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Boundary effects in non local damage model

Gilles Pijaudier-Cabot
Laboratoire des Fluides Complexes, Université de Pau et des Pays de l'Adour, Anglet, France
Frédéric Dufour
Laboratoire 3S-R, Institut National Polytechnique de Grenoble, Grenoble, France
Abbas Krayani
R&DO, Institut de Recherche en Génie Civil et Mécanique, Ecole Centrale de Nantes, Nantes, France

Interaction stresses that are at the origin of non locality are expected to vanish at the boundary of a solid, in the normal direction to this boundary. Existing models do not account for such an effect. We introduce tentative modifications of the classical non local damage model aimed at accounting for this boundary layer effect in a continuum modelling setting. Finite element simulations of size effect on notched and unnotched specimens are performed. For the same set of model parameters, including the internal length, the fracture energy derived from the size effect test method is quite different according to both approaches. Parameters in the size effect laws for notched and unnotched specimens, obtained from computation of geometrically similar bending beams, are more consistent with the modified non local model compared to the original non local formulation.

1 INTRODUCTION

Most failure models for strain softening materials involve non locality. Whether non locality is introduced in an integral or in a gradient format, an internal length is added to the material description. Such constitutive relations provide consistent continuum failure models for progressive cracking in quasi-brittle materials (see e.g. (Bažant and Jirasek 2002)) or ductile failure in alloys (see e.g. (Leblond et al. 1994)). In quasi-brittle materials at least, non locality finds its origin in the interaction between growing defects in the course of failure. When a microcracks open, stresses are released and the stress field in the neighbourhood of each crack is modified. This modification may induce further cracking elsewhere, even if the applied loads are kept constant (amplification of stresses). Interactions may be approximated following several schemes (Kachanov 1987; Fond and Berthaud 1995; Kachanov 1994) and folded into micro-mechanical damage based models (see e.g. Refs (Bažant 1994; Pijaudier-Cabot and Berthaud 1990; Pijaudier-Cabot et al. 2004)). There are at least two outcomes from these approaches: first, the weight function which is introduced in the non local averaging, along with the internal length, is recovered; second this weight function depends on the state of damage and it is direction dependent with respect to the state of stress. Cracks may shield each other or amplify the interaction stresses acting in their neighbourhood.

Boundary effects are among the pending issues in non local modelling for which very little is known from an experimental or a theoretical point of view. Nearby the boundary of the solid, interactions between defects are expected to be different compared to those observed in the bulk material. Therefore, micromechanics suggests that non locality should change in a boundary layer at the surface of solids. In continuum non local models, boundaries are, however, usually dealt with arbitrarily.

In integral models the weight function involved in the non local average is chopped off and normalized (Pijaudier-Cabot and Bažant 1987). It follows that the influence of a point A located nearby a boundary on a point B located in the bulk of the solid is not the same as the influence of B on A. Due to this truncature of the interaction domain, the weight function centered at point A and entering in the non local averaging at point A is not the same as the weight function centered at point B and entering in the non local averaging at point B. This peculiarity of integral non local models was pointed out on many occasions (see e.g. (**?**; Bažant and Pijaudier-Cabot 1988)). It is at the origin of the loss of symmetry of the tangent operator in the non local integral formulations. Because the non local interactions are changing nearby the boundary of the solid, the constitutive formulation (Pijaudier-Cabot and Bažant 1987) does not derive from a thermodynamic free energy potential. The modified symmetric non local damage theory due to Borino and coworkers (Borino et al. 2003) derives from a potential and fulfills thermodynamic principles. To this end, the weight function is modified near the boundary. The background for such a modification is the symmetry of the non local interactions and energy considerations, it is not related to some specific boundary effect expected from micromechanical analysis.

In gradient enhanced models, the normal component of the gradient of the non local variable is constrained to be zero on the boundary. Consequently, the free boundary condition on the non local variable is the same as the condition that would be induced by an axis of symmetry. The underlying assumption is that the non local interactions nearby a boundary of the solid are the same as the non local interactions that would be observed nearby an axis of symmetry. It can be hardly admitted, however, that the interaction between defects and a boundary surface is the same as the interaction between defects distributed symmetrically in a bulk material. The interaction of a small defect on a large one is not expected to be the same as the interaction of a large defect on a small one (by interation, we mean the fluctuations of the local stress and strain fields due the presence of another defect located nearby the considered defect). This non symmetry of the non local interactions seems to better correspond to the non symmetry observed in integral models, whereas it dos not exist in gradient models.

