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Stiffness estimation of RC bridges based on vehicle responses

Y. Oshima, K. Yamamoto & K. Sugiura
Civil and Earth Resources Engineering, Kyoto University, Japan

ABSTRACT: The purpose of this study is to propose and verify the estimation theory of the vibration properties of a RC bridge in time domain as well as frequency domain by using the responses of passing and parking vehicles. In this theory, the responses of several parking vehicles and one passing vehicle are used to extract the eigenfrequency of the bridge for stiffness estimation. Herein the evaluated bridge is assumed to be a simple beam with one dimensional coordinates for simplicity. The vehicles are also assumed to be mass-spring systems, which are correlated with the beam in vibration. This proposed theory was verified by numerical simulation with white noise, and it was found that the identified properties agreed with the assumed values but the estimated values in frequency domain are better than those in time domain.

1 INTRODUCTION

Stiffness of RC bridges is often evaluated in several stages of maintenance and this value can be translated into the other property such as vibration property. The estimation of vibration properties of bridges has been done for many years by several ways. Normally to detect the properties such as eigenfrequency, mode shape or damping ratio, sensors should be installed on the specific locations of the bridge, and the bridge should be vibrated by a shaker or the other ways. In the test of forced vibration with the shaker, the resonant frequencies are determined as eigenfrequencies, since the external forces are controlled to draw the Bode diagram. However, this test requires much time and labor for sensor installation and test execution, on the contrary to its high accuracy of estimation. In the other test, the proper "external force" should be used such as microtremor as a white random excitation, or free vibration should be used since the bridge vibration induced by external force is a kind of forced vibration and its response is strongly dependent on the external force. For instance, the vibration induced by passing vehicles is, of course, a forced vibration and the response of the bridge depends on the several conditions such as velocity, mass and road profiles.

Yan et al. (2004) proposed the method to estimate the vibration properties of bridges using the responses of passing vehicles. In the proposed method, the passing vehicle is used as a shaker and a receiver at the same time: the peak in the acceleration spectrum which seems to correspond to the eigenfrequency should be found. But sometimes it is quite difficult to find the peak and also the detected peak is not always "correct answer" because the spectrum is strongly dependent on the profiles and the characteristics of the passing vehicles; namely the peak is not always corresponding to the eigenfrequency. Oshima et al. (2008) also proposed the method to estimate the vibration properties using the vehicle responses but in time domain. This method opens up the possibility to estimate the bridge vibration in time domain from the responses of the passing vehicle, but the method requires several conditions to find out the bridge vibration and also the estimation accuracy should be improved for practical use.

Thus herein we propose the estimation method for eigenfrequency, i.e. stiffness of a bridge, using not only the responses of passing vehicles but also those of parking vehicles to improve the accuracy. The proposed method is based on equations of motion of a bridge and vehicles, which are expanded in time domain as well as frequency domain. In this method, several vehicles park on the bridge to measure the vibration: this situation is close to the sensor installation on the bridge but the measurement on the parking vehicle is much easier than that by the installed sensors and it can reduce the time and labor more than the other ways. This method is of course applicable to the non-random excitation by passing vehicles.

In this paper, we report on the estimation theory for bridge eigenfrequency using the responses of the passing and parking vehicles, and its verification by numerical simulation.

2 ESTIMATION METHOD

2.1 *Estimation in time domain*

Herein a simple supported beam is assumed as shown in Figure 1. In this model, unknown parameters are mass, damping and eigenfrequency of the beam. The vehicles are assumed to be a mass-spring model with one degree of freedom, and only mass should be known for estimation (the other parameters are not necessary to be known) in the time domain method. Now the acceleration responses of sprung mass of all vehicles and those of unsprung mass of the parking vehicles are measured in this method. Note that the unsprung mass system is neglected in this theory and this means that the responses of that system are identical to those of the bridge on the parking locations.

