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The influence of reinforcing bar on crack extension of concrete

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ABSTRACT: Three point bending beams with reinforcing bar placed at three different positions were used to study the influence of reinforcing steel on crack extension of concrete. A pair of forces F_s on the crack simulates the constraint exerted by steel. Cohesive force acted on the fictitious crack was considered. The whole process of crack propagation was analyzed combined with $P-CMOD$ curves obtained from the test. The process of crack initiation, stable extension, and unstable failure was investigated using resistance strain gauges that were pasted on the center of the specimens. Therefore, the initial fracture toughness K_{Ic}^{ini} and the unstable fracture toughness K_{Ic}^{un} of reinforced concrete were introduced. Double-K fracture criterion was used for describing the whole process of crack propagation of reinforced concrete three-point bending notched beams. The comparisons were made subsequently between the theory and test data, and the proposed theory agrees reasonably with experimental data.

1 INTRODUCTION

Concrete is an essential material widely used in civil engineering. A large number of experimental studies have shown that the fracture process in concrete structures includes three different stages: crack initiation, stable crack extension, and unstable failure. In order to respond to the different states in concrete fracture, the double-*K* fracture criterion (Xu & Reinhardt 1998, 1999a, b, c) were proposed. In the criterion, the two fracture parameters termed as: initial fracture toughness K_{Ic}^{ini} and unstable fracture toughness K_{Ic}^{un} are introduced; both of them are given in terms of stress intensity factor. The double-*K* fracture parameters can be measured directly in experiments by three-point bending tests (TPBT) and compact tension (CT) tests. After the unstable fracture toughness K_{Ic}^{un} is evaluated, the initial fracture toughness K_{Ic}^{ini} also can be obtained by using the relationship between double-*K* fracture parameters K_{Ic}^{ini}, K_{Ic}^{un} and the cohesive toughness K_{Ic}^{c}. An analytical approach is developed to determine the value of cohesive toughness K_{Ic}^{c}, which requires specialized numerical integration because of singularity problem at the integral boundary. Later, a simplified method was proposed using two empirical formulae to determine the double-*K* fracture parameters for TPBT (Xu & Reinhardt 2000). Compared with other fracture models, such as the fictitious crack model (Hillerborg et al. 1976), the crack band model (Bazant & Oh 1983), the two-parameter model (Jenq & Shah 1985a, b), the size effect model (Bazant & Kazemi 1990), the effect crack model (Swartz & Go 1984, Swartz & Refai 1987, Karihaloo & Nallathambi 1989, 1990), the double-*K* fracture criterion can be used to predict crack initiation, steady crack propagation and unstable fracture. Regarding practical experimental performance in the determination of fracture parameters introduced in the double-*K* criterion, one only needs to measure the ascending branch of a *P-CMOD* curve, without steady unloading procedure. It means that when a material and structural laboratory even does not have a closed-loop testing system, it can also perform the experimental measurements of the double-*K* fracture parameters K_{Ic}^{ini} and K_{Ic}^{un}. Considering the calculation of the fracture parameters, no statistical regression is necessary in the calculating procedure of the determination of the double-*K* fracture parameters.

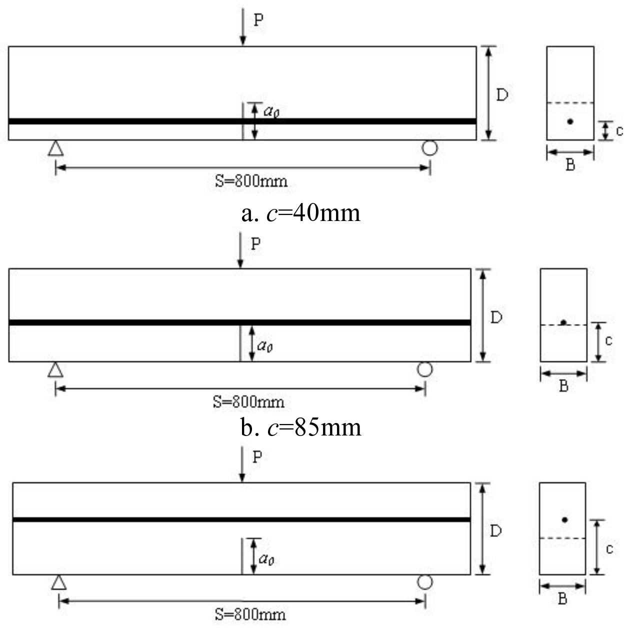
Because concrete is a heterogeneous and multiphase material consisting of hardening cement paste, aggregate and the interfacial transition zone between them, there inevitably exist gaps and micro-cracks within it, and there are some inherent characteristics of concrete, such as low tensile strength, shrinkage and creep, etc, these are the main reasons for concrete cracks. Once the concrete surface cracks, the crack will be further expansion under the influence of external factors. In recent years, the integral failure of concrete structure and durability problem caused by cracking in concrete has become a worldwide problem difficult to overcome. In order to reduce the harmful effects of the concrete cracks, suppress or delay the further expansion of the surface cracks, usually, reinforcing steel, fiber or other materials with higher tensile properties are added in concrete to enhance the tensile capacity of concrete and improve the anti-damage capability of structure.

