Fracture Mechanics of Concrete and Concrete Structures -
Recent Advances in Fracture Mechanics of Concrete - B. H. Oh, et al.(eds)
ⓒ 2010 Korea Concrete Institute, Seoul, ISBN 978-89-5708-180-8

Macroscopic probabilistic modeling of concrete cracking: First 3D results

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ABSTRACT: The paper proposes an original approach for simply modeling the complex cracking processes of cementitious composites structures. The basic idea of the model is to take into account the heterogeneous nature of concrete and the presence of initial defects as the main factors influencing cracking processes for a given stressed material volume. In this sense, the model considers volume effects, random distributions of mechanical properties and crack localization in its formulation. Therefore, the model is able to bridge the gap between the local description of the mechanisms at the material level and the global response at the structural level. 2D and first 3D simulations and validation tests are presented.

1 INTRODUCTION

The description of cracks is crucial for predicting the life expectancy of concrete structures such as dams, nuclear power plants vessels, waste (nuclear or not) storage structures, tunnels, etc. The development of models providing information on the characteristics of cracks in concrete (crack openings and spacings), for a given environment, loading and limit conditions set is still a challenging task. Moreover, a pertinent model should also take into account some characteristics being behind cracking processes for a given volume of concrete: not only scale effects, but also phenomena related to the heterogeneous nature of concrete such as initial defects in the material, cracking nucleation and propagation.

The objective of this paper is to provide a macroscopic model capable of bridging the gap between the local description of the mechanisms at the material level and the global response at the structural level. In the proposed model, finite elements are classically considered as elementary volumes of material. As far as the concrete is heterogeneous, mechanical parameters are defined via statistical distributions (requiring only two parameters) based on a large experimental campaign held at LCPC (Rossi et al. 1994). The model is also aimed to represent crack initiation, propagation and localization by a simple macroscopic probabilistic dissipative mechanism (elastic-perfectly plastic and brittle) depending on the element size.

This kind of approach allows to obtain a pertinent, statistical global response, and, simultaneously, local information (such as crack mouth opening and distribution) that can be exploited, for example, in the coupled modeling of fluids transfers.

This modeling strategy is developed in 2D and in 3D, and results are compared to original experimental tests (four point bending test and Brazilian test) performed at LCPC. The comparison will be given not only in terms of the global answer but also on cracks opening and distribution.

2 HETEROGENEITY OF CONCRETE AND CRACKING PROCESSES

Concrete is a porous multiphase material where the solid matrix is formed by cement paste and aggregates and where voids are filled with liquid and gas. In other words, concrete is, by nature, a heterogeneous material, which always contains inner defects such as pores and cracks (even under no external loads). Moreover, heterogeneity is the main cause of concrete statistical volume effects, which influence scale effects at the structural level (Bazant 2000) via the cracking processes. Heterogeneity and volume effects are aspects which are strictly correlated and that should be specifically taken into account when dealing with concrete modeling.

Rossi (Rossi et al. 1994), after having performed a huge experimental campaign on tensile behavior of concrete, interpreted cracking processes as the following: stresses develop in concrete due to the external load and aggregates tend to concentrate them in their neighborhood; on the other hand, the cement paste and also the bound between the paste and the aggregate are the places where defects are located, i.e. are places of lower strengths; cracking arises when a high stress meets a low strength, and then propagates through the cement paste; and aggregates eventually play the role of barriers or material bridges in the propagation of cracks.

Volume effects are the consequence of the impact of the heterogeneity on the behavior of the stressed volume. The closer, to the scale of the stressed volume, the scale of the local stress concentration (generated by the heterogeneity) is, the larger is its influence on the macroscopic behavior.

