in the FPZ. This value is specified by the volume of the FPZ, resulting in the spatial energy dissipation density H_f.

The progress of the quantities G_f and H_f during the fracture process is being intensively investigated at present by using the proposed method by the authors. Since there still exist some unresolved issues within the methodology, only the first part of the research described above is presented in this paper, i.e. the reconstruction of the size and shape of the FPZ during fracture process. The method is being implemented into a software tool developed as a Java application (Veselý & Frantík 2008).

4 FEM MODELLING

4.1 *Model preparation, input parameters, evaluation techniques*

The numerical analysis of the near-crack-tip stress field was performed for a case of the cube-shaped WST specimen with dimensions similar to those considered in Karihaloo et al. (2003). However, unlike their work where only the splitting force P_{sp} was assumed, our work also considers the vertical components P_v of the load transmitted to the specimen through the wedge, which influences of course also the boundary conditions at the bottom surface of the specimen. The differences in the boundary conditions are evident from the Figure 1.

For numerical analysis the following values of the dimensions were used: $W = 100$ mm, $e = 35$ mm, $f = 30$ mm, $h = 10$ mm, $d_n = 20$ mm (see Fig. 1b). The initiation notch of length c was modelled as a crack where its length varied so that parameter $\alpha = a/W$ lies in the interval from approx. 0.2 to 0.9. The material input data values for a particular case of concrete were used as follows: Young's modulus $E = 44$ GPa and Poisson's ratio $\nu = 0.2$.

A typical finite element mesh used in the computations is shown in Figure 2 (left), together with boundary conditions. A detailed view of the small region near the crack tip with a quarter point element is shown in Figure 2 (right). The size of the smallest element in the crack tip is 5×10^{-5} mm. The computations were carried out in the commercial ANSYS (Ansys 2005) FEM code

4.2 *Results of the analyses*

Numerically calculated values of the stress intensity factor K and T-stress were normalized to dimensionless functions $k(\alpha)$ and $t(\alpha)$ in order to compare them with the data from literature. In accordance with Karihaloo et al. (2003) the K-factor can be calculated using the formula

