Fracture Mechanics of Concrete and Concrete Structures -
High Performance, Fiber Reinforced Concrete, Special Loadings and Structural Applications- B. H. Oh, et al. (eds)

Fiber reinforced concrete characterization through round panel test - part I: experimental study

F. Minelli & G.A. Plizzari
University of Brescia, Italy

ABSTRACT: Standard test methods for determining the mechanical properties of Fiber Reinforced Concrete (FRC) are properly defined if they reproduce the actual structural behavior. Among many proposals, the round panel test seems to have all potentials to become an easy-to-use tool and, at the same time, a reliable procedure for the characterization of FRC, in terms of toughness and post-cracking constitutive cohesive law. A comparison between different test typologies for characterizing FRC is reported and discussed in the present paper, whit special emphasis on the different scatter that each tests produces. Tests are performed on beams as well as on panels. All specimens herein compared have the same concrete mechanical properties and fiber content. Aim of the experimental investigation is to critically discuss advantages and disadvantages of each testing procedure, focusing on the applicability of the method and on the reliability of results toward a consistent characterization of the structural behavior. A new geometry for the panel test is herein proposed and discussed in order to make the panel easier to place, handle and test, therefore avoiding one of the major drawbacks which limit an extensive utilization of the panel tests. Suitable correlations among the different fracture and energy parameters defined in the assumed standards are finally reported, resulting very useful for a harmonization of the available standards.

1 INTRODUCTION

Fiber Reinforced Concrete (FRC) is gaining an increasing interest among the concrete community for the reduced construction time and labor costs. For this reason, many structural elements are now reinforced with steel fibers as partial or total substitution of conventional reinforcement (rebars or welded mesh; di Prisco et al.2004b). Besides cost issues, quality matters are of paramount importance for a construction and FRC also fulfills these requirements since fibers allow for more distributed cracks with a smaller opening that enhances durability (Schumacher, 2006).

Construction materials require Standards for measuring their mechanical properties and for quality control. At the same time new material require new rules in building codes and special guidelines for structural design (Vandewalle, 2004; CEN, 2003). As far as FRC is concerned, design guidelines are already available in some Countries (Rilem, 2003; di Prisco et al.2004; CNR, 2006) and work is in progress for including them in the coming new *fib* Model Code (2010). In these guidelines, structural design is usually based on design values of the material parameters that are normally determined by dividing the characteristic values by a partial safety factor (γ_M).

Mechanical properties of FRC are traditionally determined from beam tests that are usually based on a three (CEN, 2003) or four point bending schemes (UNI, 2003). Early experiences with the low volume fractions of fibers that are nowadays mostly used in practice (V_f<0.8-1.0%), evidence that the characteristic values determined from beam tests (CEN, 2003; UNI, 2003) are quite low because of the high scatter in beam test results. It should be observed that this scatter is not related to the material itself by is mainly due to the small fracture areas (ranging from 160 to 180 cm^2). Such a scatter becomes particularly high when low contents (25-50 kg/m^3) of macro steel fibers (length ranging between 30 and 60 mm) are used (Sorelli et al.2005).

It is commonly accepted that FRCs with a low volume fraction of fibers are particularly suitable for structures with a high degree of redundancy where stress redistribution may occur. Because of this redistribution, large fracture areas are involved (with a high number of fibers crossing them) and, therefore, structural behavior is mainly governed by the mean value of the material properties. Furthermore, because of the large fracture areas, the scatter of experimental results from structural tests is remarkably lower than that obtained from beam tests. A typical example is shown in Figure 1, which exhibits a set of curves obtained from a standard (bending) test on notched specimens (Fig. 1a) and from structural tests on full scale slabs on grade made of the same material (Fig. 1b); the different scatter between material and structural tests is clearly evident, as a confirmation of the above presented discussion.

In order to obtain a more realistic value of the scatter of FRC material tests, specimens with larger fracture areas are needed; this suggests the use of larger beams or different specimens like slabs, where stress redistribution may also occur.

A square panel was proposed to simulate a portion of sprayed concrete in tunnel lining applications (EN 1488-5, 2004). However, since it is simply supported along the whole border, any geometrical irregularity involves that the real support may vary in different specimens; in fact, although the support can lie in a perfect (and controlled) plane, specimens are normally deformed because of shrinkage effect. The crack pattern is therefore hardly predictable (the actual three points on which any rigid body takes support are generally randomly located along the four edges) and the determination of the constitutive laws for cracked concrete becomes very difficult.

A Round Determinate Panel (RDP) test was proposed by ASTM (2004); it is a statically determinate test (a round slab having a diameter ϕ=800 mm and a thickness of 75 mm, with three supports at 120 degrees) where the crack pattern is predictable and the post-cracking material properties can be adequately determined. However, handling and placing such a specimen is quite complicated due to the large size and, consequently, high weight (91 kg). In addition, standard servo-controlled loading machines may not fit with the geometry of the panel, which is too big for many of them. The need of having a specimen easier to handle brought the Authors to come up with a proposal of a smaller round panel having a diameter of 600 mm and a depth of 60 mm, with a weight of only 40 kg.

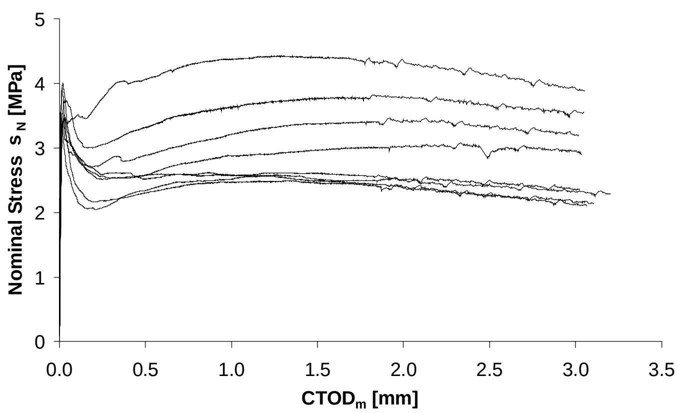

Figure 1. Experimental results from bending tests on notched beams (di Prisco et al. (1), 2004).

The present paper focuses on the comparison of different tests for FRC materials tested during the last few years at the University of Brescia. A comparison between beam and panel tests, a discussion on the smaller round panel herein proposed as well as the correlations between several fracture properties obtained from different Standards are also presented.

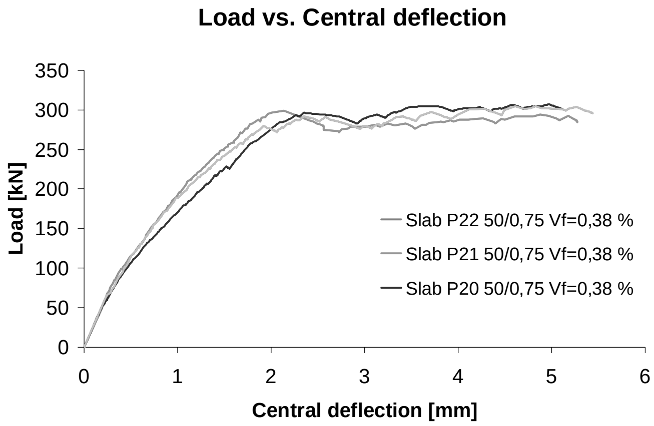

Figure 2. Experimental results from full-scale slabs on grade made of the same FRC as for Figure 1 (di Prisco et al. 2004).

