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Bearing angle model for bond of reinforcing bars to concrete

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ABSTRACT: Experimental studies have demonstrated that bond strength increases with an increase in the relative rib area bars under high confinement, but under low confinement, bond strength is independent of deformation pattern. This study is intended to explain the nature of the wedging action of reinforced bars as they interact with concrete during bond failure. Analytical expressions to predict bond resistances for splitting failure of cover by fracture and shearing failure are derived, in which the bearing angle is a key variable. As the bearing angle is decreased, the splitting bond resistance decreases while the shearing bond resistance increases. In the case of bars at a moderate level of confinement, the bearing angle is decreased to decrease the splitting resistance and to increase the shearing resistance. The bearing angle model is useful to better understand bond mechanisms between reinforcing bars and concrete.

1 INTRODUCTION

During the late 1950s and the 1960s, researchers observed two phenomena accompanied by the slip of ribbed bars: (1) concrete is split by the wedging action of the ribs and (2) concrete between the ribs is crushed (Rehm 1957, Lutz & Gergely 1967). Researchers observed that the ribs act as wedges and the concrete in front of the ribs crushes gradually, resulting in a pullout-type failure and found that the concrete in front of the ribs undergoes gradual crushing, followed by a pullout failure (Fig. 1).

A number of researchers have derived analytical expressions for bond mechanisms in splitting failure (Tepfers 1979, Cairns 1979). Bond between steel bars and concrete has been idealized in finite element analyses. For the case of splitting failure, analytical studies of interfacial bond have been performed to predict the bond strength of ribbed reinforcing bars (Choi & Lee 2002), and in this paper, the fracture of concrete cover on bond behavior is addressed.

The rib geometry of deformed bars governs bond behavior and is instrumental in guaranteeing adequate bond resistance. The influence of deformation pattern on bond performance has been studied and bond resistances have been observed to vary with the rib characteristics (Tefers 1979, Skorobogatov & Edwards 1979). Studies by Tholen & Darwin (1996) have demonstrated that bond strength increases with an increase in the relative rib area bars under high confinement, but under low confinement, bond strength is independent of deformation pattern.

With this information as background, this study is intended to explain the nature of the wedging action of ribbed bars as they interact with concrete during bond failure. Analytical expressions to determine bond resistances for splitting and shearing failures are derived and used to predict bond strength. The roles of the bearing angle, which is the key variable in the expressions, are explored. The bearing angle model is proposed for analyzing the bond behavior of ribbed reinforcing bars to concrete and improving the understanding of bond mechanisms of reinforcing steel in concrete structures.

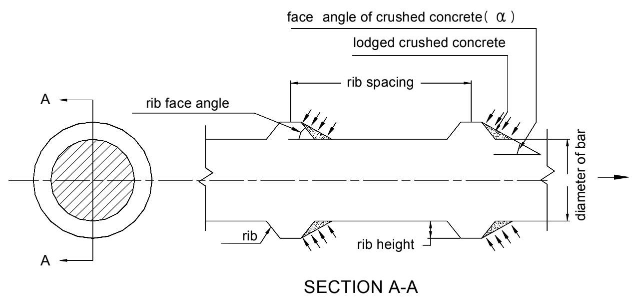


Figure 1. Flattened rib face angle by concrete crouching (Tepfers 1979).

2 BOND RESISTANCES IN SPLITTING AND SHEARING RAILURE

2.1 *Bond resistance in splitting failure*

Wedging action by the rigid steel rib of deformed bars makes it possible to resolve bond forces into normal stress σ_n and tangential shear stress τ, as shown in Figure 2. The resultant of normal components along the bar is what places the surrounding concrete in tension. When a reinforcing bar in tension P, concrete under the bearing side of a rib is placed in a state of tri-axial compression, with the major principal stress, the bearing stress, σ_q, on the rib acting parallel to the bar axis. Normal to the bearing stress, the minor principal stress σ_r acts radially around the bar. The method of analysis (presented here is a slightly revised and condensed form) has been used previously by Choi & Lee (2002) to evaluate the bond strength in splitting. The bond force equal to the sum of the bearing stress on a single rib area T, is given by

