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Effect of loading condition, specimen geometry, size-effect and softening function on double-K fracture parameters of concrete

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ABSTRACT: The paper presents numerical investigation of the influence the specimen geometry, loading condition, size-effect and softening function of concrete on the double-*K* fracture parameters. The input data needed for computation of the double-*K* fracture parameters are obtained from the well known version of Fictitious Crack Model (FCM). FCM is developed for three standard specimens: three-point bend test, compact tension specimen and four-point bend test of size range 100-600 mm at relative size of initial crack length 0.3. The analysis of numerical results shows some interesting behavior of the double-*K* fracture parameters.

1 INTRODUCTION

The application of linear elastic fracture mechanics (LEFM) concept to concrete was first attempted in early 1960s and thereafter tremendous advancement took place in the field of fracture mechanics of concrete. In the past, it has been shown that the LEFM cannot be directly applied to study the crack propagation phenomena in concrete because of a large and variable size of fracture process zone that exists ahead of a macrocrack when the specimen is loaded. This resulted in development of several nonlinear fracture models based on two basic approaches: first using numerical approach such as cohesive crack model (CCM) or fictitious crack model (FCM) (Hillerborg et al. 1976, Petersson 1981, Carpinteri 1989, Planas & Elices 1991) and crack band model (CBM) (Bažant & Oh 1983) and the second using modified LEFM concept such as two parameter fracture model (TPFM) (Jenq & Shah 1985), size effect model (SEM) (Bažant et al. 1986), effective crack model (ECM) (Nallathambi & Karihaloo 1986), K_R-curve method based on cohesive force distribution (Xu & Reinhardt 1998, 1999a), double-*K* fracture model (DKFM) (Xu & Reinhardt 1999a-b) and double-*G* fracture model (DGFM) (Xu & Zhang 2008).

The fracture models attributed to modified LEFM concept such as DKFM, DGFM and the K_R-curve method associated with cohesive force distribution capture the three important stages of crack propagation in concrete: crack initiation, stable crack propagation and unstable crack propagation while the other models under the same category such as TPFM, SEM and ECM predict the fracture loads at critical condition only. The DKFM is characterized by two material parameters: initial cracking toughness K_{IC}^{ini} and unstable fracture toughness K_{IC}^{un}. This method averts the need of closed-loop testing arrangement. Based on fracture tests, Xu & Reinhardt 1999a determined the values of K_{IC}^{ini} and K_{IC}^{un} for three-point bend test (TPBT) and compact tension (CT) specimen geometries. In the study, it was reported that both the fracture parameters K_{IC}^{ini} and K_{IC}^{un} were dependent on material properties and independent of specimen geometry and size. Xu & Reinhardt (1999b, c) developed analytical methods for TPBT and CT or wedge splitting test (WST) to determine the values of double-*K* fracture parameters. From the available experimental results, it was also shown that double-*K* fracture parameters K_{IC}^{ini} and K_{IC}^{un} were not dependent on size of the specimen. In addition, it was also shown these fracture parameters were not significantly influenced by softening function of concrete. Xu & Reinhardt (1999c) computed the material fracture parameters K_{IC}^{ini} and K_{IC}^{un} based on the fracture tests of small size wedge-splitting specimens. In the study it was observed that these fracture parameters were independent of the relative size of initial notch length and the thickness of specimens but they were slightly dependent on the specimen size. Zhao & Xu (2002) reported numerical experiments on TPBT specimens with the objective to study the influence of span/depth ratio, specimen size and strength of concrete on the double-*K* fracture parameters. In the study, it was shown that the unstable fracture toughness was independent of the span/depth ratio and dependent on the material property whereas the initial fracture toughness varied with the strength of material, span/depth ratio and depth of specimen. Zhang et al. (2007) carried out the fracture tests on the total of 43 concrete specimens (three-points bending beam and wedge splitting specimen) with the small-size aggregates being the maximum size of 10 mm for analyzing the double-*K* fracture parameters. In the study, the ratio of initial cracking load to maximum load was found to be in the range 0.67-0.71 and the ratio of initial fracture toughness to unstable fracture toughness was found to be in the range 0.45-0.50. Xu & Zhu (2009) conducted the fracture tests on different sizes of TPBT specimens made of hardening cement paste and mortar of different strengths. The test results showed that fracture behavior of all cementitious composites, even hardening cement paste and mortar was nonlinear. In addition, double-*K* fracture model was applied to calculate the fracture parameters and it was concluded that this fracture criterion was applicable to hardening cement paste and mortar.