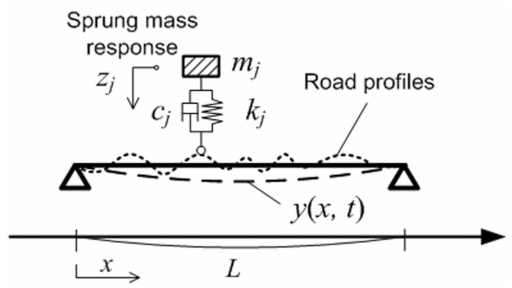

Figure 1. Assumed beam and vehicle.

Now let us assume that the displacement of the bridge is described by the modal expansion in the form:

$$y(x,t)=\sum_{i=1}^{N}\varphi_i(x)q_i(t) \tag{1}$$

where $\phi_i(x)$ is the ith shape function, $q_i(t)$ is the ith general coordinate and N is the considered degree in the expansion. N can be determined by the number of parking vehicles, and N is set to three in the following since one passing vehicle and three parking vehicles are assumed. Then the motion of equation for the beam can be given by

$$\rho\frac{\partial^2 y}{\partial t^2}+c\frac{\partial y}{\partial t}+EI\frac{\partial^4 y}{\partial x^4}=\sum_{j=1}^{m}P_j(t)\delta(x-x_j) \tag{2}$$

where ρ is unit mass, c is damping, EI is stiffness (=elastic modulus times moment of the second order) of the beam, and P_j is applying load by the jth vehicle, x_j is the position of the jth vehicle and $\delta(x)$is delta function. On the basis of modal analysis, the shape function can be given by a trigonometric function such as

$$\varphi_i(x)=\sin\left(\frac{i\pi x}{L}\right) \tag{3}$$

where i is the order of mode. Then by substituting equation (3) and (1) into (2) and integrating it from zero to L with regard to x, and the equation (2) with regard to ith order becomes

$$\alpha\ddot{q}_i(t)+\beta\dot{q}_i(t)+i^4\gamma q_i(t)=\sum_{j=1}^{m}\varphi_i(x_j)P_j(t) \tag{4}$$

where

$$\alpha=\frac{L\rho}{2},\beta=\frac{Lc}{2},\gamma=\frac{L}{2}EI\left(\frac{\pi}{L}\right)^4 \tag{5}$$

The relationships among the above parameters and the vibration properties, i.e. basic angular eigenfrequency, ω_0 mass per meter, ρ, and damping factor, h_0, of the beam are given by

$$\rho=\frac{2\alpha}{L},h_0=\frac{\beta}{2\alpha\omega_0},\omega_0=\sqrt{\frac{\gamma}{\alpha}}. \tag{6}$$

Next let us assume three parking vehicles and one passing vehicle on the beam as shown in Figure 2.

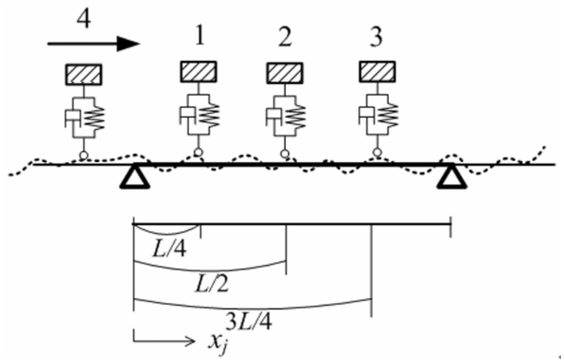

Figure 2. Location of the passing vehicle and parking vehicles.

Note that the number of 4 (j=4) is the passing vehicle and the others are parking vehicles at $L/2$, $L/4$ and $3L/4$ (j=1, 2 and 3). In this case the values of shape function up to the third degree at the location of parking vehicles are clearly determined by equation (3). Then by substituting those values into (4), the equation (4) becomes

$$\mathbf{Qs}=\mathbf{p} \tag{7}$$

Where

$$\mathbf{Q}=\begin{pmatrix}\ddot{q}_1(t) & \dot{q}_1(t) & q_1(t)\\ \ddot{q}_2(t) & \dot{q}_2(t) & 16q_2(t)\\ \ddot{q}_3(t) & \dot{q}_3(t) & 81q_3(t)\end{pmatrix},\ \mathbf{s}=\begin{Bmatrix}\alpha\\ \beta\\ \gamma\end{Bmatrix},$$

