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A model for predicting time to corrosion-induced cover cracking in reinforced concrete structures

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ABSTRACT: Concrete cover cracking induced by reinforcement corrosion is an important indication of durability limit state for reinforced concrete (RC) structures, and can ultimately determine the structural service life (SL). This paper presents a mathematical model that can predict the time from corrosion initiation to corrosion-induced cover cracking. In the present model, a relationship between the expansive pressure and the corrosion amount of steel was proposed based on mechanics of elasticity, which took account of the influence of corrosion products. Additionally, the penetration of corrosion products into radial corrosion cracks was also considered in this study. Then, Faraday's law was utilized to build the theoretical model for predicting time from corrosion initiation to corrosion cracking. Discussion of the main factors affecting the time to cracking showed that the increase of cover thickness, decrease of rebar diameter, improvement of concrete strength and control the level of oxidation were in favor of elevating the durability of RC structure. A comparison was made between the model's predictions and experimental results published in literatures and it indicated that the proposed model could give reasonable prediction for the time to cover cracking.

1 INTRODUCTION

One of the most predominant factors responsible for the structural deterioration in reinforced concrete (RC) structures is identified as corrosion of reinforcement, which may result in the damage to the structures in the form of expansion, cracking and eventually spalling of the cover concrete. Normally, concrete provides a high degree of protection to the reinforcing steel against corrosion, known as passivation of steel, by virtue of the high alkalinity in the pore solution. In aggressive environments, however, this highly desirable durability performance in concrete is usually destroyed and corrosion of reinforcement becomes a commonly encountered issue, which can be caused by two major factors: (i) concrete carbonation, and (ii) presence of chloride ions (Ahmad 2003).

When the passivation layer around reinforcement disappears, corrosion will occur as long as oxygen, water, differences in electrical potential and temperature are provided. Due to the volume expansion of corrosion products, which is about 2 to 6 times the original iron volume (Liu & Weyers 1998), a radial pressure at the steel-concrete interface is induced and the hoop tensile stresses in the surrounding concrete develop slowly which results ultimately in thorough cracking of the cover concrete. These cracks in the cover concrete may provide a path for a quicker ingress of aggressive elements to the steel bars, and accelerate the corrosion. The corrosion rate and corrosion-induced cover cracking can directly affect the decisions regarding repair, strengthening and replacement of the deteriorated RC structures to ensure their service life (SL). Therefore, it is widely accepted that the state of cover cracking induced by corrosion is identified as serviceability limit state (SLS) of RC structures (Vu & Stewart 2005, Stewart & Mullard 2007). And it is worthwhile to develop an analytical model to access the time from corrosion initiation to cover cracking.

As to the issue on cover cracking of corroded RC, a lot of laboratory tests and field investigations have been done by many researchers (Bazant 1979, Rasheeduzzafar et al. 1992, Cabrera & Ghoddoussi 1992, Andrade et al. 1993, Alonso et al. 1998, Liu & Weyers 1998, Mangat & Elgarf 1999, Vu & Stewart 2005, Maaddawy et al. 2005), and some theoretical models to predict the time of cover cracking have been proposed (Bazant 1979, Morinaga 1988, Liu & Weyers 1998, Bhargava et al. 2005, Maaddawy & Soudki 2007). These efforts had made great progress in analyzing the corrosion of reinforcement and relevant cover cracking. However, there are still some differences between the observed data and predicted values obtained from these models due to the complexity of the corrosion process itself. The reasons caused above differences may attribute to following factors: (i) the material properties of concrete itself were ignored, which might bring evident error for various strength concrete, such as Morinaga's model (Morinaga 1988), (ii) the mechanical properties of corrosion products were complicated and usually neglected, such as Liu and Weyers's model (Liu & Weyers 1998) and Maaddawy and Soudki's model (Maaddawy & Soudki 2007), and (iii) during the progress of the crack front, the ingress of corrosion products into the open radial cracks was ignored. For the last factor, the existing mathematical models all haven't taken account of it, which may cause certain difference from the practical conditions especially for natural corrosion process.

This paper firstly contributes towards a quantitative relationship between the amount of steel corrosion and concrete cover cracking, which takes the amount of corrosion products accommodated within the radial cracks into account. Then, based on Faraday's law, a mathematical model that predicts the time from corrosion initiation to corrosion-induced cover cracking is developed for reinforcements' uniform corrosion. The main factors affecting the time to cracking in the present model are analyzed, and its accuracy is demonstrated by comparing the model's predicted results with experimental data published in other literatures.

2 PROBLEM DEFINITION OF CORROSION CRACKING MODEL

2.1 *Basic assumptions*

While formulating the corrosion cracking model, the following four basic assumptions are made: (i) corrosion process is spatially uniform around the steel reinforcement which results in a uniform radial expansive pressure at the steel-concrete interface, (ii) the concrete around the steel reinforcing bar is modeled as a thick-walled cylinder and the wall thickness equals to the thinnest concrete cover, (iii) the stresses in concrete and reinforcement are induced only by the expansion of corrosion products, and (iv) the radial cracks will develop from the cylinder's inner surface to outside, and the corrosion products shall be accommodated in these cracks.

