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Fuzzy Logic Model of Fiber Concrete

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ABSTRACT: The shape of stress-strain curve of fiber concrete in uniaxial tension strongly depends on the amount and type of the used fibers. Since even a small change in dosage causes serious variation of the stress-strain curve, the numerical interpretation of the stress-strain curve should be robust enough to cover the subtle differences which in turn affect significantly the performance of the entire structure. This paper describes an alternative approach to definition of the stress-strain curve using the fuzzy logic. The proposed tools allow definition of smooth curves, whose shape can be easily corrected by experts' intervention if necessary. This approach can also help to overcome the lack of experimental data. The method is explained in examples.

1 INTRODUCTION

The mechanical properties of fiber reinforced concrete, similarly to plain concrete, are affected by the mix proportions of the basic constituents and also by the added fibers which are supposed to enhance the post-peak behavior of the rather brittle concrete. A variety of fibers ranging from steel fibers of standard shapes to synthetic fibers with very specific shapes and finishes are available and since they affect the mechanical behavior of concrete in various ways, laboratory tests need to be conducted in order to predict the fiber concrete behavior. In order to save time and money on the experimental work, numerical analyses can be performed instead. Especially when the effect of age should be also taken into account, the use of numerical, or virtual, testing should be considered.

The uniaxial stress-strain curve is one of the fundamental relations used in material modeling and numerical structural analyses. Therefore, considering the heavy dependence of the fiber concrete on various parameters, this study focuses on the development of a method for simple definition of the stress-strain curve. Since reliable experimental data are quite expensive to obtain, but on the other hand there are expert who can readily describe the fiber concrete behavior, the fuzzy set theory has been employed. The fuzzy sets (Zadeh 1965) and specifically the fuzzy controlling (Ross 1995) has been utilized in definition of the desired stress-strain curves because they proved useful in modeling of mechanical response which was difficult to describe analytically. As was shown by the authors earlier (Štemberk 2008), this approach is applicable for definition of hardening concrete. Various experimental data reported in literature were used for calibration of the stress-strain diagram of fiber reinforced concrete including the aging effect. The method is explained in examples and its applicability in numerical analyses is discussed.

2 STRESS-STRAIN DIAGRAM OF FIBER REINFORCED CONCRETE

The experimental investigation of fiber reinforced concrete is mostly run on cylindrical, cubic or beam specimens. Such specimens are subjected to either uniaxial tensile or compressive loading or to flexure. The results of experiments are recorded using the force-displacement or stress-strain diagrams, which allow detailed understanding of response of concrete to loading, and especially in the case of fiber reinforced concrete in the post-peak region when the steel fibers are activated and help to bridge the widening cracks. The results of various experimental work published in (Alhozaiamy et al. 1995, Ding & Kusterle 2000, Camps et al. 2008) are used in this study to prove applicability of the presented approach.

3 FUZZY LOGIC IN DEFINITION OF STRESS-STRAIN CURVES

Since the experimental data are rather expensive to obtain, the experimental investigation is carefully planned and only few representative amounts of fibers are selected. Then, the material engineers try to estimate the effect of intermediate amounts of fibers, where in some cases statistically supported interpolation is attempted, or more preferably the experts draw their own conclusions and try to add their ideas in the material model. The fuzzy logic can then become a very useful tool as it allows the expert in the field of material modeling to add his opinions in the material model, while the material model can be readily applied in numerical analyses, mostly based on the finite element method.

3.1 *Fuzzy Logic*

The fuzzy set theory is attributed to Lotfi A. Zadeh, who published his famous paper (Zadeh 1965) where he formulated the foundation of this theory. The fuzzy set theory proved very useful for those problems whose input parameters are imprecise or ill-defined, often represented by linguistic expressions. This is also true in the case of defining the dependencies of the stress-strain diagram on various parameters, most of which are described in linguistic terms by the experienced material scientists, using the fuzzy logic. Since its introduction the fuzzy logic proved a flexible and robust instrument for controlling a variety of processes ranging from the tiny electronic equipment to massive industrial production technology, (Klir 1997), (Qureshi 2006a), (Ross 1995). Also, in the material modeling, the fuzzy logic has been used when not enough experimental data were available, e.g. in (Akkurt 2004), (Qureshi 2006b), (Štemberk 2006), (Tanyildizi 2007), or (Unal 2007).