Recent experimental results and analysis available in the literature shows that the double-*K* fracture parameters are independent of specimen size and geometry. Several methods such as analytical method (Xu & Reinhardt 1999b) and simplified approach (Xu & Reinhardt 2000) are also available in literature for computing these fracture parameters. Recently the authors (Kumar & Barai 2008, 2009a) introduced the weight function method for determining the double-*K* fracture parameters with better computational efficiency as well as without loss of accuracy. The authors (Kumar & Barai 2008) also showed that the double-*K* fracture parameters were influenced by the specimen geometry, size-effect and the relative size of initial crack length. Prediction of size-effect from double-*K* fracture parameters was also formulized in the subsequent study (Kumar & Barai 2009b). It was further reported that the double-*K* fracture parameters marginally depended on the loading condition (Kumar & Barai 2009c). Some more results such as influence of softening function of concrete on the double-*K* fracture parameters are addressed in the present paper and comprehensive results are thoroughly analyzed.

The aim of the present study is to explore the dependency of the double-*K* fracture parameters on the various loading and geometrical parameters in a precise manner. The input data needed for computation of the double-*K* fracture parameters are obtained from the well known version of Fictitious Crack Model. For this purpose, the FCM is developed for three standard specimens: TPBT, four-point bend test (FPBT) and CT specimen of size range 100-600 mm at initial crack length/depth (a_o/D) ratio of 0.3. The effect of specimen geometry and size-effect is investigated between the TPBT and CT specimen geometries whereas the effect of loading condition on the fracture parameters is investigated between the TPBT and FPBT. Finally, effect of softening function of concrete on the double-*K* fracture parameters is studied using TPBT specimen.

2 DIMENSIONS OF TEST SPECIMENS

The present investigation is attributed to the determination of the double-*K* fracture parameters for TPBT, FPBT and CT specimen (Fig. 1) for which the input data needed for computation are obtained from the FCM.

