Fracture Mechanics of Concrete and Concrete Structures -
Recent Advances in Fracture Mechanics of Concrete - B. H. Oh, et al.(eds)

Effects of shore stiffness and concrete cracking on slab construction load

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ABSTRACT: Evaluation of construction load is a critical issue for the design and construction of flat plates which are susceptible to excessive deflections and concrete cracking. Underestimated construction load may cause structural and construction problems to flat plate slabs. The present study focused on the effects of shore stiffness and concrete cracking on the distribution and magnitude of construction load. Slabs connected by shores were idealized with a simple frame model. Based on the frame model, we developed a simplified method for the evaluation of construction load, addressing the effects of various design parameters including shore stiffness and concrete cracking. The proposed method was used to predict construction load in an example model.

1 INTRODUCTION

Recently, the use of flat-plate system for high-rise buildings has been increased due to the advantages of reduced floor height and improved constructability. However, the low slab stiffness of long-span flat-plates frequently causes excessive deflection and cracking in the slabs (Gardner & Fu 1987).

Grundy & Kabaila (1963) developed a simplified method for the evaluation of construction load. In the Grundy's method, newly superimposed construction load is distributed to the slabs, according to the ratio of the concrete stiffness of the slabs. In contrast, Mosallam and Chen (1991) developed a different method, in which total construction load is redistributed to the slabs according to the ratio of the concrete stiffness whenever the construction load is updated. However, in the existing methods, shores assumed to have infinite rigidity. The effect of concrete cracking was not considered.

In the present study, a simplified method for the evaluation of construction load was developed. To accurately evaluate construction load, the effects of the shores' stiffness and concrete cracking were considered.

2 INFLUENCE PARAMETERS

2.1 *shores' stiffness*

First, the effect of shores' stiffness was investigated. For this purpose, a beam model that is supported by continuous vertical springs and simple supports at the ends was considered (Fig. 1). Here, a slab and shores were idealized as a beam and springs, respectively. The governing equation for the deflection of the beam can be expressed as

$$EI\frac{d^4 v}{dx^4} = q - k_s v \qquad (1)$$

k_s is the spring stiffness, which can be defined by the properties of the shore as follows

$$k_s = \frac{A_s E_s}{S_1 H} \qquad (2)$$

where A_s = cross-sectional area of a shore, E_s = elastic modulus of the material used for the shore, S_l = spacing of the shores, and H = height of the shores. Defining $\beta = \sqrt[4]{k_s/4EI}$ and considering the support conditions, the deflection of the beam is be calculated as follows.

$$v=\frac{q}{k_s}\left(1-\frac{2\sin\frac{\beta L}{2}\sinh\frac{\beta L}{2}}{\cos\beta L+\cosh\beta L}\sin\beta x\cdot\sinh\beta x\right)-\frac{q}{k_s}\left(\frac{2\cos\frac{\beta L}{2}\cosh\frac{\beta L}{2}}{\cos\beta L+\cosh\beta L}\cos\beta x\cdot\cosh\beta x\right) \tag{3}$$

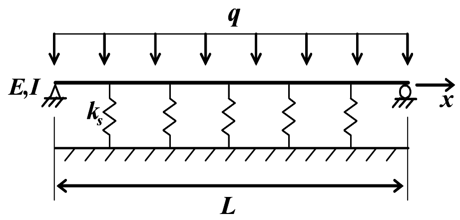


Figure 1. Beam model supported by continuous springs.

The deflection of each spring can be estimated from Equation (3). The force of each spring is expressed as $S_1 k_s v$. Thus, the sum of the spring forces LD_{sp} can be defined as

$$LD_{sp}=S_1 k_s\sum_{i=1}^{n_s} v_i \tag{4}$$

where n_s = the number of springs. The load resisted by the beam, LD_{sl}, can be calculated as

$$LD_{sl}=qL-LD_{sp} \tag{5}$$

In Equations (4) and (5), the loads that are transferred to the beam and springs are determined according to the parameters k_s, β, and L. The ratio of the loads that are resisted by the beam and the springs can be written as

$$K=\frac{LD_{sl}}{LD_{sp}} \tag{6}$$

This ratio K can be regarded as the ratio of the beam stiffness and the spring stiffness. The equivalent stiffness ratio K determines the load distribution between the slab and shores.