In displacement based gradient models (Rodriguez-Ferran et al. 2005; Jirasek and Marfia 2005), the displacements ought to be equal to the non local displacements on the boundary of the solid. This is again a different boundary condition but as we will see next, it seems to be closer to expected results from micromechanics.

In all these cases, there is very little theoretical motivation for the boundary conditions which appear in non local models. It may be argued that boundary conditions are not very important. Generally, cracks propagate inside the structure and the fracture process zone is located in the bulk material. Initiation of cracking, however, very often occurs from the boundary of a solid. The simplest situation is that of bending beams considered in this paper. It is expected that the boundary effect may have some influence on the initiation condition of cracking and this is the primary motivation for the present study. Once a crack has propagated, it forms a new, evolutive, boundary of the solid and the non local formulation should account for this additional boundary effect. This case is outside the scope of this paper, along with the issue of non local effects nearby interfaces.

2 INTUITIVE REASONING FROM MICRO-MECHANICS

Our purpose is to provide here some hint on the boundary effects induced by non locality and therefore to investigate the effect of subsequent modifications of non local averaging nearby boundaries in integral damage models. Let us start with some intuitive argument about non locality nearby the boundary of a solid (Krayani et al. 2009). Consider a finite body that contains a population of micro-cracks or micro-voids in an elastic matrix. Each defect is described explicitly and the mechanical response of the solid is the result of the deformation of the elastic matrix and of the deformation of the defects (e.g. micro-crack openings) due to the applied loads. The interaction between the defects is computed using a superposition scheme. One may choose the simple technique due to Kachanov (Kachanov 1987). According to Kachanov's superposition scheme, the state of stress and the displacements in this body containing defects is computed as the sum of two sub-problems:

- Sub-problem I: the solid is considered without any void and loaded by the remote boundary condition corresponding to σ_∞.
- Sub-problem II: The solid contains the defects. It is free from any external load / applied displacement. Inside each defect, surface forces are applied so that the stress vectors due to σ_∞ computed at the imaginary location of the defects in sub-problem I are exactly equilibrated. This is required in order to recover, after superposition, the fact that internal defect surfaces are free of any load.

This superposition is schematised in Fig. (1) in the case of an infinite body containing two voids. The remote traction σ_∞ is transformed into distributed forces $\vec{P}_1 = \vec{P}_2 = ... = \vec{P}_i = -\sigma_\infty.\vec{n}$ acting inside each defect whose inner surface is defined by the normal vector $\vec{n}$. Each distributed force $\vec{P}_i$ is the sum of an externally applied load inside defect i and the stress vectors generated by the loads applied in the other voids j computed on the location of void i. These interactions are nonlocal terms contributions typically. This superposition scheme can be applied in principle to the case of a finite body too (Fond and Berthaud 1995).The sum of the interaction forces due to the other defects, the interaction forces due to the boundary of the solid, and the applied surface forces inside this defect equilibrate the stress vectors due to σ_∞ computed at the imaginary location of the defect.

The equivalent homogeneous material which is the macroscopic counterpart of the above solid with

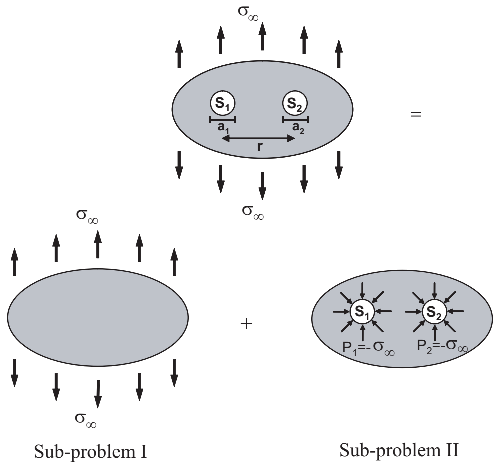

Figure 1: Infinite elastic body containing two voids.

micro-cracks is a continuum with an overall stiffness which is a function of the density, size and shape of the micro-cracks (see e.g. (Budiansky and O'Connel 1976)). The interaction stresses computed from the present superposition scheme are at the origin of non locality in a continuum description. Other frameworks, e.g. based on a generalisation of Hashin-Shtrikman variational principles (Hashin and Shtrikman 1962) as discussed by Drugan and co-workers (Drugan and Willis 1996; Monetto and Drugan 1962) provide also some useful information about the influence of interactions in upscaling techniques.