Moreover, as far as heterogeneities and defects are randomly distributed in the material, the mechanical properties are random parameters. In the experimental study (Rossi et al. 1994), authors showed, for example, an increase of the mean value and the dispersion of the tensile strength vs. a decrease of the material volume for a same concrete mix design. This effect has been found for different types of concretes, but with different magnitudes. An empirical scale effect law has been then established for the mean tensile strength $m(f_t)$ and the standard deviation $\sigma(f_t)$ as functions of easily measurable quantities such as the ratio "volume of the specimen Vs over volume of the coarsest grain of the concrete Vg" (the ratio *Vs/Vg* can be related to the size of the major heterogeneity) and the "standard compressive strength of concrete fc" (considered here as a good indicator of the cement paste quality).

3 NUMERICAL PROBABILISTIC MODELING OF CRACKING PROCESSES

The numerical modeling takes place in the general framework of the finite-element method (FEM). The underlying, and basic, idea is to consider a finite element volume like a material volume and to assume that physical mechanisms influencing the cracking processes remain the same whatever the scale of observation. Considering the heterogeneity of the material, mechanical properties are randomly distributed over the mesh: the scale laws cited in the previous section have been extrapolated to the volume of the finite element and used as input data in a numerical modeling based on a probabilistic approach.

Considering the modeling of cracks – initiation, propagation and localization – two strategies are here presented: a discrete explicit model and an original continuum based approach.

3.1 *A discrete Approach*

Rossi (Rossi et al. 1992) originally presents a probabilistic model implemented via a discrete-explicit approach in which interface elements are used to describe the discontinuities. The volume of the massive elements which are adjacent to the considered interface element, acts as the reference (material) volume. The mechanical properties of the interface elements (tensile strength) and of the massive elements (young modulus) are considered as randomly distributed variables. As cited above, distribution characteristics (mean value and standard deviation) are obtained from an extrapolation of the empirical formulas (see section 2). It should be pointed out that the model is aimed at explicitly representing localized crack patterns in concrete taking into account volume effects. The model is considered as probabilistic, but after the random distribution of mechanical properties over the mesh, the computation remains deterministic. It is then necessary to follow a Monte Carlo method and therefore to perform a large number of computations for statistically validating the results. Scale effects are effectively taken into account and the model is auto-coherent in the sense that data at the local scale are coherent with results at the global scale since a generic law taking into account volume effects can define concrete mechanical properties at each scale. Although locally no energy is dissipated (the failure of the elementary volume remains elastic-perfectly brittle), the model allows to statistically representing a global dissipation of energy through inelastic residual strains, softening behaviors.

According to the local and probabilistic character of the approach, the volume of the element has to be sufficiently small when compared to the volume of the meshed structure or to the zone size where stress gradients can develop (i.e. the fracture process zone). This can lead to very small ratios *Vs/Vg* which fall out of the domain of validity supported by the experimental campaign (Rossi et al. 1994). An inverse analysis (Tailhan et al. 2007) has then been used to determine the extrapolation of the empirical formulas to the small ratios *Vs/Vg* domain. The original size-effect law is therefore updated and will be used in the finite element analysis.

Nevertheless, this modeling strategy has, however, some shortcomings. Some questions arise in the applicability of the discrete-explicit model. Firstly, the main criticism, which can be made, is that cracks inevitably depend on the orientation of the contact elements, even if the random distribution of the mechanical properties tempers this effect. Secondly, for more global approaches, at the scale of a whole structure for example, such model leads to prohibitive computational costs as the use of contact elements doubles the number of nodes. This is even more sensitive in the case of 3D modeling.

These considerations justify an enhancement towards a continuum based approach. Such a model seems more adequate in many situations and in particular when dealing with real structures. If compared to a discrete model, a continuum model does not require contact elements, i.e. no pre-oriented cracks (any crack direction is favored).