Reinforcing bar has characteristics as high tensile strength and good plastic deformation. Due to the closing action of reinforcing bar added in concrete, crack initiation will be delayed, crack speed will be slow down, and carrying capacity will be improved. However, the process of crack propagation in reinforced concrete more complex than that in concrete. The main objective of the paper is to study the fracture process of concrete three-point bending beams with reinforcing bar placed at three different positions, and determine the fracture criterion for describing the state of crack propagation.

2 THEORETICAL ANALYSIS

2.1 *Specimen configuration*

Configurations of concrete three-point bending beams with reinforcing bar placed at three different positions are shown in Figure 1. Where B is beam width, D is depth, S is span length and S=4D, a_0 is preformed crack length, *d* is bar diameter, *c* is the distance between the center of bar and beam bottom. According to different value of *c*, there are three kinds layout of reinforcing bar.



a. *c*=40mm

b. *c*=85mm

c. *c*=120mm

Figure 1. Arrangement of reinforcing bar in concrete three-point bending beam.

2.2 *Fundamental assumption*

(1) In the whole fracture process, the interface slippage between concrete and steel is neglected;
(2) The relationship between cohesive force and crack opening displacement use nonlinear softening relation;
(3) A pair of concentrated forces on the crack simulates the constraint exerted by steel;
(4) An ideal elastic-plastic model is used for describing the constitutive relation of steel.

2.3 *Analysis of crack propagation process of reinforced concrete*

For concrete three-point bending beams with reinforcing bar, with the load increases, the stress intensity factors at initial crack tip K_I also increase gradually. When the net K_I at the crack tip obtained by superposing the stress intensity factors produced at the crack tip by the load K_{Ip} and the closure forces exerted by steel K_{Is} reaches the initial fracture toughness K_{Ic}^{ini} of concrete, concrete at the crack tip begins to crack. With the load further increases, the bridging force from steel F_s is growing. When the net K_I at the crack tip reaches the unstable fracture toughness K_{Ic}^{un} of concrete, crack begins to expand unstably, and concrete gradually withdraws from the work.

The analysis of crack propagation indicates fracture process of reinforced concrete can be divided into three stages: crack initiation, stable extension and unstable failure. Therefore, after the initial fracture toughness K_{Ic}^{ini} and the unstable toughness K_{Ic}^{un} of reinforced concrete are redefined, the double-K fracture criterion is used for describing the whole process of crack propagation of reinforced concrete three-point bending notched beams:

$K_I < K_{Ic}^{ini}$, the crack does not extend;
$K_I = K_{Ic}^{ini}$, the crack begins to crack initially;
$K_{Ic}^{ini} < K_I < K_{Ic}^{un}$, the crack extend steadily;
$K_I = K_{Ic}^{un}$, the crack begins to crack unstably;
$K_I > K_{Ic}^{un}$, the crack extend unsteadily.

Due to the constraint exerted by steel, calculated methods of the initial fracture toughness K_{Ic}^{ini} and the unstable fracture toughness K_{Ic}^{un} of reinforced concrete are different from plain concrete. Here, the initial fracture toughness is measured from the plain concrete using the same material and specimen form with reinforced concrete, while the unstable toughness is obtained from the initial fracture toughness and the actual cohesive toughness of reinforced concrete according to the three parameters law (Xu & Reinhardt 2000).

$$K_{Ic}^{un} = K_{Ic}^{ini} - K_{Ic}^{c} \tag{1}$$

The nonlinear softening constitutive relation (Reinhardt et al. 1986) is used for calculating the cohesive toughness.

$$\frac{\sigma}{f_t}=\left\{1+(c_1\frac{w}{w_0})^3\right\}\exp(-c_2\frac{w}{w_0})-\frac{w}{w_0}(1+c_1^3)\exp(-c_2) \tag{2}$$