Let us focus attention on the boundary of the solid. Overall, it remains always free from any load, at least in subproblem II in which interactions between defects and between defects and the boundary are computed. It means the sum of the stress vector due to the interactions, computed normal to the boundary, always cancels whatever the distribution of defects. *If non locality of the constitutive response of the homogenised solid is understood as the consequence of upscaling interactions towards a continuum setting, non locality should vanish on the free surface*, *in the normal direction to this surface*. Close to the boundary, and in the normal direction to the boundary, there should be a layer in which non locality increases going farther from the boundary in order to reach the non locality expected in an infinite solid.

This simple qualitative reasoning implies, in a quite general context, that on the surface of the solid (along the normal of this surface), local and non local quantities should be equal. This provides a rational ground for implementing the corresponding boundary conditions in existing non local models. This is rather straightforward in displacement based gradient models but it remains to be discussed in non local integral or gradient models. We shall discuss a modified phenomenological non local integral model that satisfies the condition of local material response on the boundary of the solid only, in the normal direction to the boundary. We will then compare the original and modified formulation in structural analyses and we will show that the modified non local model provides a more consistent fit of the size effect laws on both type of specimens than the original non local model.

3 DAMAGE MODEL

Before considering boundary effects, let us recall the main equations involved in the damage model used in the present study.

3.1 *Local damage model*

The classical stress-strain relation for this type of model reads:

$$\sigma_{ij} = (1-D)C_{ijkl}\varepsilon_{kl} \tag{1}$$

where σ_{ij} and ε_{kl} are the components of the stress and strain tensors, respectively $(i,j,k,l \in [1,3])$ and C_{ijkl} are the components of the fourth-order elastic stiffness tensor. The damage variable D represents a measure of material degradation which grows from zero (undamage material with the virgin stiffness) to one (at complete loss of integrity). The material is isotropic, with E and ν the initial Young's modulus and Poisson's ratio respectively.

For the purpose of defining damage growth, a scalar equivalent strain ε_{eq} is introduced, which quantifies the local deformation state in the material in terms of its effect on damage. In this contribution, Mazars' definition of the equivalent strain is used (Mazars 1984):

$$\varepsilon_{eq} = \sqrt{\sum_{i=1}^{3} \left(\langle \varepsilon_i \rangle_+\right)^2} \tag{2}$$

where $\langle \varepsilon_i \rangle_+$ are the positive principal strains. Damage growth is governed by the loading function:

$$f(\varepsilon, k) = \varepsilon_{eq}(\varepsilon) - k \tag{3}$$

k equals the damage threshold ε_{D_0} initially, and during the damage process it is the largest ever reached value of ε_{eq}. The evolution of damage is governed by the Kuhn-Tucker loading-unloading condition:

$$f(\varepsilon, k) \le 0, \quad \dot{k} \ge 0, \quad \dot{k} f(\varepsilon, k) = 0 \tag{4}$$

The damage variable D is determined as a linear combination of two damage variables D_t and D_c, that represent tensile damage and compressive damage respectively, by the help of two coefficients α_t and α_c which depend on the type of stress state (Mazars 1984):

$$D = \alpha_t D_t + \alpha_c D_c \tag{5}$$

$$D_{t,c} = 1 - \frac{1 - A_{t,c}}{\varepsilon_{eq}} + \frac{A_{t,c}}{\exp\left(B_{t,c}\left(\varepsilon_{eq} - \varepsilon_{D_0}\right)\right)} \tag{6}$$

Standard values of the model parameters in the damage have been given in Ref. (Mazars 1984).

