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Prediction equation of drying shrinkage of concrete

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ABSTRACT: In this study, in order to obtain the prediction equation for expressing the drying shrinkage of concrete with higher accuracy, the authors' earlier prediction equation based on two-phase composite model is changed to one based on three-phase model, and is extended to the form taking into account of the effects of member size and shape, and relative humidity of environment. In addition, for the case of no data concerned with aggregate properties, which are input data for the authors' equation, another prediction equation is shown. Furthermore, the proposed prediction equation is discussed in comparison with past data of drying shrinkage.

1 INTRODUCTION

The prediction equations of drying shrinkage of concrete were proposed by many researchers (AIJ 2003), and recent representative examples are such as CEB-FIP-1990 model code equation (CEB-FIP 1990), Bazant's equation (RILEM TC 107 equation) (Bazant & Baweja 1995), Gardner's equation (Gardner 2000), JSCE equation (JSCE 2002), and JSCE equation for high strength concrete (JSCE 2002). These can predict a drying shrinkage strain on the basis of relatively easy to obtain data and are easy to be used at the design stage without much information about the used concrete. However, the prediction accuracy of these equations is not high (JCI 2001). In particular, although drying shrinkage strain of aggregate varies widely as the measurement examples are shown in Figure 1 (Tatematsu et al. 2001), that effect is not taken into account in the existing prediction equations, and it is considered that this is the major cause of reduction of the prediction accuracy.

With this background, the authors proposed the prediction equation for expressing the drying shrinkage of concrete based on a composite model previously proposed (Eguchi & Teranishi 2005). This equation improves the prediction accuracy by inputting drying shrinkage strain and Young's modulus of aggregate, and is used for the material selection and the mix design of concrete. In this paper, the authors' prediction is developed further.

2 MODIFICATION AND EXTENSION OF PREDICTION EQUATION IN THIS PAPER

2.1 *Authors' earlier prediction equation*

The prediction equation of drying shrinkage of concrete shown in previous our paper (Eguchi & Teranishi 2005) is as follows.

$$\varepsilon_{sc}(t)=\varepsilon_{sm}(t)\frac{\left[1-\left(1-m'_g n'_g\right)V_g\right]\left[n'_g+1-\left(n'_g-1\right)V_g\right]}{n'_g+1+\left(n'_g-1\right)V_g} \tag{1}$$

$$\varepsilon_{sm}(t)=\varepsilon_{sp}(t)\frac{\left[1-V_g-\left(1-m_s n_s\right)V_s\right]\left[\left(n_s+1\right)\left(1-V_g\right)-\left(n_s-1\right)V_s\right]}{\left[\left(n_s+1\right)\left(1-V_g\right)+\left(n_s-1\right)V_s\right]\left(1-V_g\right)} \tag{2}$$

$$\varepsilon_{sp}(t)=\frac{t}{\alpha W/C+\beta+t}\left(\lambda W/C+\delta\right) \tag{3}$$

$$E_p=\frac{100}{W/C}\gamma+\eta \tag{4}$$

where $n_s = E_s/E_p$; $n'_g = E_g/E_m$; $m_s = \varepsilon_{ss}(t)/\varepsilon_{sp}(t)$; $m'_g = \varepsilon_{sg}(t)/\varepsilon_{sm}(t)$; ε = drying shrinkage strain (×10^{-6}); E = Young's modulus (GPa); V = aggregate volume ratio; t = drying period (day); W/C = water-cement ratio (%); and α, β, λ, δ, γ, h = constants which depend on type of cement (obtained by Table 1).