$$\mathbf{p}=\begin{Bmatrix}\frac{1}{\sqrt{2}}P_1(t)+P_2(t)+\frac{1}{\sqrt{2}}P_3(t)+\sin\left(\frac{\pi vt}{L}\right)P_4(t)\\ P_1(t)-P_3(t)+\sin\left(\frac{2\pi vt}{L}\right)P_4(t)\\ \frac{1}{\sqrt{2}}P_1(t)-P_2(t)+\frac{1}{\sqrt{2}}P_3(t)+\sin\left(\frac{3\pi vt}{L}\right)P_4(t)\end{Bmatrix} \tag{8}$$

The load of jth vehicle, P_j, can be described using the acceleration responses of sprung mass, $\ddot{z}_j$, the gravity, g, and sprung mass, m_j, as

$$P_j(t) = m_j(g - \ddot{z}_j) \tag{9}$$

From the fundamental assumption, the acceleration responses are known and then the vector of **p** becomes a known vector. Furthermore, since the responses of unsprung mass, a_j, are assumed to be identical to those of the bridge on the vehicle location, the following equation can be formed:

$$a_j = \frac{d^2 y(t, x_j)}{dt^2} = \sum_{i=1}^{N} \varphi_i(x_j)\ddot{q}_i(t) \tag{10}$$

Herein the values of shape function are determined by location and also the acceleration responses are given as measured data. Then the general coordinates, $\ddot{q}_j$, (j=1, 2 and 3) can be determined by equation (10). Then by numerical integration of acceleration of the general coordinates, the velocity and displacement can be also obtained. Note that of course the accuracy decreases by numerical integral and this degradation is improved by the other method, i.e. frequency domain method. In this method, the accuracy associated with numerical integral is improved by using the time difference of equation (7) since that equation is valid for all time and this process eliminate the error accumulation of velocity.

Next, letting us build the following matrix **A** and vector **b** by $\Delta\mathbf{Q}$ and $\Delta\mathbf{p}$, which are the differences of **Q** and **p**, respectively.

$$\mathbf{A} = \begin{pmatrix} \Delta\mathbf{Q}(t_1) \\ \Delta\mathbf{Q}(t_3) \\ \vdots \\ \Delta\mathbf{Q}(t_{N/2-1}) \end{pmatrix}, \mathbf{b} = \begin{Bmatrix} \Delta\mathbf{p}(t_1) \\ \Delta\mathbf{p}(t_3) \\ \vdots \\ \Delta\mathbf{p}(t_{N/2-1}) \end{Bmatrix} \tag{11}$$

to have

$$\mathbf{As} = \mathbf{b}\,. \tag{12}$$

Then to obtain the optimized values of vector **s**, let the objective function J be defined as

$$J = (\mathbf{As} - \mathbf{b})^T \mathbf{W}(\mathbf{As} - \mathbf{b}) \tag{13}$$

where **W** (= 3(N/2-1) x 3(N/2-1)) is a weight matrix given by

$$\mathbf{W} = \begin{pmatrix} \mathbf{\Sigma} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \mathbf{\Sigma} \end{pmatrix}, \mathbf{\Sigma} = \begin{pmatrix} w_1 & 0 & 0 \\ 0 & w_2 & 0 \\ 0 & 0 & w_3 \end{pmatrix} \tag{14}$$

where w_1, w_2 and w_3 are determined according to the accuracy and should be determined beforehand. In this study, several values of weights were assumed according to the reliability of measurement. Finally by minimizing the objective function on the basis of least square method, the optimized vector can be obtained by the following equation.

$$\hat{\mathbf{s}} = (\mathbf{A}^T\mathbf{WA})^{-1}\mathbf{A}^T\mathbf{Wb} \tag{15}$$

Using the above optimized values, the unknown parameters, i.e. basic angular eigenfrequency, ω_0 mass per meter, ρ, and damping factor, h_0, are obtained by equation (6).