3.2 *Non local formulation*

In the integral-type non local damage models, the local equivalent strain is replaced by its weighted average:

$$\bar{\varepsilon}_{eq}(\mathbf{x}) = \int_\Omega \Psi(\mathbf{x}, \xi)\, \varepsilon_{eq}(\xi)\, d\xi \tag{7}$$

with Ω the volume of the structure and $\Psi(\mathbf{x}, \xi)$ the weight function. It is required that the non local operator does not alter the uniform field, which means that the weight function must satisfy the condition:

$$\int_\Omega \Psi(\mathbf{x}, \xi) d\xi = 1 \qquad \forall x \in \Omega \tag{8}$$

For this reason, the weight function is recast in the following form (Pijaudier-Cabot and Bažant 1987):

$$\Psi(\mathbf{x}, \xi) = \frac{\Psi_0(\mathbf{x} - \xi)}{\Omega_r(\mathbf{x})} \tag{9}$$

with

$$\Omega_r(\mathbf{x}) = \int_\Omega \Psi_0(\mathbf{x} - \xi) d\xi \tag{10}$$

where $\Omega_r(\mathbf{x})$ is a representative volume and $\Psi_0(\mathbf{x} - \xi)$ is the basic non local weight function which is often taken as the polynomial bell-shaped function (Bažant and Jirasek 2002), or here as the Gauss distribution function:

$$\Psi_0(\mathbf{x} - \xi) = \exp\left(-\frac{4\,\|\mathbf{x} - \xi\|^2}{l_c^2}\right) \tag{11}$$

l_c is the internal length of the non local continuum. Preserving the uniform field in the vicinity of the boundary makes the averaging in Eq. (9) not symmetric with respect to its arguments **x** and ξ. This lack of symmetry leads to the non-symmetry of the tangent operator (Bažant and Pijaudier-Cabot 1988; **?**; Jirasek and Patzàk 2002). A symmetric non local formulation can be devised (Borino et al. 2003). The weight function becomes:

$$\Psi(\mathbf{x}, \xi) = \left(1 - \frac{\Omega_r(\mathbf{x})}{\Omega_\infty}\right) \delta(\mathbf{x} - \xi) + \frac{\Psi_0(\mathbf{x} - \xi)}{\Omega_\infty} \tag{12}$$

where $\delta(\mathbf{x} - \xi)$ is the Dirac function and Ω_∞ is the representative volume in the infinite solid where it has a constant value. The first term is local, it vanishes for points far from the boundary and the original weight function in Eq. (9) is recovered. According to this modified formulation, the computation of the non local equivalent strain becomes:

$$\bar{\varepsilon}_{eq}(\mathbf{x}) = \varepsilon_{eq}(\mathbf{x}) + R \tag{13}$$

with the non local residual R defined as:

$$R = \frac{1}{\Omega_\infty} \int_\Omega \Psi_0(\mathbf{x} - \xi)\left(\varepsilon_{eq}(\xi) - \varepsilon_{eq}(\mathbf{x})\right) d\xi \tag{14}$$

The integrand in the right hand-side term of this equation involves the difference between the local equivalent strain at the considered point and the local equivalent strain in its neighborhood. It is clearly a non local contribution to the quantity (non local equivalent strain) that controls damage. This non local distribution, however, does not vanish on the boundary of the solid. To this extend, this modified formulation is not consistent with the boundary effects discussed in the previous section, although it derives from a potential and it is symmetric.

3.3 *Modified non local model*

This modification of the non local damage model has been proposed by (Krayani et al. 2009). It is based on the original non local formulation (Equation (9)), in which a transformation of the coordinate system is introduced. Within a 2D setting, the following mapping is defined in the weight function:

$$\|\mathbf{x} - \xi\| = l_c \sqrt{(\mathbf{x_1} - \xi_1)^2 / a^2 + (\mathbf{x_2} - \xi_2)^2 / b^2} \tag{15}$$

in a coordinate system where subscript 1 refers to a vector that is normal to the closest boundary of the solid and subscript 2 refers to the orthogonal direction. a is the minimum between the internal length and the distance from the point to the closest boundary and b is assumed to be the minimum between the internal length and the distance to the boundary of the solid in the orthogonal direction. When the non local average is computed at points close to a boundary, namely at a distance which is smaller than the internal length, the weight function is modified. The relative weight of the points located in the neighborhood of point **x** at which the average is computed are decreased as **x** is getting close to the boundary. If **x** is located on the boundary exactly, the weight function is a Dirac-delta function. The material response becomes local which is consistent with micromechanics. This transformation may be extended to a 3D formulation without difficulties.