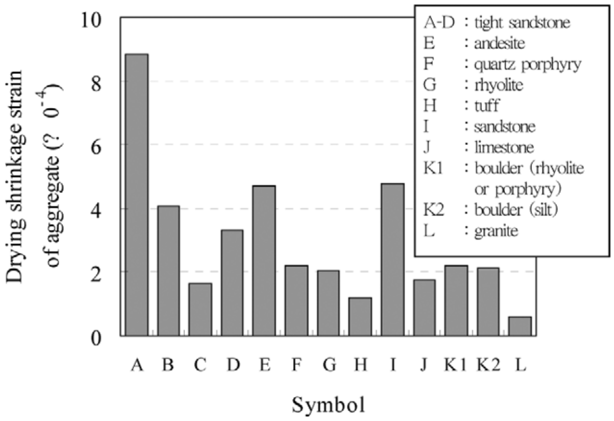


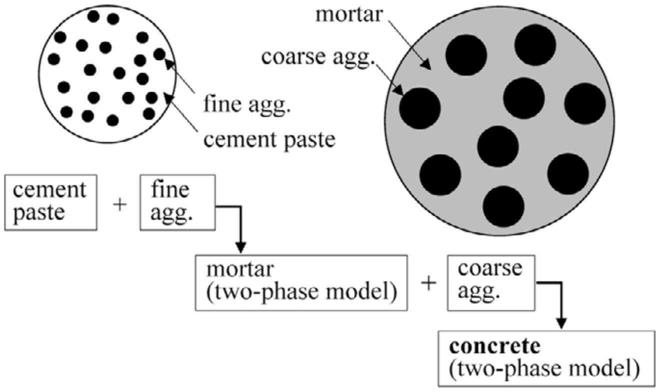
Figure 1. Drying shrinkage strain of various aggregates.

The suffixes c, m, p, s, g stand for the concrete, mortar, cement paste, fine aggregate, and coarse aggregate, respectively (this is the same hereinafter).

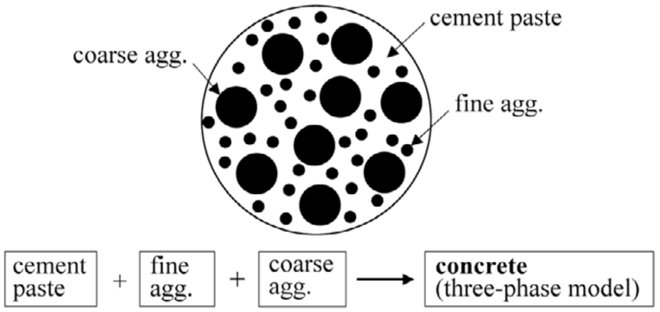
Above series of equations is the prediction equation based on Baba's equation (Kishitani & Baba 1975), which is based on one of the composite model of drying shrinkage. Here, the composite models of drying shrinkage are the theoretical model which expresses the drying shrinkage strain of multiphase materials such as concrete and mortar by the properties (drying shrinkage, Young's modulus) and the volume ratio of each phase composing the material. However, it cannot take into account of two types of aggregate in concrete, i.e., fine aggregate and coarse aggregate, simultaneously, because this is generally applied to the two-phase material composed of aggregate and matrix. For this reason, the authors' prediction equation is the form that the composite model is applied to mortar component in concrete and concrete step-by-step as shown in Figure 2a. Furthermore, this equation predicts the drying shrinkage under the limited condition of relative humidity of 60 % and member size of 10 × 10 × 40 cm.

Table 1. Value of constants in Equations 3 and 4.

Type of cement	α	β	λ	δ	γ	η
N	0.322	4.77	86.3	54	5.9	4.2
FB	0.518	-4.72	67.8	581	6.9	0.2
BB	0.608	-10.77	143.7	-1408	6.9	-0.9



(a) Equation based on two-phase model.



(b) Equation based on three-phase model.

Figure 2. Model of prediction equation.

2.2 *Modifying and extending matters*

In this paper, the authors' earlier prediction equation is modified and extended as follows.

(1) Equations 1 and 2, which are the core of prediction equation, are changed from two-phase model to three-phase model composed of cement paste, fine aggregate and coarse aggregate as shown in Figure 1b in order to simplify the structure of the prediction equation (Chapter 3).

(2) The prediction equation is extended to the form taking into account of member size and shape (Chapter 4), and relative humidity of environment (Chapter 5), so that the prediction equation can be applied to a member level.

(3) For the case of no data concerned with aggregate properties (drying shrinkage and Young's modulus), another prediction equation which assumes that the aggregate with average properties is used for concrete is shown (Section 6.1).

3 CHANGE OF PREDICTION EQUATION TO ONE BASED ON THREE-PHASE MODEL

In this chapter, Baba's equation is rewritten to one based on three-phase model in order to change the core of the authors' earlier prediction equation based on two-phase model, i.e., Equations 1 and 2, to a simpler form.