$$K_I = \sigma\sqrt{2\pi W}\,k(\alpha) = a_1\sqrt{2\pi} \tag{3}$$

and the functions $k(\alpha)$ and $t(\alpha)$ are defined as

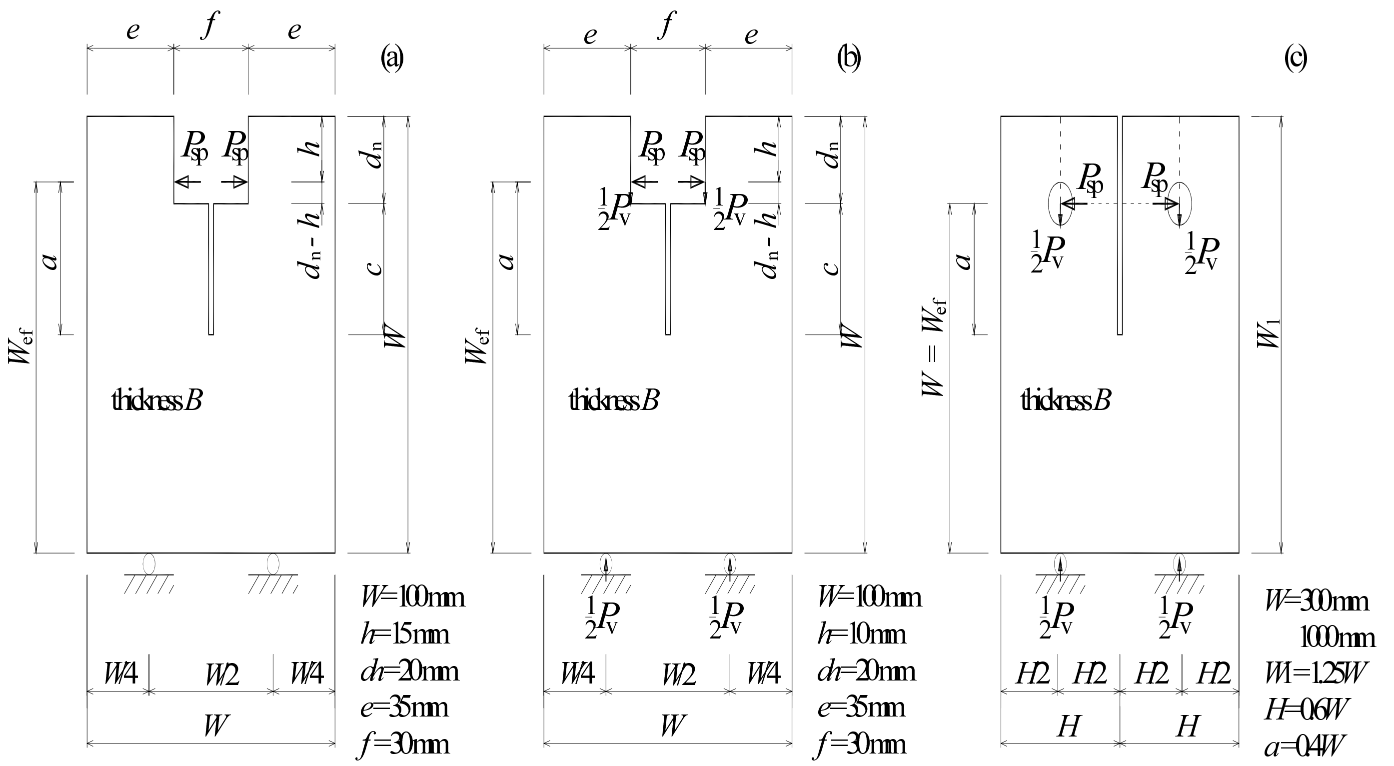

Figure 1. WST geometry – boundary conditions and dimensions from: a) Karihaloo et al. (2003), b) Seitl et al. (2009a,b), Xu et al. (2007).

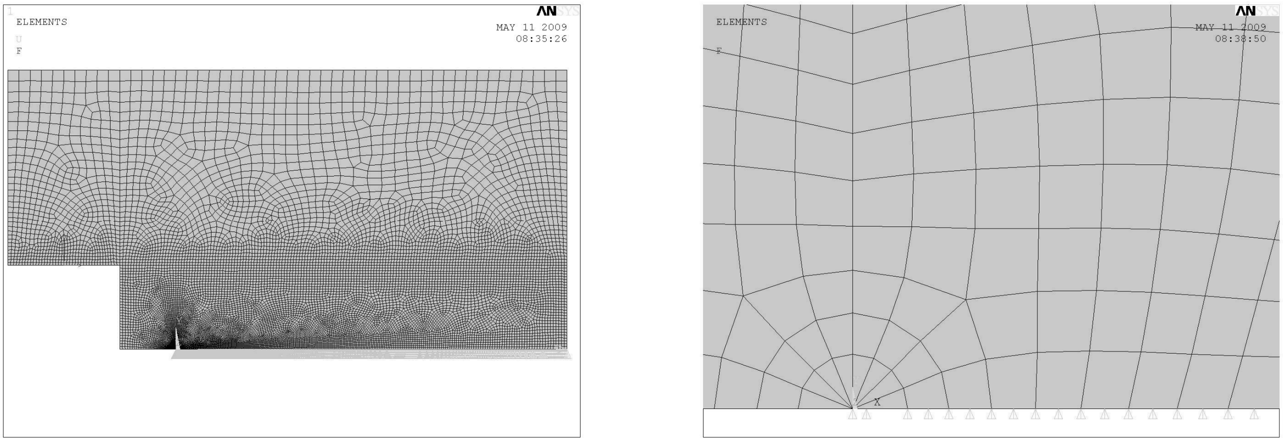

Figure 2. The finite element mesh used in the computations: left – one half of the WST specimen, right – detailed view of the small region near the crack tip (quarter-point crack-tip element was used).

$$k(\alpha) = \frac{a_1}{\sigma\sqrt{W}}\,, \qquad t(\alpha) = \frac{T}{\sigma} = \frac{4a_2}{\sigma}\,, \tag{4}$$

where $\sigma = P_{sp}/BW$, BW = cross-section area of the specimen, A_1, A_2 = coefficients of the first and second term of Williams' series. Comparisons of the functions $k(\alpha)$ and $t(\alpha)$ from our own computations corresponding to Figure 1b) and data by Karihaloo et al. (2003) corresponding to Figure 1a) are depicted in Figures 3 and 4, respectively.