2.2 *Analytical method*

Bazant (1979) proposed a physical model for steel corrosion in concrete sea structures, and believed that the time from corrosion initiation to cover cracking should be mainly dependent on the corrosion rate, cover thickness, spacing between steel reinforcement, diameter of the reinforcement and properties of cover concrete. When the spacing s is large enough, the cover concrete will expand, crack and spall along the longitudinal reinforcement, as shown in Figure 1a. While the cover thickness c, or the ratio of cover thickness to diameter c/d, is domi-nant, the delamination of cover concrete will develop at the surface of steel's layer, see Figure 1b. In these two cases, it is affirmative that the corrosion-cracking will reach to the exterior of thinnest concrete cover before the spalling and delamination occurs. So, it is feasible to adopt thick-walled cylinder to analyze the issue of corrosion-induced cover cracking.

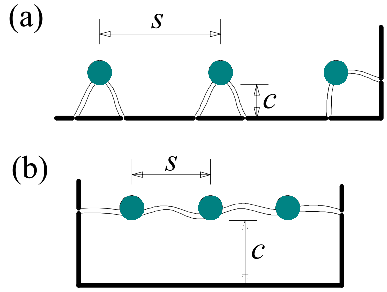


Figure 1. Spalling and delamination of cover concrete caused by reinforcement corrosion.

Corrosion-induced cracks will occur firstly at the steel-concrete interface when the hoop tensile stress at every part of inner circumference reaches to the tensile strength of concrete and then expand to the external surface of cover gradually. The specific position and amount of cracks, however, are uncertain and random. Here, smeared cracking approach is adopted and the relevant formulations are written in terms of average stresses and strains (Pantazopoulou & Papoulia 2001, Bhargava et al. 2005).

To simplify the problem, the total amount of corroded reinforcement existing at the onset of entire cracking of the concrete cover may be assumed to be calculated with two components:

(i) The amount of corrosion products which can result in entire cracking of cover without taking account of the ingress of corrosion products into corrosion cracks. The corresponding radial loss of reinforcement and corrosion time are marked as δ_{s1} and t_1, respectively, and

(ii) The amount of corrosion products which accumulates in the open radial cracks during the progress of the crack front. The needed radial loss of reinforcement and the corrosion time are marked as δ_{s2} and t_2, respectively.

3 THEORETICAL MODEL OF TIME T_1

3.1 *Relationship between ratio of steel mass loss and radial pressure*

For an initial unrestrained RC specimen with the bottom clear cover c and original diameter of reinforcing bar d, the thick-walled concrete cylinder of a homogeneous material is shown in Figure 2a. And around the steel-concrete interface there is a porous zone caused by various reasons viz. transition from cement paste to steel, entrapped/entrained air voids, etc. (Liu & Weyers 1998, Bhargava et al. 2005, Maaddawy & Soudki 2007). For the sake of simplicity, the porous zone is assumed to be uniform and its thickness is indicated by δ_0.

Table 1. Physical properties of various corrosion products.

Corrosion products	α	n	$\gamma = \alpha n$
Fe	1	1	1
FeO	0.777	1.80	1.40
Fe_3O_4	0.724	2.00	1.45
Fe_2O_3	0.699	2.20	1.54
$Fe(OH)_2$	0.622	3.75	2.33
$Fe(OH)_3$	0.523	4.20	2.20
$Fe(OH)_3{\cdot}3H_2O$	0.347	6.40	2.22

* α = ratio of molecular weight of iron to the molecular weight of the corrosion product.
n = ratio of volume of corrosion product to the volume of consumed iron.
γ = ratio of density of iron to the density of corrosion production.

The uniform corrosion products must first fill this porous zone before their volume expansion (see Table 1) starts to create uniform radial pressure q_r around the surrounding concrete, due to which the concrete gets an internal radial displacement δ_c as shown in Figure 2b. Meanwhile, the change in the diameter of the steel reinforcing bar and the deformation of corrosion products are shown in Figure 2c. In Figure 2c, the combined diameter of uncorroded steel plus free-expansive corrosion products is indicated by d_1 with the radial loss of reinforcement δ_s. Under the same expansive radial pressure q_r, the corrosion products are compacted and the corresponding radial displacement is indicated by δ_r. When $r_0=d/2+\delta_0$ and $r_1=d_1/2$, the deformation compatibility equation at steel-concrete interface is given by:

$$r_0 + \delta_c = r_1 - \delta_r \tag{1}$$

For the displacement of concrete δ_c, it can be easily obtained from the books of mechanics of elasticity and can be expressed as:

$$\delta_c = \frac{r_0}{E_{cef}}\left[\frac{(r_0+c)^2 + r_0^2}{(r_0+c)^2 - r_0^2} + \upsilon_c\right]\cdot q_r \tag{2}$$