2.2 *Estimation in frequency domain*

On the contrary to the time domain method, only the response of sprung mass is needed for estimation in the frequency domain method, but the properties of parking vehicles should be known beforehand. Now the equation of motion of a parking vehicle is given by

$$\begin{aligned} m_i\ddot{z}_i(t) + c_i\dot{z}_i(t) + k_i z_i(t) = \\ c_i\dot{y}(x_i, t) + k_i y(x_i, t) \end{aligned} \tag{16}$$

where z_i is the displacement response of sprung mass (i=1, 2 and 3), m_i, c_i and k_i are the mass, damping and spring of the sprung mass, respectively, and y is the response of beam. Dot means differential in time domain. Then the Fourier transform of the equation (15) leads to

$$\frac{-\omega^2 m_i + j\omega c_i + k_i}{j\omega c_i + k_i} Z_i(\omega) = Y_i(\omega) \tag{17}$$

where ω is angular frequency, j is an imaginary unit and $Z_i(\omega)$ and $Y_i(\omega)$ is the Fourier transform of $z_i(t)$ and $y(x_i, t)$, respectively. Note that we assumed the initial values are zero in the Fourier transform and thus some errors are inevitably caused by this assumption. Then by multiplying ω^2 by both sides of the equation (17), we obtain

$$\frac{-\omega^2 m_i + j\omega c_i + k_i}{j\omega c_i + k_i}\left(-\omega^2 Z_i(\omega)\right) = -\omega^2 Y_i(\omega) \tag{18}$$

Unlike the method in time domain, herein we just need the acceleration of the sprung mass, not the unsprung mass. Thus in this method, we obtain the Fourier transform directly by the response of sprung mass since "$-\omega^2 Z_i(\omega)$" is the Fourier transform of acceleration of z_i.

Considering the right side of the equation (18) is equal to the Fourier transform of the acceleration of $y(x_i, t)$, the right side can be expressed by modal expansion in the form

$$-\omega^2 Y_i(\omega)=\sum_{k=1}^{N}\phi_k(x_i)\left\{-\omega^2 Q_k(\omega)\right\} \tag{19}$$

where $Q_i(\omega)$ is the Fourier transform of the kth general coordinate. In this study, the shape function is assumed by the equation (3), and also the assumed number of parking vehicles is equal to the expansion order of (1). For instance, when the order of 3 is assumed, the following relationship is obtained.

$$\mathbf{Y}=\mathbf{\Phi Q} \tag{20}$$

where

$$\mathbf{Y}=\begin{Bmatrix}-\omega^2 Y_1(\omega)\\-\omega^2 Y_2(\omega)\\-\omega^2 Y_3(\omega)\end{Bmatrix},\ \mathbf{Q}=\begin{Bmatrix}-\omega^2 Q_1(\omega)\\-\omega^2 Q_2(\omega)\\-\omega^2 Q_3(\omega)\end{Bmatrix},$$
$$\mathbf{\Phi}=\begin{pmatrix}\phi_1(x_1)&\phi_2(x_1)&\phi_3(x_1)\\\phi_1(x_2)&\phi_2(x_2)&\phi_3(x_2)\\\phi_1(x_3)&\phi_2(x_3)&\phi_3(x_3)\end{pmatrix} \tag{21}$$

Since $\mathbf{Y}$ is known by equation (17), then $\mathbf{Q}$ can be obtained by multiplying the inverse matrix of $\mathbf{\Phi}$ in the form

$$\mathbf{Q}=\mathbf{\Phi}^{-1}\mathbf{Y} \tag{22}$$

Meanwhile, the right side of equation of (4) can be determined by substituting the measurable data such as the acceleration of sprung mass of passing and parking vehicles. Now letting us define the function $f_i(t)$ as the right side terms of the equation (4), i.e.

$$f_i(t)=\sum_{j=1}^{m}\varphi_i(x_j)P_j(t). \tag{23}$$

Then the Fourier transform of (4) is given by

$$\left(-\omega^2\alpha+j\omega\beta+i^2\gamma\right)Q_i(\omega)=F_i(\omega) \tag{24}$$

where $F_i(\omega)$ is the Fourier transform of $f_i(t)$. Then the following equation is obtained by dividing the above equation by $Q_i(\omega)$ and multiplying $-\omega^2$ by both denominator and numerator.