From these figures it is evident that the loading arrangements and the supports on the bottom side of the specimen influence the values of $k(\alpha)$ only negligibly (apart from the region of very short ligament lengths where the results are burdened by a considerable error), in contrast to the function $t(\alpha)$ where the influence is more pronounced, especially in the interval $\alpha \in (0.3;\ 0.8)$. Note that the interval is commonly used within the measurement of fracture parameters in the case of the mentioned WST geometry. It is expected that higher order terms of Williams' series will be affected even more (the higher term the higher influence).

5 EXAMPLE

The above-mentioned method has already been tested on cases of the three-point bending of notched beams (Veselý et al. 2009, Veselý & Frantík, in prep.). As an illustration of the developed procedure functioning two selected tests of a rather extensive experimental WST campaign carried out by Xu et al. (2007) were chosen. The tests were performed on several sizes W varying from 200 mm to 1000 mm. The shape of the specimens is depicted in Figure 1c) where the dimensions of the selected specimens can also be found whose fracture process was reconstructed by the developed procedure in this paper.

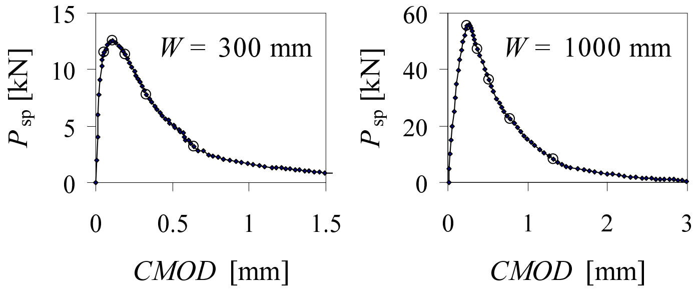

Figure 5. Two selected load–displacement diagrams from experimental campaign carried out by Xu et al. (2007).

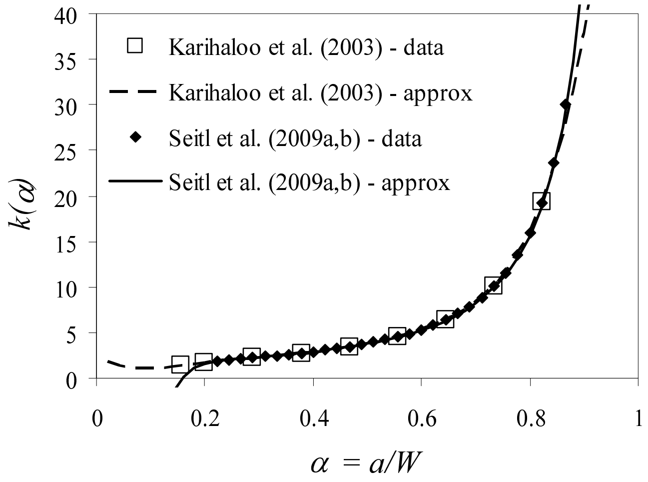

Figure 3. Dimensionless function $k(\alpha)$.