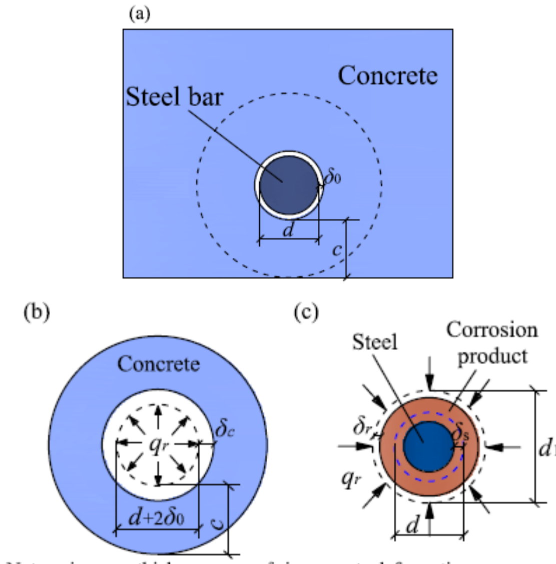
where υ_c = Poisson's ratio of concrete; and E_{cef} = effective modulus of elasticity of concrete which can be calculated by Equation (3) (Bhargava et al. 2005, Maaddawy & Soudki 2007).

$$E_{cef} = E_c/(1.0+\varphi) \tag{3}$$

where E_c = elastic modulus of concrete; and φ= creep coefficient of concrete.

When the uncorroded reinforcement is assumed to be rigid, the displacement of corrosion products δ_r can be given as follows (Wang et al. 2008):

$$\delta_r = \frac{r_1}{E_r}\cdot\frac{(1-\upsilon_r^2)(r_1^2 - d_n^2/4)}{(1-\upsilon_r)d_n^2/4 + (1+\upsilon_r)r_1^2}\cdot q_r \tag{4}$$



Note: c is cover thickness; d is steel diameter; δ_0 is porous zone depth; δ_c is concrete deformation; δ_r is corrosion product deformation; δ_s is radial loss depth of steel;

Figure 2. Idealization of cover concrete as thick-walled cylinder and deformations caused by corrosion: (a) Initial unrestrained RC specimen; (b) Deformation of concrete; (c) deformation of corrosion products.

where E_r = elastic modulus of corrosion products; υ_r = Poisson's ratio of corrosion products; and d_n= diameter of uncorroded steel bar, d_n=d-2δ_s.

Let ρ denotes the ratio of corroded steel mass M_{loss} to the original steel mass M_s per unit length, namely:

$$\rho = \frac{M_{loss}}{M_s} = 1 - \left(\frac{d_n}{d}\right)^2 \tag{5}$$

Based on Equation (5), the parameters of d_n and d_1 can be rewritten as:

$$d_n = d\sqrt{1-\rho} \tag{6}$$

$$d_1 = 2r_1 = d\sqrt{1+(n-1)\rho} \tag{7}$$

where n = ratio of volume expansion of corrosion products (see Table 1).

Substitution of Equation (6) and Equation (7) into Equation (4) will yield the following expression:

$$\delta_r = \frac{d}{2E_r} \cdot \frac{n\rho\left(1-\upsilon_r^2\right)\sqrt{1+(n-1)\rho}}{\left[(1+\upsilon_r)n-2\right]\rho+2} \cdot q_r \tag{8}$$

Then, from Equation (1) using the relationships given in Equation (2) and Equation (8), the relationship between the radial pressure q_r and the ratio of steel mass loss ρ can be expressed as:

$$q_r = \frac{\sqrt{1+(n-1)\rho}-1-2\delta_0/d}{\frac{1}{E_{cef}}\left[\frac{(r_0+c)^2+r_0^2}{(r_0+c)^2-r_0^2}+\upsilon_c\right]+\frac{1}{E_r}\frac{n\rho\left(1-\upsilon_r^2\right)\sqrt{1+(n-1)\rho}}{\left[(1+\upsilon_r)n-2\right]\rho+2}} \tag{9}$$

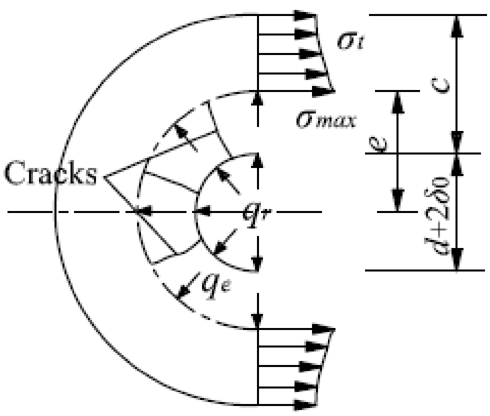


Figure 3. Stress distribution of un-cracking concrete cover.

3.2 *Critical ratio ρ_c for entire cracking of cover*

At failure, cracks induced by the expansive pressure have reached the concrete surface. If it is assumed that the tensile stress in the cover concrete is uniform, the critical pressure $q_{r,c}$ at that time may be represented as (Liu & Weyers 1998, Maaddawy & Soudki 2007):

$$q_{r,c} = 2\frac{c}{d} f_{ct} \tag{10}$$

where f_{ct} = tensile strength of concrete.