3.2 *A continuum Approach*

The continuum based approach is defined at a macroscopic scale where stress and strain states are defined. At this scale, it is theoretically possible to establish a constitutive relationship between stress and strain defining the macroscopic behavior of the material. Cracking processes can be then taken into account by considering a dissipative mechanism at the material scale. In this meaning, and strictly speaking, cracks must be considered as sufficiently small (micro cracks) and diffused in the whole material representative elementary volume. Two important facts have to be pointed out:

- Usually, the identification of the material behavior is performed on laboratory samples which size has to be larger than the Representative Elementary Volume (REV) in order to properly take into account the material heterogeneity. However, when dealing with concrete this size is not often in accordance with the size of the finite elements used in the modeling. It is thus necessary to perform an extrapolation of the identified experimental behavior to the scale of the finite element. This requires taking into account scale changes, i.e. volume effects must be considered at this stage.
- The localization of cracks, generally occurring at the peak, has to be carefully taken into account. Before localization, material integrity is quite preserved even if the material is severely damaged. After localization, material integrity fails such that it is impossible to consider the post-peak softening behavior as representative of the behavior of the material. In other words, after the peak we shift from a material behavior to a structural behavior (Rossi 1998). Numerical translations of these problems are mostly leading to strong mesh sensitivities and non objective responses (Bazant & Jirasek 2002). Additional assumptions must be done to solve this problem.

The model takes into account at the finite element level these aspects as follows:

- Firstly, it is assumed that it is possible to define macroscopic quantities whatever the size of the finite element, whether it is material representative or not. It is then supposed that the mechanical behavior of the finite element depends on its size and position, i.e. the behavior of each finite element is prone to random variations, thus taking account the material heterogeneity.
- The mechanical behavior of the finite element (pre- and post-localization) is replaced by an equivalent material behavior. Since it is considered as a material behavior, this equivalent behavior does not have a softening branch after the peak. A dissipative mechanism is chosen to represent the whole cracking process, pre- and post-localization. The equivalent behavior is defined via an equivalence in deformation energy. It can be argued that the local dissipative mechanism is not representative of the local energy amount really dissipated by the material during cracking. At the end of the cracking process, when the total amount of available energy is dissipated, failure of the finite element is assumed to be brittle.

The dissipative mechanism is represented via perfect plasticity. This choice is justified by the simplicity of the approach together with the well-established theoretical framework and the robust numerical implementations. The principle of the energy equivalence is depicted in Figure 1. Details are given in (Tailhan 2009).

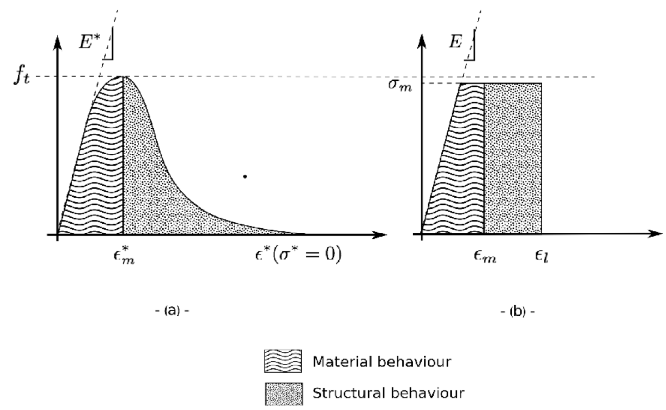

Figure 1. Principle of the equivalence for a uniaxial tensile behavior.

As far as the uniaxial behavior depends on the stressed volume of material and presents some randomness, the area under the curves is also a random quantity, influenced by volume effects. Consecutively, σ_m and the dissipated energy (W_d) can be considered as random parameters of the elastic-plastic equivalent model (also influenced by volume effect). The general laws, defining the characteristics of the probabilistic distributions for σ_m and W_d vs. the parameters of the model (i.e. *fc* and *V/Vg*, see section 2), have to be identified via an experimental campaign or via a numerical campaign using the discrete approach presented above.