2 MATERIALS, SET UP, EXPERIMENTAL RESULTS AND DISCUSSION

Among more than 100 comparative experiments carried out on beams and panels (60 tests on small round panels and 52 on large round panels), the following discussion will deal with a number of tests performed on members containing either 20 or 30 kg/m^3 of hooked-end steel fibers having a length of 50 mm and a diameter of 1 mm (the aspect ratio L/ϕ is 50). Fibers have a circular cross section and a tensile strength of 1100 MPa. Besides the FRC specimens, plain concrete beams and panels were also made.

Table 1 reports the main geometrical characteristics of the five testing typologies studied herein; the weight of each specimen, assuming a density of 24 kN/m^3, is also outlined.

In order to study the behavior of all specimens up to failure, including any possible unstable branch after cracking, a displacement controlled testing method was adopted. The equipment shown in Figure 3 was utilized for all beam tests, whereas the dimensions of the round panel specimens, according to ASTM, as already mentioned, required utilizing a different equipment, exhibited in Figure 4. In the first case, an INSTRON 1274 machine was used (having a closed loop and a maximum load of 300 kN). In the second case, the displacement was imposed by adopting an electro-mechanical screw jack (having a maximum load of 500 kN and a stroke of 300 mm) placed into a steel frame (Fig. 4); no closed loop was provided in this case. More recently, a steel supporting and loading system was designed for performing tests on small round panels using the INSTRON machine. Figure 5 and Figure 6 show a small round panel ready to test and the steel supporting system to be used in the servo-controlled machine.

A CMOD (Crack Mouth Opening Displacement)-controlled procedure was adopted for the notched beams (UNI and CEN prescribe a notch with a different depth) whereas a displacement-controlled test (screw control of the electro-mechanical jack) was performed with the big round panel tests, resulting in an instability of the response immediately after the peak of the concrete matrix. For more details concerning the geometry, tests set-up and instrumentation, one can refer to Marinoni et al. (2007).

Table 1. Main geometric characteristics of specimens. *Large Round Panel refers to the standard test of ASTM (2004).

Specimen	Length [mm]	Height [mm]	Notch [mm]	Weight [kg]	Loading Scheme
Beam UNI	600	150	25	32.4	3 points
Beam CEN	550	150	45	29.7	4 points
	Dimensions [mm]		Thickness [mm]	Weight [kg]	Loading Scheme
Large Round Panel* (RPL)	800 (radius)		75	90.5	Central point load
Small Round Panel (RPS)	600 (radius)		60	40.7	Central point load
Square Panel (SP)	600 (side)		100	86.4	Central point load

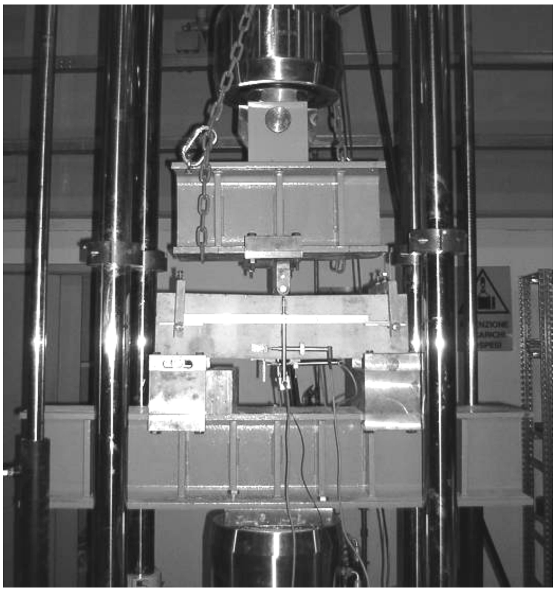

Figure 3. Set up for performing beam tests.

In the latest small round panels, tested with the servo-controlled machine available in the laboratory, a fictitious CMOD was set in a region close to the midpoint in the bottom panel surface. By imposing a very small opening of this point, a much more refined and stable control of the test was possible, without the sudden load drop experienced in all previous panels (both large and small) tested with a simple screw jack. This recent improvement in the test set-up guaranteed a helpful refinement of the test, especially concerning the crack monitoring and its use for material characterization, as shown in the second part of this research report.

After the peak load, once the load path was stabilized, the test was conducted under a stroke-control procedure, as for beam tests.

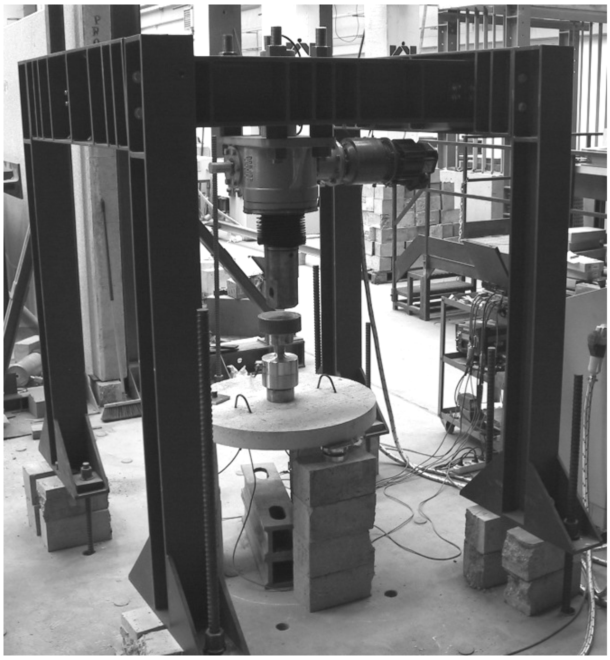

Figure 4. Set up for performing Large Panel tests.

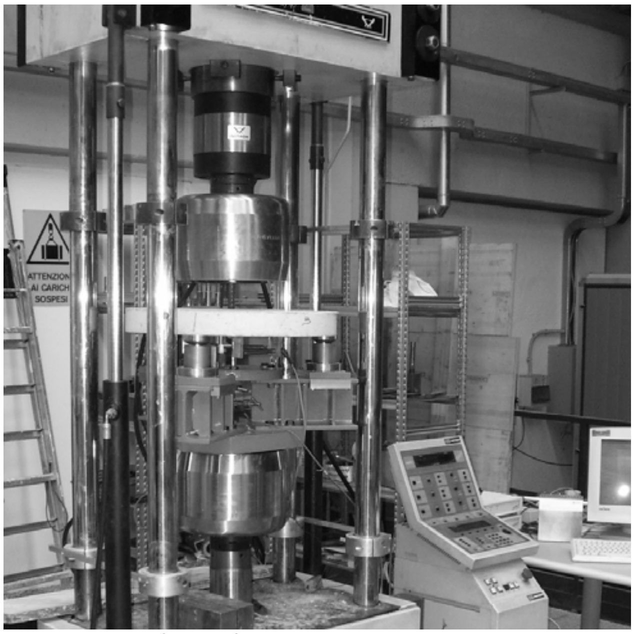

Figure 5. Set up for performing Large Panel tests.