3.1 *Derivation of equation of drying shrinkage based on three-phase model*

Baba adopted capillary tension theory as generating mechanism of drying shrinkage, and expressed the capillary tension which acts on the pores in concrete by Laplace-Kelvin equation as follows (Kishitani & Baba 1975).

$$\Delta p = -\frac{RT\rho}{M}\ln\frac{p}{p_0} \tag{5}$$

where Δp = capillary tension (Pa); p = vapor pressure (Pa); p_0 = saturated vapor pressure (Pa); R = gas constant (J/K•mol); T = absolute temperature (K); ρ = density of water (kg/m^3); and M = molecular weight of water (kg/mol).

Furthermore, he expressed as follows the shrinkage in a pore due to capillary tension on the basis of the elastic theory of thick-walled spherical shell. Moreover, similar equations were obtained for cement paste, fine aggregate and coarse aggregate.

$$\varepsilon_{sc} = \frac{3(1-\mu_c)}{2E_c} W_{ec}\cdot\Delta p \tag{6}$$

where μ = Poisson's ratio; and W_e = volume water content.

In addition, the following equation is obtained regarding the volume water content W_e.

$$W_{ec} = W_{ep}\left(1-V_s-V_g\right)+W_{es}V_s+W_{eg}V_g \tag{7}$$

Above equations are rearranged for ε_{sc} and $\mu_c = \mu_p = \mu_s = \mu_g = 0.2$ is assumed and ε_s is rewritten as a function of time, so that the following equation is obtained.

$$\varepsilon_{sc}(t) = \varepsilon_{sp}(t)\frac{1-(1-m_s n_s)V_s-(1-m_g n_g)V_g}{n_c} \tag{8}$$

where $n_c = E_c/E_p$; $n_g = E_g/E_p$; and $m_g = \varepsilon_{sg}(t)/\varepsilon_{sp}(t)$.

3.2 *Derivation of equation of Young's modulus based on three-phase model*

In order to delete E_c from Equation 8, it is expressed by the composite model of Young's modulus in imitation of Baba. For this reason, here, the composite model of Young's modulus for three-phase material is derived from that of bulk modulus for multiphase material proposed by Hashin (Hashin 1965). According to Hashin, the bulk modulus of composite material with *k* types of inclusion is shown as follows.

$$\frac{K_C}{K_M} = 1+3(1-\mu_M)\sum_{i=1}^{k}\frac{\left(\frac{K_P^{(i)}}{K_M}-1\right)V_i}{2(1-2\mu_M)+(1+\mu_M)\left[\frac{K_P^{(i)}}{K_M}-\left(\frac{K_P^{(i)}}{K_M}-1\right)V\right]} \tag{9}$$

where $V = \sum_{i=1}^{k} V_i$; K_C = bulk modulus of composite material (GPa); K_M = bulk modulus of matrix (GPa); $K_P^{(i)}$ = bulk modulus of number *i* inclusion (GPa); and μ_M = Poisson's ratio of matrix.

Above equation is rewritten for the three-phase material composed of cement paste, fine aggregate and coarse aggregate, and $K = E/3(1-2\mu)$ is substituted for it. Furthermore, it is rearranged for Young's modulus and $\mu_c = \mu_p = \mu_s = \mu_g = 0.2$ is assumed, so that Young's modulus based on three-phase model is obtained as the following equation.

$$n_c = \frac{E_c}{E_p} = 1+\frac{2(n_s-1)V_s}{n_s+1-(n_s-1)(V_s+V_g)}+\frac{2(n_g-1)V_g}{n_g+1-(n_g-1)(V_s+V_g)} \tag{10}$$

Equations 8 and 10 obtained above are based on the model extended from Baba's equation for three-phase material, and the core of author's earlier equations, i.e., Equations 1 and 2, are replaced with those.