In the sequel, we shall denote as ERF the modified formulation and RWF the original non local formulation. We are going to compare these two formulations on structural computations of three point bend specimens.

4 THREE POINT BENDING SIZE EFFECT TESTS

A salient characteristics of non local modelling is structural size effect, understood here as the dependence of nominal strength on the structural size. The response of geometrically similar specimens is not geometrically similar. The size of the fracture process zone (FPZ) is controlled by the internal length (Mazars and Pijaudier-Cabot 1996). When the size of the structure changes, the ratio between the size of the FPZ, which is constant, and the size of the structure is changing too. This produces differences in energy release during the propagation of a crack and size effect on the nominal strength. Detailed explanations can be found in the textbook by Bažant and Planas (Bažant and Planas 1998), along with comparisons with experimental data on quasi-brittle materials such as concrete.

Our aim is here to compare the original (RWF) and modified (ERF) non local damage formulations. We will start first with unnotched specimens and consider three point bending tests with three geometrically similar sizes (see Figure 2). The specimens with various height $D = 80$, 160, 320 mm are referred to as small, medium and large beam respectively. Simulations are 2D plane stress calculations. The element size was kept constant in the FPZ, and small enough compared to the internal length (at least 3 times smaller). The model parameters used for these simulations are: $E = 3.85 \times 10^4$ MPa, $\nu = 0.24$, $A_t = 0.95$, $A_c = 1.25$, $B_t = 9200$, $B_c = 1000$ and $\varepsilon_{D_0} = 3.0 \times 10^{-5}$. The internal length is equal to 10 mm.

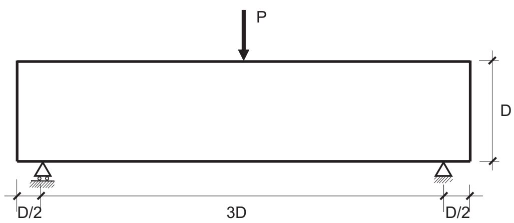

Figure 2: Three point bending test: Geometry and loading
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4.1 *Size effect on unnotched specimens*

The nominal strength now is computed according to the formula:

$$\sigma_N = \frac{9}{2}\frac{P_u}{bD} \qquad (16)$$

It is the maximum tensile stress computed at peak load according to the elastic beam theory. b is the thickness of the beam (unit thickness in our computations). The nominal strengths are listed in Table 1.

Size	D (mm)	σ_N (MPa) (RWF)	σ_N (MPa) (ERF)
small	80	3.70	3.62
medium	160	3.48	3.45
large	320	3.38	3.38

Table 1: Numerical results for three different sizes with RWF and ERF ($l_c = 10$ mm)

For this set of specimens, we have also run the calculations with an internal length of 40 mm. The results are shown in Table 2.

Size	D (mm)	σ_N (MPa) (RWF)	σ_N (MPa) (ERF)
small	80	4.91	4.46
medium	160	4.06	3.95
large	320	3.69	3.61

Table 2: Numerical results for three different sizes with RWF and ERF ($l_c = 40$ mm)

We notice an influence of the type of nonlocal formulation and of the internal length. The difference between the ERF and RWF decreases with decreasing internal length (smaller non local effect).

As pointed out by Bažant, there is first a layer of distributed damage that forms at the bottom of the beam in the tensile part. The thickness of this boundary layer should be a function of the internal length. At peak load, a crack forms in the center of the beam. The crack is perpendicular to the distributed damage layer, it starts from this layer, inside the beam and not directly from the boundary. Therefore, the effect of the boundary on the inception of the crack is small and peak loads are similar.