The choice of a numerical support for the representation of the crack localization is important as it should combine the relative simplicity of implicit models (which are particularly suitable for being used in the description of large structures) together with the capacity of giving some extra information necessary for a proper crack description. Three finite element approaches have been tested on different configurations in order to evaluate the eventual stress locking and mesh dependence: a Rashid-like (Rashid 1968) model (in which element stiffness is reduced to zero as soon as an energy threshold is reached), a fixed crack model (Droz 1987) and an embedded formulation (Alfaiate 2003). According to our test results, the Rashid-like model did not exhibit stress locking and together with the proposed probabilistic approach proved to be mesh independent. For these reasons, this model has been retained for the further probabilistic analysis.

4 EXPERIMENTAL VALIDATION

The continuum modeling presented in the previous sections has been firstly compared to an original experimental test performed at LCPC. The experiment consists of a four point displacement-controlled bending test on a plain concrete beam. The beam geometry is a 70x20x15cm prism. The span is 60 cm long. The constant moment zone is 20 cm long. The concrete used is an ordinary concrete (E=35GPa, fc=50MPa, ft=3MPa, values experimentally determined). Displacements (are measured on the front face via 6 LVDTs, and the bending vertical displacement is also recorded.

The numerical probabilistic approach is performed according the following steps:

- 30 computations are performed with the discrete approach for simulating the uniaxial tensile behavior of the concrete. A mean behavior is deduced from these results.
- An inverse analysis is done on the mean behavior to determine the parameters of the continuum approach
- These parameters are used to the modeling of the bending behavior. Again 30 computations are performed.

The beam has been modeled via T3 regular elements (see Figure 3); the Rashid-like model has been used. Results are given Figure 2.

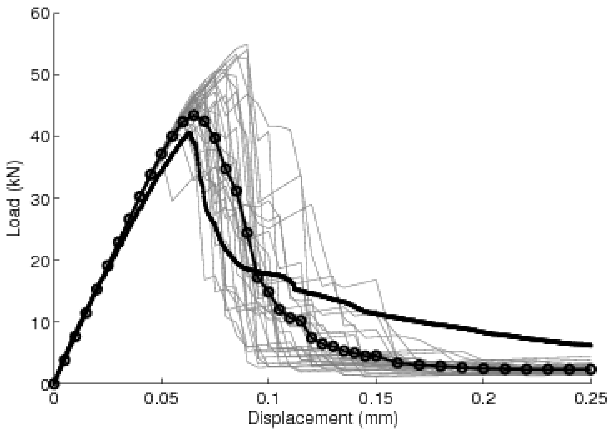

Figure 2. Global behavior: experimental (bold), numerical answer (grey) and mean (circles).

The correlation between the experimental result and the mean curve is quite good as the experimental result is contained in the set of the numerical answers and is very close to the mean answer. The macroscopic model provides not only a global answer but also some local information on cracks openings and distribution. The Figure 3 shows a typical crack pattern and the crack opening curves are presented in Figure 4. It is interesting to observe that not only a main macro crack is represented but also the multi-cracking character of the global failure is represented.

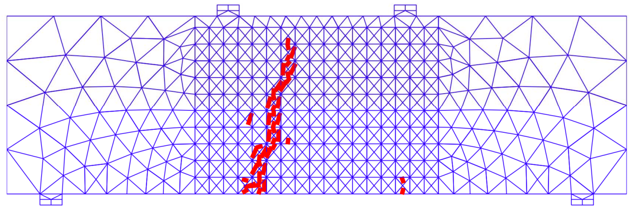

Figure 3. Typical crack pattern at the end of the simulation.

