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# Relationship between $G_F$ and $G_f$

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ABSTRACT: The present study is an attempt to understand the relationship between $G_F$ from work of fracture and $G_f$ from size effect methods. The $G_F$ increases with an increase in the compressive strength and also with an increase in the notch- depth ratio, while $G_f$ decreases slightly with an increase in compressive strength. Therefore the relationship between the fracture energy measured by the work of fracture method, $G_F$ and by the size effect method ($G_f$) is not constant but is influenced by the compressive strength.

## 1 INTRODUCTION

Fracture behavior of plain concrete is the basis for all the studies on behavior of reinforced concrete and prestressed concrete structures via fracture mechanics. Experimental studies have been conducted to ascertain the effect of aggregate on the fracture behavior of concrete. It is reported that an increase in the maximum size of aggregate decreases the brittleness of hardened concrete and increases the fracture energy as well as fracture toughness Amparano et al. (2000), Appa Rao & Raghu Prasad (2002), and Strange & Bryant (1979). As pointed out by Bazant & Pfeiffer (1987), Bazant & Kazemi (1990), the fracture energies (GF , Gf) are two different material characteristics. The total fracture energy GF is obtained by the area under the complete load-deflection curve, and the fracture energy Gf represents the area under the initial tangent of the softening curve. It has been shown by using statistical analysis on 230 samples that the fracture energy Gf Bazant & Giraudon (2002) increases with increasing compressive strength and the same has been pointed out by other researchers Ta-Peng & Mei-Miao (1996) and Perdikaris & Romeo (1995). Many investigators have also obtained the ratio GF/Gf viz. the ratio of the fracture energy measured by the work-of-fracture method to that by size effect model (SEM), as shown in Table1.

On the contrary, in a later paper by Bazant & Pfeiffer (1987) and Alexander (Karihaloo & Nallathambi 1991), it is shown that $G_f$ decreases as compressive strength increases. Hence, it has become a contentious topic in the fracture mechanics of concrete, which needs further investigation.

Table 1. Values of GF/Gf (from literature).

| Reference | $G_F$ /$G_f$ |
|---|---|
| Bazant & Giraudon (2002) and Bazant & Becq-Giraudon (2001) | 2.5 |
| Bharatkumar et al. (2005) | 2.6 |
| Einsfeld & Velasco (2006) | 2.8 |
| Navalurkar & Hsu (2001) | about 2.0 |

If the ratio $G_F/G_f$ has to be constant or nearly constant, the functional relationship of $G_F$ & $G_f$ independently with the compressive strength has to be the same, which does not appear to be so when one looks at the values reported in literature.

## 2 RESEARCH SIGNIFICANCE

Although numerous studies on relationship between $G_F/G_f$ are reported in literature, still it appears to be open for discussion, and particularly in self consolidating concrete (SCC) as microstructure of SCC is much different compared to that of normal concrete (NC) or high performance concrete (HPC). Therefore a study on the ratio $G_F/G_f$ will be needed. With that aim, the objectives of the present work are; study of variation on $G_F$ and $G_f$ with compressive strength of concrete based on the present experimental investigation and compare with the values reported in literature.

## 3 EVALUATION OF FRACTURE CHARACTERISTICS

### 3.1 *Size effect model (SEM)*

As a consequence, different values for the fracture energy are obtained for specimens of different sizes. In an alternative method proposed by Bazant & Pfeiffer (1987), the fracture energy is determined from the size effect law. If geometrically similar beams are loaded up to rupture and extrapolated to a beam of infinite dimensions, the fracture energy must have one single value, regardless of the type, size or shape of the specimen. Bazant & Pfeiffer suggested the following relationship

$$\sigma_N = \frac{Bf_t}{(1+b/b_0)^{1/2}} \tag{1}$$

where $\sigma_N = C_n \frac{P_u}{bd}$ is the nominal stress at failure, $f_t$ is tensile strength, $P_u$ is ultimate load and $C_n$ is a coefficient introduced for convenience. The coefficients B and $b_0$ are determined by linear regression, b & d are width and depth of the specimen respectively.