Let us now use Bažant's size effect law (Bažant and Planas 1998) for the case of unnotched beams:

$$\sigma_N = f_{r\infty}\left(1 + \frac{D_b}{D}\right) \qquad (D \gg D_b) \qquad (17)$$

where D_b and $f_{r\infty}$ are constants. The first one is the thickness of the boundary layer in which damage is distributed prior to localised crack inception and the second one is the strength for a specimen of infinite size. This is the modulus of rupture of a infinitely large specimen. D_b and $f_{r\infty}$ are obtained by fitting our computed values of the nominal strengths with the size effect law (Tables 3 and 4).

One can observe in this table that $f_{r\infty}$ is almost constant and does not depend on the internal length.

	RWF	ERF
	$l_c = 10$ mm	$l_c = 10$ mm
$f_{r\infty}$ (MPa)	3.27	3.30
D_b (mm)	10.34	7.82

Table 3: $f_{r\infty}$ et D_b for $l_c = 10$ mm, original and modified weight functions.

	RWF	ERF
	$l_c = 40$ mm	$l_c = 40$ mm
$f_{r\infty}$ (MPa)	3.26	3.35
D_b (mm)	40.16	26.62

Table 4: $f_{r\infty}$ et D_b for $l_c = 40$ mm, original and modified weight functions.

The tensile strength of the material is, according to the constitutive relations, equal to 2 MPa. Here, the modulus of rupture $f_{r\infty}$ computed from the FE analyses of specimens of different sizes is approximately 1.65 times the tensile strength. This is close to the usual ratio of 1.5 between tensile strengths measured by 3 point bending test and direct tension respectively

According to dimensionnal analysis, D_b should proportional to the internal length (Bažant and Planas 1998). This proportionality is almost recovered if we compare table 3 and table 4.

In the original RWF formulation, D_b is approximately equal to the internal length. Due to the boundary effect, it decreases in the ERF formulation. This is again consistent with the fact that non local effects are decreased nearby the bottom face of the beam.

4.2 *Size effect on notched specimens*

The notch, now located at mid-span, is $0.2D$ high. We shall focus here on computations with an internal length of 10 mm but trends are the same for other values of the internal length.

The ultimate loads P_u for the three sizes with the two approaches are obtained form the finite element calculation and the corresponding nominal strengths are computed according to the following equation:

$$\sigma_N = \frac{9}{2}\frac{P_u}{Db(0.8^2)} \tag{18}$$

It is the maximum tensile stress in the beam computed at the tip of the notch according to the elastic beam theory. At the peak, damage occurs nearby the tip of the notch only and this is where the modification of the non local model is important. Results, showing a great sensitivity of the mechanical response of specimens to the weight function, have already been obtained, e.g. by Jirasek and co-workers (Jirasek et al. 2003). Table 5 shows the nominal strength computed for the three sizes and according to the two non local formulations.The numerical results are now interpreted with Bažant's size effect law for notched beams (Bažant 1984):

$$\sigma_N = Bf_{r\infty}(1 + D/D_0)^{-1/2} \tag{19}$$

B is a dimentionless geometry-dependent parameter and D_0 is a characteristic size. For each formulation,

Size	D (mm)	σ_N (MPa) (RWF)	σ_N (MPa) (ERF)
small	80	3.72	3.24
medium	160	2.83	2.54
large	320	2.14	1.99

Table 5: Numerical results for three different sizes of the notched beam with RWF and ERF ($l_c = 10$ mm)

D_0 and $Bf_{r\infty}$, identified from a linear regression as explained in (Bažant and Planas 1998) are reported in the Table 6.

	RWF	ERF
$Bf_{r\infty}$ (MPa)	6.25	4.64
$D_0 (mm)$	42.42	71.42

Table 6: $Bf_{r\infty}$ et D_0 of the notched beam of different sizes for $l_c = 10$ mm

There is a decrease of the maximum carrying capacity with the ERF formulation compared to the original non local formulation. This decrease results in the size effect law into a decrease of $Bf_{r\infty}$, that is a decrease of $f_{r\infty}$ because B is constant and related to the shape of the specimen only.