Figure 2 shows the arrangement of shores supporting a slab. In the figure, S_2 is the shore's space perpendicular to the span L, and h is the slab's thickness. I used for the calculation of $\beta=\sqrt[4]{k_s/4EI}$ in Equation (3) can be defined as the moment of inertia of the slab ($=S_2h^3/12$).

Figure 3 shows a simplified discrete model that replaces the continuum beam-spring model. In the model, the continuous springs representing shores is replaced with a concentrated spring. In the figure, k_{sl} is the equivalent stiffness of each slab, and k_{sp} is the equivalent stiffness of the concentrated spring, which represents the sum of the stiffness of all shores in a story. For the idealized discrete system in

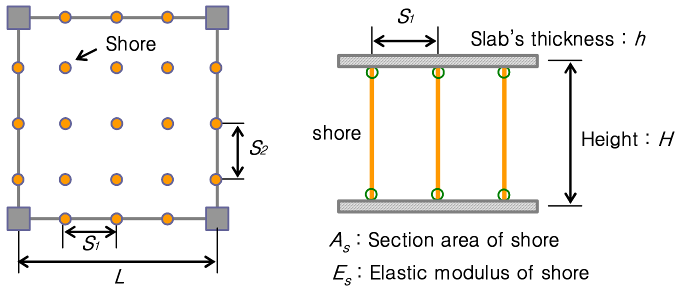
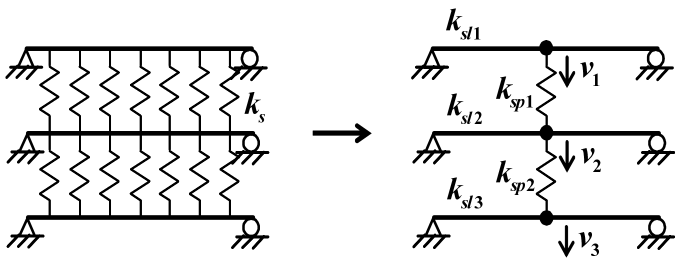


Figure 2. Shoring system for flat plate under construction.



(a) continuous shoring system (b) Idealized discrete system

Figure 3. Idealized discrete model for slabs supported by shores.

Boundary condition	Effective length factor ψ
Continuous at both ends	ψ = 0.6; ψL; $0.67M_0$; $0.33M_0$
Continuous at one end, Pinned at the other end	ψ = 0.8; $0.75M_0$; ψL; $0.63M_0$
Pinned at both ends	ψ = 1.0; ψL; M_0

Figure 4. Effective length factor ψ for slabs with various boundary restraints.

Figure 3(b), the stiffness matrix K_{ff} can be written as

$$\underline{K}_{ff}=\begin{pmatrix} k_{sl1}+k_{sp1} & -k_{sp1} & 0 \\ -k_{sp1} & k_{sl2}+k_{sp1}+k_{sp2} & -k_{sp2} \\ 0 & -k_{sp2} & k_{sl3}+k_{sp2} \end{pmatrix} \tag{7}$$

Using the equivalent stiffness ratio $K = k_{sl} / k_{sp}$ which is defined as the value in Equation (6), Equation (7) can be redefined as

$$\underline{K}_{ff}=\begin{pmatrix} k_{sl1}+\frac{k_{sl1}}{K_1} & -\frac{k_{sl1}}{K_1} & 0 \\ -\frac{k_{sl1}}{K_1} & k_{sl2}+\frac{k_{sl1}}{K_1}+\frac{k_{sl2}}{K_2} & -\frac{k_{sl2}}{K_2} \\ 0 & -\frac{k_{sl2}}{K_2} & k_{sl3}+\frac{k_{sl2}}{K_2} \end{pmatrix} \tag{8}$$

Though Equation (8) was developed for a simply supported slab, the slab stiffness can vary with the boundary conditions. Figure 4 shows the effective length factor ψ for addressing various boundary conditions. Using the effective length ψL instead of L in Equations (3) through (8), the stiffness matrix K_{ff} for various boundary conditions can be estimated (see 4. application of proposed method).