4 CONSIDERATION OF EFFECT OF MEMBER SIZE AND SHAPE IN PREDICTION EQUATION

In most of past studies concerned with the effects of member size and shape on drying shrinkage strain, these effects were represented by *V/S* (volume to surface area ratio) (Almudaiheem & Hansen 1987, Inoue et al. 2002). Meanwhile, according to authors' past experimental result (Hanzaka & Teranishi 2005), in case the drying shrinkage of cement paste at any drying period is expressed by Equation 11 (this equation is the same form with Equation 3), the effect of *V/S* on $\varepsilon_{sp\infty}$ in the equation is small as shown in Figure 3. Furthermore, N_s in the equation is expressed approximately by Equation 12, which is the function of *V/S*, as shown in Figure 4.

$$\varepsilon_s(t) = \frac{t}{N_s+t}\varepsilon_{s\infty} \tag{11}$$

$$N_s = 55.26\log(V/S)+19.7 \tag{12}$$

where $\varepsilon_{s\infty}$= ultimate value of drying shrinkage strain ($\times 10^{-6}$); N_s = constant which represents progression rate of drying shrinkage (day); and *V/S* = volume to surface area ratio (cm).

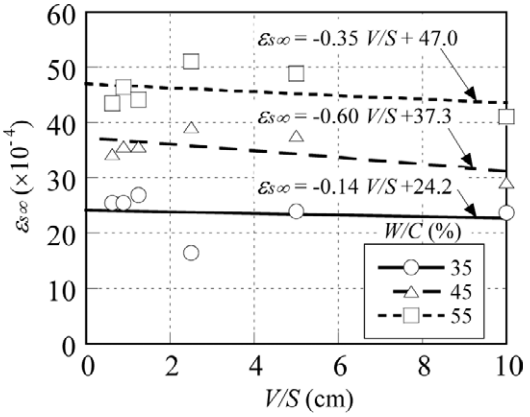


Figure 3. Relationship between $\varepsilon_{s\infty}$ and *V/S* of cement paste.

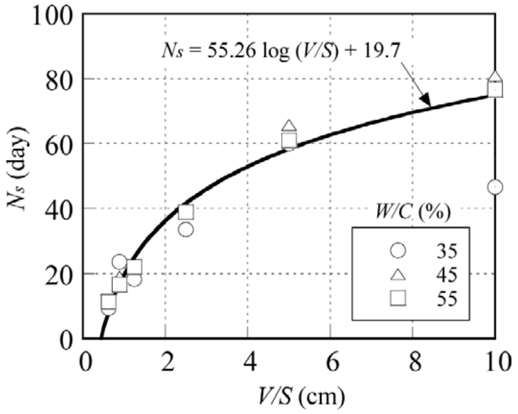


Figure 4. Relationship between N_s and *V/S* of cement paste.

Here, the constants α, β, λ, δ shown in Table 1 are the values obtained from specimens of 4 × 4 × 16 cm, and the value of N_s for the *V/S* = 0.89 cm of this specimen is 16.82 days according to Equation 12. Namely, Equation 3 represents the drying shrinkage strain of cement paste in case of N_s = 16.82 days. Consequently, in this prediction equation, the effect of *V/S* is reflected in the progression rate of drying shrinkage as the ratio to the case of N_s = 16.82 days. For this reason, Equation 3 is modified to Equation

13 from comparison with Equation 11, and Equation 12 is rewritten to Equation 14.

$$\varepsilon_{sp}(t) = \frac{t}{R_s(\alpha W/C + \beta) + t}(\lambda W/C + \delta) \tag{13}$$

$$R_s = \frac{N_s}{16.82} = 3.29\log(V/S) + 1.17 \tag{14}$$

5 CONSIDERATION OF EFFECT OF RELATIVE HUMIDITY IN PREDICTION EQUATION

In this study, the effect of relative humidity of environment is introduced to the authors' prediction equation based on the following procedure.

(1) The relative humidity is introduced to the equation for cement paste as in the case of *V/S*.

(2) Most of existing prediction equations of drying shrinkage consider that relative humidity affects only the ultimate value of drying shrinkage strain and does not affect the progression rate of drying shrinkage (CEB-FIP 1990, Bazant & Baweja 1995, Gardner 2000, JSCE 2002). Therefore, this prediction equation also follows such consideration.