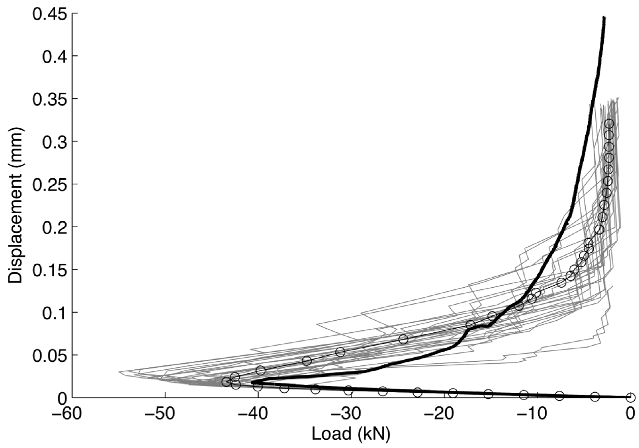

Figure 4. Crack opening (numerical -grey-, mean answer -circles- and experimental -bold-).

The second validation test concerns the simulation of a Brazilian test, performed at LCPC as well. The experiment consists in applying a compressive load on an 11cm diameter concrete cylinder. The evolution of the cylinder diameter is recorder on both faces of the specimen, and their mean value is used as the loading parameter of the testing device avoiding, in this way, loading instabilities.

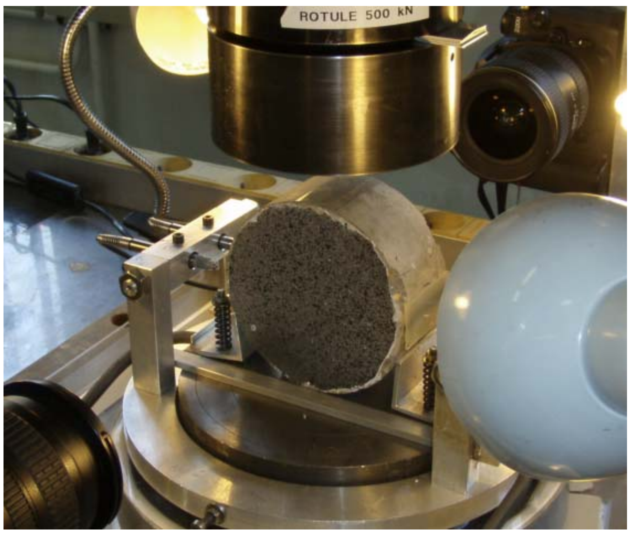

Figure 5. Experimental device for the Brazilian test.

The concrete used is the same as described above. And the modeling strategy is also the same as the one previously depicted. It is underlined, that the computation is driven in the same way as the experiment: the diametric expansion is also here the indirect loading parameter.

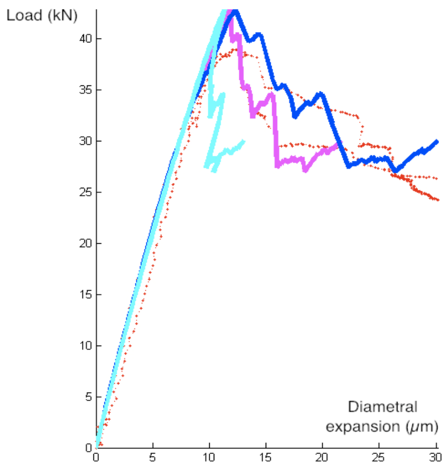

Figure 6. Applied load vs. diametric expansion curves. Comparison between experimental results (dotted red curves) simulation (blue curves – the magenta curve representing their mean value).

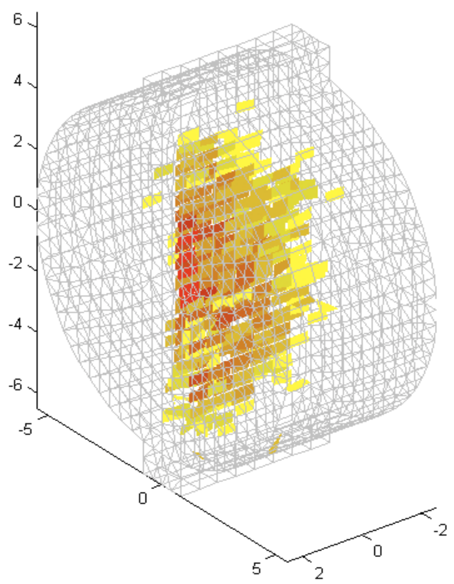

Figure 7. 3D crack pattern at the end of the simulation.