2.2 *effective stiffness of slab*

In the present study, Bischoff's equation (Bischoff & Scanlon 2007) was used to calculate the effective moment of inertia(I_e) of slabs. The Bischoff's equation gives good predictions for flexural members with low reinforcement ratios less than 1%, which is typical for slabs. In the Bischoff's equation, the effective moment of inertia(I_e) is defined as

$$I_e = \frac{I_{cr}}{1-\left(\dfrac{M_{cr}}{M_a}\right)^2\left(1-\dfrac{I_{cr}}{I_g}\right)} \le I_g \tag{9}$$

where I_g is the moment of inertia of the gross section, and I_{cr} is the moment of inertia of the cracked section. M_{cr} is the cracking moment of the slab defined with f_c(t) (concrete strength at age t). M_a is the maximum moment applied to the slab, which can be expressed with the construction load. Using the definitions of M_{cr} and M_a,

$$\begin{aligned}\frac{M_{cr}}{M_a} &= \frac{0.63\sqrt{f_c}\cdot S_2\cdot h^2/6}{w\cdot(\psi L)^2/8} \\ &= \frac{0.63\sqrt{f_c}\cdot S_2\cdot h^2/6}{23.5S_2\cdot h\cdot(LR)\cdot(\psi L)^2/8/10^6} \\ &= 0.357\cdot 10^5\,\frac{\sqrt{f_c}\cdot h^2}{(LR)\cdot(\psi L)^2}\end{aligned} \tag{10}$$

where the slab's unit weight w is 23.5kN/m³, and LR is the ratio of the construction load to the slab's self-weight.

I_{cr} varies with various design parameters. Generally, the reinforcement ratios in flat-plates range 0.5%~1.0%, concrete's elastic modulus 15000MPa~35000MPa, and concrete cover 20mm~30mm. With the parameters of this range, I_{cr} is 12%~35% of I_g. In the present study, I_{cr} is assumed to be 25% of I_g for simplicity in calculations.

Inserting Equation (10) and $I_{cr} = 0.25 I_g$, Equation (9) can be simplified as

$$\frac{I_e}{I_g} = \frac{1}{4 - C\left(\dfrac{\sqrt{\beta_{cc}}}{LR}\right)^2} \tag{11}$$

where $C = 3.82\times 10^9\left(f_{cu}\cdot t^2/(\psi L)^4\right)$, f_{cu} is the concrete's compression strength at 28 day, and β_{cc} = $exp\left\{0.25\left[1-\sqrt{28/t}\right]\right\}$. In Equation (11), the effective moment of inertia can be defined as the function of $LR/\sqrt{\beta_{cc}}$. Figure 5 shows the variations of the effective moment of inertia with $LR/\sqrt{\beta_{cc}}$ for a slab.

$\left(LR/\sqrt{\beta_{cc}}\right)_{cr}$ indicates the critical load corresponding to initial concrete cracking, which can be defined as $\left(LR/\sqrt{\beta_{cc}}\right)_{cr} = \sqrt{C}/\sqrt{3}$.

The average effective stiffness of a slab varies with the boundary conditions. I_e in Equation (11) can be used for simply supported beams. By applying the effective length factor ψL, the variations of the slab's effective stiffness can be approximately addressed. ACI committee 435 (1978) reported that the deflection of cracked member depends largely on the effective stiffness at the center of the span. Thus, the average effective stiffness of continuous slabs as well as simply supported slabs can be approximately determined by the effective stiffness at the mid-span.

3 PROPOSED METHOD FOR CALCULATING CONSTRUCTION LOAD

In the multi-story slabs connected by shores, generally, additional construction load is newly superimposed at the time when new concrete is cast at the top floor, and at the time when shores are removed at the bottom floor. Generally, the construction live load excluding the slab's self weight account for 50 percent of the slab's self weight D (ACI 347, 2005). Thus, the total superimposed load at the top floor becomes 1.5D, which indicates that the load ratio LR at this construction stage is 1.5.

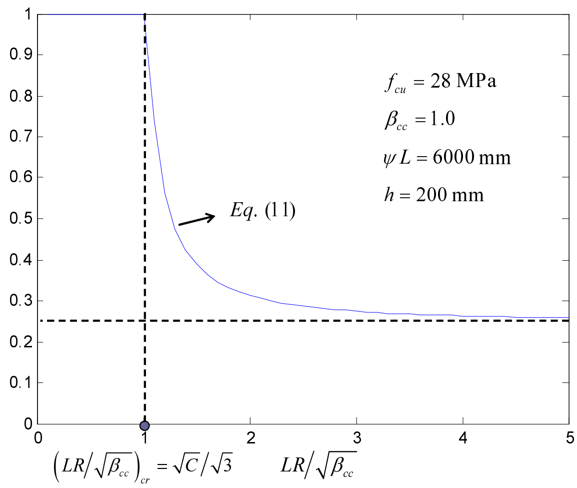


Figure 5. Relationship between load ratio and effective moment of inertia.