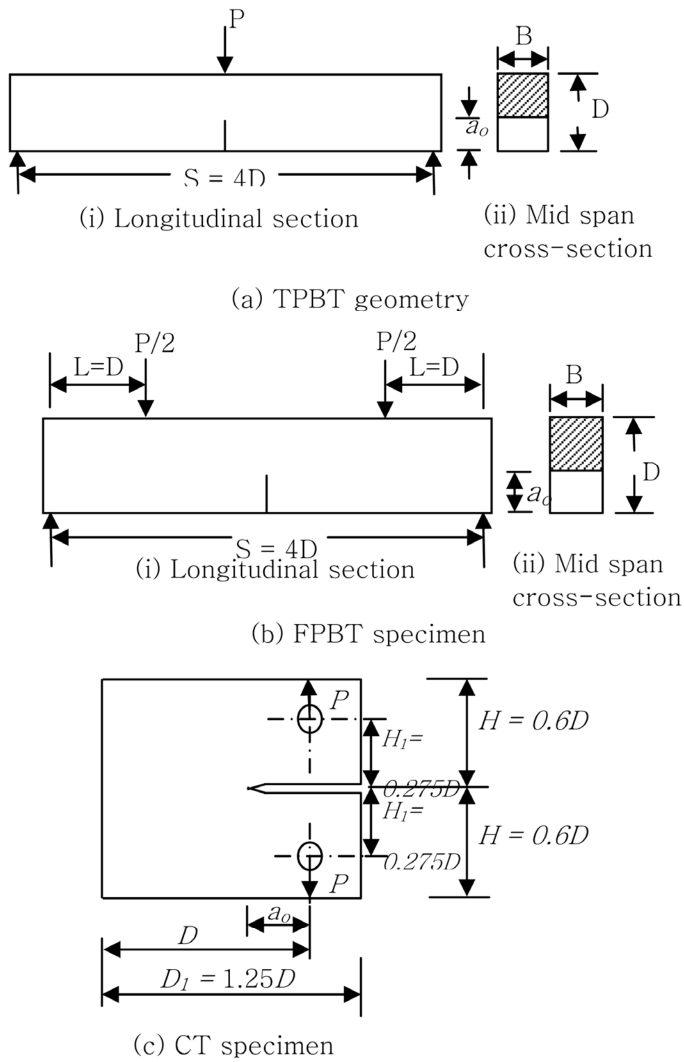

Figure 1. Dimensions and loading schemes.

The standard dimensions of the TPBT (RILEM Technical Committee (50-FMC) 1985) are shown in Figure 1 (a) in which the symbols: *B, D* and *S* are the width, depth and span respectively for TPBT geometry with $S/D = 4$. The study for the effect of loading condition on the fracture parameters can be carried out by comparing the gained results between the TPBT and FPBT. In this case, two different loading conditions with the same specimen geometry in all respect can be obtained for precise comparison of loading effect on the fracture parameters. The loading and support conditions along with the dimensions of the FPBT considered in the study are shown in Figure 1(b); in which, the symbols: *B, D* and *S* are the width, depth and support span respectively with $S/D = 4$ and moment arm $L=D$. The load span is taken to be 2D. This four-point bend specimen resembles exactly the same geometrical properties as of the standard three-point bending test geometry except for only difference in loading condition. The dimensions and configuration of standard CT specimen according to the ASTM standard E-399 (2006) are shown in Figure 1(c). The dimensioning should comply $D_1 = 1.25D$, $H = 0.6D$, $H_1 = 0.275D$ and the specimen thickness $B = 0.5D$. The effect of specimen geometry on the fracture parameters can be investigated by comparing the results gained between the TPBT and CT specimen.

3 MATERIAL PROPERTIES AND FICTITIOUS CRACK MODEL

Three material properties such as modulus of elasticity E, uniaxial tensile strength f_t, and fracture energy G_F are required to describe the FCM or CCM (Hillerborg et al. 1976, Petersson 1981, Carpinteri 1989, Planas & Elices 1991). The FCM for TPBT, FPBT and CT specimens is developed in the present study. In this method, the governing equation of crack opening displacement (COD) along the potential fracture line is written. The influence coefficients of the COD equation are determined using linear elastic finite element method. Four nodded isoparametric plane element is considered for finite element calculation. The COD vector is partitioned according to the enhanced algorithm introduced by Planas and Elices (1991). Finally, the system of nonlinear simultaneous equation is developed and solved using Newton-Raphson method. The concrete mix for which $f_t = 3.21$ MPa, $E = 30$ GPa, $G_F = 103$ N/m, maximum size of aggregates d_a=16 mm and Poisson's ratio $\nu = 0.18$ along with quasi-exponential softening function (Planas & Elices, 1990) are taken into account for the present investigation. For standard TPBT, FPBT and CT specimens with $B = 100$ mm having size range D=100-600 mm, the finite element analysis is carried out for which the half of the specimens are discretized due to symmetry as shown in Figure 2 and 80 numbers of equal isoparametric plane elements are considered along the dimension D.

Two well-known relations of FCM *i.e.*, the characteristic length $l_{ch} = EG_F/f_t^2$ and the critical value of stress intensity factor $K_C = \sqrt{(G_F E)}$ are also used in comparison of numerical investigation.