$$-\omega^2\alpha+j\omega\beta+i^2\gamma=\frac{-\omega^2 F_i(\omega)}{-\omega^2 Q_i(\omega)} \tag{25}$$

In this equation, the denominator is already known from equation (22). As for the numerator, this Fourier transform can be calculated by using the obtained data with equation (3) and (9), i.e. (23). Since the imaginary number is only used in the damping term, the following equations can be obtained in the actual number.

$$-\omega^2\alpha+i^2\gamma=\mathrm{Re}\left\{\frac{-\omega^2 F_i(\omega)}{-\omega^2 Q_i(\omega)}\right\}\equiv h_i(\omega) \tag{26}$$

Then using two values of ω, the following equations can be built.

$$\mathbf{Ws}=\mathbf{h} \tag{27}$$

where

$$\mathbf{W}=\begin{pmatrix}-\omega_1^2 & i^2\\-\omega_2^2 & i^2\end{pmatrix},\mathbf{s}=\begin{Bmatrix}\alpha\\\gamma\end{Bmatrix},\ \mathbf{h}=\begin{Bmatrix}h_i(\omega_1)\\h_i(\omega_2)\end{Bmatrix} \tag{28}$$

Then multiplying the inverse matrix of $\mathbf{W}$ to the left side of the equation, the vector $\mathbf{s}$ can be obtained in the form

$$\mathbf{s}=\mathbf{W}^{-1}\mathbf{h} \tag{29}$$

In this study, we adopted the value of ω giving a local maximum of $-\omega^2 Q_i(\omega)$ in the ith order and its adjacent value for the second value, because it was found empirically that the estimation becomes stable using these values. Additionally, the first and second orders are considered for estimation and the values of $\mathbf{s}$ were obtained for two orders respectively.

3 NUMERICAL VERIFICATION

3.1 *Numerical simulation of bridge vibration*

The proposed method was verified by the numerical simulation of one-dimensional simple beam with one passing vehicle and three parking vehicles. We herein assumed two different masses and velocities of passing vehicle and three levels of noise, while the properties of the bridge and parking vehicles were fixed. The assumed properties of a bridge (RC bridge) and vehicles and the analyzed series and cases are listed in Table 1 and Table 2, respectively.

In this simulation the equation of motion with modal expression with 5th degrees was integrated by Newmark-β method with the increment of 1/500 second. The road profiles were also obtained by the Mote Carlo simulation based on the standard spectrum given by

$$S(\Phi)=\frac{a}{b^n+\Phi^n} \tag{30}$$

where a=0.001 $(\mathrm{cm}^2)^2$(cycle/m), b=0.05(cycle/m) and Φ is wavelength (cycle/m). The simulated profile is shown in Figure 3. The passing vehicle started 5m before the bridge to produce stable vibration, and also we set additional time enough to eliminate the vibration of parking vehicles due to gravity. To evaluate the noise influence, the white noise of 1% and 5% of the maximum amplitude was added to the simulated values.

Table 1. The assumed properties of the bridge and parking vehicles.

Type	Bridge	Parking vehicles
Eigenfrequency	3.80Hz	2.50Hz
Damping	0.05	0.05
Mass	30 kN/m	19.6kN
Length	30m	-

Table 2. The assumed cases for numerical verification.

Series	Case	Noise	Mass	Velocity
	00	0%		
A	01	1%	196 kN	15 m/s
	05	5%		
	00	0%		
B	01	1%	196 kN	5 m/s
	05	5%		
	00	0%		
C	01	1%	98 kN	15 m/s
	05	5%		
	00	0%		
D	01	1%	98 kN	5 m/s
	05	5%		

*Eigenfrequency and Damping of the passing vehicle is 1.50Hz and 0.3, respectively

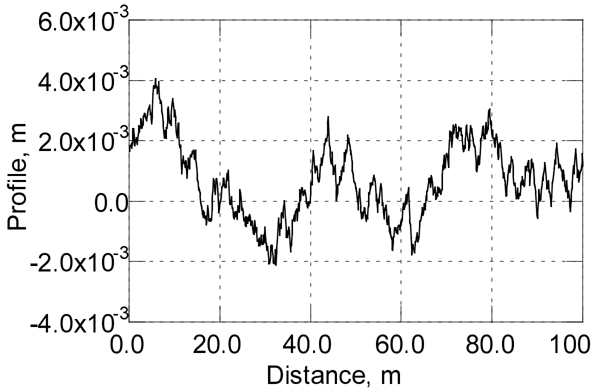

Figure 3. Simulated road roughness profile.