$$T = A_r\,\sigma_q \tag{1}$$

in which A_r = projected area of rib parallel to the bar axis, approximated by $A_r = \pi d_b h_r$ where h_r is the average rib height, σ_q = bearing stress on the bar rib acting parallel to the bar axis. The frictional force between the concrete and the steel on the inclined surface of the rib may be represented using the Mohr-Coulomb relation.

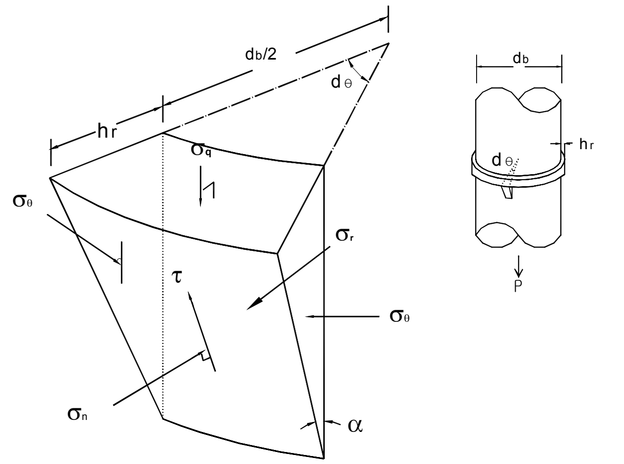


Figure 2. Stresses acting on rib of bar (Cairns 1979).

Suppose that the stresses along an interface with an angle of α, defined as bearing angle, are in equilibrium with the sliding stress by σ_q and the normal stress by σ_n. The stress σ_q, is given by

$$\sigma_q = \left(\sigma_r \frac{(1+\mu\cot\alpha)}{(1-\mu\tan\alpha)} + \frac{c}{\sin\alpha(\cos\alpha-\mu\sin\alpha)} \right) \tag{2}$$

Equation (3) is substituted into Equation (1) to obtain

$$T = A_r \left(\sigma_r \frac{(1+\mu\cot\alpha)}{(1-\mu\tan\alpha)} + \frac{c}{\sin\alpha(\cos\alpha-\mu\sin\alpha)} \right) \tag{3}$$

where σ_r acts radically around the bar axis on the concrete cover. The radial stress σ_r acts over a distance of dF_x below the rib, and exerts a bursting force on the concrete around the bar. Figure 2 shows the force, $h_r \cot\alpha$ exerted by σ_r on one rib over a short length of the bar circumference. The component of force in the x-direction and the summation of the component force on the perimeter is given by

$$F_x = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} dF_x = \sigma_r \cot\alpha h_r d_b \tag{4}$$

Equation (6) is substituted in to Equation (4), resulting in the final equation to predict bond resistance, which is expressed as follows.

$$T_{split} = F_x \pi \tan\alpha \frac{(1+\mu\cot\alpha)}{(1-\mu\tan\alpha)} + A_r \frac{c}{\sin\alpha(\cos\alpha-\mu\sin\alpha)} \tag{5}$$

where F_x is the confining force by fracture of concrete cover or transverse reinforcement.

2.2 *Bond resistance in shearing failure*

Deformed bars bear against the concrete in front of the ribs, thus increasing shearing stress on the concrete key. Shear may cause failure, and the potential failure plane can be established for such cases along which shear stresses are high, as shown in Figure 3. The location of shear failure surface along the possible shear crack depends on the rib geometry and the levels of vertical force (confining force) and horizontal force (bond force). Failure occurs when the shear strength of the concrete key is overcome. From the force boundary conditions, an angle α is made along the shear failure surface, where the tangential stresses and the radial stresses are in equilibrium. Based on a study by Birkeland & Birkeland (1966), for cracks in monolithic concrete, shear strength should not be assumed greater than $0.2f'_c A_c$ as shown in Equation (6).

$$V_n = 0.2 f'_c A_c \tag{6}$$

where A_c is the area of cracked surface.