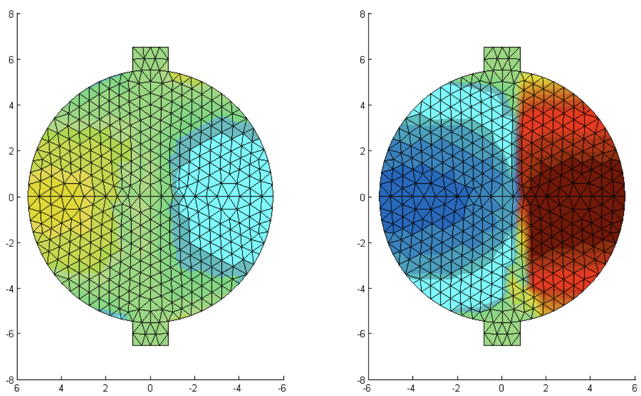

Figure 8. Distribution of horizontal displacements on both faces of the specimen.

Again, the correlation between the experimental result and the simulation is quite good, as far as only one result is shown here (Figure 6 to Figure 8). Nevertheless, the simulation clearly shows the 3D character of the cracking process in the specimen, leading to a strong dissymmetry between both faces. The crack pattern and the horizontal displacement are also clearly dissymmetric Figure 7 and Figure 8. These facts have been also experimentally observed.

5 CONCLUSIONS

In this paper, a model representing cracking processes in concrete through a robust continuum approach coupled with a simple numerical modeling of discontinuities is presented. 2D and 3D validation tests are depicted. One should not forget that the simplicity of the numerical modeling is meaningful only if the approach is coupled with the statistical distribution of properties and the given scale laws. This solution strategy allows to properly take into account scale effects and the heterogeneous nature of concrete, providing a reliable global answer as well as local information such as crack patterns and crack opening. The enhancement of the model towards 3D is a necessary step to take into accounts the complex three dimensional nature and geometry of cracks and giving a satisfactory description of the local-global behavior of a structure.

REFERENCES

Alfaiate, J. and Simone, A. and Sluys, L.J. 2003. Non-homogeneous displacement jumps in strong embedded discontinuities, Int.J.Solids Struct., 40(21), 5799-5817.

Bazant, Z.P. 2000. Size Effect, Int.J.Sol.Struct., 37 :69-80.

Bazant, Z. and Jirasek, M. 2002. Nonlocal integral formulations of plasticity and damage: survey of progress, J.Eng.Mech., 11: 1119-1149.

Droz, P. 1987. Modèle numérique du comportement non-linéare d'ouvrages massifs en béton non armée, EPFL,Lausanne.

Rashid, Y.R. 1968. Analysis of prestressed concrete pressure vessels, Nucl.Eng.Des., 7, 334-344.

Rossi, P. & Wu, X. 1992. Probabilistic model for material behaviour analysis and appraisement of concrete structures. Mag. Of Concr. Res., 44(161): 271-280.

Rossi, P., Wu, X., Le Maou, F. & Belloc, A. 1994. Scale effects on concrete in tension. *Materials and Structures*, 27 (172): 437-444.

Rossi, P., 1998. Réflexions sur l'utilisation des modéles continus non linéaires pour la prise en compte de la fissuration des structures en béton, BLPC, 217, 85-89.

Tailhan, J.-L., Rossi, P., Lombart, J. & Guerrier, F. 2007. From a discontinuous to a macroscopic modelling of the mode I cracking behaviour of cement-based composites. FRAMCOS 6, *Catania, Italy*.

Tailhan, J.-L., Dal Pont, S. & Rossi, P. 2009. From local to global probabilistic modeling of concrete cracking, submitted to *Annals of solids and structural mechanics*.