For the thick-walled concrete cylinder, the tensile stress in the concrete is usually non-uniform (see Figure 3). Zhao and Jin (2006) believed that the entire cracking of the concrete cover occurs when e=0.5(c+d/2), and the corresponding critical pressure $q_{r,c}$ at failure can be expressed as:

$$q_{r,c} = \left(0.3+0.6\frac{c}{d}\right) f_{ct} \tag{11}$$

When $q_r = q_{r,c}$, the governing equation about critical ratio of ρ_c at cover cracking can be determined from Equation (9) and (11) as:

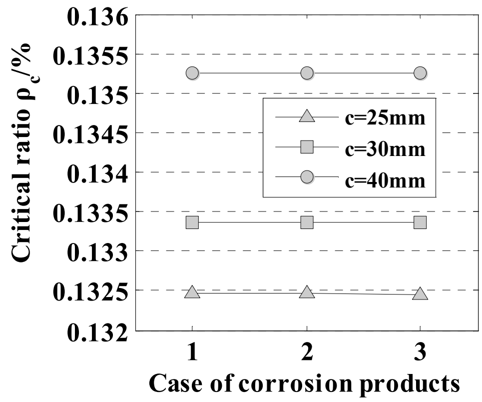
$$\frac{\sqrt{1+(n-1)\rho_c}-1-2\delta_0/d}{\frac{1}{E_{cef}}\left[\frac{(r_0+c)^2+r_0^2}{(r_0+c)^2-r_0^2}+\upsilon_c\right]+\frac{1}{E_r}\frac{n\rho_c\left(1-\upsilon_r^2\right)\sqrt{1+(n-1)\rho_c}}{\left[(1+\upsilon_r)n-2\right]\rho_c+2}} = \left(0.3+0.6\frac{c}{d}\right) f_{ct} \tag{12}$$

Usually, the corrosion products in concrete may be a complex admixture combined by various oxides of iron. The components of corrosion products depend on the level of oxidation and show obvious diversity under different conditions. Bhargava (2005) considered the mechanical properties of corrosion products to be same as that of reinforcement, viz. υ_r=υ_s=0.3 and E_r = E_s=210GPa. Molina (1993) studied the performance of corrosion products in concrete and assumed that υ_r=0.49 and E_r =6000(1- 2υ_r) =120GPa. Here, the influence of material properties of corrosion products on critical ratio ρ_c is analyzed under three following conditions:

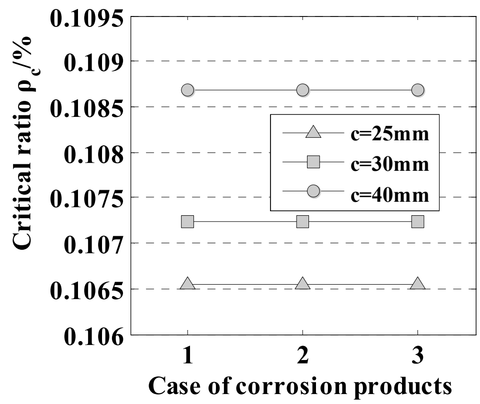
Case 1: referring to Bhargava (2005), υ_r=0.3 and E_r =210GPa.

Case 2: referring to Molina (1993), υ_r=0.49 and E_r =120GPa, and

Case 3: neglecting the influence of reinforcement's deformation, that is, δ_r=0.



(a) f_{ct}=1.43MPa and d=16mm



(b) f_{ct}=1.43MPa and d=20mm

Figure 4. Relationship between properties of corrosion products and critical ratio ρ_c.

Figure 4 shows the results of the influence of corrosion products on critical ratio ρ_c with different RC elements. From Figure 4, it can be known that the influence of material properties of corrosion products on critical ratio is negligible and the results of Case 1 and 2 are approximately equal to that of Case 3. The main reason may attribute to that the displacement of corrosion products δ_r is inverse proportionate to the modulus of E_r, and the elastic modulus of corrosion products, even E_r =120GPa in Case 2, is several times higher than that of concrete. So, for the sake of simplicity, the deformation of corrosion products is neglected in following analysis, just like Case 3. Then, the detailed expression of critical ratio ρ_c can be obtained from Equation (12) as:

$$\rho_c = \frac{\left\{\left(0.3+0.6\frac{c}{d}\right)\cdot\frac{f_{ct}}{E_{cef}}\left[\frac{(r_0+c)^2+r_0^2}{(r_0+c)^2-r_0^2}+\upsilon_c\right]+1+\frac{2\delta_0}{d}\right\}^2-1}{n-1} \tag{13}$$

The relevant radial loss of reinforcement δ_{s1} can be expressed as:

$$\delta_{s1} = \frac{d-d_{n,c}}{2} = \frac{d}{2}\left(1-\sqrt{1-\rho_c}\right) \tag{14}$$

3.3 *Time to cover cracking of t_1*

The mass of consumed steel is related to the amount of current that flows through the electrochemical corrosion cell. The corrosion process can be formulated using Faraday's law as (Morinaga 1988, Andrade et al. 1993, Maaddawy & Soudki 2007):

$$M_{loss} = \frac{MI_{corr}}{zF}t \tag{15}$$

where t = corrosion time (s); M_{loss} = mass of steel consumed in time t; I_{corr} = corrosion current (A); M = atomic mass of the metal (M=56g); z = ionic valency (z=2 for $Fe(OH)_2$ and z=3 for $Fe(OH)_3$); and F = Faraday's constant (F=96,500 As).