3.2 *Simulated responses*

The acceleration responses obtained by the numerical simulation in the case of B-00 where the passing vehicle is 196kN at 15m/s without noise are shown in Figure 4, and their power spectrums are also shown in Figure 5.

From these figures, it is found that the peaks in the spectrums, even of the bridge, do not correspond to the eigenfrequency, i.e. 3.8Hz. Those spectrums as well as the peaks may vary due to the velocity and the other factors. Especially, the first peak of the bridge seems to split into two peaks, i.e., 3.3Hz and 4.7Hz, which may be attributed to the effect of a moving load. Thus it can be said that the eigenfrequencies cannot be determined just by detecting the peaks in the obtained spectrum when the vibration is induced by the passing vehicle. Additionally the damping ratio as well as mass of the bridge cannot be detected by just picking the peaks in the spectrums.

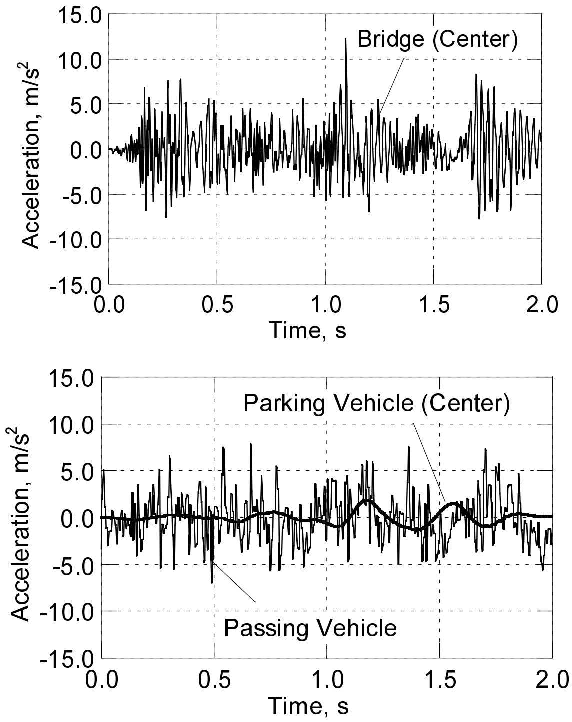

Figure 4. Acceleration responses of the bridge (top) and the vehicles. (bottom).

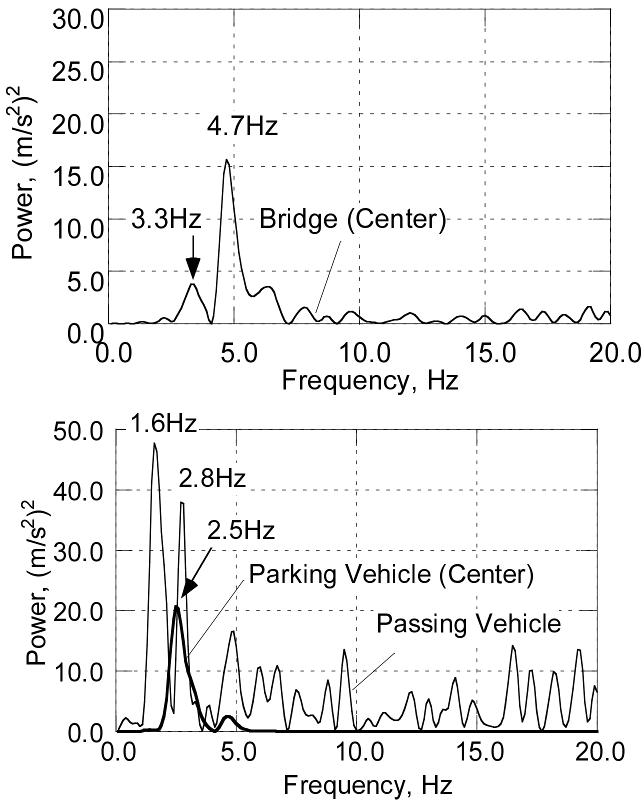

Figure 5. Acceleration power spectrums of the bridge (top) and the vehicles. (bottom).

