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Prediction of size dependent strength of reinforced concrete deep beams using strut-and-tie model

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ABSTRACT: This paper reports on development of a refined strut-and-tie model for determining the size dependent shear strength of reinforced concrete deep beams. The predicted model is based on Siao's Model for size independent shear strength of deep beams, Modified Bazant's size effect law and the experimental data reported by several researchers. The proposed model predicts the shear strength of deep beams as a function of compressive strength of concrete, longitudinal and web reinforcements steel ratios, shear span-to-depth ratio and the effective depth of beam.

1 INTRODUCTION

1.1 *Size effect in ordinary beams*

The decrease in shear strength with increasing the beam depth is prominently described as the size effect in concrete structures. The evidence of the size effect has been confirmed through several research studies conducted on normal beams ($a/d \geq 2.5$). Some predictive models are reported for evaluating the size dependent shear strength of such beams.

1.2 *Size effect in deep beams*

Many studies have been reported to understand the behavior of deep beams without web reinforcement *e.g.* Kani (1967), Shioya et al. (1989) for which the size effect was observed to be severe. Though, the size effect is present in short and deep beams with web reinforcement, from the scarcely available test data, there is lack of simple predictive model.

1.3 *Need for size dependent strength*

The aim of the size dependent model for evaluation of the shear strength of reinforced concrete structures is not only to safe guard the structures against the reduction of strength with size but also to ensure uniform margin of safety irrespective of the size of structure. In high-strength concrete, which is relatively brittle material, fracture mechanics based size dependent strength is inevitable.

1.4 *Aspects of size effect*

Among the two aspects of size effect viz., statistical and deterministic, the statistical aspect is based on randomness of the material strength. This type of size effect affects the strength only prior to the peak stress. But, concrete on the other hand possesses ample reserve strength after the peak stress. The other deterministic aspect is the dominant size effect generally observed after the peak stress and is caused by the release of the stored elastic energy through crack front and propagating type of failure.

2 RESEARCH SIGNIFICANCE

A simple model for predicting the shear strength of reinforced concrete deep beams ($a/d \leq 1$) with web reinforcement is proposed using refined strut-and-tie model, size effect law and the available experimental data. Prediction of the shear strength based on this model agrees well with the existing experimental data base.

3 FAILURE MECHANISM

The failure mechanism in short beams ($1 < a/d \leq 2.5$) and deep beams ($a/d \leq 1$) differs very much from that of the slender beams due to the strut-and-tie action that enhances the shear capacity. Hence the available size effect equations for the slender beams can not be used for short and deep beams.

4 STRUT-AND-TIE MODEL

In strut-and-models, the compressive forces are assumed to be carried by a portion of the concrete known as struts and the tensile forces are considered to be carried by the steel reinforcement bars called ties. These models are mainly applied to the regions of beams like geometrical discontinuities, loading points and also to deep beams, corbels and pile caps where the conventional beam theory is not appropriate. The applicability of these models to structural components was done by Schlaich et al. (1987) by considering uniformly inclined concrete struts between cracks in the entire length of the beam.

5 SIZE EFFECT LAWS

5.1 *Bazant's size effect law*

Based on the dimensional analysis of the strain energy release rate, Bazant (1984) showed that the stress at failure varies as $(1+d/(d_a\lambda_0))^{-1/2}$, in which d = beam depth, d_a = maximum size of coarse aggregate, and λ_0 = constant. This indicates that the nominal stress decreases with increasing size.

5.2 *Modified Bazant's size effect law*

Even though the size effect law proposed by Bazant as indicated above is in good agreement with test results, there exists some discrepancy reported by Kim & Park (1996) between the prediction by Bazant's law and the test data, particularly for large-sized specimens. To reduce this discrepancy, they suggested the modified size effect law proposed by Kim & Eo (1990) based on the concept of dissimilar initial cracks as given by

$$\sigma_N = \frac{k_1\sigma_r}{(1+k_4 d)^{1/2}} + k_2\sigma_r \qquad (1)$$

in which σ_N is the nominal strength of beam at failure, σ_r is the nominal strength of the beam with reference to size, k_1, k_2, and k_4 are empirical constants. For the purpose of dimensional balancing and to keep the originality of Bazant's size effect law, the modified Bazant's law (Equation 1) can be written as

$$\begin{aligned}\sigma_N &= \frac{k_1\sigma_r}{(1+d/(d_a\lambda_0))^{1/2}} + k_2\sigma_r \\ &= k_1\sigma_r\left(\frac{k_2}{k_1} + \frac{1}{(1+(1/\lambda_0)(d/d_a))^{1/2}}\right)\end{aligned} \qquad (2)$$

5.3 *Refined strut-and-tie model for deep beams*

Based on the concept of refined strut-and-tie model, Siao (1993) had formulated the size independent shear strength of deep beams. The internal forces in the refined strut and tie in a deep beam on the application of external forces are shown in Figure 1.