Several LVDTs (Linear Variable Displacement Transducers) were used in each test to measure the vertical displacements (under the load points and in other locations) and the crack openings (including the Crack Tip Opening Displacements in notched beams). In the beam tests, instruments were placed for measuring the point load displacement/s in the front and rear face, the crack width (CTOD) both in the front and back face, and the CMOD gauge.

Concerning the round panels, besides the central

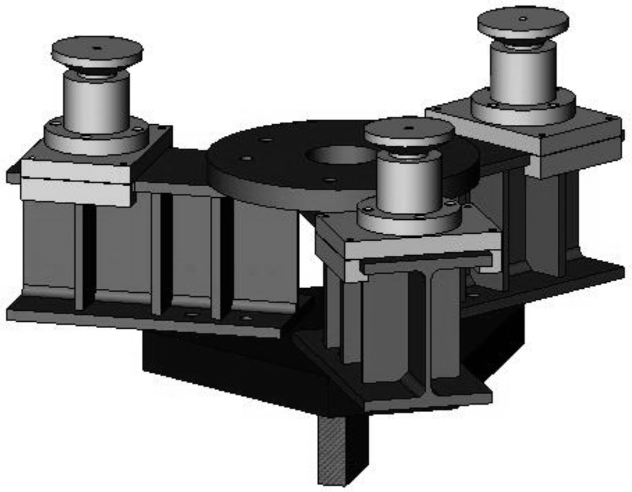

Figure 6. Steel supporting frame for performing Small Panel tests.

displacement of the bottom side and the CMOD, as already reported, 3 LVDTs were also disposed for the measurements of the three cracks. The three instruments were placed at a distance of 120 mm from the midpoint, to make their placement easy. The measurement length was 150 mm, which generally allowed intercepting the crack. Figure 7 shows a picture with the instrumentation on a round panel small prior testing.

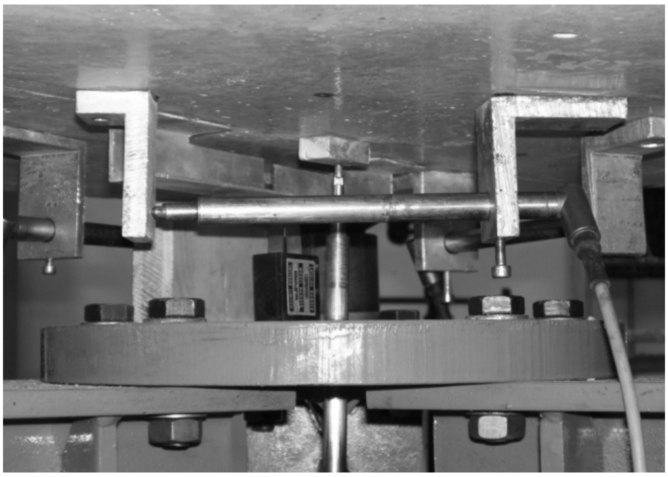

Figure 7. Instrumentation, bottom side of a small round panel.

Figure 8-Figure 11 show the experimental curves of a series of experiments made of different specimens from the same concrete batch. Figure 8 and Figure 9 report the Nominal Stress (according to a linear stress distribution in the cracked section) versus the CTOD for both CEN and UNI beam tests whereas Figure 10 and Figure 11 exhibit the load vs. the central vertical displacement of both large and small round panels made of the identical material. As expected, the post-peak behavior is similar for the two types of beam tests, which are characterized by a rather large scatter. A smaller dispersion can be seen in the panel plots: this is further confirmed from Figure 12, which shows the load-displacement curve of small round panels cast with High Strength Concrete and with two fiber contents: 30 and 60 kg/m^3.

Figure 14 and Figure 15 exhibit the coefficient of variation from the dispersion of previous figures, it results that the coefficient is definitely much smaller in panels than in the corresponding beam tests, both for large and small round panels.

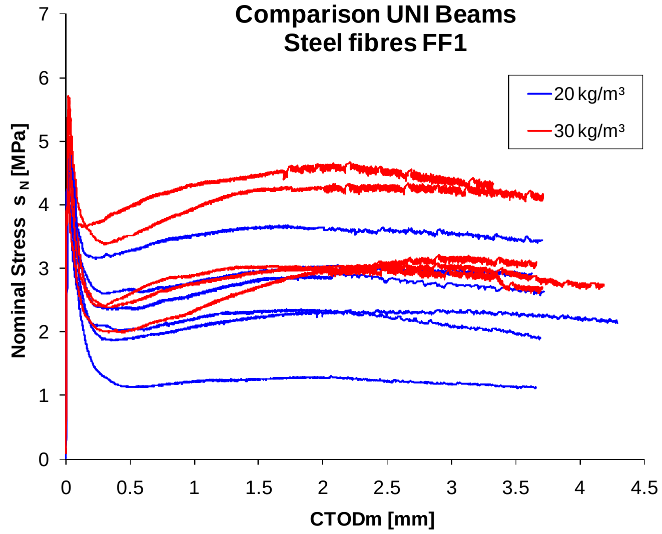

Figure 8. Nominal stress-CTOD of Beam tests according to UNI, for FRC with 20 and 30 kg/m^3.

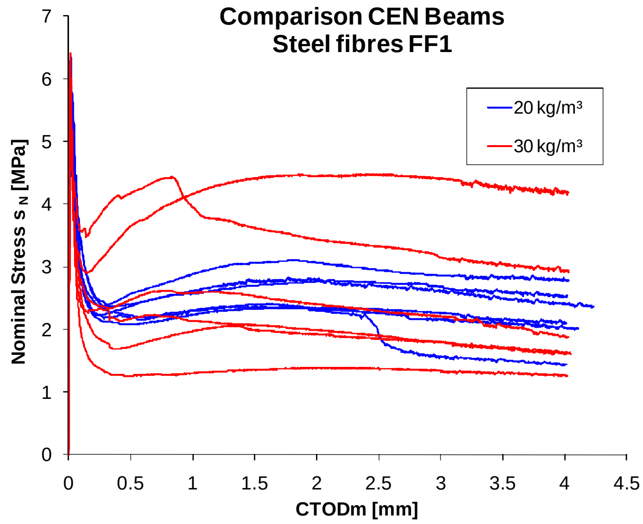

Figure 9. Nominal stress-CTOD of Beam tests according to CEN, for FRC with 20 and 30 kg/m^3.