3.3 *Results and discussion*

Figure 6 shows the relative errors between the estimated and assumed frequency based on time domain with three unknown parameters such as eigenfrequency, damping and mass. Note that weights were given such as (w_1, w_2, w_3) = (0.001, 0.999, 0.00001) and of course the influence of the weight on the estimation is not ignorable but herein the weights were fixed for evaluation of noise influence. From this figure, it is found that the accuracy decreases as noise increases. It is also found that in the case of 00, i.e. 0% of noise, the estimated values agree well with the assumed ones for all series, especially for the series of B and D with lower velocity. This may be attributed to the fact that in the series of lower velocity the recorded time is longer than that of higher, which gives high reliability. Comparing with the results of 1% noise for all series, that of series D is relatively higher than those of the others. Light mass may induce higher frequencies of vibration and smaller amplitude of the responses, which normally decreases the accuracy. However, in this verification the allocated weight for optimization is highest at the second order (i.e. w_2=0.999) and this allocation may lead to emphasis on the higher frequencies in estimation and give higher accuracy to the series of D. Due to the numerical integration of general coordinates, there must be some errors of calculation but the results show good agreement in the cases of 00. Note that when we input the exact values of the general coordinates used in our theory instead of integration, the exact values of the assumed properties are obtained.

When the unit mass is known, the accuracy of estimated frequencies were slightly improved as shown in Figure 7. Especially for the series of B with the noise of 1% the estimation accuracy is much improved from 21% to about 3%. Note that the accuracy is not improved as much as expected when both of the mass and damping are known at the same time, and the accuracy is almost identical to that with the above case. Note also that the weight set of (0.98, 0.019, 0.001) gives the most exact values for the cases without noise for all series, but the accuracy for the cases with noise decreases. This indicates that the information from the first order equations have lower robustness than those from the second order equations in (4).

On the other hand, Figure 8 shows the relative errors between the estimated and assumed frequency based on frequency domain with the first order of equation of (4). Note that the peak values of ω used in estimation depend on the series because ω giving a local maximum value is different from each other. The substituted frequencies ($=\omega/2\pi$) are listed in Table 3. From Figure 8, it is clearly found that the estimated values by the proposed method based on frequency domain agree well with the assumed values for all series and noise. Even when the data contains 5%, the eigenfrequency is obtained within 2% error for all cases. Thus it can be said that the proposed method based on frequency domain has high robustness against noise.

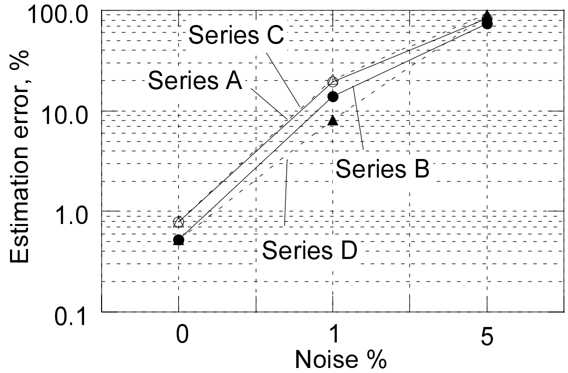

Figure 6. Estimation error vs. noise % based on time domain method.