The corrosion current density i_{corr} is defined as the corrosion current per unit steel surface. If the unit length L_0 equals to 1cm and the unit of diameter d is mm, a relationship between I_{corr} (A) and i_{corr} (μA/cm^2) can be established as:

$$I_{corr} = 1\times\pi\frac{d}{10}\times i_{corr}\cdot 10^{-6} = \pi\cdot d\cdot i_{corr}\cdot 10^{-7} \tag{16}$$

With ρ_s =7.85 g/cm^3 and L_0=1cm, the Equation (6) can be rewritten as follows:

$$M_{loss} = \rho M_s = \rho\cdot\frac{\pi}{4}\left(\frac{d}{10}\right)^2\rho_s = 0.0616\rho d^2\,(\text{g}) \tag{17}$$

Combining Equation (15), (16) and (17) with z=2.5 (mean value for Fe^{2+} and Fe^{3+}), the corrosion time t (h) can be expressed with ρ and i_{corr} (μA/cm^2) as follows:

$$t = \frac{zFM_{loss}}{MI_{corr}} = \frac{2.5\times 96500\times 0.0616\rho d^2}{56\pi d i_{corr}\times 10^{-7}\times 3600} = 234762\frac{\rho d}{i_{corr}} \tag{18}$$

In case of $\rho=\rho_c$, the time from corrosion initiation to entire cracking of cover without taking account of the ingress of corrosion products into cracks, t_1 (h), can be obtained from Equation (13) and (18) as:

$$t_1 = 234762\times \frac{d\left\{\left(0.3+0.6\frac{c}{d}\right)\cdot\frac{f_{ct}}{E_{cef}}\left[\frac{(r_0+c)^2+r_0^2}{(r_0+c)^2-r_0^2}+\upsilon_c\right]+1+\frac{2\delta_0}{d}\right\}^2-d}{(n-1)\cdot i_{corr}} \tag{19}$$

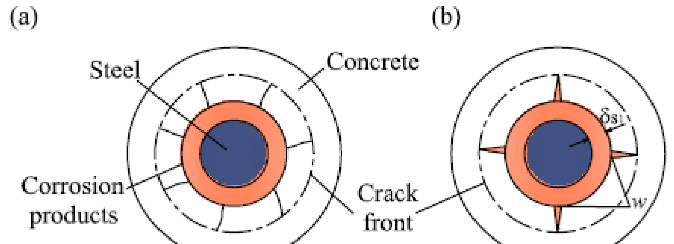


Figure 5. Schematic diagram of corrosion crack and its filling: (a) Development of crack front; (b) Rust deposited within triangular cracks.

4 THEORETICAL MODEL OF TIME T_2

As explained previously, corrosion-induced cracks will occur firstly at inner surface of the thick-walled concrete cylinder and then expand to the external surface of cover gradually. During the progress, the corrosion products will penetrate into these radial cracks under radial pressure until cover cracking. With smeared cracking approach, the schematic diagram of crack front is shown in Figure 5(a). Assuming the shape of corrosion crack to be a triangle, as shown in Figure 5(b), the radial loss of steel δ_{s2}, with which the corresponding corrosion products can fully fill in the total volume of cracks, can be expressed as (Zhao and Jin 2006):

$$\delta_{s2} = \frac{d}{2}-\sqrt{\left(\frac{d}{2}\right)^2-c\delta_{s1}-(n-1)\left(\frac{c}{d}\right)^2\delta_{s1}^2} \tag{20}$$

Usually, the radial loss δ_{s1} (see Equation (14)) is a very small decimal, and the square of δ_{s1} will become smaller. So, the third item in the square root in

Equation (20) can be neglected, and the Eq. (20) can be rewritten as:

$$\delta_{s2} = \frac{c}{d}\delta_{s1} \tag{21}$$

For long-term natural corrosion of reinforcement in concrete, it is inevitable for corrosion products to penetrate and fill in corrosion cracks, which is in accordance with the results of field inspection for RC structures. For short-term accelerated corrosion, however, the extent of filling of rust deposited within cracks depends on the producing speed of corrosion products, which greatly lies on the intensity of accelerated corrosion current. Larger the current is, smaller the corrosion products penetrate into radial cracks. So, some researcher (Liu & Weyers 1998, Bhargava et al. 2005, Maaddawy & Soudki 2007) believed that no amount of corrosion products should be accommodated within the open cracks during the progress of the crack front. Herein, taking account of above two corrosion conditions synthetically, the Equation (21) is mended by a coefficient *k*, as follows:

$$\delta_{s2} = k\frac{c}{d}\delta_{s1} \tag{22}$$

where k = modified coefficient which depends on corrosion condition of steel. For accelerated corrosion, k=0.15~0.30; while for natural corrosion, k=0.8~1.0.