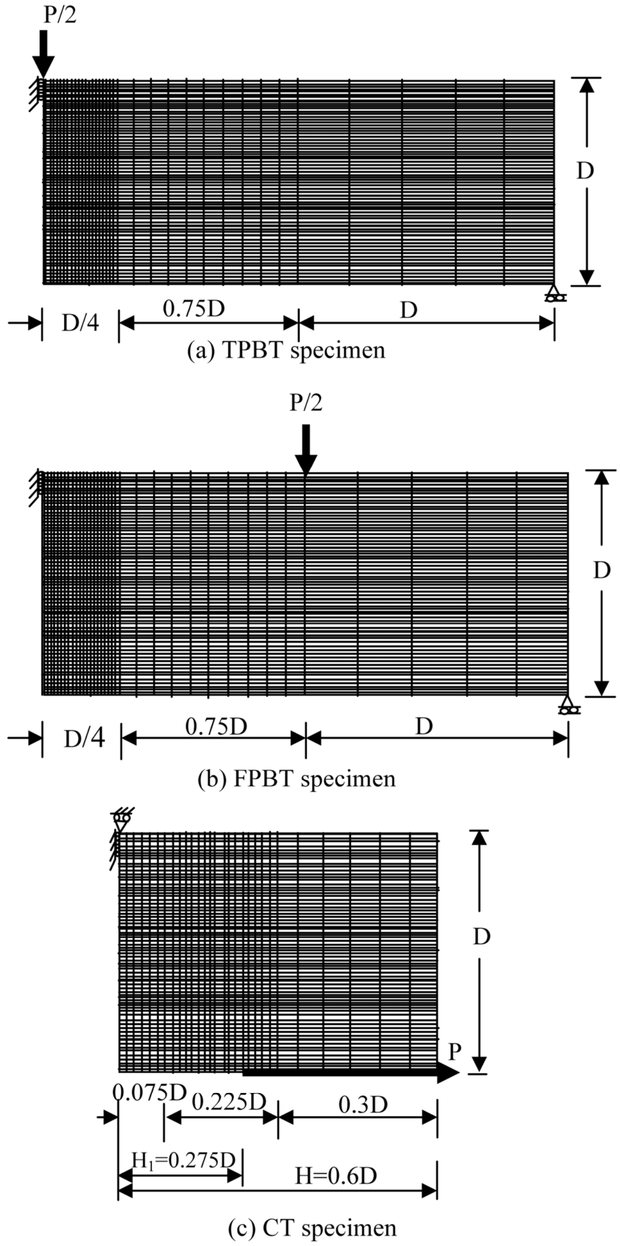

Figure 2. Finite element discretization.

4 NUMERICAL COMPUTATIONS

For the precise numerical investigation, the effect of self-weight in TPBT and FPBT geometries can be accounted for at all computation stages whereas this effect cannot be obviously considered for the CT specimen. In the TPBT and FPBT specimens, the input data are gained using the developed FCM which accounts for the effect of self-weight of the beam. Therefore, the FCM yields the external load P without inclusion of the self-weight whereas the crack mouth opening displacement (CMOD) consists of the contribution of the self-weight. Hence, in addition to the external load P acting on the TPBT and FPBT, the influence due to a concentrated load equal to $w_g.S/2$ (w_g being the self-weight of the beam per unit length) and acting at the mid span, is also effective during all stages of the computations. This yields precise comparison amongst the fracture

Table 1. The peak load and corresponding crack opening displacement gained using FCM.

D (mm)	TPBT geometry		FPBT geometry		CT specimen	
	P_u (kN)	$CMOD_c$ (mm)	P_u (kN)	$CMOD_c$ (mm)	P_u (kN)	COD_c (mm)
100	3934.5	0.0411	7428.3	0.0427	4378.1	0.0457
200	6571.4	0.056	12319	0.0602	7427	0.0645
400	10405	0.0797	19383	0.0808	12212	0.0926
600	13034	0.101	24224	0.1019	16105	0.117

parameters between the two specimen geometries (TPBT and CT) and the two loading conditions (TPBT and FPBT) considered in the study. The details of calculation procedure are outlined elsewhere (Kumar & Barai 2008, 2009c).