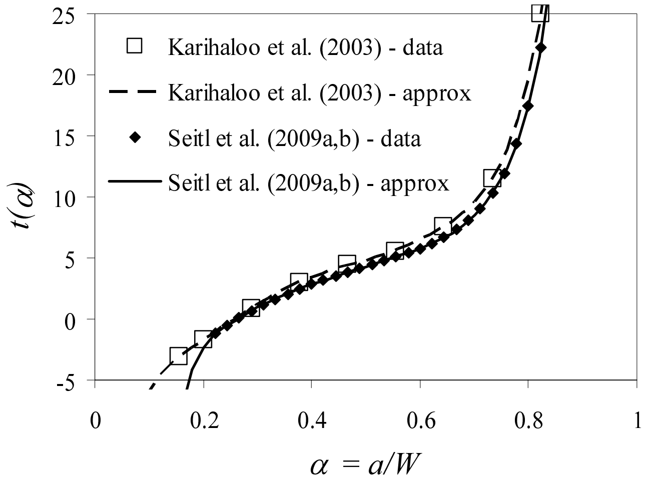

Figure 4. Dimensionless function $t(\alpha)$.

The *P–CMOD* diagrams recorded during the two selected tests are displayed in Figure 5. In each graph there are five points emphasized which correspond to the values of relative equivalent elastic crack α equal to 0.5, 0.6, 0.7, 0.8, and 0.9. Note that the relative notch length α_0 was 0.4. For the reconstruction of the FPZ at individual stages of the fracture the following parameters/models were used:

- 5 terms of Williams' series for the stress field approximation (2 by Seitl et al. 2009a,b, the other 3 by Karihaloo et al 2003 – chosen because of better appropriateness even though being inconsistent);
- Rankine failure criterion, tensile strength $f_t = 4$ MPa (corresponding approximately to compressive strength $f_c = 53.3$ MPa reported in Xu et al. 2007);
- Exponential tension softening law $\sigma(w)$, fracture energy $G_f = 85$ Jm^{-2} (corresponding approximately to $G_F/2.5$ where G_F reported by Xu et al. 2007 was equal to 205 and 284 Jm^{-2} for the small and the large specimens, respectively).

Figure 6 displays a sequence of the FPZs evolving during the fracture in the two studied WST specimens. Stages corresponding to the points highlighted in the *P–CMOD* curves drawn in Figure 5 are depicted including the representation of the cohesive stress intensity. The stepwise (layered) character of the FPZs is caused by the relatively sparse sequence of points in the *P–CMOD* diagrams.

6 DISCUSSION OF RESULTS

6.1 *Description of near-crack-tip stress field in WST specimens*

The results of the performed computations were compared with the data found in literature. The basic conclusion following from this analysis is that the influence of the vertical compressive component of the loading force and the reaction from the supports at the bottom surface of the specimen are much stronger on the values of *T*-stress than on the *K*-factor (the influence on the *K*-factor is negligible). Based on this fact it may be expected that the higher order terms of Williams' series will be influenced even more. Therefore, a proper numerical analysis of the stress field must be performed in the case of any change of the specimen shape or boundary conditions in order to approximate the stress field in the testing specimens precisely.

6.2 *Estimation of the FPZ extent*

Applications of the developed method on testing results of the three point bent notched beams and the WST specimens provided satisfactory results concerning the estimation of the FPZ extent. However, particular issues that are worth deeper investigation must be addressed. The procedure does not (still) take into account the possible redistribution of stresses within the technique of estimating the current zone of the material failure. Therefore, the FPZ width is underestimated in the initial stages of the fracture and overestimated in stages when a substantial part of the cohesive zone has already been formed. Particular attention must also be paid to the number of terms of Williams' series taken into account for the FPZ evaluation as well as to the selection of an appropriate failure criterion. The tensile strength and parameters of the considered cohesive law also play a significant role within this issue.

7 CONCLUSIONS

The finite element analysis of the near-crack-tip stress field for a particular variant of the cube-shaped WST specimen was performed by means of