Combining Equation (5), (14) and (22), the ratio of mass loss ρ_2 relating with radial loss δ_{s2} may be established as:

$$\rho_2 \approx 2k\frac{c}{d}\left(1-\sqrt{1-\rho_c}\right) = 2k\frac{c}{d}\frac{\rho_c}{1+\sqrt{1-\rho_c}} \approx k\frac{c}{d}\rho_c \tag{23}$$

From Equation (18), it can be known that the corrosion time is proportionate to the corrosion ratio ρ. So, the corrosion time t_2 (h) can be expressed as:

$$t_2 = k\frac{c}{d}t_1 \tag{24}$$

5 THEORETICAL MODEL OF TIME TO CORROSION CRACKING

5.1 *Model establishment*

Combining Equation (19) and (24), the total time from corrosion initiation to cover corrosion cracking, t_{cr} (h), can be given by:

$$t_{cr} = t_1 + t_2 = \left(1+k\frac{c}{d}\right)t_1 = 234762(d+kc)\times$$
$$\frac{\left\{\left(0.3+0.6\frac{c}{d}\right)\cdot\frac{f_{ct}}{E_{cef}}\left[\frac{(r_0+c)^2+r_0^2}{(r_0+c)^2-r_0^2}+\upsilon_c\right]+1+\frac{2\delta_0}{d}\right\}^2-1}{(n-1)\cdot i_{corr}} \tag{25}$$

In above Equation (25), the Poisson's ratio of concrete υ_c is 0.2. The thickness of the porous zone δ_0 is typically in the range of 10~20μm (Maaddawy & Soudki 2007), and the mean value of 15μm is adopted in this present model. The ratio of volume expansion of corrosion products n is generally between 2 and 4 (Liu & Weyers 1998), and n=2.5~3.0 is practical for normal corrosion of steel bars.

5.2 *Discussion of influencing factors*

Now assume that a RC member has f_{ct} =1.43MPa, E_c=30GPa, d=20mm and c=35mm. And the corrosion condition is an accelerated corrosion with i_{corr}=100μA/cm^2 (k = 0.20). Figure 6 indicates the relationship between time to cover cracking t_{cr} and volume expansive ratio n. It is clearly seen that the time t_{cr} decreases sharply with the increment of value of n. So, controlling the level of oxidation is helpful to obtain lower value of n, and it finally can prolong the cracking time. Using the same parameters' values with n=2.7, Figure 7 shows the relationship between the time t_{cr} and cover thickness c. It is understandable that the cracking time increases when the cover thickness increases. Thick cover concrete can not only protect the reinforcement from rusting, but also prolong the corrosion cracking time. So, increasing the cover thickness is good for structural durability.

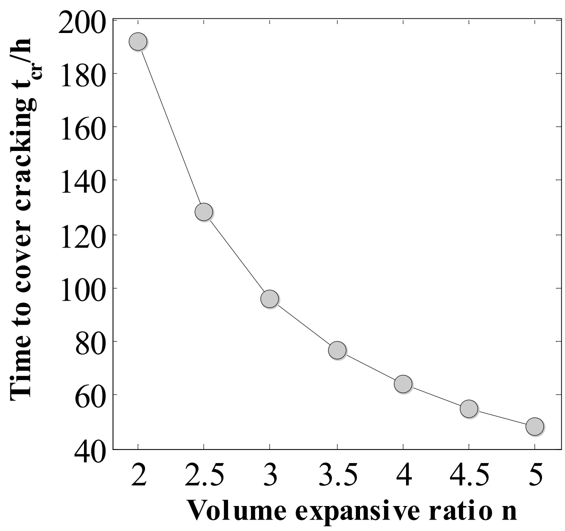


Figure 6. Relationship between time to cover cracking and volume expansive ratio.

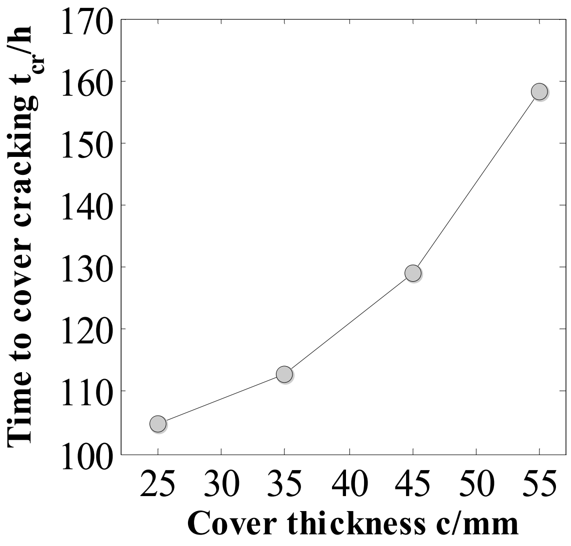


Figure 7. Relationship between time to cover cracking and cover thickness.

Based on the same parameters' values, Figure 8 displays the relationship between time to cover cracking and the diameter of reinforcement. It is found that with c=35mm the cracking time has a significant decline in the range of d=14~18mm, then the cracking time reduce slowly with the increment of steel diameter. In fact, the ratio of cover thickness to steel diameter, viz. c/d, can be used to synthetically estimate the influence on cover cracking time. Keeping the invariable values of parameters, Figure 9 reveals the relationship between the time to cover cracking and the tensile strength of concrete. It is obvious that the cracking time increases when the concrete's tensile strength increases, but the increment is slight. The reason caused above consequence is that increasing the tensile strength of concrete will