For this purpose, Equation 1 applicable to geometrically similar specimens of different sizes, could be algebraically rearranged to a linear regression plot Y= AX + C, in which

$$Y = (\frac{1}{\sigma_N})^2 \text{ ; X =b; b0 =C / A; B =} 1/\sqrt{C} \tag{2}$$

The plastic region around the concrete fracture zone for infinite size is relatively small. In this case, the fracture energy ($G_f$) and fracture process zone ($c_f$) respectively for infinitely large specimens are calculated as:

$$G_f = \frac{g(\alpha_0)}{AE} \tag{3}$$

$$C_f = \frac{g(\alpha_0)C}{g'(\alpha_0)A} \tag{4}$$

$$g(\alpha_0) = (S/b)^2 \pi\alpha[1.5g_1(\alpha_0)]^2 \tag{5}$$

$$g'(\alpha_0) = dg(\alpha_0)/da$$

where E is the Young's modulus of elasticity of the concrete, A is the angular coefficient of the linear regression plot, $g'(\alpha_0)$ is the non-dimensional energy release rate calculated Where E is the Young's modulus of elasticity of the concrete, A is the angular coefficient of the linear regression plot, $g'(\alpha_0)$ is the non-dimensional energy release rate calculated according to LEFM and $\alpha_0$ is the relative notch-depth ratio (a/b).

### 3.2 *Fracture energy (GF) from work-of-fracture*

Many methods have been recommended to determine the fracture energy and characteristic length, using simple three points bend test Mindess (1984), Petersson (1980a), Petersson (1980b), Tang et al. (1996), Belhamel et al. (2002), Elices et al. (1997), NT Build 491 (1999); Guinea et al. (1992) and Elices et al. (1992). One can apply the recommendation of the Technical Committee RILEM FMC-50 (1985) to perform three-point bend tests in notched beams. The Fracture energy is defined as the amount of energy necessary to create a crack of unit surface area projected in a plane parallel to the crack direction. As the beam is split into two halves, the fracture energy can be determined by dividing the total dissipated energy by the total surface area of the crack. According to the RILEM FMT 89 (1990) fracture energy can be calculated as

$$G_F = \frac{W_0 + 2mg\delta_0}{t(b-a)} \tag{6}$$

where $G_F$ is the fracture energy (N/m), $W_0$ is the area under the load-deflection curve, m is the weight of the beam between supports (kg), t is the thickness, b is the depth, $\delta_0$ is the displacement corresponding to almost zero load in the softening portion of load-displacement curve and 'a' is the initial notch of the beam.

## 4 STUDY BASED ON LITERATURE

### 4.1 *Influence of compressive strength on $G_F$*

In finite element analysis of fracture problems, the fracture energy obtained from the load deflection curve is being used as one of the inputs in addition to the compressive strength of concrete. Following the recommendations of RILEM, it would be much easier to obtain fracture energy based on work of fracture. As is well known, $G_F$ is size dependent. However, the relationship between $G_F$ and compressive strength has been direct. It has been found from 20 extensive research work that $G_F$ increases by 10% for an increase of 30% in compressive strength Gettu et al. (1990) compared GF of high strength concrete with normal concrete and concluded that $G_F$ increases by 12% for an increase of 160% in compressive strength. Investigations by many researchers Appa Rao & Raghu Prasad (2002), Bazant & Giraudon (2002), Bharatkumar et al. (2005), Einsfeld & Velasco (2006), Gettu et al. (1990) and Wu et al. (2001) have shown that increase in compressive strength of concrete results in higher fracture energy $G_F$.

### 4.2 *Influence of compressive strength on $G_f$*

As pointed out by Bazant and Giraudon (2002) and other researchers Ta-Peng & Mei-Miao (1996) and Perdikaris & Romeo (1995), the fracture energy $G_f$ increases with increase in compressive strength. On the contrary, Bazant & Pfeiffer (1987), Alexander (Shah et al. 1995) and Einsfeld & Velasco (2006) have obtained the values of $G_f$ which show a decrease with increase in compressive strength as shown in Table 2.

Interestingly it may be observed from the expression given by Bazant for $G_f$ from SEM, viz $G_f = g(\alpha_0)/AE$ that $G_f$ may decrease or increase with compressive strength depending on the values of A and E. In the above expression $g(\alpha_0)$ is a geometric parameter depending on notch to depth and span to depth ratios:

A is slope of the regression plot of $(1/\sigma_N)^2$ vs. depth b, E is the Elastic modulus of concrete.

Assuming $g(\alpha_0)$ to be a constant, for different strengths of concrete, in specimens with geometric similarity, the product AE can vary differently. E increases with $f_c$ while the slope A of the regression plot may not have a definite trend of variation with the compressive strength although it is clear that the intercept on the vertical axis decreases or increases with the increase or decrease of the compressive strength respectively.

Consequently the value $G_f$ may decrease or increase with compressive strength as further detailed below:

$$(G_f)_i = g(\alpha_0)/(AE)_i$$

here i indicates different situations.