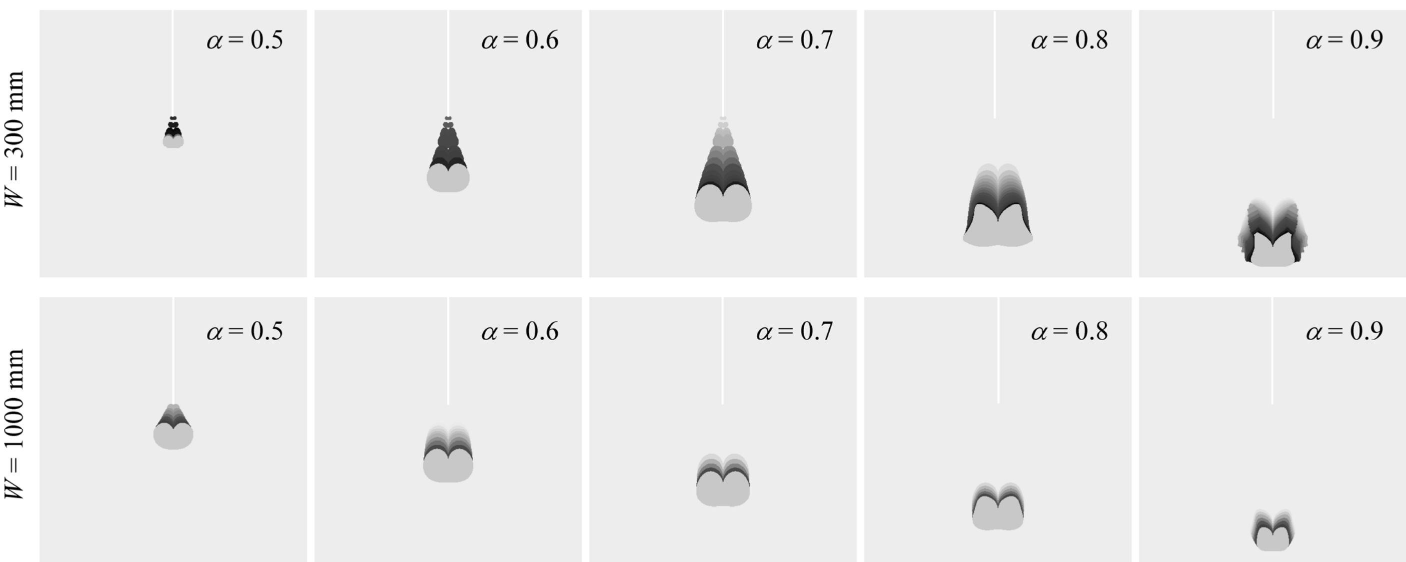

Figure 6. Evolution of the FPZ during fracture through the ligament of WST specimens – two selected cases from experimental campaign carried out by Xu et al. (2007) (intensity of the cohesive stress within the FPZ is depicted).

the constraint-based two-parameter fracture mechanics approach. The compressive component of the loading force (apart from the splitting one) together with corresponding reactions from supports on the opposite side of the specimen was considered, which refines the parameters reported for the testing geometry in literature.

In the paper a method is also sketched which provides estimation of the size and shape of the fracture process zone which is a typical feature accompanying the fracture process in quasi-brittle materials. This method employs a combination of various approaches from different fields of the theory of fracture mechanics and plasticity. This technique is being developed in order to create/refine procedures which enable the determination of fracture parameters of quasi-brittle materials independent of the size, shape and boundary conditions of laboratory test specimens. The procedure relates the energy dissipated in the FPZ to its volume and is currently under intensive development, investigation and testing by the present authors.

ACKNOWLEDGEMENTS

The work was supported by the grant project of the Grant Agency of Academy of Sciences of the Czech Republic KJB200410901, and the project of the Czech Scientific Foundation GA ČR 103/08/0963.

REFERENCES

Anderson T.L. 2004. Fracture mechanics: Fundamentals and Applications, Third Edition. Boca Raton: CRC Press.

ANSYS Users manual version 10.0 2005. Swanson Analysis System, Inc., Houston.

ASTM 2000. Standard E 647-99: Standard Test Method for Measurement of Fatigue Crack-Growth Rates, Annual Book of ASTM Standards, Vol. 03.01 591-630.

Ayatollahi, M.R., Pavier, M.J. & Smith, D.J. 1998. Determination of T-stress from finite element analysis for mode I and mixed mode I/II loading. Int. J. Fract. 91: 283–298.

Bažant, Z.P. 1996. Analysis of work-of-fracture method for measuring fracture energy of concrete. J. Eng. Mech. 122(2):138-144.