The basis of fuzzy logic, whose core is the decision-making, consists of three basic steps. Firstly, the fuzzification is performed, when the input variables are assigned a degree of membership to a fuzzy set. Secondly, the decision-making itself is run, when the output variables are given a degree of membership to its fuzzy set according to the expert's rules. Thirdly, through defuzzificaton, a single real value is obtained. For the fuzzy logic, it is crucial to decide a proper number of fuzzy sets for both the input and output variables and the shape of their membership functions so that the performance of the fuzzy logic system is both acceptably precise and computationally inexpensive.

Two fuzzy-logic approaches to definition of the stress-strain curve of fiber concrete are described in the following.

3.2 *Classical Fuzzy-logic Approach to Definition of Stress-strain Diagram*

The first proposed model uses the classical fuzzy logic techniques, when each combination of the fuzzy sets describing the input variables is assigned a fuzzy set of the output variable. Specifically, the classical procedure used for defining the stress-strain curve of fiber reinforced concrete runs as follows.

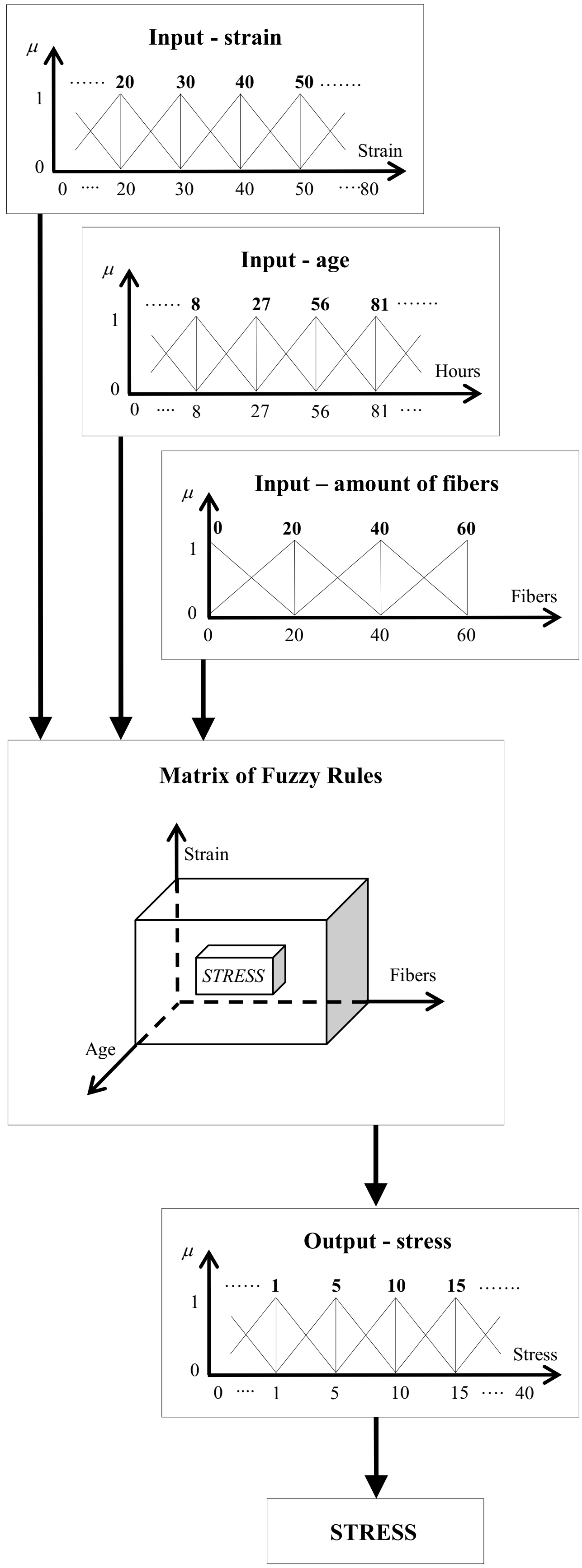

Figure 1. Flowchart of definition using classical fuzzy logic procedure.