The tensile stress, f_t acting normal to AB is

$$f_t = \frac{V}{2\sin\alpha}\left(\frac{\sin\alpha}{bz}\right) = \frac{V}{2bz}$$

Hence

$$V = 2f_t bz \qquad (3)$$

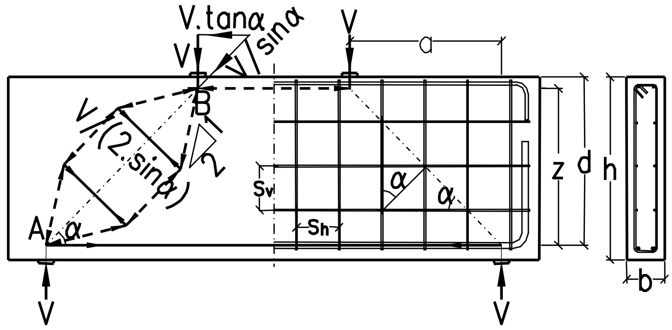

Figure 1. Refined Strut-and-tie model.

where V is the shear force in beam, b is the breadth of the beam, and z is the lever arm between centroids of main steel reinforcement near the bottom of beam and the compression strut near the top beam.

The tensile resistance of concrete, f_{ct} across the diagonal, AB shown in Figure 1 shall be taken as

$$f_{ct} = 0.5\sqrt{f_c'}$$

if A_h is the area of horizontal web reinforcement provided in each level at a vertical spacing of s_v, the tensile resistance offered by the horizontal web reinforcement shall be calculated as

$$\begin{aligned}f_{sth} &= \left(n\frac{A_h}{bs_v}0.5\sqrt{f_c'}\sin\alpha\right)\sin\alpha \\ &= 0.5\sqrt{f_c'}\left(n\rho_h\sin^2\alpha\right)\end{aligned}$$

similarly, if A_v is the area of the vertical web reinforcement provided at a horizontal spacing of s_h, the tensile resistance offered by the vertical web reinforcement can be expressed as

$$\begin{aligned}f_{stv} &= \left(n\frac{A_v}{bs_h}0.5\sqrt{f_c'}\cos\alpha\right)\cos\alpha \\ &= 0.5\sqrt{f_c'}\left(n\rho_v\cos^2\alpha\right)\end{aligned}$$

Now the total resistance, f_{tR} offered by concrete and web reinforcement shall be obtained as

$$\begin{aligned}f_{tR} &= f_{ct} + f_{sth} + f_{stv} \\ &= 0.5\sqrt{f_c'}\left[1+n\left(\rho_h\sin^2\alpha + \rho_v\cos^2\alpha\right)\right]\end{aligned} \qquad (4)$$

in which f_c' is the characteristic cylindrical strength of concrete in *MPa.*, n is the modular ratio of steel to concrete, ρ_h & ρ_v are the ratios of horizontal and vertical web reinforcements respectively, and α is the angle between the line joining the points of load and reaction and the horizontal axis.

5.4 *Effect of steel ratio*

The lever arm, z in Equation (3) can be expressed as $z=j_0 d$ in which j_0 is a constant, defining the location of the compression resultant at the end of shear span, a. According to the classical bending theory of RC beams with only tensile reinforcement and with a negligible tension capacity of concrete, we have

$$j_0 = 1 - \frac{k}{3}, \quad k = \sqrt{(n\rho)^2 + 2n\rho} - n\rho \tag{5}$$

where ρ is the main reinforcement ratio, and k is the ratio of the depth of neutral axis, c, to the effective depth, d. Equation (5) may be replaced by a simpler form as $j_0 = A_1/\rho^{a1}$.

In practice, the range of n varies between 5 and 10, and that of ρ is between 0.005 and 0.05. By plotting the actual values of j_0 for the above range of n and ρ, we can obtain the range of values for the coefficients A_1 and a_1 based on the actual trend of the curve.

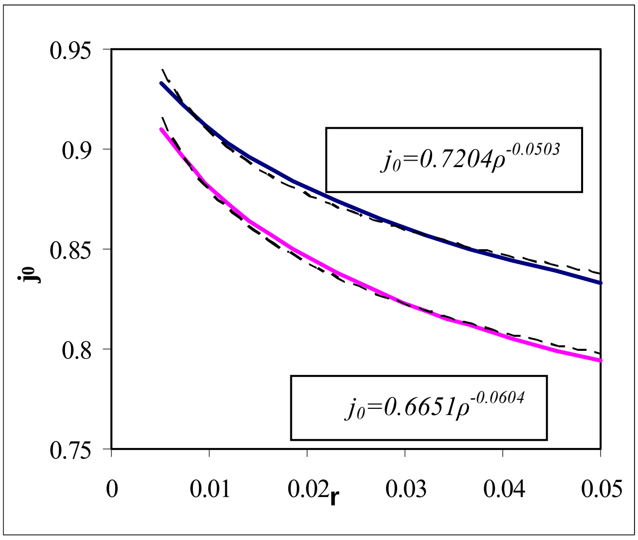

Figure 2. Comparison between the actual values of j_0 and the simplied values from the expression.