(3) The authors' earlier prediction equation predicts the drying shrinkage strain under the environment of relative humidity of 60 % as above, and this equation is extended to take relative humidity into account. For this reason, the effect of relative humidity is expressed by the ratio to the case of relative humidity of 60 %.

Figure 5 shows the relationship between the drying shrinkage strain (ultimate value) of cement paste, which is expressed by the ratio to the case of relative humidity of 60 % (hereinafter referred to as R_h), and relative humidity measured by Nagamatsu et al. (Nagamatsu et al.1992). Furthermore, in the representative existing prediction equations of drying shrinkage, the effect of relative humidity on the drying shrinkage strain of concrete is expressed by the expressions shown in Table 2, and the relationships by these expressions are also shown in the figure. As will be noted from the figure, the effect of relative humidity varies with curing period. However, because the curing period at actual works is generally short, here, the effect of relative humidity is expressed by the same expression as that of CEB-FIP-1990 equation and Bazant's equation. For this reason, Equation 13, which expresses the drying shrinkage strain of cement paste, is modified again to Equation 15, and the term expressing ultimate value in the equation is multiplied by R_h shown by Equation 16, which expresses the ratio to the case of relative humidity of 60 %.

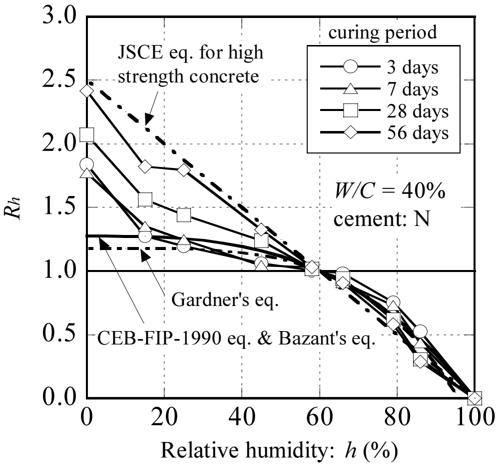


Figure 5. Relationship between drying shrinkage and relative humidity of cement paste.

Table 2. Terms which express effect of relative humidity in existing prediction equation.

Prediction equation	CEB-FIP-1990 eq. and Bazant's eq.	Gardner's eq.	JSCE eq. for high strength concrete
Term*	$1-\left(\frac{h}{100}\right)^3$	$1-1.18\left(\frac{h}{100}\right)^4$	$1-\frac{h}{100}$

* The ultimate value of drying shrinkage strain is multiplied by these terms in each prediction equation.

Table 3. Outline of data used for verification of prediction equation.

Type of cement	N (65), BB (1)
W/C (%)	60 ~ (6), 50 ~ 60 (47), 40 ~ 50 (8), 30 ~ (5)
Unit water content (kg/m^3)	190 ~ 200 (2), 180 ~ 190 (38), 170 ~ 180 (17), 160 ~ 170 (9)
28 day comp. strength (MPa)	20 ~ 40 (16), 40 ~ 60 (14), 60 ~ (6), no data (30)
Specimen shape (cm)	10×10×40 (58), 10×10×50 (3), 10×10×60 (2), ϕ10×20 (3)
Curing condition	Temperature of 20 ˚C and Relative humidity of 60 % (66)

* The values in parentheses show the number of cases.
* The case with special aggregate, admixture and special curing is not included.

$$\varepsilon_{sp}(t) = \frac{t}{R_s(\alpha W/C + \beta) + t}R_h(\lambda W/C + \delta) \tag{15}$$

$$R_h = 1.28\left\{1-\left(\frac{h}{100}\right)^3\right\} \tag{16}$$

where h = relative humidity (%).

6 VERIFICATION OF PREDICTION EQUATION BY EXISTING DATA

In this chapter, using the measured data of drying shrinkage reported to Summaries of technical papers of AIJ in 1997-2003 as an outline is shown in Table

3, authors' prediction equation of drying shrinkage (i.e., the combination of Equations 8, 10, 15, 14, 16 and 4) is verified.