The area of cracked surface A_c defined by the area of a cone with the angle of α,

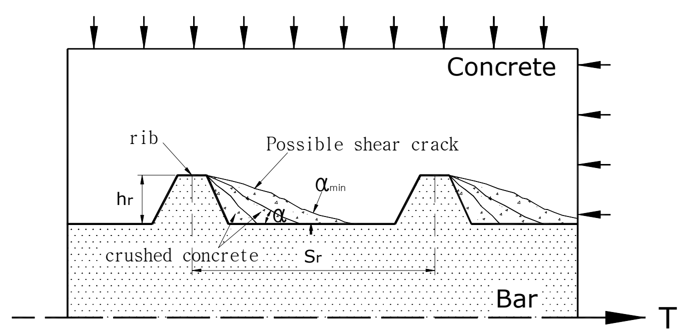


Figure 3. Shear cracks by the concrete key between bar ribs.

$$A_c = \frac{\pi d_b h_r}{\sin\alpha} \tag{7}$$

The concrete in contact with the bearing side of a rib is in a state of triaxial compression and is subjected to very high compression from the confining force F_x. This triaxility of stress increases the shear strength of the concrete. The high compression is also beneficial to increase the shear strength, since the high compressive stress modifies the magnitude and direction of principal stress and increases the cracking load. Two parameters accounting for the increased shear strength from the tri-axial state and the high compression, κ_1 and κ_2 are proposed.

Using Equation (6) and (7) and the two parameters, the bond resistance in shearing failure is proposed by

$$T_{shear} = \kappa_1 \kappa_2 \frac{0.2 f'c \pi d_b h_r}{\sin\alpha} \tag{8}$$

where κ_1 = triaxial state parameter and κ_2 = high compression parameter. Information on these two quantities shall be obtained from the results of future analytical or experimental studies.

3 BEARING ANGLE MODEL

The friction coefficient μ is one of the key variables to determine the bond resistance. Bond resistance increases as the friction coefficient increases. The contribution from cohesion to bond resistance is small and diminishes as bars slip. The confinement force F_x, provided by fracture of concrete cover or transverse reinforcement, is proportional to the bond force. The capacity of the confinement force is made up of the splitting resistance by concrete cover or by transverse reinforcement, thus the confinement force has a limitation. When the confinement is determined by the structure itself, the bearing angle is the only variable in Equation (5) corresponding to the change of bond resistance. The bearing angle of the failure surface of the concrete in front of the ribs may be varied.

As in Equation (8), the shearing resistance is obtained by the concrete key which would be sheared off, forming a cone with a length equal to several times the rib height. The bearing angle is, again, the key variable since the length of the cone is a function of the bearing angle. The bearing angle tends to be decreased to a smaller value, to increase the shearing bond resistance. There might be a lower limit on the bearing angle and the minimum value of the bearing angle can be obtained by the ratio of the rib spacing to the rib height.

Bond strength is determined along the interface at a state of resistance equilibrium under any failure condition. Normally, the weaker mode of the two failures, splitting and shearing failure, is considered to govern bond strength, but both failures control bond strength because two failures appears to occur simultaneously. In these cases, the bearing angle is decreased to decrease in the splitting resistance and increase in the shearing resistance. As the bearing angle reaches a certain value of the angle, then, the concrete key is sheared off. The bearing is determined so that the splitting resistance can be equal to the shearing resistance, and finally the resistance itself becomes bond strength T_{bond}. Thus,

$$T_{split} = T_{shear} = T_{bond} \tag{9}$$

Equation (9) can be solved for the bearing angle α. The solution for the bearing angle to determine bond strength by the bearing angle model is schematically illustrated in Figure 4. As in cases of moderate or high confinement, when the splitting resistance is higher than the shearing resistance, the splitting resistance decreases with decreasing the bearing angle. As in cases of low confinement, when the shearing resistance is higher than the splitting resistance, the shearing resistance tends to be minimized and the splitting resistance tends to be maximized keeping the bearing angle as high as possible.