Bažant, Z.P. & Kazemi, M.T. 1990. Determination of fracture energy, process zone length and brittleness number from size effect, with application to rock and concrete. Int. J. Fract. 44: 111-131.

Bažant, Z.P., Oh, B.H. 1983. Crack band theory for fracture of concrete. Mater. Struct. 16: 155-177.

Bažant, Z.P. & Planas, J. 1998. Fracture and size effect in concrete and other quasi-brittle materials. Boca Raton: CRC Press.

Brühwiler, E., Wittmann, F.H. 1990. The wedge splitting test, a new method of performing stable fracture mechanics test. Engrg. Fract. Mech. 35: 117-125.

Duan, K., Hu, X.-Z. & Wittmann, F.H. 2003a. Boundary effect on concrete fracture and non-constant fracture energy distribution, Eng. Fract. Mech. 70: 2257-2268.

Duan, K., Hu, X.-Z. & Wittmann, F.H. 2003b. Thickness effect on fracture energy of cementitious materials. Cem. Concr. Res. 33: 499-507.

Duan, K., Hu, X.-Z. & Wittmann, F.H. 2006. Scaling of quasi-brittle fracture: Boundary and size effect. Mech. Mater. 38: 128-141.

Guinea, G.V, Elices, M., Planas, J. 1996. Stress intensity factors for wedge-splitting geometry. Int.J.Fract. 81: 113-124.

Hillerborg, A., Modéer, M. & Petersson, P-E. 1976. Analysis of crack formation and crack growth in concrete by means of fracture mechanics and finite elements. Cem. Concr. Res. 6: 773-782.

Hu, X.-Z. & Duan, K. 2004. Influence of fracture process zone height on fracture energy of concrete. Cem. Concr. Res. 34: 1321-1330.

Hu, X.-Z. & Wittmann, F.H. 1992. Fracture energy and fracture process zone. Mater. Struct. 25: 319-326.

Hu, X.-Z. & Wittmann, F.H. 2000. Size effect on toughness induced by crack close to free surface. Eng. Fract. Mech. 65: 209-221.

Karihaloo, B.L. 1995. Fracture mechanics and structural concrete. New York: Longman Scientific & Technical.

Karihaloo, B.L., Abdalla, H.M. & Imjai, T. 2003. A simple method for determining the true specific fracture energy of concrete. Mag. Concr. Res. 55: 471-481.

Karihaloo, B.L., Abdalla, H., Xiao, Q.Z. 2003. Coefficients of the crack tip asymptotic field for wedge splitting specimens. Engrg. Fract. Mech. 70: 2407-2420.

Kim, J.K. & Kim,Y.Y. 1999. Fatigue crack growth of high-strength concrete in wedge-splitting test. Cem. Concr. Res. 29: 705-712.

Knésl, Z. & Bednář, K. 1998. Two-parameter fracture mechanics: Calculation of parameters and their values (in Czech). Brno: Institute of Physics of Materials, Czech Academy of Sciences, 1998.

Knésl, Z., Bednář, K., Radon, J.C. 2000. Influence of T-stress on the rate of propagation of fatigue crack. Physical Mesomechanics: 5-9.

Linsbauer, H.N. & Tschegg, E.K. 1986. Fracture energy determination of concrete with cube-shaped specimens. Zement und Beton 31: 38-40.

Löfgren, I., Stang, H., Olesen, J.F. 2005. Fracture properties of FRC determined through inverse analysis of WS and TPB tests. J. Adv. Concr. Techn. 3: 423-434.

Murakami, Y., et al., 1987. Stress Intensity Factor Handbook I, II, III. Oxford: Pergamon Press.

Nallathambi, P. & Karihaloo, B.L. 1986. Determination of specimen-size independent fracture toughness of plain concrete. Mag. Concr. Res. 38: 67-76.

Østergaard, L. 2003. Early-age fracture mechanics and cracking of concrete. Experiments and modelling. PhD Thesis, Department of Civil Engineering, DTU, Lyngby, Denmark.