To make the result more precise, the constants c_1, c_2, w_0 in equation (2) are recalculated as follow based on the actual characteristics of concrete.

$$\begin{aligned}
&f_{cm}=0.4f_{cu}{}^{7/6}\\
&f_{ck}=f_{cm}-8\\
&f_t=1.4(f_{ck}/f_{ck0})^{2/3}\\
&G_F=(0.0204+0.0053d_{\max}^{0.95}/8)(f_{cm}/f_{cm0})^{0.7}\\
&\lambda=10-[f_{ck}/(2f_{ck0})]^{0.7}\\
&c_1=(d_{\max}/8)^{0.75}\\
&c_2=(0.92-d_{\max}/400)\lambda\\
&\alpha_F=\lambda-d_{\max}^{0.9}/8\\
&w_0=\alpha_F G_F/f_t
\end{aligned} \tag{3}$$

On crack initiation, the stress intensity factor produced at the crack tip by the load is (Tada et al. 1985)

$$K_{Ip}^{ini}=\frac{3(2P_{ini}+mg)S}{4D^2B}\sqrt{a_0}\,F_1\left(\frac{a_0}{D}\right) \tag{4}$$

$$F_1\left(\frac{a_0}{D}\right)=\frac{1.99-\left(\frac{a_0}{D}\right)\left(1-\frac{a_0}{D}\right)\left[2.15-3.93\frac{a_0}{D}+2.7\left(\frac{a_0}{D}\right)^2\right]}{\left(1+2\frac{a_0}{D}\right)\left(1-\frac{a_0}{D}\right)^{3/2}} \tag{5}$$

where P_{ini} is the initial cracking load, mg is the self – weight of the specimen between supports.

On crack initiation, the stress intensity factor produced by the concentrated steel force is (Tada et al. 1985)

$$K_{Is}^{ini}=-\frac{2F_s^{ini}}{B\sqrt{\pi a_0}}F_2\left(\frac{c}{a_0},\frac{a_0}{D}\right) \tag{6}$$

$$\begin{aligned}
F_2\left(\frac{c}{a_0},\frac{a_0}{D}\right)=&\frac{3.52(1-c/a_0)}{(1-a_0/D)^{3/2}}-\frac{4.35-5.28c/a_0}{(1-a_0/D)^{1/2}}+\\
&\left\{\frac{1.30-0.30(c/a_0)^{3/2}}{\left[1-(c/a_0)^2\right]^{1/2}}+0.83-1.76c/a_0\right\}\{1-(1-c/a_0)a_0/D\}
\end{aligned} \tag{7}$$

where F_s^{ini} is the steel force when crack starts to open. As reinforcement force acting on the crack is a closed force, so K_{Is}^{ini} is a negative value.

On crack unstable failure, the stress intensity factor produced at the crack tip by the load is (Tada et al. 1985)

$$K_{Ip}^{un}=\frac{3(2P_{un}+mg)S}{4D^2B}\sqrt{a_c}\,F_1\left(\frac{a_c}{D}\right) \tag{8}$$

$$F_1\left(\frac{a_c}{D}\right)=\frac{1.99-\left(\frac{a_c}{D}\right)\left(1-\frac{a_c}{D}\right)\left[2.15-3.93\frac{a_c}{D}+2.7\left(\frac{a_c}{D}\right)^2\right]}{\left(1+2\frac{a_c}{D}\right)\left(1-\frac{a_c}{D}\right)^{3/2}} \tag{9}$$

where P_{un} is the maximum load, a_c is the critical effective crack length.

On crack unstable failure, the stress intensity factor produced by the concentrated steel force is (Tada et al. 1985)

$$K_{Is}^{un}=-\frac{2F_s^{un}}{B\sqrt{\pi a_c}}F_2\left(\frac{c}{a_c},\frac{a_c}{D}\right) \tag{10}$$

$$\begin{aligned}
F_2\left(\frac{c}{a_c},\frac{a_c}{D}\right)=&\frac{3.52(1-c/a_c)}{(1-a_c/D)^{3/2}}-\frac{4.35-5.28c/a_c}{(1-a_c/D)^{1/2}}+\\
&\left\{\frac{1.30-0.30(c/a_c)^{3/2}}{\left[1-(c/a_c)^2\right]^{1/2}}+0.83-1.76c/a_c\right\}\{1-(1-c/a_c)a_c/D\}
\end{aligned} \tag{11}$$

where F_s^{un} is the steel force when crack starts to expand unsteadily.