With the aforementioned three LVDTs, crack widths greater than 20 mm were measured with a post-cracking load higher than one third of the peak load. Such an instrumentation is not required by the ASTM Standard, which states that one should only calculate the energy absorption that is defined by the vertical load and the vertical displacement. By monitoring the crack widths, whose location is predictable because of the statically determinate support system, it was also possible to come up with nominal stress (from elastic analysis) vs. crack width plots, as a suitable tool for the mechanical characterization of

FRC materials and for the definition of simplified stress-crack width cohesive constitutive laws.

Figure 13 reports a typical experimental curve of the crack width vs. load. The three crack locations are in most of cases in good agreement with the expectations (120°) and their values are rather similar one to each other.

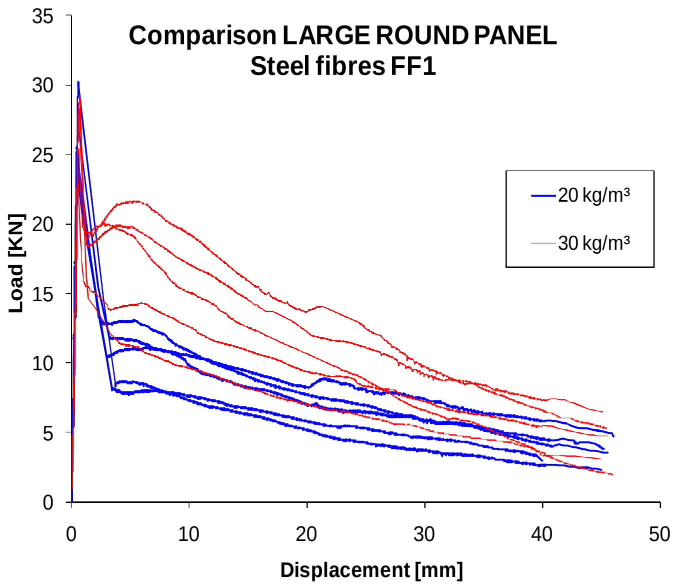

Figure 10. Load-Displacement curve of Large Round Panel Tests according to ASTM, for FRC with 20 and 30 kg/m^3.

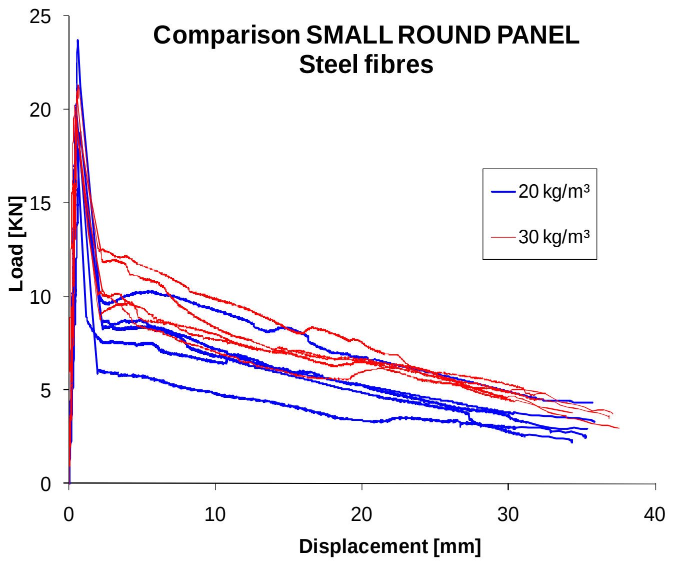

Figure 11. Load-Displacement curve of Small Round Panel Tests, for FRC with 20 and 30 kg/m^3.

The coefficient of variation, as an indicator of the test-result scatter, was calculated for all properties and indexes defined in the different standards, both for quantities which refer to the serviceability limit states (SLS, Fig. 14) and ultimate limit states (ULS, Fig. 15). Once again, a significant lower coefficient of variation can be outlined for panel tests in comparison with beam tests.

It seems worth using the low scatter of panel tests for the determination of more suitable fracture properties consistent with the ones determined from the standard beam tests (i.e. finding residual stresses or post-cracking strengths as stated by beam standard but using round panel tests). To this aim, the following procedure was undertaken:

1. Find correlation between the different parameters required by different standards using experimental results;
2. Given the correlations, calculate average value of the equivalent (UNI) or local (CEN) post-cracking strength (beam tests) from the experimental values of energy absorptions from panels;
3. Calculate the characteristic values of the post-cracking strength from panels accounting for a lower experimental scatter;
4. Compare the values of equivalent post-cracking strength determined from beam tests (direct method) with those from large and small panel tests.

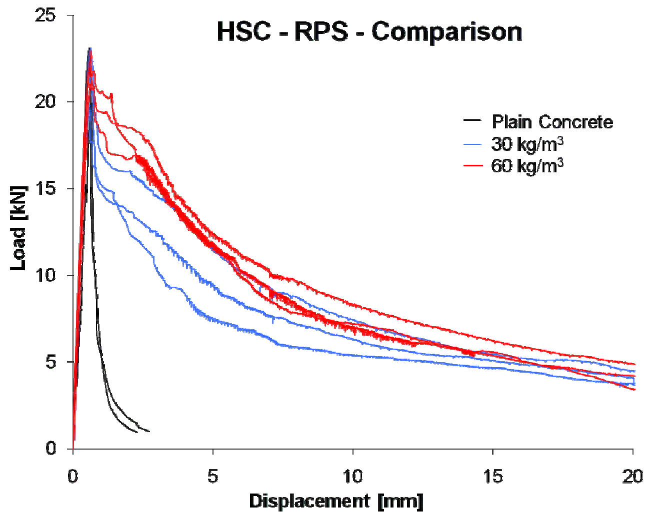

Figure 12. Load-Displacement curve of Small Round Panel Tests, HSC panels, FRC with 30 and 60 kg/m^3.

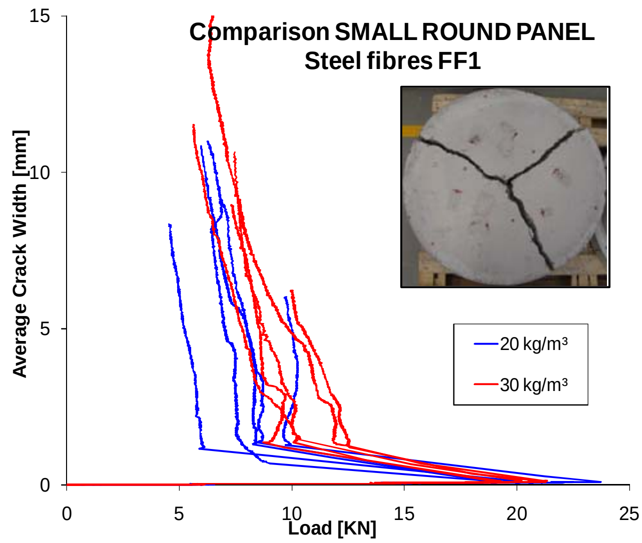

Figure 13. Crack width-Load curve of Small Round Panel Tests, HSC panels, FRC with 30 and 60 kg/m^3.