From Figure 2, the difference between the actual values j_0 and the values obtained from simplified expression for j_0 is almost indistinguishable.

For convenience substituting $R= (a/d)$, the terms in Equation (4) viz., n, $\sin^2\alpha$ and $\cos^2\alpha$ can be written as

$$n = \frac{42.26}{\sqrt{f_c'}} \tag{6}$$

$$\sin^2\alpha = \frac{1}{(1+(a/z)^2)} = \frac{1}{(1+(R/j_0)^2)} \tag{7}$$

$$\cos^2\alpha = \frac{(a/z)^2}{(1+(a/z)^2)} = \frac{(R/j_0)^2}{(1+(R/j_0)^2)} \tag{8}$$

Using Equations. (6), (7), & (8), Equation (4) can now be expressed as

$$f_{tR} = 0.5\sqrt{f_c'} + \left(\frac{21.13}{(1+(R/j_0)^2)}\right)\left(\rho_h + (R/j_0)^2\rho_v\right) \tag{9}$$

The above equation does not take in to account the size effect and hence the stress obtained from this equation is called the size independent stress. Substituting f_{tR} from Equation (9) for σ_r in Equation (2) we would get the size dependent stress

$$\sigma_N = \left[0.5\sqrt{f_c'} + \left(\frac{21.13}{(1+(R/j_0)^2)}\right)\left(\rho_h + (R/j_0)^2\rho_v\right)\right] k_1\left(\frac{k_2}{k_1} + \frac{1}{\sqrt{1+(1/\lambda_0)(d/d_a)}}\right) \tag{10}$$

On substituting σ_N from Equation (10) for f_t in Equation (3), we finally get the size dependent shear strength of deep beams as

$$V = 2bj_0 d\left[0.5\sqrt{f_c'} + \left(\frac{21.13}{(1+(R/j_0)^2)}\right)\left(\rho_h + (R/j_0)^2\rho_v\right)\right] k_1\left(\frac{k_2}{k_1} + \frac{1}{\sqrt{1+(1/\lambda_0)(d/d_a)}}\right) \tag{11}$$

in which the coefficients A_1 & a_1 in j_0 and k_1, k_2, & λ_0 are to be determined using non-linear regression analysis. For the evaluation of these coefficients the experimental data consisting of 80 deep beam tests reported by Smith & Vantsiotis (1982), Kong et al. (1970), de Paiva & Siess (1965), Tan et al. (2003), and Oh & Shin (2001) are used. Using nonlinear optimization (Levenberg-Marquardt algorithm), and on trial and error approach the coefficients are obtained. Equation (11) can now be expressed using the coefficients as

$$V = bj_0 d\left[\sqrt{f_c'} + \left(\frac{42.26}{(1+(R/j_0)^2)}\right)\left(\rho_h + (R/j_0)^2\rho_v\right)\right]\left(0.63 + \frac{1}{\sqrt{1+(1/22)(d/d_a)}}\right) \tag{12}$$

in which $j_0 = (0.7/\rho^{0.05})$, and $R = (a/d)$.

In Figure 3, the following expressions are used for plotting the graph.

$$\upsilon = (V_{test}/(bd))$$

$$C = j_0\left[\sqrt{f_c'} + \frac{42.26}{(1+(R/j_0)^2)}(\rho_h + (R/j_0)^2\rho_v\right] \tag{13}$$

$$K = 0.63$$

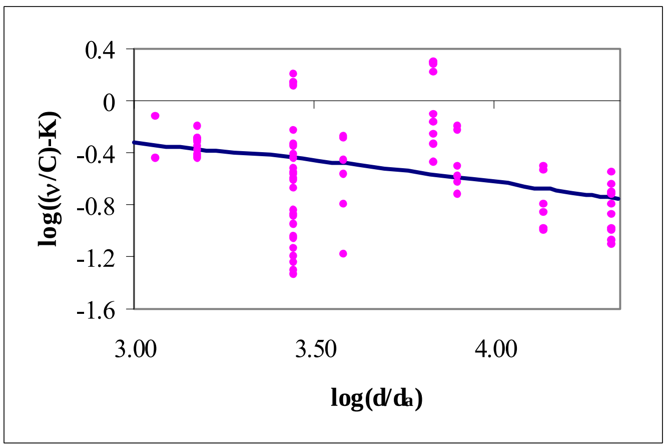

Figure 3. Comparison with existing test data for beams of different sizes.