Detailed calculated method of the critical effective crack length of reinforced concrete may be seen elsewhere (Zhu 2009).

2.4 *Determination of the forces from steel*

For c=40mm, when concrete begins to crack initially, the steel deformation is small and still within the regime of elastic deformation. Therefore, the steel force can be calculated by the Hooke's law.

$$\sigma_s^{ini}=E_s\varepsilon_s^{ini} \tag{12}$$

$$F_s^{ini}=\sigma_s^{ini}A_0 \tag{13}$$

For c=85mm and c=120mm, when concrete starts to open, the steel almost no distortion, so the constraint exerted by steel can be ignored. To simplify the computations, it is assumed that the stress intensity factor at initial crack tip produced by steel is zero.

When the crack begins to crack unstably, if the steel yields, stress of the steel is the yield strength; if the steel not yields, the steel force is also calculated by the Hooke's law.

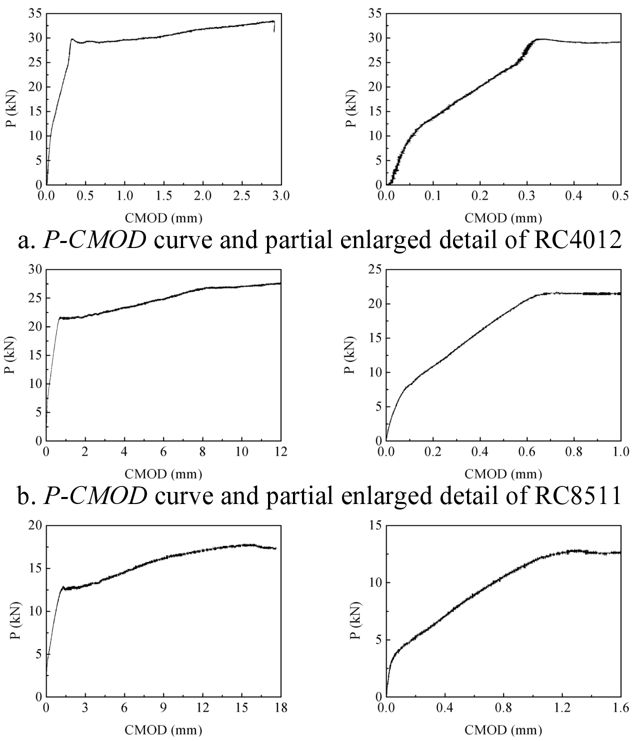
$F_s^{un} = \sigma_s^{un} A_0 = f_y A_0$, the steel yields;

$F_s^{un} = \sigma_s^{un} A_0 = E_s \varepsilon_s^{un} A_0$, the steel not yields.

3 TEST RESULTS

3.1 *Test curves*

The compressive strength of concrete used in the test is 44.68MPa, elastic modulus is 34.28GPa, and steel yield strength is 428.55MPa. B is 100mm, D is 200mm, S is 800mm, a_0 is 80mm, and d is 10mm. The typical load and crack mouth opening displacement (*P-CMOD*) curves of each group are shown in Figure 2.



a. *P-CMOD* curve and partial enlarged detail of RC4012

b. *P-CMOD* curve and partial enlarged detail of RC8511

c. *P-CMOD* curve and partial enlarged detail of RC1203

Figure 2. Typical $P-CMOD$ curve of each group obtained from the test.

As can be seen from Figure 2, no matter what position of steel in concrete, all the *P-CMOD* curves show the same pattern. At initial loading stage, with the increase of *CMOD*, the load is almost linearly increased, and the load is increased faster than the *CMOD*. When the concrete at crack tip cracking, the increased rate of the load becomes slow, and the *CMOD* is increased faster, the fictitious crack extension stage of concrete begins. Concrete on the fictitious crack surface will subject to the closed action caused by cohesive force, load and crack mouth opening displacement begins to show nonlinear relationship, which is similar to the ascending curve of plain concrete. However, as load continues to increase, the curve of reinforced concrete show different characteristic from plain concrete. Due to crack mouth opening displacement constantly increases, the constraint action exerted by steel becomes reinforcement. When load, cohesive force as well as the constraint action exerted by steel achieves a relative balanced state, the crack propagates stably once again, and the *P-CMOD* curve presents a linear state. When crack starts to expand unstably, concrete gradually withdraws from the work; the external load will be fully borne by the steel.