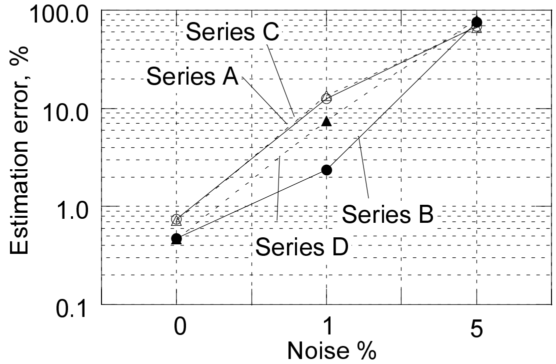

Figure 7. Estimation error vs. noise % (known mass) based on time domain method.

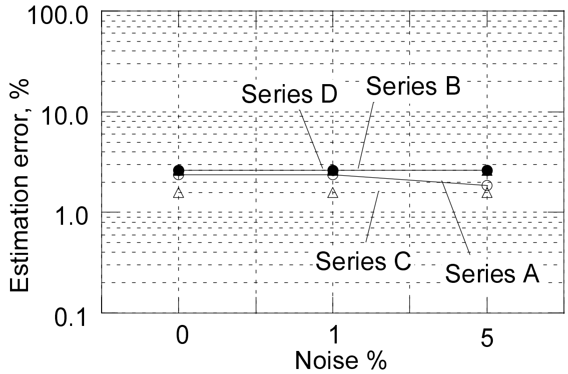

Figure 8. Estimation error vs. noise % based on frequency domain method. (First order).

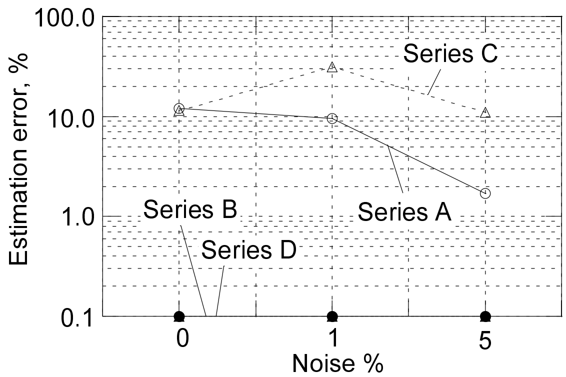

Figure 9. Estimation error vs. noise % based on frequency domain method. (Second order).

Table 3. The substituted values of ω into the equation (26).

Series	1st order	2nd order
A	4.00Hz (4.50Hz	15.01Hz (15.51Hz)
B	4.00Hz (4.10Hz)	15.34Hz (15.50Hz)
C	4.00Hz (4.50Hz)	15.01Hz (15.51Hz)
D	4.00Hz (4.10Hz	15.34Hz (15.50Hz)

*Values in bracket is the other value used in the estimation

Moreover, Figure 9 shows the relative errors when the estimation is based on the second order. Unlike the first order, it is found that the errors of Series B and D are significantly lower than those of Series A and C, which indicates that the accuracy decreases as velocity increases. This may be attributed to the fact that the length of the data of low velocity is larger than that of high velocity, which gives high reliability. It is also found from the comparison of Series A and C that the heavier vehicle gives the higher accuracy. This may be caused by the larger amplitude due to heavier vehicle, which also gives high reliability.

Finally it can be said that the proposed method based on frequency domain enables us to estimate the eigenfrequency with high accuracy as well as high robustness against noise, comparing with that on time domain.

4 CONCLUSIONS

Herein we proposed and verified the estimation theory for the vibration properties of bridges using the responses of passing and parking vehicles. On the basis of numerical simulation, the followings were drawn:

The vibration property such as eigenfrequency can be determined using the proposed theory, even for the non-stationary vibration. This method may have advantage against that based on the spectrum since the spectrum may be influenced by the external factors such as velocity of the vehicles.

1. In the method based on time domain, the accuracy of estimation significantly decreases as the noise increases. Additionally, the accuracy is slightly improved when the unit mass is known parameter.
2. Unlike the method based on time domain, it can be said the proposed method based on frequency domain enables us to estimate the eigenfrequency with high accuracy as well as high robustness against noise, comparing with that on time domain.

Then stiffness of RC bridge can be effectively estimated by the method on frequency domain.

For further research, the estimation theory should be verified using the actual data obtained on site, and the theory must extend to three dimensional when it is applied to the actual data.

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