Figure 16, as an example, describes the correlation between the equivalent post-cracking strength of the UNI standard and the energy absorption of the

ASTM panels at SLS. One should note that the energy absorption relevant for the SLS was conventionally defined by the authors as the energy measured at 5 mm displacement for the classical round panels, whereas at 3.75 mm (just by scaling the dimensions) for the small panels. These two values refer to crack stages which are significant for general situation related to serviceability limit states (compare, as an example, Fig. 11 and Fig. 13).

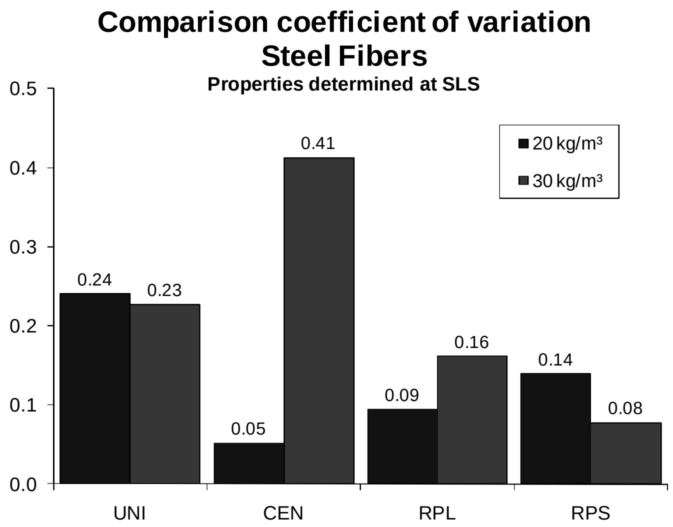

Figure 14. Coefficient of variation calculated for all parameters required by the standards considered in the present experimental campaign.

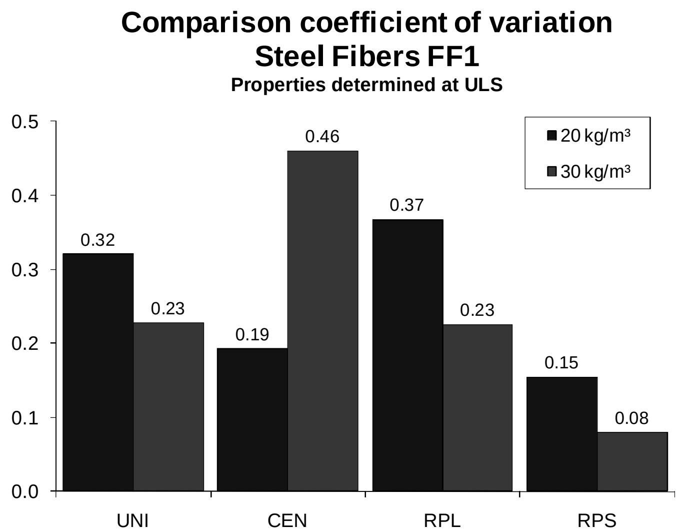

Figure 15. Coefficient of variation calculated for all parameters required by the standards considered in the present experimental campaign.

Thirteen comparative studies are reported with different points in the plots, each one referring to the average of at least three round panels per series. Different fiber contents, fiber materials, fiber "cocktails", fiber geometry and concrete classes are considered in these plots. Figure 17 shows the identical correlation calculated with the parameters at ULS. From these two plots, one can notice that a linear regression between the two quantities represents well the trend and that the coefficient of variation R^2 is quite small.

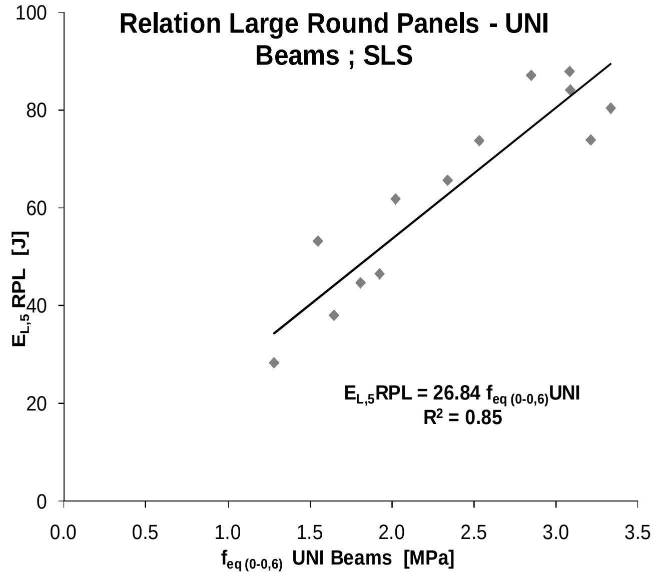

Figure 16. Correlations between energy absorption and equivalent post cracking strength: Classical Large Round Panel tests ASTM vs. Beam Tests UNI, serviceability limit states.

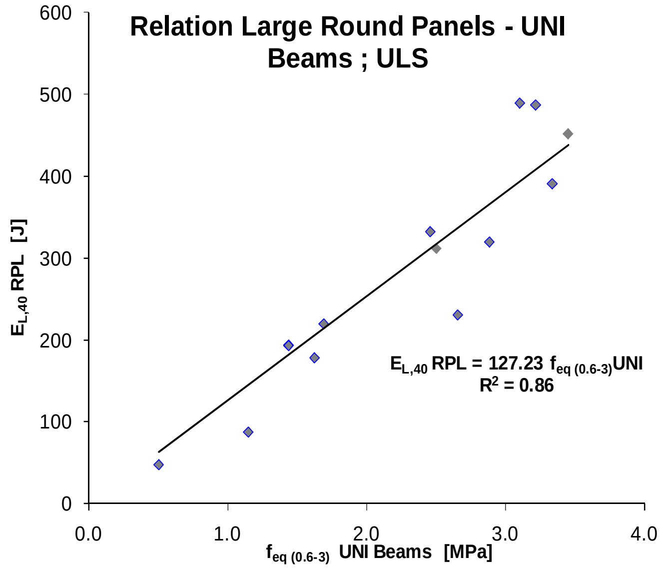

Figure 17. Correlations between energy absorption and equivalent post cracking strength: Classical Large Round Panel tests ASTM vs. Beam Tests UNI, ultimate limit states.

The two relationships found are as follows:

$$E_{L,5} = 26.84\, f_{eq(0-0.6)}$$
$$E_{L,40} = 127.23\, f_{eq(0.6-3)} \qquad (1)$$

where:

$E_{L,5}$ is the energy absorption for the large round panel up to a vertical displacement of 5 mm (set by the authors);

$E_{L,40}$ is the energy absorption for the large round panel up to a vertical displacement of 40 mm (defined in the ASTM Standard);

$f_{eq(0\text{-}0.6)}$ is the equivalent post-cracking strength calculated for a CTOD range varying from 0 to 0.6 mm (SLS) included in the UNI Standard.

$f_{eq(0.6\text{-}3.6)}$ is the equivalent post-cracking strength calculated for a CTOD range varying from 0.6 to 3 mm (ULS) included in the UNI Standard.