Assuming $g(\alpha_0)$ = constant for any two different compressive strengths $f_{c1}$, $f_{c2}$ such that $f_{c1} < f_{c2}$ and therefore $E_1 < E_2$. Any of the following three situations are possible

i) $(G_f)_1 = (G_f)_2$ if $A_1 > A_2$ such that $(AE)_1 = (AE)_2$

ii) $(G_f)_1 > (G_f)_2$ if $A_1 \leq A_2$ such that $(AE)_1 < (AE)_2$

iii) $(G_f)_1 < (G_f)_2$ if $A_1 > A_2$ such that $(AE)_1 > (AE)_2$

Therefore the question is whether A depends on fc and if so, in what manner?. From the investiga-

Table 2. Values of $G_f$ from three points bend test.

| Reference | S (mm) | b (mm) | t (mm) | fc (MPa) | E (GPa) | a0/b | A | Gf (N/m) |
|---|---|---|---|---|---|---|---|---|
| Alexander | 400 | 100 | 100 | 29 | 32.5 | 0.4 | 0.027 | 71.06 |
| Alexander | 800 | 200 | 100 | 29 | 32.5 | 0.4 | 0.027 | 71.06 |
| Alexander | 2000 | 500 | 100 | 29 | 32.5 | 0.4 | 0.027 | 71.06 |
| Alexander | 400 | 100 | 100 | 26.3 | 32 | 0.2 | 0.00709 | 94.06 |
| Alexander | 800 | 200 | 100 | 26.3 | 32 | 0.2 | 0.00709 | 94.06 |
| Alexander | 1200 | 300 | 100 | 26.3 | 32 | 0.2 | 0.00709 | 94.06 |
| Alexander | 2000 | 500 | 100 | 26.3 | 32 | 0.2 | 0:00709 | 94.06 |
| Alexander | 3200 | 800 | 100 | 26.3 | 32 | 0.2 | 0:00709 | 94.06 |
| Bazant-Pfeiffer | 95 | 38 | 38 | 34.1 | 27.7 | 0.167 | 0.00598 | 36.6 |
| Bazant-Pfeiffer | 191 | 76 | 38 | 34.1 | 27.7 | 0.167 | 0.00598 | 36.6 |
| Bazant-Pfeiffer | 381 | 152 | 38 | 34.1 | 27.7 | 0.167 | 0.00598 | 36.6 |
| Bazant-Pfeiffer | 762 | 305 | 38 | 34.1 | 27.7 | 0.167 | 0.00598 | 36.6 |
| Bazant-Pfeiffer | 95 | 38 | 38 | 47.6 | 32.9 | 0.167 | 0.00873 | 23.67 |
| Bazant-Pfeiffer | 191 | 76 | 38 | 47.6 | 32.9 | 0.167 | 0.00873 | 23.67 |
| Bazant-Pfeiffer | 381 | 152 | 38 | 47.6 | 32.9 | 0.167 | 0.00873 | 23.67 |
| Bazant-Pfeiffer | 762 | 305 | 38 | 47.6 | 32.9 | 0.167 | 0.00873 | 23.67 |
| Einfeld-Velasco | 95.25 | 38.1 | 38.1 | 65 | 33.7 | 0.167 | 0.00343 | 52.44 |
| Einfeld-Velasco | 190.5 | 76.2 | 38.1 | 65 | 33.7 | 0.167 | 0.00343 | 52.44 |
| Einfeld-Velasco | 381 | 152.4 | 38.1 | 65 | 33.7 | 0.167 | 0.00343 | 52.44 |

tions available in literature there is no definite conclusion about the manner in which A varies. There is scope for further work on the above. However, until such a conclusion, it is better to conclude that the ratio GF /Gf depends on the compressive strength in the case of high strength and high performance concretes as well as SCC, besides it depending on type and size of aggregate, age of concrete and the rate of loading.

## 5 PRESENT EXPERIMENTAL STUDY

The cement used in the present study is 53 MPa [7685 psi] grade, and crushed granite aggregates of maximum size 16 mm [0.63 in] were used. The specific gravity of the sand was 2.62 and the fineness modulus was 2.48. Class F Fly ash from the thermal power plant near Raichur, India, was used. The quantities of different materials for various mixes of SCC are listed in Table 3.

Table 3. Quantities of material for SCC kg/$m^3$.

| Materials | SCC1 | SCC2 | SCC3 |
|---|---|---|---|
| Cement (kg) | 240 | 400 | 360 |
| Water (kg) | 220 | 180 | 190 |
| Fine Agg. (kg) | 900 | 900 | 900 |
| Coarse Agg. (kg) | 830 | 830 | 830 |
| Fly ash (kg) | 184 | 200 | 196 |
| Silica fume (kg) | 12 | 36 | 29 |
| HRWR(litre) | 2 | 4 | 3.5 |
| AEA(litre) | 0.2 | 0.24 | 0.4 |

Note: (1 kg = 2.20462 lb, HRWR= high –range water reducing admixture, Fine Agg. = Fine aggregate, Coarse Agg. = coarse aggregate, AEA= air entraining admixture).