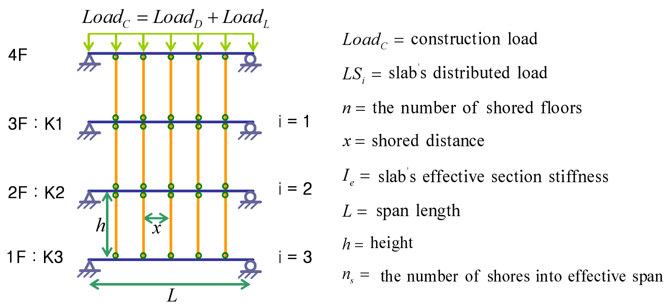


Figure 6. Slab-shore model at the time when new concrete is cast at the top floor.

3.1 *Concrete casting at top floor*

Figure 6 shows three floor slabs supported by shores. In the figure, concrete is cast at the top floor (4F). The floors from 3F to 1F resist the newly superimposed load. The number of the slabs resisting the superimposed load $n = 3$. Variable i indicates the order of the floors resisting the construction load, from the top to the bottom: for the 3F slab, $i = 1$. As mentioned, the magnitude of the newly superimposed load ($Load_c$) at the top floor is assumed to be $1.5D$.

From Equation (8), the stiffness matrix for the slab-shore system can be expressed as

$$\underline{K}_{ff} = \begin{pmatrix} k_{sl1} + \dfrac{k_{sl1}}{K_1} & -\dfrac{k_{sl1}}{K_1} & \cdots & 0 \\ -\dfrac{k_{sl1}}{K_1} & k_{sl2} + \dfrac{k_{sl1}}{K_1} + \dfrac{k_{sl2}}{K_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & -\dfrac{k_{sln-1}}{K_{n-1}} \\ 0 & 0 & -\dfrac{k_{sln-1}}{K_{n-1}} & k_{sln} - \dfrac{k_{sln-1}}{K_{n-1}} \end{pmatrix} \tag{12}$$

E_c and I_e of each slab are used to calculate k_{sli} and K_i where E_c is the slab's elastic modulus at its age, and I_e is the slab's effective moment of inertia.

With the stiffness in Equation (12), the equilibrium equation for the slab-shore system can be defined as

$$\underline{K}_{ff} \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix} = \begin{pmatrix} Load_c \\ 0 \\ \vdots \\ 0 \end{pmatrix} \tag{13}$$

where v_i is the deflection increment developed by the superimposed load ($Load_c$). It should be noted that v_i is not the deflection of actual slabs, but the deflection of the equivalent discrete model in Figure 3. The deflections of actual slabs can be calculated using Equation (3) and (13). Using v_i resulting from Equation (13), the construction loads distributed to the slabs can be calculated.

$$\begin{pmatrix} LS_1 \\ LS_2 \\ \vdots \\ LS_n \end{pmatrix} = \underline{K}_{fsl} \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix} = \underline{K}_{fsl} \underline{K}_{ff}^{-1} \begin{pmatrix} Load_c \\ 0 \\ \vdots \\ 0 \end{pmatrix} \tag{14}$$

$$\text{where} \quad \underline{K}_{fsl} = \begin{pmatrix} k_{sl1} & 0 & \cdots & 0 \\ 0 & k_{sl2} & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & k_{sln} \end{pmatrix} \tag{15}$$

When the effective moment of inertia I_e is considered, the construction load varies with the values of I_e, and thus, iterative calculations are required. In the present study, to avoid iterative calculations, the I_e values calculated at the previous construction stage were used.

3.2 *Removal of shores at bottom floor*

When shores are removed at the bottom floor, the load, which is transmitted to the bottom floor through the shores, is redistributed to the upper slabs. This procedure is the same as that described in "3.1 *Concrete casting at top floor*."

Figure 7 shows the slab-shore model at the time when shores are removed at the bottom floor. The order i of the slabs resisting the redistributed load is shown in Figure 7. In this construction stage, the top floor can provide resistance to the construction load. Designating the superimposed load caused by the removal of shores as $Load_{sh}$, the load distributed to the upper slabs can be calculated as follows.

$$\begin{pmatrix} LS_1 \\ LS_2 \\ \vdots \\ LS_n \end{pmatrix} = \underline{K}_{fsl} \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix} = \underline{K}_{fsl} \underline{K}_{ff}^{-1} \begin{pmatrix} 0 \\ 0 \\ \vdots \\ Load_{sh} \end{pmatrix} \tag{16}$$

where v_i is the deflection increments developed by the load $Load_{sh}$.