Firstly, the input strain, the input age and the input amount of fibers are fuzzified. Secondly, the fuzzy sets representing the stress corresponding to the fuzzy sets expressing the strain, the age and the amount of fibers are selected, and thirdly, the stress is defuzzified and obtained in the form of a single value. The flow of the classical fuzzy logic procedure is shown in Figure 1. The entries in the matrix of the fuzzy rules were selected to give the minimum error at a selected number of points. This was done manually as the number of stress-strain curves was low. Similarly, the fuzzy rules were defined in (Štemberk 2008), (Tanyildizi 2007) and (Unal 2007). Mathematically, this model represents the linear interpolation between the points with the degree of membership equal to unity.

The drawback of the classical fuzzy logic method when linear membership functions are used is a rather rough shape of the stress-strain curve and the discontinuous first derivative. Of course, the shape of the resulting stress-strain curve can be smoother if a larger number of fuzzy sets is used. This however means a much larger number of fuzzy rules, which need to be decided and which would make this method very difficult for use by material engineers who have poor background in fuzzy logic. Or, the engineers would not be able to define the subtle differences in the behavior of the stress-strain diagram, which would unnaturally emerge with the finely refined strains and stresses. Therefore, it seems more reasonable to modify the fuzzy logic model so that the generation and optimization of the numbers and shapes of fuzzy sets is done automatically and only the decision-making defined by just a few representative combinations of strains, age, amounts of fibers and stresses remains to be decided by the material engineer.

3.3 *Modified Fuzzy-logic Approach to Definition of Stress-strain Diagram*

The main difference of the modified model from the classical fuzzy logic one is represented by the separation of the decision-making from definition of the known stress-strain curves, when in the classical approach all the known stress-strain curves are defined by the same number of fuzzy sets representing the strains. Obviously, the minimum number of fuzzy sets necessary for description of the stress-strain curves at given accuracy depends on complexity of its shape and of course it may differ for different stress-strain curves. In the modified approach, the optimization is performed automatically independently of the user, which means the user does not have "to define the most representative points in the stress-strain curve according to the experimentally obtained stress-strain curves and the user's expertise," which is necessary in the classical fuzzy logic modeling. The representative points are represented by the entries in the decision-making matrix, which correspond to the n-tuples of fuzzy sets expressing the input variables.