### 5.1 *Fracture energy (GF, $G_f$)*

To study the size effect, beam specimens cast with SCC were employed and tested under three point bend condition. The beam specimens had two dimensional geometrical similarity viz.; (i) the ratio of the span to the depth of the beam (s/b) was 4 for all the specimens, (ii) the ratio of the notch length $a_0$ to the beam depth b was 0.33 for all the specimens and (iii) the ratio of the total length to the beam span was 1.2. Geometrically similar specimens with three different sizes were used in order to allow the fracture energy evaluation through the SEM besides the work-of fracture method. The specimens had depths (b) of 50, 100 and 200 mm [1.97, 3.94 and 7.88 in]. All the specimens had the same thickness (t) of 50 mm [1.97 in], same length equal to 4.2b, and same span equal to 4b. The maximum size of the aggregate was (16 mm [0.63 in]) to make sure that the thickness of the beams is kept equal to three times larger than the maximum aggregate size. A total of 40 specimens, divided into three series of concrete batches (SCC1 to SCC3) were cast. Each series contained six specimens for each of the batches. The equipment used to test the beam specimens was the same viz. 'DARTEC' servo controlled machine with three channels for data acquisition and a load cell of 50 kN [11250 lb] capacity. All the tests were conducted under the CMOD control. The notch was pre made using an acrylic plate with the thickness of 2 mm [0.079 in] in the time of casting. Figure 1 shows typical load-deflection and load-CMOD diagrams for a typical specimen.

The fracture energy, $G_f$, can be calculated by using Equation (3) together with the values of B, $b_0$, and E of each group of specimens with a specific $f_c$. The non dimensional energy release rate $g(\alpha_0)$ is obtained from Equation (5) for g(0.33).

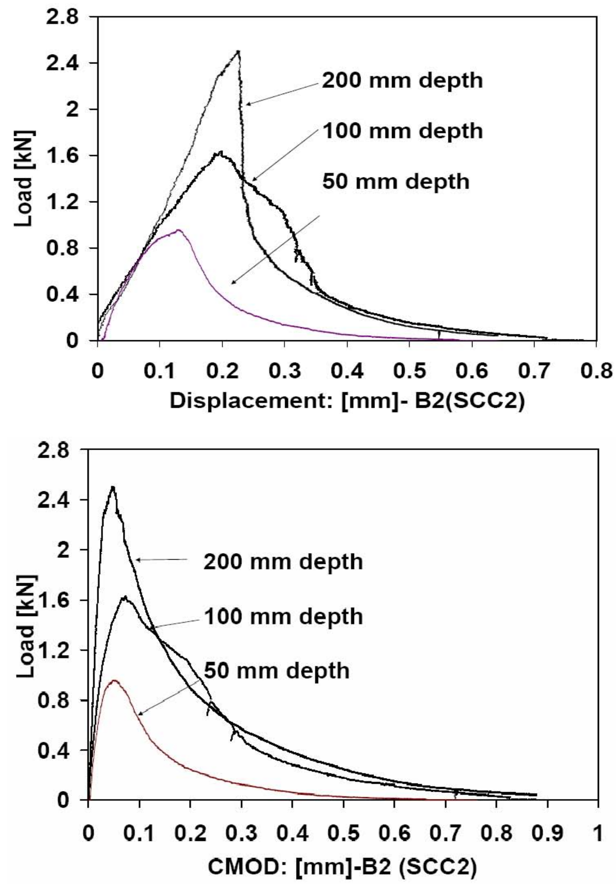


Figure 1. Typical load vs CMOD/Deflection for mix B2.

Fracture energy ($G_F$,$G_f$) according to the work of fracture and size effect law methods are listed in Table 4.

The standard requirements concerning the limiting values of the coefficient of variation of the slope of the regression line ($w_A$) and the relative width of scatter-band (m) and the values of $w_A$ and m should not exceed 0.10 and 0.20 (Bazant & Kazemi (1990)) respectively, have been satisfied.