3.2 *Test parameters*

From the analysis of *P-CMOD* curves in Figure 2, one also can find that the process of crack propagation in reinforced concrete can be divided into three stages: crack initiation, stable extension and unstable failure, which coincides with the above analysis. According to the characteristic points of crack initiation and unstable failure in *P-CMOD* curves, the initial cracking load (P_{ini}), the maximum load (P_{max}), and the corresponding other parameters are measured, which are listed in Table 1.

As can be seen from the Table 1, due to the enhancement of steel added in concrete three-point bending beam, the initial cracking load, the maximum load and other parameters improved greatly compare with those of plain concrete, especially for the specimens of c=40mm. When the crack begins to crack initially, the strain of steel for c=85mm and c=120mm is very small, and the acting force of steel is also small, which can be neglected. Therefore, the

Table 1. Fracture parameters of specimens.

Category designation	P_{ini} (kN)	P_{un} (kN)	ε_s^{ini} ($\mu\varepsilon$)	σ_s^{ini} (MPa)	F_s^{ini} (kN)	F_s^{un} (kN)	$CMOD_c$ (mm)	a_c (mm)
RC4012	6.975	25.24	465	93	7.301	33.641	0.272	146.658
RC4013	6.37	25.08	341	68.2	5.354	33.641	0.421	143.203
RC4014	6.03	25.16	365	73	5.731	33.641	0.285	145.984
RC4015	6.22	24.92	367	73.4	5.762	33.641	0.306	141.166
Average	**6.399**	**25.1**	**384.5**	**76.9**	**6.037**	**33.641**	**0.321**	**144.253**
RC8511	3.36	18.49	18.6	3.712	0.291	33.641	0.504	157.147
RC8512	3.48	18.41	28.7	5.74	0.451	33.641	0.467	157.33
RC8514	3.08	18.18	29.3	5.86	0.46	33.641	0.524	154.761

RC8515	2.82	18.26	26	5.2	0.408	33.641	0.538	154.564
Average	**3.185**	**18.335**	**25.6**	**5.128**	**0.403**	**33.641**	**0.508**	**155.95**
RC1202	2.8	12.144	12.87	2.574	0.202	33.641	0.77	154.595
RC1203	3.334	12.303	6.65	1.33	0.104	33.641	0.967	157.305
RC1204	2.91	12.382	13.5	2.7	0.212	33.641	0.838	158.365
RC1205	2.86	12.223	11.36	2.272	0.178	33.641	0.887	158.309
Average	**2.976**	**12.263**	**11.09**	**2.219**	**0.174**	**33.641**	**0.866**	**157.143**
CON1	2.818	4.683					0.054	100.487
CON2	3.255	5.194					0.0658	103.956
CON3	2.659	4.921					0.0585	101.637
CON4	4.167	5.318					0.0536	95.806
CON5	3.215	5.159					0.0603	101.093
CON6	2.624	4.524					0.0631	107.122
Average	**3.123**	**4.967**					**0.0592**	**101.684**

assumption that the stress intensity factor at initial crack tip produced by steel is zero is feasible. All stress added in concrete yield when crack begins to propagate unstably.

As can be seen from the Table 1, due to the enhancement of steel added in concrete three-point bending beam, the initial cracking load, the maximum load and other parameters improved greatly compare with those of plain concrete, especially for the specimens of c=40mm. When the crack begins to crack initially, the strain of steel for c=85mm and c=120mm is very small, and the acting force of steel is also small, which can be neglected. Therefore, the assumption that the stress intensity factor at initial crack tip produced by steel is zero is feasible. All stress added in concrete yield when crack begins to propagate unstably.

According to the fracture parameters listed in Table 1, the net stress intensity factors at initial crack tip on crack initiation and unstable failure are calculated. The calculated results are compared with the initial fracture toughness K_{Ic}^{ini} and the unstable fracture toughness K_{Ic}^{un} of reinforced concrete. All results are listed in Table 2 and 3.

Table 2. Calculated results of initial fracture toughness.