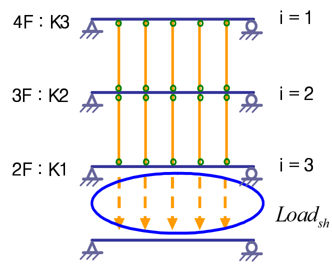
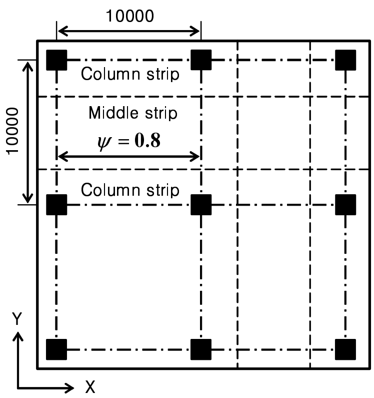


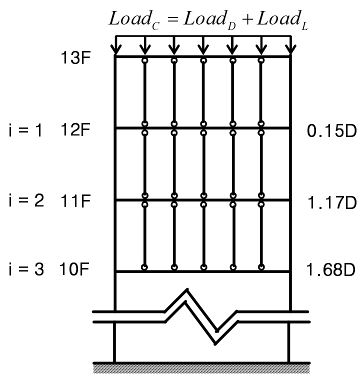
Figure 7. Slab-shore model at the time when shores are removed at the bottom floor.

4 APPLICATION OF PROPOSED METHOD

The proposed method was used to evaluate the construction load of an example structure shown in Figure 8. The construction period per floor was 6 days. The bottom slab's shores were removed 3 days after concrete casting of the top floor. The slab span was 10000mm in the X- and Y-directions. Construction load was calculated for the middle strip. ψ =0.8 was used for the slab's effective length factor, considering the boundary condition of the middle strip. The material properties and other conditions are presented in Table 1. Construction load was calculated at the time when new wet concrete was cast at the level 13. Figure 8(b) shows the construction load right before the concrete casting at level 13.



(a) Floor plan



(b) Slab construction load before concrete casting of level 13

Figure 8. Example structure for application of proposed method.

Table 1. Geometry and material properties of example structure.

Slab			
28day Strength	Span length	Thickness	Number of shored slabs
36MPa	10000mm	300mm	3
Shore			
Elastic modulus	Section area	Spacing	Height
200000MPa	576㎟	1000mm	3000mm

At this construction stage, the concrete's age of the slabs were 15, 9, and 3 days for the levels 10, 11 and 12, respectively. Using the time-dependent concrete strength $f_c(t) = exp\left\{0.25\left(1-\sqrt{28/t}\right)\right\} f_{cu}$ for normal weight concrete, the critical load for concrete cracking can be calculated as follows.

$$LR_{cr,12} = 3.6\times10^4 \times \frac{\sqrt{f_{c,3}}\cdot h}{(\phi L)^2} = 3.6\times10^4 \times \frac{\sqrt{21.5\text{MPa}}\cdot 300\text{mm}}{(8000\text{mm})^2} = 0.78D \quad (17)$$

$$LR_{cr,11} = 3.6\times10^4 \times \frac{\sqrt{f_{c,9}}\cdot h}{(\phi L)^2} = 0.91D$$

$$LR_{cr,10} = 3.6\times10^4 \times \frac{\sqrt{f_{c,15}}\cdot h}{(\phi L)^2} = 0.96D$$

At floor 12, the slab construction load was $0.15D$ in Figure 8 (b). Since the construction load was less than the critical load $LR_{cr,12}$ in Equation (17), concrete cracking did not occur. At the floors 10 and 11, on the other hand, concrete cracking occurred because of the greater construction loads. The shore's spring coefficient can be calculated as follows.

$$k_s = \frac{A_s E_s}{S_1 H} = \frac{576\text{mm}^2 \times 200000\text{MPa}}{1000\text{mm}\times 3000\text{mm}} = 38.4\,\text{N/mm}^2 \quad (18)$$

Using Equation (11), I_e for the floor 10 can be calculated as follows($E_c = 31949$MPa , $t = 18$ day).

$$\frac{I_e}{I_g} = \frac{1}{4-\left(0.38\times10^{10}\times\dfrac{f_{c,15}\cdot 300^2}{8000^4}\right)\Big/1.68^2} = 0.33 \quad (19)$$

In Equation (19), the concrete' age should correspond to the time when concrete cracking occurred. For the floor 10, concrete cracking occurred at concrete age of 15 days. Thus, I_e was calculated for the concrete age of 15days.