The peak load P_u and corresponding CMOD ($CMOD_c$) or crack opening displacement (COD) (COD_c) for a_o/D ratio 0.3 and specimen size range 100-600mm obtained using FCM for the TPBT, FPBT and CT specimen geometries as presented in Table 1 are employed to evaluate the double-K fracture parameters.

The double-K fracture parameters are computed using the weight function approach having five terms (Kumar & Barai 2008, 2009c). A trial-and-error method is used for calculation of critical value of effective crack extension a_c for all the three types of specimens using linear asymptotic superposition assumption. The same quasi exponential softening function is employed in whole calculation for which a comprehensive computer program is developed using MATLAB.

5 INFLUENCE OF SPECIMEN GEOMETRY AND SIZE-EFFECT ON THE FRACTURE PARAMETERS

The values of unstable fracture toughness K_{IC}^{un}, cohesive toughness K_{IC}^{C} and initial cracking toughness K_{IC}^{ini} are calculated using five terms weight function approach for size ranging between 100-600mm for TPBT and CT test specimen geometries at a_o/D ratio of 0.3. Variations of non-dimensional parameters K_C/K_{IC}^{un}, K_C/K_{IC}^{C} and K_C/K_{IC}^{ini} with l_{ch}/D for both the specimen shapes are plotted through Figures 3-5 respectively.

It is observed from the figures that values of fracture parameters are marginally dependent on the types of specimen geometry. Both the parameters K_{IC}^{un} and K_{IC}^{C} increase with increase in specimen size whereas the values of K_{IC}^{ini} are relatively less dependent on the sizes ranging 100-400 mm. However, beyond the size range 400 mm, a decrease in the value of K_{IC}^{ini} is observed. Furthermore, it is also found that the values of K_{IC}^{un} for CT specimen are less than those obtained for TPBT specimen by about 2.33, 1.12, 1.33 and 1.71% for the specimen sizes 100, 200, 400 and 600 mm respectively. The values of cohesion toughness determined show that

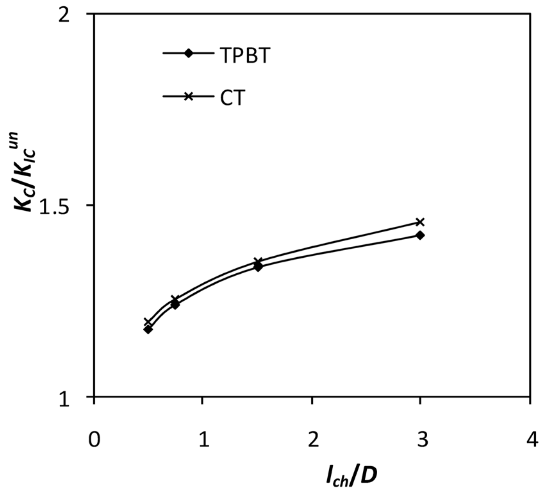

Figure 3. Effect of specimen geometries on K_{IC}^{un} for different specimen sizes.

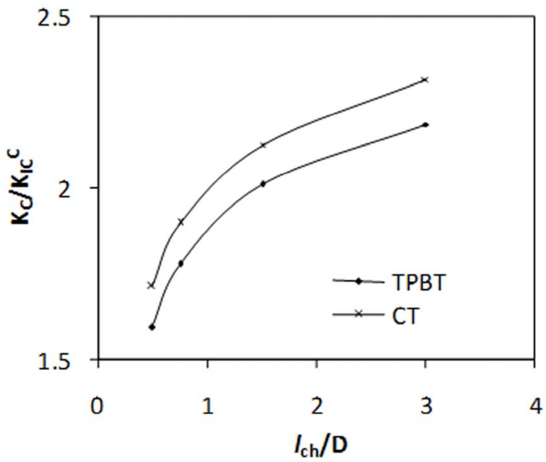

Figure 4. Effect of specimen geometries on K_{IC}^{C} for different specimen sizes.