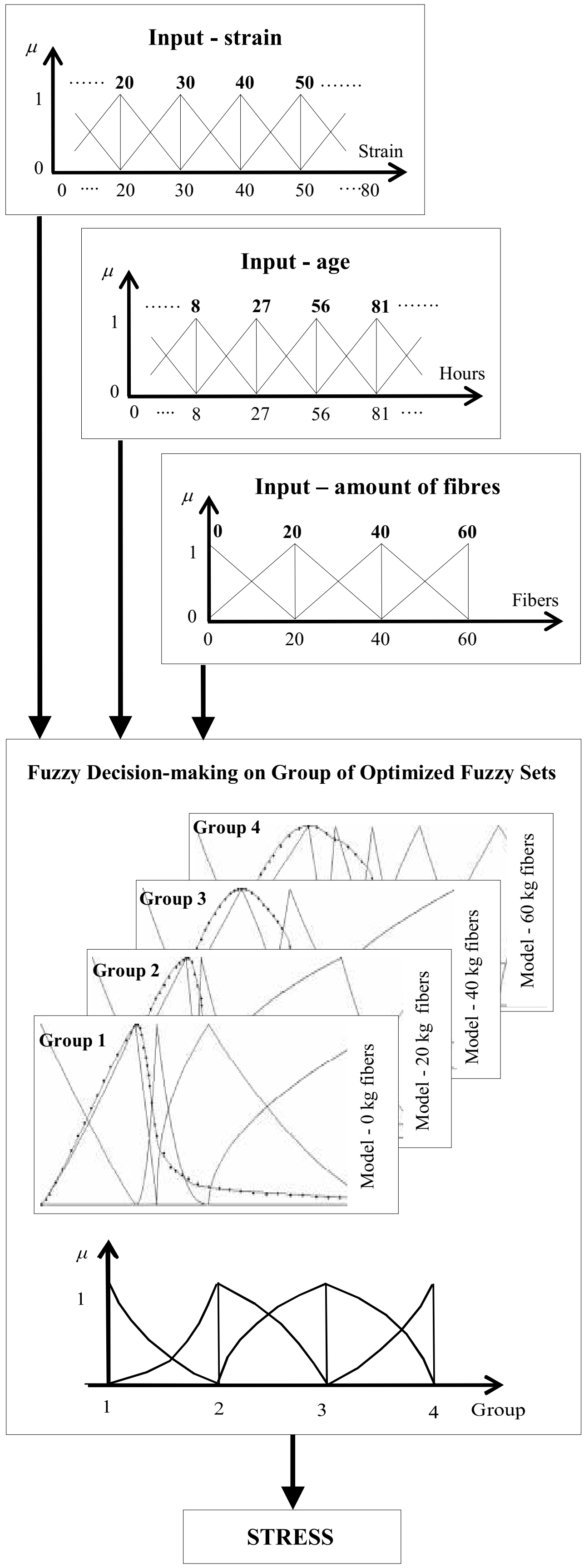

Figure 2. Flowchart of definition using modified fuzzy logic procedure.

In the modified approach, the user's task is "to decide the number of representative intervals of strain and age, and on each interval to decide according to his/her expertise how the stress-strain curve for a given amount of fibers is close to any of the known stress-strain curves." For example, at the first quarter of the stress-strain curve, the constructed stress-strain curve should follow a certain known curve. In the second quarter, it should be slightly above the same known curve, and in the remaining part of the stress-strain diagram it should be identical to the same curve. This approach of definition of the fuzzy logic model is natural for experienced material engineers. The flow of the modified approach is shown in Figure 2. Also, it helps to utilize the exact shape of the known stress-strain curves, which would be lost when using the classical approach with a limited number of fuzzy sets. An example of results obtained by this approach is documented by any of the gray curves in Figures 3 to 9 (the explanation is given in the following section). If the user does not specify the effect of the amount of fibers on the shape of the stress-strain curve by using nonlinear membership functions, which describe the transition between the known stress-strain curves, and only two fuzzy sets are used for the entire range of strain, the result is a linear interpolation between the two known neighboring stress-strain curves. However, as was pointed out above, the user can also refine the effect of the amount of steel fibers on the stress-strain curve by dividing the range of strain into a number of representative intervals. Then the user can easily define whether an amount of steel fibers makes the concrete behave similarly (or how much similarly) to a known stress-strain curve or curves on each interval.

4 RESULTS AND DISCUSSION

The result presented and discussed in this section has been obtained using the modified approach of definition of stress-strain curves.

The experimental data shown in Figs. 3 to 5 were taken from (Ding & Kusterle 2000), where only the compressive stress-strain curves for the ages of 9 and 81 hours for plain concrete and steel fiber concrete were presented. The only accompanying information on the evolution of the material over time is given by a table comprising the evolution of the compressive strength of the ages 8, 10, 18, 30, 48 and 72 hours and for the steel fiber concrete mixes with 0, 20, 40 and 60 kg of steel fibers. The task in this case was to define the intermediate stress-strain curves based on the provided experimental data, which were rather scarce. Fig. 3 shows the definition of the stress-strain curves for concrete at the age of 9 hours with the amount of steel fibers of 0, 20, 40 and 60 kg. The stress-strain curves for the amounts of 0 and 60 kg were calibrated using the available experimental data while the stress-strain curves for the intermediate amounts of steel fibers were defined by the modified fuzzy-logic algorithm.

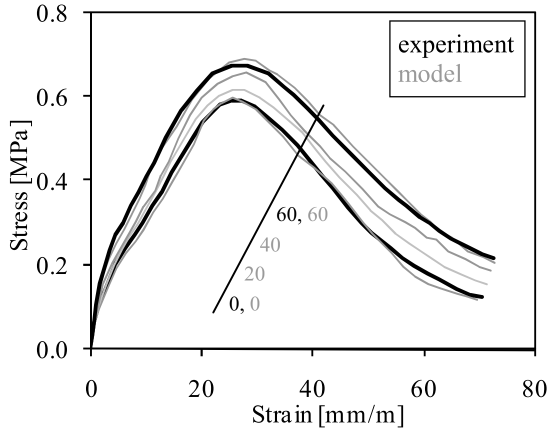

Figure 3. Stress-strain curves constructed by modified approach from experimental data at 9 hours of age.