6.1 *Derivation of prediction equation in case of no data concerned with aggregate*

The authors' prediction equation tries to improve its accuracy by inputting data concerned with aggregate properties as above. However, the drying shrinkage strain and Young's modulus of aggregate can be determined by only extracting a core sample from a raw stone in case of crushed stone and crushed sand, and the method to determine these properties easily is not established at this time. Therefore, here, for the case of no data concerned with aggregate properties, another prediction equation (hereinafter referred as to easy-to-use equation) which assumes that the aggregate with average properties is used for concrete is derived.

For derivation of the easy-to-use equation, firstly, 60 GPa which is the average value of Young's modulus of aggregate (Tatematsu et al. 2001) is input for E_s and E_g in Equations 8 and 10. Furthermore, considering that the drying shrinkage of aggregate in concrete progresses keeping a certain relationship with that of cement paste component, it is assumed that $\varepsilon_{ss}(t)$ and $\varepsilon_{sg}(t)$ in Equation 8 (these are the factors of m_s and m_g, respectively) can be expressed by the following equations.

$$\varepsilon_{ss}(t)=\frac{t}{R_a\cdot R_s(\alpha W/C+\beta)+t}\varepsilon_{ss\infty} \tag{17}$$

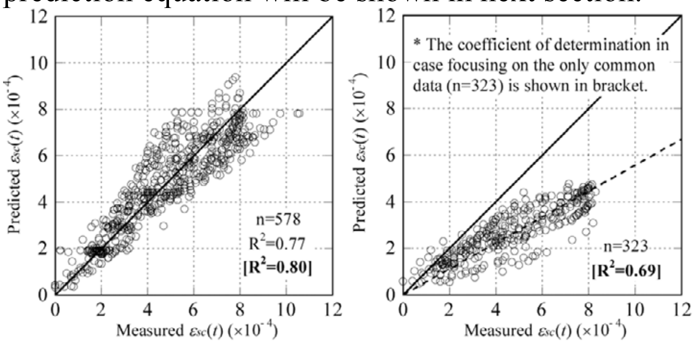
$$\varepsilon_{sg}(t)=\frac{t}{R_a\cdot R_s(\alpha W/C+\beta)+t}\varepsilon_{sg\infty} \tag{18}$$

where R_a = the constant which represents the ratio of progression rate of drying shrinkage of aggregate in concrete to that of cement paste component.

The multiple regression of the verification data by authors' prediction equation was performed based on above assumption, and as a result $R_s = 0.22$, $\varepsilon_{ss\infty} = 337 \times 10^{-6}$ and $\varepsilon_{sg\infty} = 180 \times 10^{-6}$ were obtained. The authors' equation to which these equations and values are input in advance is the easy-to-use equation. This equation can be utilized as an ordinary prediction equation of drying shrinkage without consideration of aggregate properties.

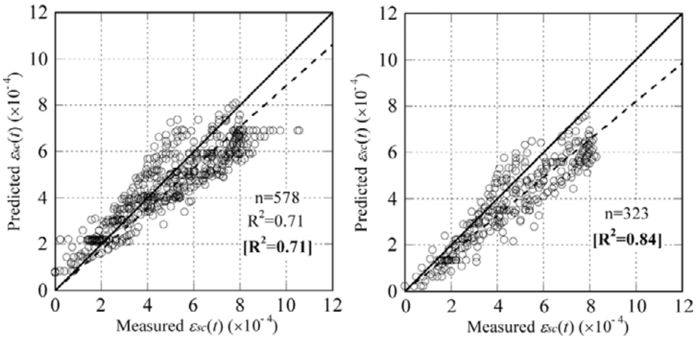
6.2 *Verification of prediction accuracy in case without consideration of aggregate properties*

Generally, when the drying shrinkage test of concrete was carried out, aggregate properties have hardly ever been determined simultaneously. Therefore, first, the prediction accuracy of the easy-to-use equation is verified in this section, and the effectiveness of consideration of aggregate properties in the prediction equation will be shown in next section.



(a) Authors' prediction equation (easy-to-use equation).

(b) CEB-FIP-1990 prediction equation.



(c) JSCE prediction equation.

(d) JSCE prediction equation for high strength concrete.