Other correlations were also determined between panels (small and large) and beams (CEN and UNI).

Figure 18 and Figure 19 reports the correlations between UNI beam test and Small Round Panel, obtained using 12 experimental points. The coefficient of variation is again considerably good and the linear approximation is also consistent, leading to the following relations:

$$E_{S,3.75} = 15.00\, f_{eq(0-0.6)}$$
$$E_{S,30} = 70.84\, f_{eq(0.6-3)} \qquad (2)$$

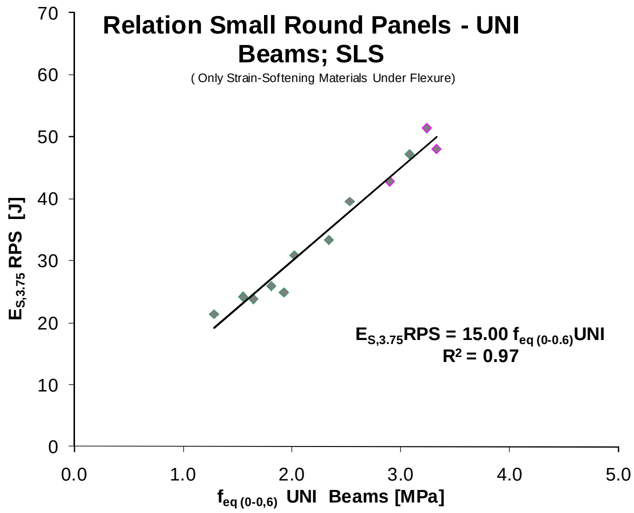

Figure 18. Correlations between energy absorption and equivalent post cracking strength: Small Round Panel tests vs. Beam Tests UNI, serviceability limit states.

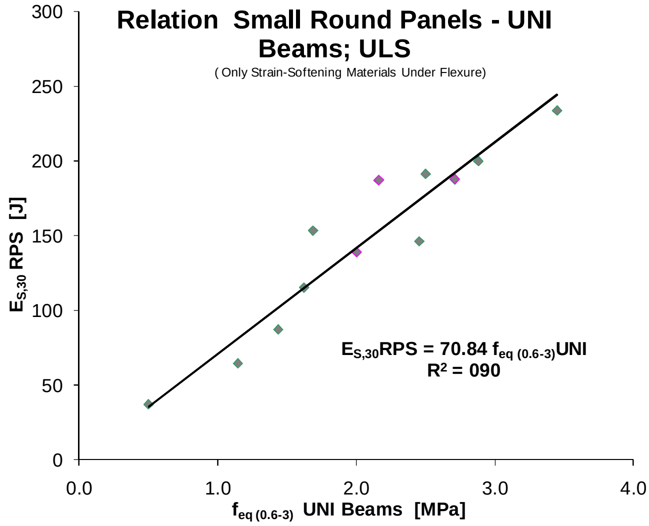

Figure 19. Correlations between energy absorption and equivalent post cracking strength: Small Round Panel tests vs. Beam Tests UNI, ultimate limit states.

Figure 20, Figure 21 and Figure 22 exhibit the correlation between the residual strengths and equivalent post-cracking stresses defined in the CEN and UNI standard beam tests. Even though a smaller database is available (only 7 experimental points are plotted, each one referring to the average values of at least three beam tests per series), the coefficient of variation is quite good and the relation already consistent. Further comparative studies or collection of experimental comparative experimental data worldwide would be useful for a refinement of these relationships. Note that all correlations were set as lines and force to cross the axes origin. This in fact seems to be in good agreement with the experimental results.

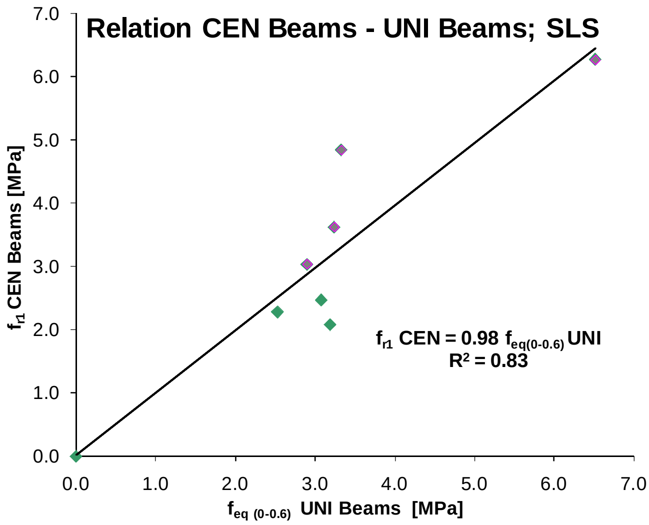

Figure 20. Correlations between residual stress and equivalent post cracking strength: Beam Test CEN vs. Beam Tests UNI, serviceability limit states.

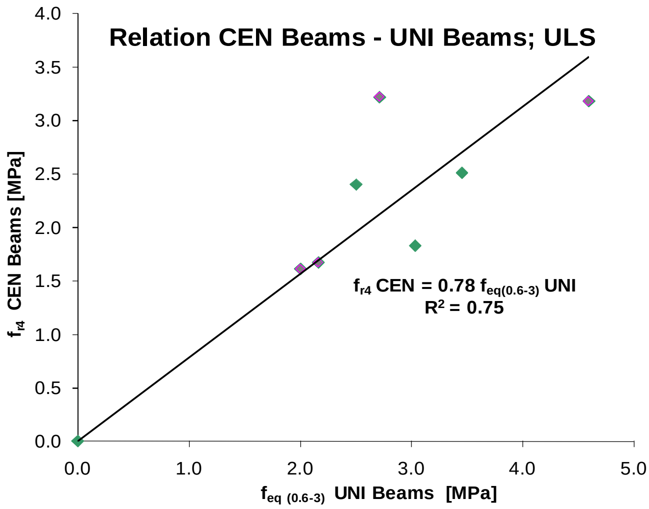

Figure 21. Correlations between residual stress and equivalent post cracking strength: Beam Test CEN vs. Beam Tests UNI, ultimate limit states.

Once the correlations are found, it is possible to determine, for example, beam toughness properties from panel tests, using the corresponding lower dispersion. As an example, the characteristics values of the equivalent post-cracking strength, suitable for design purposes, were calculated by using the corresponding experimental scatter respectively of UNI beam, large (ASTM) and small (our proposal) round panels. In doing so, Figure 23 and Figure 24 exhibit a comparison between the average and the characteristic values of the fracture parameter $f_{eq(0\text{-}0.6)}$, for FRC with 20 (Fig. 23) and 30 kg/m^3 of steel fibers (Fig. 24). Due to a rather high scatter, using the beam test the characteristic values turn out to be at least 40% lower than the average one whereas, in the case of panel test, the reduction varies from 15 to 30%; this lower difference represents a beneficial effect on the values adopted in the design process and, consequently, on the structural dimensions. The design advantages are, therefore, doubtless.