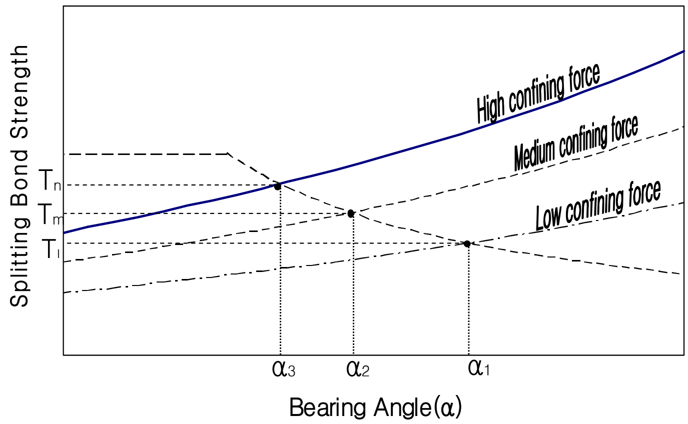


Figure 4. Schematic for determination of bond strength by bearing angle model (different confinement).

4 DISCUSSIONS

The bearing angle model is proposed for analyzing the bond behavior of ribbed reinforcing bars to concrete. Bearing angle may be reduced so that splitting strength is maintained to be less than pullout strength. The bearing angle is determined so that the splitting resistance can be equal to the shearing resistance and the resistance itself becomes bond strength. Bearing angle is only a single variable to relate failure modes.

Table 1. Confinement effects for bars with the same rib height.

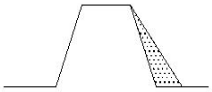
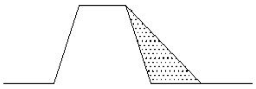
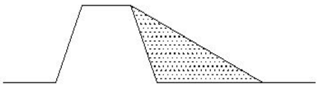
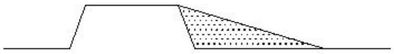
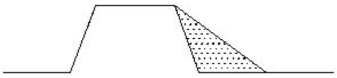
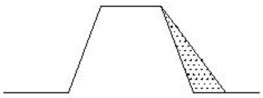
Cases	Crushing Shape	Mode	α	F_x	Bond Strength
Low Conf.		Splitting	High	Low	Low
Med Conf.		Splitting	Med	Med	Med
High Conf.		Pullout	Low	High	High

Table 2. Rib height effects for bars with high confinement.

Cases	Crushing Shape	Mode	α	Bond Strength
Low Conf.		Splitting	High	Low
Med Conf.		Splitting	Med	Med
High Conf.		Pullout	Low	High

As confinement increases, bearing angles reduced as illustrated in Table 1. When pullout resistance is constant, bearing angle decreases as confinement increases. When splitting resistance is constant, bearing angle increases as pullout resistance increases as in Table 2. Behavior matches experimental observations that high rib face angle is flattened by crushed concrete wedge. The bearing angle model is useful to simulate ribbed bars-concrete interface behavior and response.

5 CONCLUSIONS

Analytical expressions to determine the bond resistances for splitting and shearing failures are derived where the bearing angle is a key variable. As the bearing angle is decreased, the splitting bond resistance decreases while the shearing bond resistance increases. In the case of bars at a moderate level of confinement, which represents the practice, the bearing angle is decreased to decrease the splitting resistance and to increase the shearing resistance, until reaching a certain value of angle. Bearing angle model is useful to simulate ribbed bars-concrete interface behavior and response.

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