Category designation	K_{Ip}^{ini} (MPam$^{1/2}$)	K_{Is}^{ini} (MPam$^{1/2}$)	K_{I}^{ini} (MPam$^{1/2}$)	K_{Ic}^{ini} (MPam$^{1/2}$)	Relative error (%)
RC4012	1.269	-0.723	0.546	0.586	-6.826
RC4013	1.161	-0.53	0.631	0.586	7.679
RC4014	1.101	-0.567	0.534	0.586	-8.873
RC4015	1.135	-0.571	0.564	0.586	-3.754
Average	**1.166**	**-0.598**	**0.569**	**0.586**	**-2.901**
RC8511	0.628	0	0.628	0.586	7.167
RC8512	0.649	0	0.649	0.586	10.751
RC8514	0.578	0	0.578	0.586	-1.365
RC8515	0.532	0	0.532	0.586	-9.215
Average	**0.597**	**0**	**0.597**	**0.586**	**1.877**
RC1202	0.529	0	0.529	0.586	-9.727
RC1203	0.623	0	0.623	0.586	6.314
RC1204	0.548	0	0.548	0.586	-6.485
RC1205	0.539	0	0.539	0.586	-8.020
Average	**0.56**	**0**	**0.56**	**0.586**	**-4.437**

Table 3. Calculated results of unstable fracture toughness.

Category rdesignation	K_{Ip}^{un} (MPam$^{1/2}$)	K_{Is}^{un} (MPam$^{1/2}$)	K_{I}^{un} (MPam$^{1/2}$)	K_{Ic}^{c} (MPam$^{1/2}$)	K_{Ic}^{un} (MPam$^{1/2}$)	Relative error (%)
RC4012	15.897	-13.583	2.313	-1.763	2.349	-1.533
RC4013	14.365	-12.179	2.186	-1.565	2.151	1.627
RC4014	15.548	-13.29	2.258	-1.723	2.309	-2.209
RC4015	13.532	-11.451	2.08	-1.489	2.075	0.241
Average	**14.836**	**-12.626**	**2.21**	**-1.635**	**2.221**	**-0.495**
RC8511	16.258	-13.15	3.108	-2.479	3.065	1.403
RC8512	16.293	-13.25	3.043	-2.499	3.085	-1.361
RC8514	14.731	-11.932	2.799	-2.268	2.854	-1.927
RC8515	14.698	-11.839	2.859	-2.252	2.838	0.740
Average	**15.495**	**-12.543**	**2.952**	**-2.374**	**2.961**	**-0.304**
RC1202	9.834	-6.872	2.961	-2.245	2.831	4.592
RC1203	10.932	-7.807	3.124	-2.472	3.058	2.158
RC1204	11.427	-8.216	3.211	-2.581	3.167	1.389
RC1205	11.259	-8.194	3.065	-2.573	3.159	-2.976
Average	**10.863**	**-7.772**	**3.09**	**-2.468**	**3.054**	**1.179**

The relative error in Table 2 and 3 is (the net stress intensity factors at initial crack tip K_I - the initial fracture toughness K_{Ic}^{ini} or the unstable fracture toughness K_{Ic}^{un}) / the initial fracture toughness K_{Ic}^{ini} or the unstable toughness $K_{Ic}^{un}\times 100\%$. Negative sign indicates the value of fracture toughness is greater than that of the net stress intensity factors.

The results in Table 2 indicate the net stress intensity factors at initial crack tip K_I^{ini} on crack initiation judged from *P-CMOD* curve is close to the initial fracture toughness K_{Ic}^{ini}. For a single specimen, the relative error is less than 10%; for the average, the relative error is no more than 5%. The results in Table 3 indicate the net stress intensity factors at initial crack tip K_I^{un} on unstable failure judged from *P-CMOD* curve is close to the unstable fracture toughness K_{Ic}^{un}. For a single specimen, the relative error is less than 5%; for the average, the largest relative error is only 1.179%. Therefore, after the initial fracture toughness and the unstable fracture toughness are redefined, the double-*K* fracture criterion can be used for judging the status of crack propagation in reinforced concrete.

4 CONCLUSIONS

Fracture problems of reinforced concrete are studied using concrete three point bending beams with reinforcing bar placed at three different positions. A pair of forces on the crack simulates the constraint exerted by steel, and cohesive force acted on the fictitious crack was considered. The whole process of crack propagation was analyzed combined with $P-CMOD$ curves obtained from the test. The results show that fracture process of reinforced concrete is similar to plain concrete which can be divided into three stages: crack initiation, crack stable extension and crack unstable failure. Therefore, the initial fracture toughness and the unstable fracture toughness suitable for reinforced concrete are introduced; the double-*K* fracture criterion is used for describing the whole process of crack propagation in reinforced concrete. The applicability of the double-*K* fracture criterion has been verified by test, and the theoretical results are in good agreement with the test results, which indicate the double-*K* fracture criterion can be used as theoretical model for judging the state of crack expansion of reinforced concrete, after the fracture parameters in it are redefined.

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