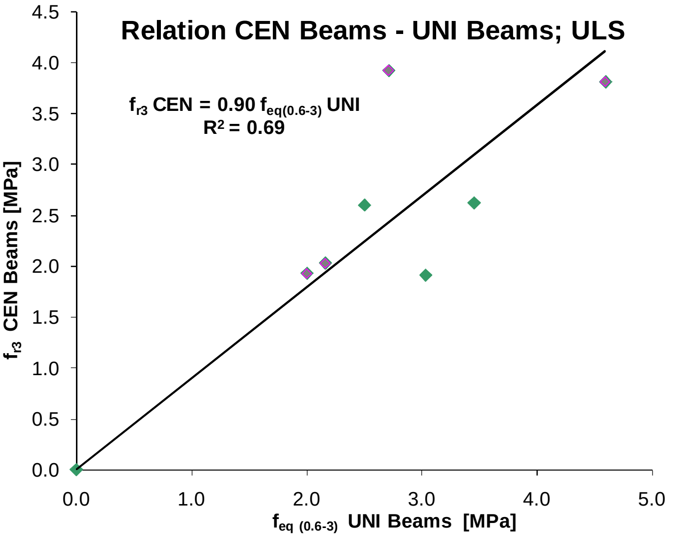

Figure 22. Correlations between residual stress and equivalent post cracking strength: Beam Test CEN vs. Beam Tests UNI, ultimate limit states.

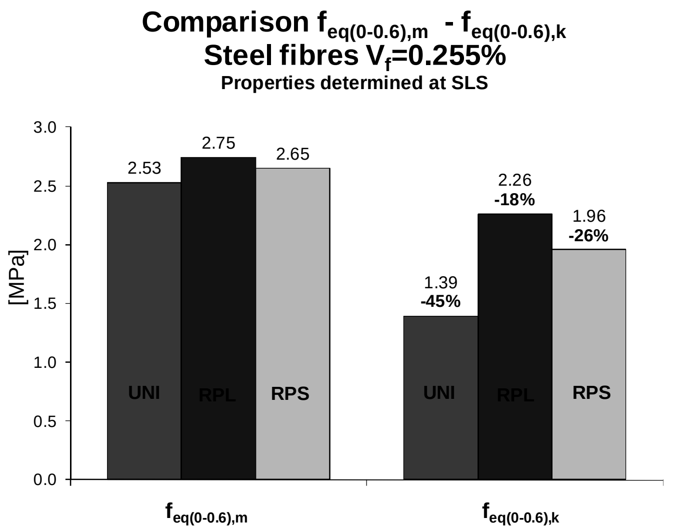

Figure 23. Average vs. characteristic $f_{eq(0\text{-}0.6)}$ using the scatter of different standards: Round Panel tests vs. Beam Tests UNI.

One should notice that the three average values are almost identical, as a further proof of the strength and consistency of the aforementioned correlations.

As a further confirmation of the abovementioned trend, one should also look at the results plotted in Figure 25 and Figure 26, showing the calculation of the characteristic value of the equivalent post-cracking strength $f_{eq(0.6\text{-}3)}$ for the two fiber contents considered. The scatter of round panels, small in these cases, is even lower than that of classical panels ASTM, even though, as a general observation, based on the total experimental program (more than 100 panel tests) carried out during the last 4 years in Brescia, the dispersion of results in rather similar between the two panels.

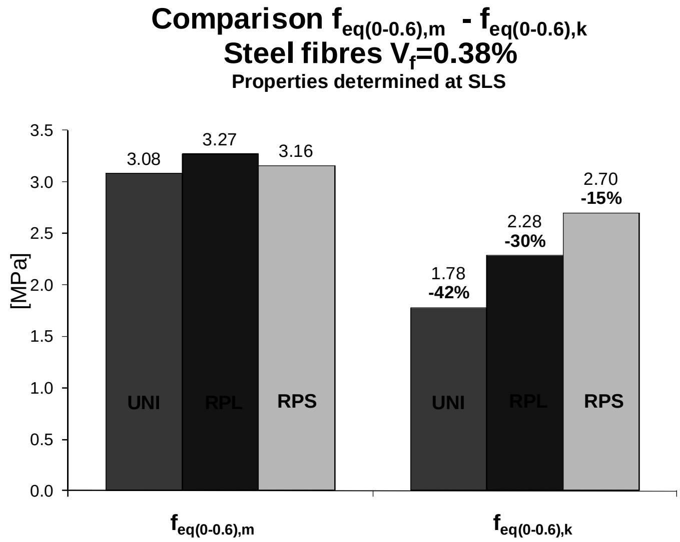

Figure 24. Average vs. characteristic $f_{eq(0\text{-}0.6)}$ using the scatter of different standards: Round Panel tests vs. Beam Tests UNI.

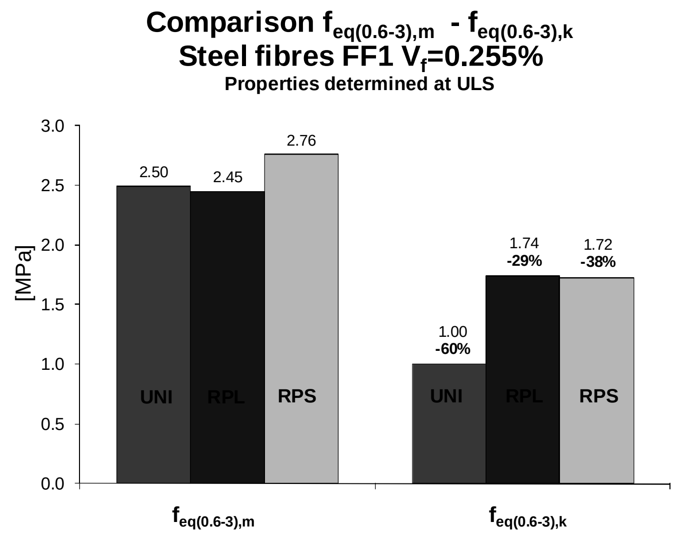

Figure 25. Average vs. characteristic $f_{eq(0.6\text{-}3)}$ using the scatter of different standards: Round Panel tests vs. Beam Tests UNI.

3 CONCLUDING REMARKS

A comparative study between beam and panel tests was discussed in the present paper. Results show that the high experimental scatter generally present in beam tests is definitely caused by the small geometry and fracture area involved in the tests. It does not represent, in general, the actual structural behavior where much larger fracture areas are involved and, consequently, a lower dispersion occurs.

Panel tests are therefore more suitable for representing the actual behavior of FRC materials. Provided that the test is performed under a close-loop control with crack width measurements, the round panel test can be adopted for the characterization of FRC as it implies much lower dispersion of experimental results.