We may now compare the values of the modulus of rupture $f_{r\infty}$ obtained from size effect on unnotched specimens to those obtained with notched specimens. B is calculated first according to Rilem recommandations (Rilem 1990) (B=1.11). Then, we take $f_{r\infty}$ obtained for the unnotched specimens, multiply by B, and compare the result with the value of the fit obtained from size effect on notched specimens. Ideally, the two values should be the same. Table 7 shows that the relative error on the prediction of $Bf_{r\infty}$ is three times smaller with the ERF formulation than with the RWF one. In view of the high sensitivity of the size effect parameters to the values of the peak loads (Le Bellégo et al. 2003), an error of the order of 30% is quite reasonable and the modified ERF formulation provides consistent results for both type of size effects whereas it is not the case for the original non local formulation.

We may also observe in Table 6 that D_0 is increasing in the ERF calculations compared to the RWF calculations. It should be pointed out that the size effect formula in Eq. (19) holds for sizes that are large compared to D_0. D is close to D_0 and we may consider

$Bf_{r\infty}$ (MPa)	RWF	ERF
computed from unnotched	3.63	3.66
fit from notched	6.25	4.64
Relative error	72%	27%

Table 7: $Bf_{r\infty}$ computed from notched and unnotched specimens

that it is more appropriate to implement here the universal size effect formula proposed by Bažant instead of two separate formulae for notched and unnotched specimens:

$$\sigma_N = Bf_{r\infty}(1+\frac{D}{D_0})^{-1/2}.$$

$$(1+[(\eta+\frac{D}{D_b}).(1+\frac{D}{D_0})]^{-1}) \tag{20}$$

where η is taken here equal to 1. In order to obtain the parameters in this formula, we consider first the case of unnotched specimens. Eq. (20) reduces exactly to Eq. (17) and the corresponding constants can be fitted. We use then the data computed for notched specimens and the corresponding value of B computed above and we look for the value of D_0 that provides the best fit with the data. Figure 3 shows these fits obtained for both non local formulations.

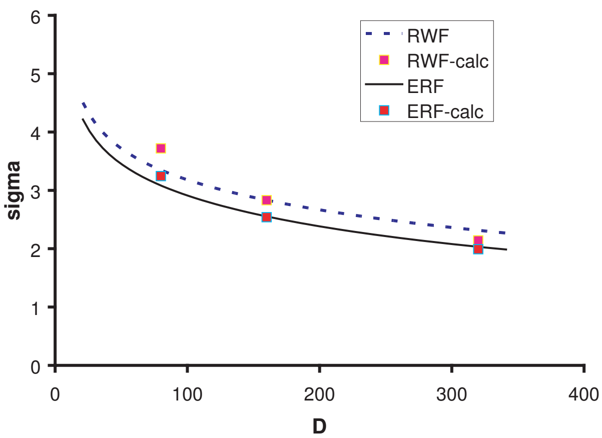

Figure 3: Fits of the universal size effect law on notched specimens according to the RWF and ERF formulations.

These fits are obtained for D_0= 210 mm with the RWF formulation and D_0= 140 mm with the ERF formulation. Note that the agreement is again better with the modified non local formulation than with the original non local one. Finally, it is possible to compute the fracture energy from the size effect law.

$$G_f = Bf_{r\infty}^2.D_0.g/E \tag{21}$$

where E is the Young's modulus and g is related to the geometry of the specimen (here g=1.1116 according to (Rilem 1990)). For the RWF formulation, we obtain G_f=80 N/m and for the ERF formulation we obtain G_f=54 N/m. The fracture energies computed according to the original and modified non local formulation, and defined according to the size effect method, are very different.

5 CONCLUSION

We have discussed a modification of the original non local damage model that is consistent with the intuitive argument that non locality should vanish at the boundary of a solid (in the direction normal the the boundary) and should grow getting inside the solid. The model remaps the coordinate system defined in the weight function when the average is computed close to the boundary.

The simulation of notched and unnotched specimens of different sizes have been interpreted with the help of Bažant's size effect laws. With the modified ERF formulation, it is possible to fit data for notched and unnotched bending beams with the same set of model parameters. It is not possible to achieve the same results with the original RWF formulation. The modulus of rupture needs to be changed in a range beyond possible calibration errors. The fracture energies predicted according to the size effect technique are also quite different with the modified and original non local damage models.

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