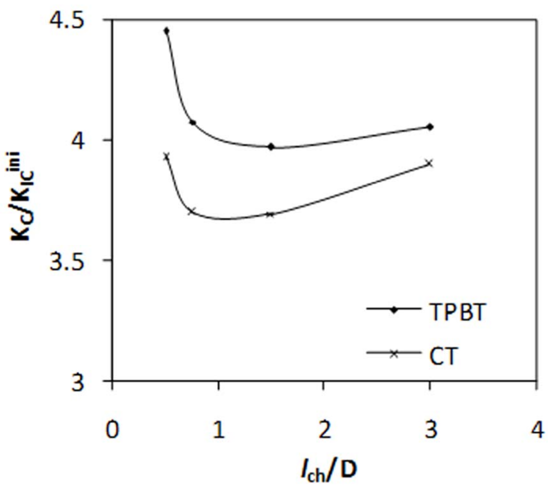

Figure 5. Effect of specimen geometries on K_{IC}^{ini} for different specimen sizes.

the these values for CT specimen are less than those obtained for TPBT geometry by about 5.68, 5.47, 6.32 and 7.07% for specimen sizes 100, 200, 400 and 600 mm respectively. Consequently, the values of K_{IC}^{ini} determined as such for TPBT geometry are less than those obtained for CT specimen by nearly 3.47, 6.93, 9.19 and 11.71% for specimen sizes 100, 200, 400 and 600 mm respectively.

A similar investigation was presented by the authors (Kumar & Barai 2008) in which the fracture parameters were calculated at different a_o/D ratios ranging 0.3-0.5 for each specimen size. It was observed that these fracture parameters were marginally affected by the value a_o/D ratio hence the mean values of the fracture parameters K_{IC}^{un}, K_{IC}^{C} and K_{IC}^{ini} are were obtained neglecting the influence of a_o/D ratio for a particular specimen size. From the study it was reported that the values of K_{IC}^{un} for CT specimen were less than those obtained for TPBT specimen by about 2.99, 1.69, 1.20 and 1.16% for sizes 100, 200, 400 and 600 mm respectively. The mean values of K_{IC}^{C} for CT specimen were less than those obtained for TPBT geometry by about 4.59, 4.43, 5.01 and 5.82% for specimen sizes 100, 200, 400 and 600 mm respectively. As a result, the average values of K_{IC}^{ini} for TPBT geometry were less than those obtained for CT specimen by nearly 0, 3.33, 6.55 and 9.63% for specimen sizes 100, 200, 400 and 600 mm respectively.

6 INFLUENCE OF LOADING CONDITION AND SIZE-EFFECT ON THE FRACTURE PARAMETERS

The gained values of K_{IC}^{un}, K_{IC}^{C}, and K_{IC}^{ini} using five terms weight function approach for different specimen sizes ranging between 100-600mm for TPBT and FPBT loading conditions for a_o/D ratio 0.3 are plotted in Figures 6-8 respectively.

It is observed from the figures that values of fracture parameters are marginally dependent the type of loading conditions. It is found that the absolute difference in the values of K_{IC}^{un} between the two loading conditions is about 1.26, 3.26, 0.30 and 0.08% for sizes 100, 200, 400 and 600 mm respectively. The values of cohesion toughness determined show that the absolute difference in the values of K_{IC}^{C} for these loading conditions are about 1.42, 4.80, 0.90 and 1.85% for specimen sizes 100, 200, 400 and 600 mm respectively. Consequently, the values of K_{IC}^{ini} determined as such for the two loading conditions are differed by nearly 0.95, 0.06, 1.12 and 1.86% for specimen sizes 100, 200, 400 and 600 mm respectively.