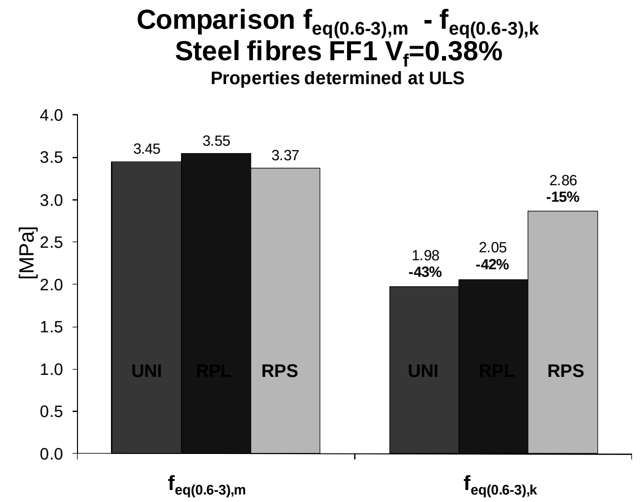

Figure 26. Average vs. characteristic $f_{eq(0.6-3)}$ using the scatter of different standards: Round Panel tests vs. Beam Tests UNI.

The proposed smaller specimens for panel tests does not affect the low scatter of the standard ASTM panel; moreover, it allows for an easier placing and handling (lower weight and smaller geometry that fits with many servo-controlled testing machines). In fact, results from round panels with a diameter of 600 mm and a thickness of 60 mm, are consistent, reliable and provide a repeatable and predictable crack pattern with a consistently lower dispersion than classical beam tests.

The correlation among fracture parameters found in the experimental program are in general very promising and allow engineers to analytically determine the fracture parameters of different tests from performing only one or few test typologies. These correlations, that were somehow expected since standard tests always refer to fracture properties of the same material, may be useful in practice since they can give immediate indications of the fracture properties without performing expensive tests. In addition, they are useful for exchanging results from a broad database of beam and panel test available worldwide form different laboratories and universities.

Finally, the round panel test could be considered as a complete test for the characterization of FRC once suitable range of crack widths will be defined. From these ranges, the corresponding equivalent (or residual) post-cracking strengths can be defined (from σ-w plots) following the same procedure as done for beam tests.

The second part of the paper will focus on the definition of a simplified cohesive constitutive law.

4 ACKNOWLEDGMENTS

The research was partially co-funded either by Magnetti Building (Carvico, Italy) and Officine Maccaferri (Bologna, Italy) whose support is gratefully acknowledged.

The Authors would like to thank also Engineers Moira Conti, Giovanni Flelli, Yuri Marinoni, Raoul Martin Milla and Nicola Vezzoli for their assistance in performing the experiments and in data processing.

REFERENCES

ASTM C1550-04, "Standard test method for flexural toughness of fiber reinforced concrete (using centrally loaded round panel), pp. 9, 2004.

CEN/TC 229 - PrEN 14651-5, "Precast Concrete Products – Test method for metallic fibre concrete – Measuring the flexural tensile strength", Europ. Standard, 2003, pp.15.

CNR DT 204/2006. 2006. "Guidelines for the Design, Construction and Production Control of Fibre Reinforced Concrete Structures", National Research Council of Italy.

di Prisco, M., Failla, C., Plizzari, G.A. and Toniolo, G. (2004), "Italian guidelines on SFRC", in Fiber Reinforced Concrete: from theory to practice, di Prisco, M. and Plizzari, G.A. Eds., Proceedings of the International Workshop on advances in Fibre Reinforced Concretes, Bergamo (Italy), September 24-25, pp. 39-72.

di Prisco, M., Felicetti, R. and Plizzari, G.A. Eds. (2004b), Proceedings of the 6th RILEM Symposium on Fibre Reinforced Concretes, BEFIB 2004, RILEM PRO 39, 1514 pp.

Eurocode 2. 2003. Design of Concrete Structures, prEN 1992 Ver. December 2003.

fib SAG 5. 2010. Draft proposal the new fib Model Code, in preparation.

Lambrechts, A.N., "The variation of steel fibre concrete characteristics. Study on toughness results 2002-2003", Proceeding of the International Workshop "Fiber Reinforced Concrete. From theory to practice", Bergamo, September 24-25, 2004, pp. 135-148.

Marinoni, Y., Minelli, F., Plizzari, G.A., and Vezzoli, N.: "Prove di frattura su calcestruzzi fibrorinforzati in accordo con le principali normative internazionali", Technical Report, University of Brescia, Department DICATA, 2007.

Meda, A., Minelli, F., Plizzari, G.A. and Riva, P. 2005. Shear Behavior of Steel Fibre Reinforced Concrete Beams. *Materials and Structures*, 38, 343-351.

Minelli, F. 2005. Plain and Fiber Reinforced Concrete Beams under Shear Loading: Structural Behavior and Design Aspects. *Ph.D. Thesis, Department of Civil Engineering, University of Brescia, Starrylink Editor*, pp. 430.

Minelli, F. & Plizzari, G.A. 2006. Steel Fibers as Shear Reinforcement for Beams. *Proceedings of The Second Fib Congress, Naples, Italy, 5-8 June, 2006, abstract on page 282-283, full length paper available on accompanied CD, pp.12*.

Minelli, F. & Vecchio, F.J. 2006. Compression Field Modeling of Fiber Reinforced Concrete Members under Shear Loading. *ACI Structural Journal*, 103(2), 244-252.

Minelli, F., Cominoli, L., Meda, A., Plizzari, G.A., and Riva, P. 2005. Full Scale Tests on HPSFRC Prestressed Roof Elements Subjected to Longitudinal Flexure. *Proceedings of the International Workshop on High Performance Fiber Reinforced Cementitious Composites in Structural Appli-*

cations, Honolulu, Hawaii, USA, May 23-26, 2005, 323-332.
PrEN 14488-5-CEN/TC 104, "Testing sprayed concrete – Part 5: Determination of energy absoprtion capacity of fibre reinforced slab specimens", Europ. Standard, 2004, pp.1-7.
RILEM Final Recommendation TC-162-TDF, "Test and Design Methods for Steel Fibre Reinforced Concrete; $\sigma-\varepsilon$ Design Method", Materials and Structures, Vol 36, October 2003, pp. 560-567.
RILEM TC162-TDF: "Test and Design Methods for Steel Fibre Reinforced Concrete: Recommendations. Bending test", Materials and Structures, 33, (225), (2000) 3-5.
Schumacher, P., "Rotation capacity of self-compacting steel fiber reinforced concrete", Ph.D Thesis, Delft University of Technology, The Netherlands, 2006
Sorelli, L., Meda, A. and Plizzari, G.A., "Bending and uniaxial tensile tests on concrete reinforced with hybrid steel fibers", ASCE Materials Journal, Vol.15(5), 2005, pp. 519-527.
UNI 11039, "Steel Fiber Reinforced Concrete - Part I: Definitions, Classification Specification and Conformity - Part II: Test Method for Measuring First Crack Strength and Ductility Indexes", Italian Board for Standardization, 2003.
Vandewalle, L., "Test and design methods for steel fibre reinforced concrete proposed by RILEM TC 162-TDF", Proceeding of the International Workshop "Fiber Reinforced Concrete. From theory to practice", Bergamo, September 24-25, 2004, pp. 3-12.