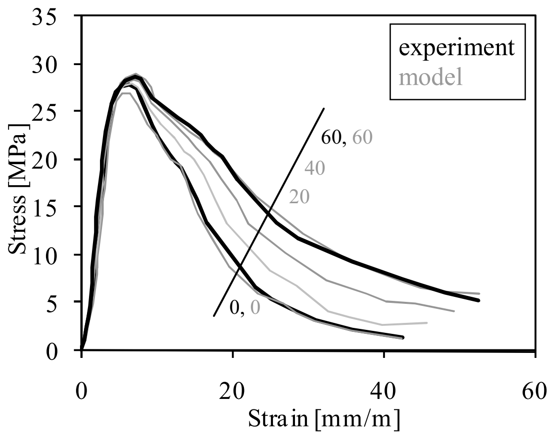

Figure 4. Stress-strain curves constructed by modified approach from experimental data at 81 hours of age.

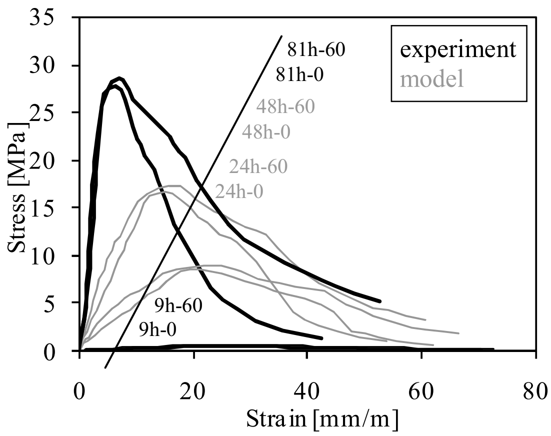

Figure 5. Stress-strain curves constructed by modified approach for ages of 24 and 48 hours.

Since no additional information for the evolution of the stress-strain curves was available, a simple linear interpolation between the known stress-strain curves was assumed. The resulting stress-strain curves obtained by the fuzzy model (gray curves) in Figs. 4 and 5 were obtained similarly. Of course, if more detailed information is available, the transition between the know curves can readily reflect that information, which would result most probably in non-linear transitions.

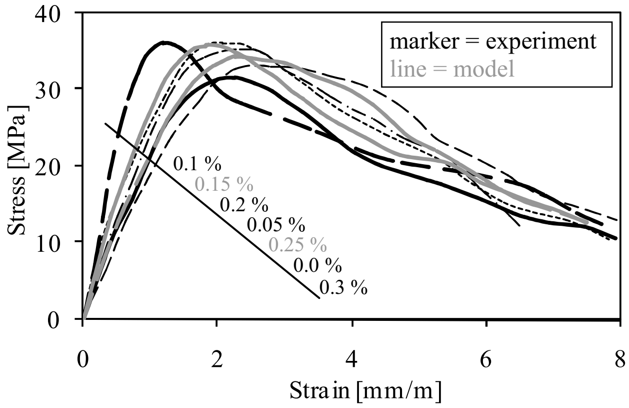

Figure 6. Stress-strain curves constructed by modified approach for intermediate fiber amount of 0.15 % and 0.25 %.

In Fig. 6, the missing stress-strain curves of the amounts of 0.15 % and 0.25 % in (Alhozaiamy 1995) were added using the modified fuzzy model. The reasoning in the decision-making procedure considered the inconsistency in the experimental data when the average strain-strain curves for the couples “0 % and 30 %” and “10 % and 20 %” were not identical. Therefore, non-linear transition between the known stress-strain curves was decided with the additional knowledge on the similarity of the stress-strain curve for various amounts of fibers at different parts of the stress-strain curve, which also commonly used in fracture models of concrete, e.g. (Stahli & van Mier 2007).