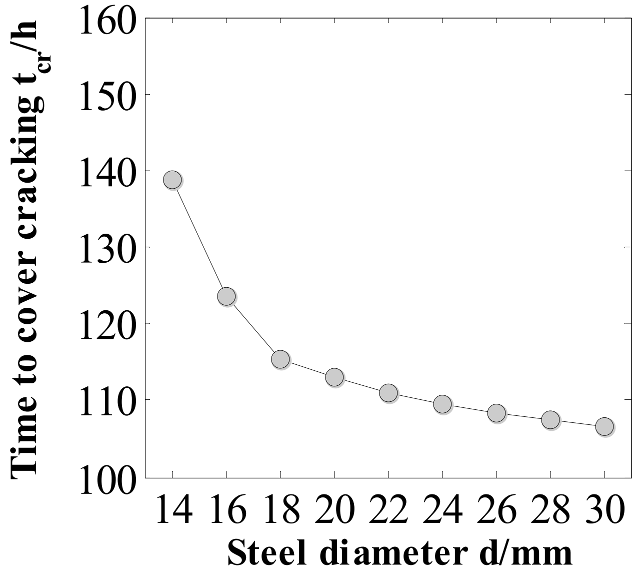


Figure 8. Relationship between time to cover cracking and steel diameter.

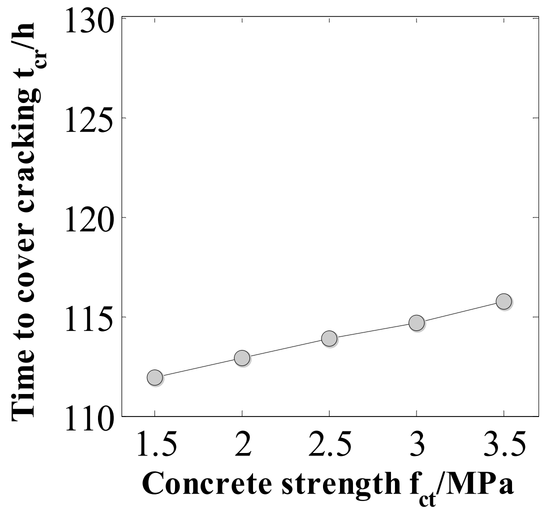


Figure 9. Relationship between time to cover cracking and tensile strength of concrete.

simultaneously improve the value of elastic modulus of concrete, so the ratio of f_{ct}/E_{cef} will be increased slightly.

6 EXAMINATION OF THE PROPOSED MODEL'S ACCURACY

To evaluate the accuracy of the proposed model, some experimental results published in literatures are compared with the results predicted by the mathematical model. A summary of test parameters of the experimental studies used here is shown in Table 2. Usually, the time to cover cracking mainly depends on the density of corrosion current (see Equation (25)). In order to shorten the period of testing, a high current density, i_{corr}=100μA/cm^2 or more, was used by many researchers for accelerated corrosion. So, the corresponding experiments are regarded as short-term accelerated experiments. While, the current density in the test done by Liu and Weyers (1998) was less than 4μA/cm^2, and the relevant test period continued for several years. Therefore, the test belongs to long-term experiment, which needs to take account of the influence of creep of concrete.

To predict the cracking time with Equation (25), other parameters except for which listed in Table 2 are assumed as: n=2.7, υ_c=0.2, δ_0=15μm and k=0.15 (accelerated experiment) or 0.90 (long-term experiment). The predicted results are also given in Table 2. From Table 2, it can be concluded: (i) for accelerated experiments, most of the predicted times from corrosion initiation to corrosion cracking agree with the observed times obtained from experiments, and their errors are less than 10%. However, some errors of the predictions are slightly higher than 10%, such as the results observed by Cabrera (1992). The reasons caused above differences might be ascribed to two aspects: one is the model's error and another result from test error.

Table 2. Comparison between experimental and predicted results.

Researcher	d (mm)	c (mm)	c/d	i_{corr} (μA/cm^2)	f_{ct} (MPa)	E_c[2] (GPa)	Time to cover cracking t_{cr} (h/y)		
							Tested	Predicted	Error
Accelerated experiments:									
Andrade (1993)	16	20	1.25	100	3.55	22	96.4h	105.7h	9.6%
Alonso (1998)	16	20	1.25	100	3.85	22	113h	111.8h	-1.1%
	16	50	3.13	100	3.85	22	208h	201.1h	-3.3%
	16	70	4.38	100	3.85	22	264h	255.8h	-3.1%
	16	70	4.38	10	3.85	22	2643h	2558.1h	-3.1%
Cabrera (1992)	12	69	5.75	244	6.97[1]	33	108h	128.5h	18.9%
Mangat (1999)	10	20	2.00	800	6.30[1]	30	14.4h	15.3h	6.2%
Vu and Stewart (2005)	16	25	1.56	100	3.44	20	134.0h	117.8h	-12.1%
	16	50	3.13	100	3.44	20	194.7h	200.5h	3.0%
	16	25	1.56	100	5.05	29	116.0h	118.0h	1.7%
Maaddawy (2005)	16	33	2.06	150	4.9[1]	28	95h	87.1h	-8.3%
Long-term experiments:									
Liu and Weyers (1998)	16	27	1.69	3.75	3.3	27	0.72y	0.75y (0.70y)[3]	4.2% (-6.7%)
	16	48	3.00	2.41	3.3	27	1.84y	1.89y (1.66y)[3]	2.7% (-9.8%)
	16	70	4.38	1.79	3.3	27	3.54y	3.61y (3.06y)[3]	2.0% (-13.6)

* [1] Data got from literature (Maaddawy & Soudki 2007);
[2] Calculated value based on equation E_c=4500(f_c')$^{1/2}$, where f_c' is the compressive strength of concrete;
[3] Data in bracket was calculated with φ=2.0 (Liu & Weyers 1998).