The test results of SCC are plotted in Figure 2 and are compared with those of HPC resulting from the work of Bharatkumar (Bharatkumar et al. 2005), for beams with depths (b) equal to 50, 100 and 200

Table 4. Fracture characterization obtained from the Size effect law, S=4b.

| Series | b | a | fc | E | a/b | P 0 | GF | Gf | AB | wA | m |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | mm | mm | MP a | GP a | | N | N/m | N/m | mm-1Mpa-2 | | |
| SCC1 | 50 | 16.5 | 15 | 17.4 | 0.33 | 720 | 67 | 39 | 0.044 | 0.086 | 0.102 |
| SCC1 | 100 | 33 | 15 | 17.4 | 0.33 | 1360 | 82 | 39 | 0.044 | 0.086 | 0.102 |
| SCC1 | 200 | 66 | 15 | 17.4 | 0.33 | 2250 | 90 | 39 | 0.044 | 0.086 | 0.102 |
| SCC2 | 50 | 16.5 | 30 | 24.06 | 0.33 | 880 | 88 | 34 | 0.045 | 0.4132 | 0.095 |
| SCC2 | 100 | 33 | 30 | 24.06 | 0.33 | 1590 | 95 | 34 | 0.045 | 0.4132 | 0.095 |
| SCC2 | 200 | 66 | 30 | 24.06 | 0.33 | 2580 | 103 | 34 | 0.045 | 0.4132 | 0.095 |
| SCC3 | 50 | 16 | 45 | 30.02 | 0.33 | 1035 | 93 | 32 | 0.045 | 0.5049 | 0.153 |
| SCC3 | 100 | 33 | 45 | 30.02 | 0.33 | 1920 | 105 | 32 | 0.045 | 0.5049 | 0.153 |
| SCC3 | 200 | 66 | 45 | 30.02 | 0.33 | 2800 | 108 | 32 | 0.045 | 0.5049 | 0.153 |

[1.97, 3.94 and 7.88 in] mm and compressive strength of 47 MPa [6815 psi]. The plot Figure 2 reveals that high performance concrete (Bharatkumar et al. 2005) of 47 MPa [6815 psi] is relatively more brittle than SCC of 45 MPa [6815 psi].

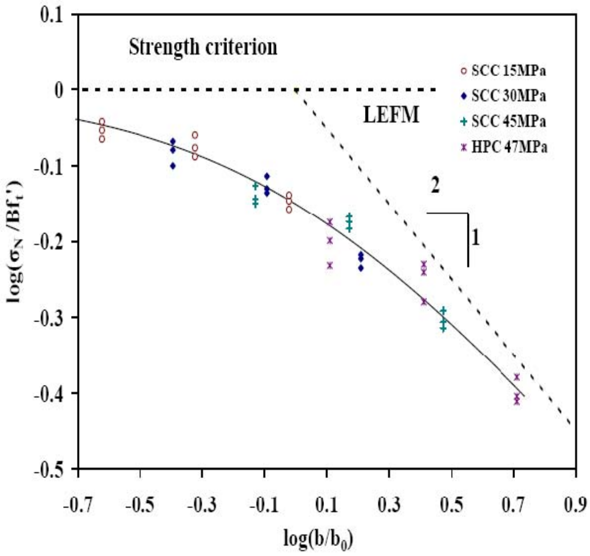


Figure 2. Comparison of size effect plot for SCC and HPC (Bharatkumar 2005) [1 MPa=145 psi].

The point pertaining to HPC is to the right of SCC. It can be interpreted that, the SCC is less size dependent than HPC. By contrast, Figure 2 gives rather systematic results and the regular shift to the right with increasing strength nicely documents increasing brittleness. From Table 4, it can be observed that $G_F$ increases with an increase in depth of the beam as well as with the compressive strength $f_c$ of concrete. On the contrary the fracture energy from SEM viz $G_f$ increases as the compressive strength $f_c$ decreases, as earlier observed by Bazant & Pfeiffer (1987).

## 6 CONCLUSIONS

Besides reconfirming the earlier conclusion for HPC and HSC, for SCC also, i.e. the fracture energy $G_F$ increases with the depth of the beam as well as with the compressive strength of concrete a few other conclusions which could be considered as contribution from this work are:

1) When GF of HPC is compared with GF of SCC, it can be seen that SCC being more ductile, has a value of GF more than that of HPC for the same strength. In literature $G_F/G_f$ of HSC has been reported and not that of SCC. This work is an attempt towards that direction.

2) The fracture energy GF increases with compressive strength more at lower strengths than at higher strengths, while the fracture energy $G_f$ uniformly decreases with the compressive strength from low to high.

3) Because of the above, the ratio $G_F/G_f$ can not be considered as a constant, particularly for SCC. It is appropriate to consider it as dependent on the compressive strength.

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