Kumar & Barai (2009c) carried out a similar study of the effect of loading condition on the double-K fracture parameters for the specimen size

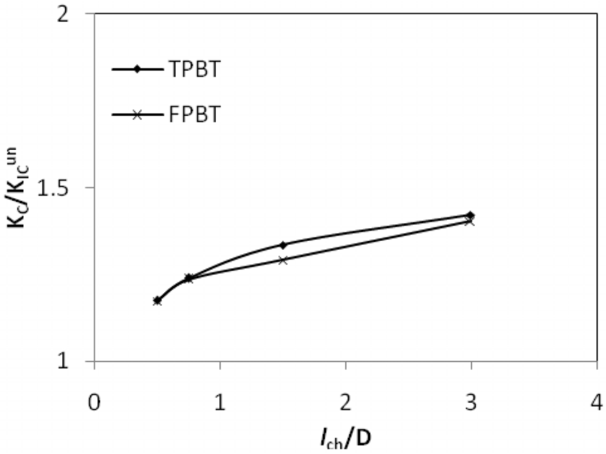

Figure 6. Effect of loading condition on K_{IC}^{un} for different specimen sizes.

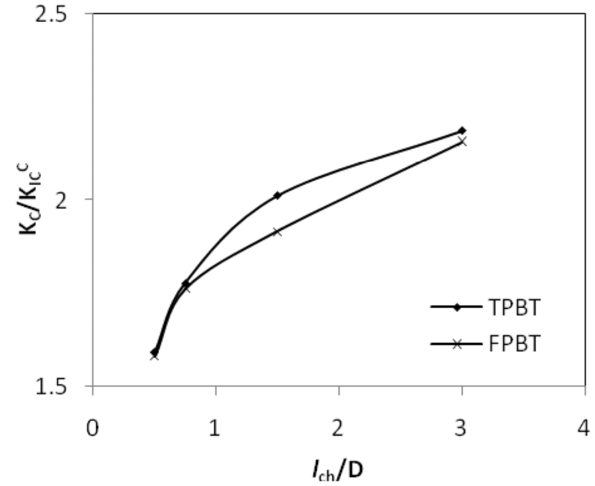

Figure 7. Effect of loading condition on K_{IC}^{C} for different specimen sizes.

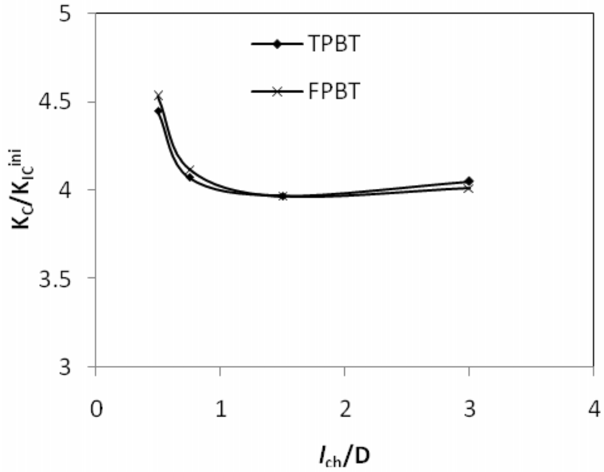

Figure 8. Effect of loading condition on K_{IC}^{ini} for different specimen sizes.

range 100≤D≤400 mm and a_o/D ratio ranging 0.3-0.5. It was observed that the double-K fracture parameters were influenced by size-effect, initial crack-length/depth ratio and type of loading condition. Neglecting the effect of a_o/D ratio, the mean values of K_{IC}^{un} and K_{IC}^{ini} for TPBT and FPBT specimens for a particular specimen size were computed. It was found that the maximum absolute difference in the value K_{IC}^{un} between the two loading conditions was less than 2% and that in the value K_{IC}^{ini} was less than 3.5% for specimen size-range 100-400 mm.