(ii) for long-term experiment, two predicted results calculated with φ=0 and φ=2 respectively are given. The anterior predictions are slightly higher than the observations, while the latter values are lower than those. So, it can be concluded that the influence of creep coefficient of concrete φ on the time to corrosion-induced cover cracking is obvious for long-term corrosion of reinforcement and that it is crucial to get a reasonable value of φ to obtain precise predictions. In conclusion, considering the complexity of the corrosion process for accelerated and long-term corrosion, it is evident that the proposed model can give reasonable prediction for the time form corrosion initiation to corrosion cracking.

7 CONCLUSIONS

In this paper a mathematical model that can predict the time from initiation to corrosion cracking was proposed. In the present model, the concrete around the reinforcement bar was modeled as a thick-walled cylinder with a wall thickness equal to the thinnest concrete cover. The model accounted for two key issues about corrosion-induced cracking as follows: (i) the deformation of corrosion products induced by radial pressure was neglected because its influence on critical steel corrosion ratio was very little; and (ii) the amount of corrosion products deposited in radial open cracks was taken into account and the characteristics of accelerated and natural corrosion should be considered to determine the extent of filling of rusts in radial cracks. Then, Faraday's law was then utilized to predict the time from corrosion initiation to cover cracking.

Discussion of the main influencing factors of the prediction model shows that increasing the cover thickness, reducing the bar diameter, improving the concrete strength and controlling the level of oxidation can prolong the time to cover cracking, which will be good to structural durability. A comparison of our proposed model's predictions with experimental results published in literatures, including accelerated and long-term experiments, indicates that the proposed model could give reasonable prediction for the time to cover cracking. It is practical to use the proposed model to predict the corrosion-induced cracking time and analyze the service life of field RC structures.

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REFERENCES

Ahmad, S. 2003. Reinforcement corrosion in concrete structures, its monitoring and service life prediction-a review. *Cement & Concrete Composites* 25(4-5): 459-471.

Alonso, C. et al. 1998. Factors controlling cracking of concrete affected by reinforcement corrosion. *Materials and Structures* 31: 435-441.

Andrade, C. et al. 1993. Cover cracking as a function of rebar corrosion: Part I-experimental test. *Materials and Structures* 26: 453-464.
Bazant, Z. P. 1979. Physical model for steel corrosion in concrete sea structures-application. *Journal of the Structural Division* 105(6): 1155-1166.
Bhargava, K. et al. 2005. Modeling of time to corrosion-induced cover cracking in reinforced concrete structures. *Cement and Concrete Research* 35(11): 2203-2218.
Cabrera, G.J. & Ghoddoussi, P. 1992. The effect of reinforcement corrosion on the strength of the steel/concrete bond. In: International conference on bond in concrete, Riga, Lativa; 10-24.
Liu, Y.P. & Weyers, R.E. 1998. Modeling the time to corrosion cracking in chloride contaminated reinforced concrete structures. *ACI Materials Journal* 95(6): 675-681.
Maaddawy T.E. et al. 2005. Long-term performance of corrosion-damaged reinforced concrete beams. *ACI Structural Journal*, 102(5): 649-656.
Maaddawy, T.E. & Soudki, K. 2007. A model for prediction of time from corrosion initiation to corrosion cracking. *Cement and Concrete Composites* 29 (3): 168-175.
Mangat, P.S. & Elgarf, M.S. 1999. Bond characteristics of corroding reinforcement in concrete beams. *Materials and Structures*, 32: 89-97.
Molina, F.J. et al. 1993. Cover cracking as a function of bar corrosion: Part II-numerical model. *Materials and Structures*, 26: 532-548.
Morinaga, S. 1988. Prediction of service lives of reinforced concrete buildings based on rate of corrosion of reinforcing steel. Report No. 23, Shimizu Corp, Japan: 82-89.
Pantazopoulou, S.J. & Papoulia, K.D. 2001. Modeling cover-cracking due to reinforcement corrosion in RC structures. *Journal of Engineering Mechanics* 127(4): 342-351.
Rasheeduzzafar et al. 1992. Corrosion cracking in relation to bar diameter, cover, and concrete quality. *Journal Material and Civil Engineering* 4(4): 327-343.
Steward, M.G. & Mullard, J.A. 2007. Spatial time-dependent reliability analysis of corrosion damage and the timing of first repair for RC structures. *Engineering Structures* 29 (7): 1457-1464.
Vu, K.A.T. & Stewart, M.G. 2005. Predicting the likelihood and extent of reinforced concrete corrosion-induced cracking. *Journal of Structural Engineering* 131(11): 1681-1689.
Wang, H.L., Jin, W.L. and Sun, X.Y. 2008. Fracture model for protective layer cracking of reinforced concrete structure due to rebar corrosion. *Journal of Hydraulic Engineering* 39(7): 863-869. (in Chinese)
Zhao, Y.X. & Jin W.L. 2006. Modeling the amount of steel corrosion at the cracking of concrete cover. *Advances in Structural Engineering*, 9(5): 687-696.