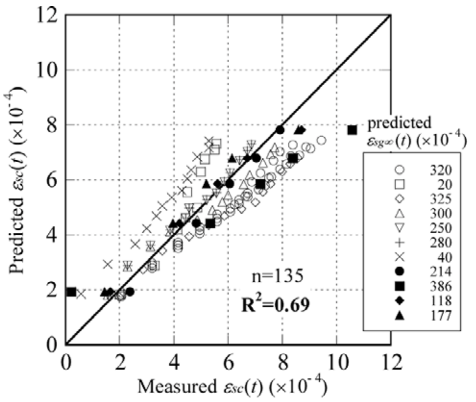
Figure 6. Comparison between prediction value by various equations and measured value.

Figure 6 shows the comparison between $\varepsilon_{sc}(t)$ predicted by the easy-to-use equation and that measured, and it also shows that predicted by CEB-FIP-1990 equation (CEB-FIP 1990), JSCE equations (JSCE 2002), and JSCE equation for high strength concrete (JSCE 2002). It is noted that non-Japanese prediction equations can be applied to the concrete using Japanese cement. Therefore, there is little point in evaluating merely the fitness of the predicted value to the measured value to compare accuracy between prediction equations. For this reason, here, the coefficient of determination (the case focusing on the only common data) shown in Table 6 is focused, so that the dispersion of the predicted value of even the easy-to-use equation, i.e., authors' prediction equation without consideration of aggregate properties, proved to be the smallest next to that of JSCE equation for high strength concrete.

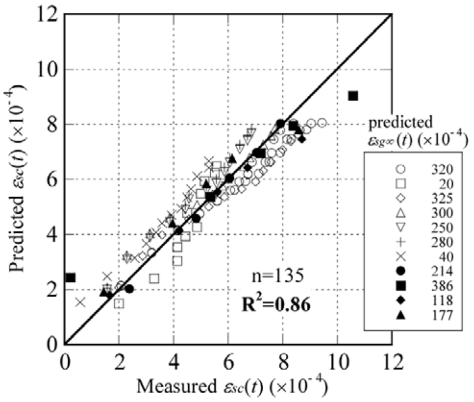
6.3 *Effect of consideration of drying shrinkage of aggregate*

The cases (Fujimoto et al. 2003, Imamoto et al. 2000) that the drying shrinkage strain of coarse aggregate was determined simultaneously were extracted from the verification data shown in Table 3, and using these data, the effect of consideration of aggregate properties on the accuracy of prediction equation was investigated. Figure 7 shows the result. Here, Figure 7a shows the comparison between $\varepsilon_{sc}(t)$

predicted by the easy-to-use equation and that measured, i.e., it shows a portion of data of that shown in Figure 6a. Figure 7b shows the same comparison in case that the measured data was input to $\varepsilon_{sg\infty}$ in Equation 18 (the input values except for $\varepsilon_{sg\infty}$ are the same as those in case of Figure 7a).



(a) Case without consideration of aggregate properties (easy-to-use equation).



(b) Case that measured data is input to $\varepsilon_{sg\infty}$.

Figure 7. Effect of consideration of ε_{sg}.

From comparison with these two figures, it turns out that the coefficient of determination increases and the prediction accuracy is improved considerably with consideration of drying shrinkage of coarse aggregate in the authors' prediction equation. Additionally, in case the measured $\varepsilon_{ss\infty}$, E_s, E_g are obtained, further improvement of prediction accuracy is expected.

7 CONCLUSIONS

In this study, in order to obtain the prediction equation of drying shrinkage of concrete with higher accuracy, the authors' earlier prediction equation was changed to one based on three-phase model, and was extended to the form taking into account of member size and shape, and relative humidity of environment. Furthermore, for the case of no data concerned with aggregate properties, another prediction equation was proposed. The findings throughout this study are listed as follows.

(1) Drying shrinkage strain of concrete members is predicted with high accuracy by the prediction equation proposed in this study (i.e., the combination of Equations 8, 10, 15, 14, 16 and 4), which takes the effects of member size and shape, and relative humidity of environment into account.

(2) In case of no data concerned with aggregate properties, another prediction equation proposed in this study, which assumes that the aggregate with average properties is used, can be utilized. Furthermore, its accuracy is not lower than that of other existing prediction equations, however, is much lower than that of the case that the data concerned with aggregate properties is input.

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