β_i was calculated using I_e for the floor 10.

$$\beta_3 = \sqrt[4]{\frac{k_s}{4E_c I_e}} = 0.00079\,\text{N/mm} \quad (20)$$

v is calculated in Equation (3), considering L =8000mm and shores' positions x =-3000mm, -2000mm, -1000mm, 0mm, 1000mm, 2000mm, and 3000mm. In Equation (4), the shore force is calculated as follows.

$$L_{sp,3} = S_1 k_s \sum_{i=1}^{n_s} v_i = 6.614 S_1 q \quad (21)$$

The shore's stiffness and load at floor 11 is calculated in the same manner ($E_c = 30849$MPa , $t = 12$).

$$\frac{I_e}{I_g} = \frac{1}{4 - \left(0.38 \times 10^{10} \times \frac{f_{c,9} \cdot 300^2}{8000^4}\right) \Big/ 1.17^2} = 0.46 \tag{22}$$

$$\beta_2 = 0.00072 \text{ N/mm} \tag{23}$$

$$L_{sp,2} = S_1 k_s \sum_{i=1}^{n_s} v_i = 6.534 S_1 q \tag{24}$$

The shore's stiffness and force at the floor 12 is calculated as follows($E_c = 28503$MPa , $t = 6$). β_i is calculated using $I_e = I_g$ for the floor 12 without concrete cracking.

$$\beta_1 = 0.00064 \text{ N/mm} \tag{25}$$

$$L_{sp,1} = S_1 k_s \sum_{i=1}^{n_s} v_i = 6.314 S_1 q \tag{26}$$

Using Equation (6), the equivalent stiffness ratio K_i of each slab is calculated as follows.

$$K_1 = \frac{L_{sl,1}}{L_{sp,1}} = \frac{8S_1q - 6.314S_1q}{6.314S_1q} = 0.267 \tag{27}$$

$$K_2 = \frac{L_{sl,2}}{L_{sp,2}} = \frac{8S_1q - 6.534S_1q}{6.534S_1q} = 0.224 \tag{28}$$

The stiffness matrix of the slabs connected by shores is calculated using Equation (12)~(15).

$$\begin{aligned} k_{sl1} &= 28503 I_g \\ k_{sl2} &= 30849\left(0.46 I_g\right) \\ k_{sl3} &= 31949\left(0.33 I_g\right) \end{aligned} \tag{29}$$

$$\underline{K}_{ff} = I_g \begin{pmatrix} 135210 & -106710 & 0 \\ -106710 & 184040 & -63160 \\ 0 & -63160 & 73720 \end{pmatrix} \tag{30}$$

$$\underline{K}_{fsl} = I_g \begin{pmatrix} 28503 & 0 & 0 \\ 0 & 14170 & 0 \\ 0 & 0 & 10560 \end{pmatrix} \tag{31}$$

Finally, using Equation (14), the load distribution of each slab can be calculated as follows.

$$\begin{aligned} \begin{pmatrix} LS_1 \\ LS_2 \\ LS_3 \end{pmatrix} &= \underline{K}_{fsl} \underline{K}_{ff}^{-1} \begin{pmatrix} Load_c \\ 0 \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} 0.5992 & 0.4921 & 0.4216 \\ 0.2446 & 0.3100 & 0.2656 \\ 0.1562 & 0.1979 & 0.3128 \end{pmatrix} \begin{pmatrix} 1.5D \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.90D \\ 0.37D \\ 0.23D \end{pmatrix} \end{aligned} \tag{32}$$

In order to calculate the total construction load for each slab, this construction load increment is added to the existing construction load. When the bottom slab's shores are removed, the construction load distribution can be calculated in the similar manner.

5 CONCLUSIONS

In the present study, a simplified method for the evaluation of construction load in flat-plates was developed. In the proposed method, unlike existing methods, the effects of concrete cracking and flexible shores were addressed.

ACKNOWLEDGEMENT

This research was financially supported by the Housing Eng. Team of Samsung Engineering & Construction, and the authors are grateful to the authorities for their support.

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