7 INFLUENCE OF SOFTENING FUNCTION ON THE FRACTURE PARAMETERS

It is clear that the cohesive stress distribution is not involved in computation of K_{IC}^{un} of double-K fracture criterion hence these values are unaffected by the choice of the softening functions of concrete. Also, it has been reported in the literature that the influence of shapes of softening relations on the calculation of the cohesion toughness are not observable. In the present study, some of commonly used softening functions such as: Petersson (1981) bilinear, Wittmann et al. (1988) bilinear, modified bilinear (Xu & Reinhardt 1999b), nonlinear softening functions (Reinhardt et al. 1986) and quasi-exponential (Planas & Elices 1990) are employed to obtain the K_{IC}^{C} and K_{IC}^{ini} for the TPBT specimen at a_o/D ratio 0.3 for size range 100-600mm. The non-dimensional forms of the gained results of K_{IC}^{C} and K_{IC}^{ini} are plotted in Figures 9 and 10 respectively.

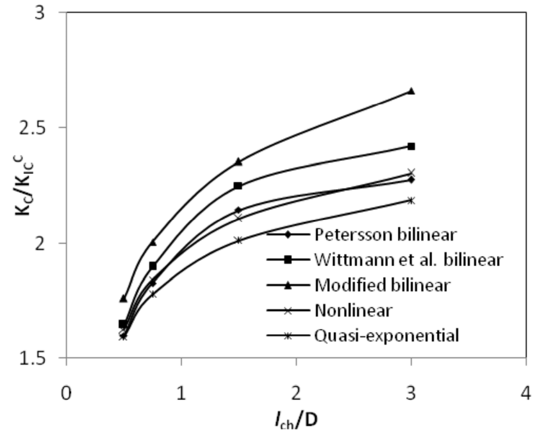

Figure 9. Effect of softening function on K_{IC}^{C} for different specimen sizes for TPBT.

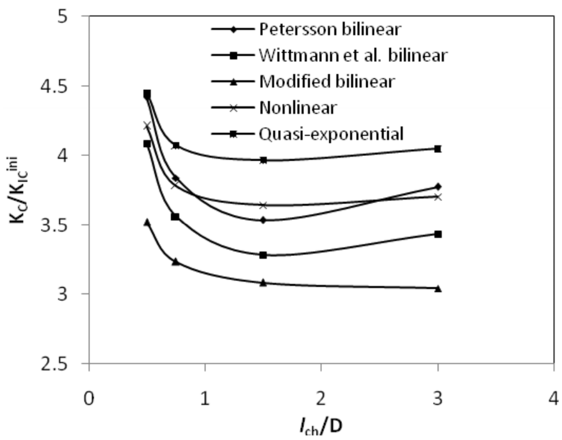

Figure 10. Effect of softening function on K_{IC}^{ini} for different specimen sizes for TPBT.

The figures show that these fracture parameters are somewhat influenced by the softening function of concrete.

8 CLOSING REMARKS

A precise numerical study was carried out in the present work to show the effect of various parameters on the double-K fracture parameters. These parameters are mainly involved while characterizing the material fracture parameters. From the investigation, the summary of the present work done can be highlighted as given below:

· The double-K fracture parameters are influenced by many factors such as specimen geometry, loading condition, size-effect and softening function of concrete. The influence of shape of test specimen and type of loading condition is relatively less than the size-effect on the values of fracture parameters.

· For specimen size range (100≤D≤600mm) considered in the study, the maximum difference in the values of K_{IC}^{un}, K_{IC}^{C} and K_{IC}^{ini} between the two specimen geometries TPBT and CT specimen are about 2, 7 and 12% whereas those difference between the two loading conditions TPBT and FPBT are about 3, 5 and 2% respectively.

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