RILEM Committee FMT 50 1985. Determination of the fracture energy of mortar and concrete by means of three-point bend test on notched beams, Mater. Struct. 18: 285-290.

RILEM Report 5 1991. Fracture Mechanics Test Methods for Concrete. Shah, S.P. et al. (eds.). London.

Řoutil, L., Veselý, V., Keršner, Z., Seitl, S. & Knésl, Z. 2008. Fracture process zone size and energy dissipated during crack propagation in quasi-brittle materials. In Pokluda, J. et al. (eds.), Proc. of 17th European Congress on Fracture – ECF 2008 (book of abstracts + CD-ROM), Brno, Czech Republic: 97 + CD 8 p. Brno: Vutium.

Seitl, S., Dymáček, P., Klusák, J., Řoutil, L., Veselý, V. 2009a. Two-parameter fracture analysis of wedge splitting test specimen. In Topping, B.H.V. et al. (eds.), Proc. of the 12th Int. Conf. on Civil, Structural and Environmental Engineer-

ing Computing, Funchal, Portugal. Stirling: Civil-Comp Press.
Seitl, S., Hutař, P., Veselý, V., Keršner, Z. 2009b. T-stress values during fracture in wedge splitting test geometries: a numerical study. In Brandt, A. & Olek, J. (eds.): Proc. of Brittle Matrix Composites 9, Warsaw, Poland. Cambridge: Woodhead Publishing Ltd. 419-428.
Shah, S. P., Swartz, S. E. & Ouyang, C. 1995. Fracture mechanics of structural concrete: applications of fracture mechanics to concrete, rock, and other quasi-brittle materials. New York: John Wiley & Sons.
Tan, C.L., Wang, X., 2003 The use of quarter-point crack-tip elements for T–stress determination in boundary element method analysis. Engrg Fract. Mech. 70: 2247-2252.
Tada, H., Paris, P. C. & Irwin, G. R. 2000. The stress analysis of cracks handbook, 3rd ed. Bury St. Edmunds: Professional Engineering Publishing, Ltd.
Trunk, B. & Wittmann, F. H. 2001. Influence of size on fracture energy of concrete. Mater. Struct. 34: 260-265.
van Mier, J. G. M. 2007. Fracture Processes of Concrete: Assessment of Material Parameters for Fracture Models. Boca Raton : CRC Press.
Veselý, V. & Frantík, P. 2008. ReFraPro – Reconstruction of Fracture Process, Java application.
Veselý, V. & Frantík, P. (in prep.). Reconstruction of fracture process zone during tensile failure of quasi-brittle materials. Submitted to journal Computational Mechanics.
Veselý, V., Frantík, P. & Keršner, Z. 2009. Cracked volume specified work of fracture. In Topping, B.H.V. et al. (eds.), Proc. of the 12th Int. Conf. on Civil, Structural and Environmental Engineering Computing, Funchal, Portugal. Stirling: Civil-Comp Press.
Veselý, V., Řoutil, L. & Keršner, Z. 2007. Structural geometry, fracture process zone and fracture energy. In Carpinteri, Al. et al. (eds.): Proc. of FraMCoS 6, Catania, Italy: vol. 1, 111-118. Taylor & Francis/Balkema.
Williams, M. L. 1957. On the stress distribution at the base of stationary crack. ASME J. Appl. Mech. 24: 109-114.
Xiao, Q.Z., Karihaloo, B.L. 2007. An overview of a hybrid crack element and determination of its complete displacement field. Engrg Fract. Mech. 74: 1107-1117.
Xu, S., Bu, D., Gao, H., Yin, S., Liu, Y. 2007. Direct measurement of double-K fracture parameters and fracture energy using wedge-splitting test on compact tension specimens with different size. In Carpinteri, Al. et al. (eds.): Proc. of FraMCoS 6, Catania, Italy: vol. 1, 271-278. Taylor & Francis/Balkema.
Yang, B., Ravi-Chandar, K. 1999. Evaluation of elastic T-stress by the stress difference method. Engrg Fract. Mech. 64: 589-605.