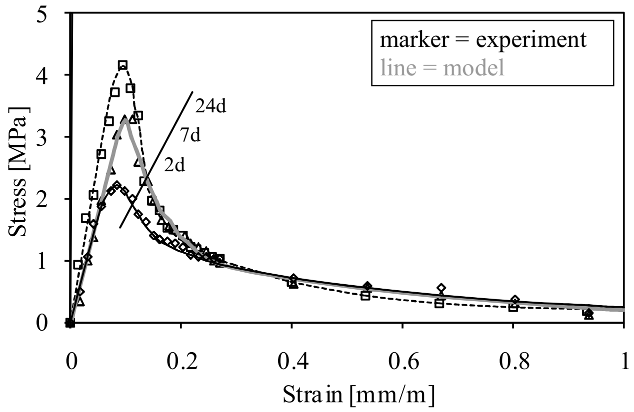

Figure 7. Evolution of tensile stress-strain curve of plain concrete.

Figs. 7 to 9 contain the experimental data on evolution of tensile strength of fiber concrete, specificall the stress-strain curves for plain and fiber concrete at the ages of 2, 7 and 28 days published in (Camps et al. 2008). Figs. 7 to 9 try to document the capability of the modified fuzzy approach to copy precisely the experimentally obtained mechanical behavior of fiber concrete in tension.

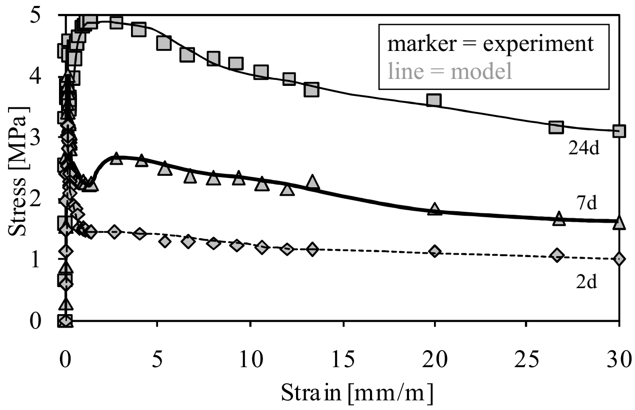

Figure 8. Evolution of tensile stress-strain curve of steel-fiber-reinforced concrete (entire strain range).

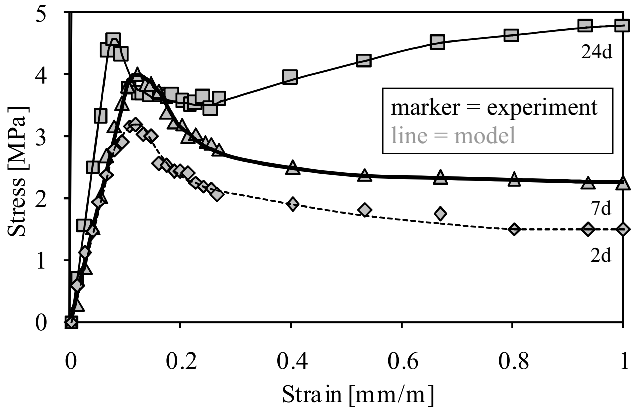

Figure 9. Evolution of tensile stress-strain curve of steel-fiber-reinforced concrete (close-up).

5 CONCLUSION

Two approaches to definition of the stress-strain curve of fiber reinforced concrete were proposed. In the first approach, which is based on the classical fuzzy-logic control, the material engineer is expected to define the number of the fuzzy sets for description of strain, age of concrete, the amount of fibers and stress, and to define the fuzzy rules, which are collected in the decision-making matrix. Since this approach, especially when only fuzzy sets with linear membership functions are used, requires a large number of fuzzy sets, which subsequently requires a large number of fuzzy decisions to be set, it is believed that the material engineer may find such definition of material model rather tedious, even though this approach gives him considerable freedom in definition of the material behavior.

Therefore, a modified approach was proposed which performs the optimization of the number of fuzzy sets and the shape of the fuzzy sets automatically. This means that this part of the model definition is performed independently of the material engineer which reduces the user's workload and ensures objectivity. Then, the only task left for the material engineer is the very natural duty of deciding the ranges of strains and ages where the shapes of the stress-strain curves should be refined and to which material the behavior of the investigated specimen is more similar.

Since the fuzzy-logic material model on the outside is identical to standard functions defining the material models, which means the input parameters of the function and the outputs are identical, the fuzzy-logic material model can be readily used in existing finite element codes.

6 ACKNOWLEDGEMENT

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