Even though a large scatter is observed in Figure 3, from the predictions, the size effect is clearly demonstrated. The reason for such scatter is due to the fact that the beams are not geometrically similar in respect of shear span-to-effective depth ratio, size, reinforcement ratio, cover to reinforcement, pressence of web reinforcement and vast variation of material properties and testing methods adopted at various laboratories. In spite of all such variations, the general trend of decrease in the shear strength with increasing the beam size is clearly observed.

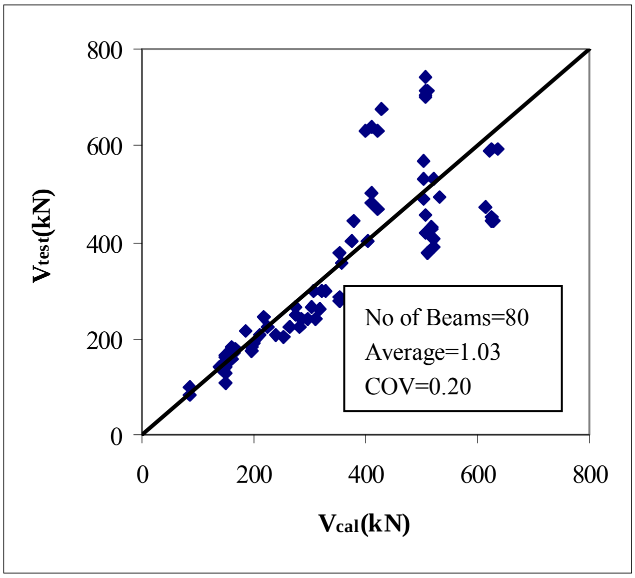

Figure 4. Comparison between experimental data and calculated data by using the proposed equation.

From Figure 4, it can be noticed that the proposed equation predicts the size dependent shear strength of deep beams provided with web reinforcements well. If some of the outliers are deleted from the data, the correlation coefficient can still be improved.

6 EXISTING SIZE EFFECT MODELS

6.1 *Tan and Cheng's Model*

Tan et al. (2001, 2003a, & 2003b) proposed a general strut-and-tie model for deep beams with web reinforcements and also without web reinforcements. In this model, the stress field was assumed as uniform across the width of the strut and also along its length. A hydrostatic state of stress was assumed in the nodal zone.

Later on Tan & Cheng (2006) modified the above general strut-and-tie model to account for the size effect from strut geometry viz., width and length of the strut and also the strut boundary conditions like spacing and diameter of the web reinforcement intercepted by the inclined strut. This model is found to predict the size dependent shear strength of deep and short beams.

6.2 *Zhang and Tan's Model*

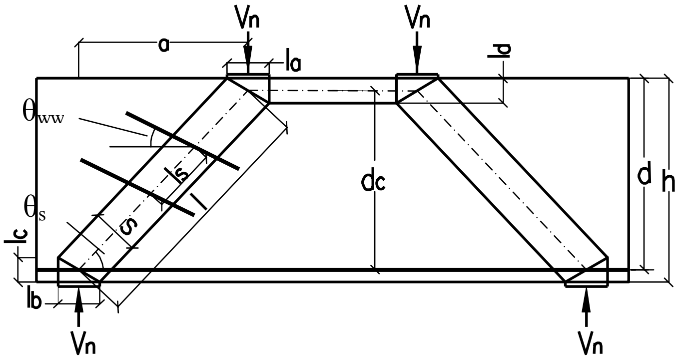

Figure 5. Strut-and-tie model for RC deep.

In 2007, Zhang & Tan, made some modification to the model proposed earlier as above to account for the strength of concrete after cracking in terms of the concrete strain at cracking and the tensile strain in the longitudinal steel. In this model, the ultimate shear strength of RC deep beams considering the size effect is given as

$$V_n = \frac{1}{\dfrac{2\sin 2\theta_s}{f_t A_c} + \dfrac{\sin\theta_s}{\nu f_c' A_{str}}} \tag{14}$$

where θ_s is the angle of inclination of diagonal strut as shown in Figure 5; $f_c^{'}$ is the cylindrical compressive strength; A_c is the cross-sectional area of the deep beams; A_{str} is the cross sectional area of the diagonal strut and f_t is the maximum tensile capacity of the bottom nodal zone computed as

$$f_t = \frac{4A_s f_y \sin\theta_s}{A_c/\sin\theta_s} + \sum \frac{f_{yw} A_{sw}\sin(\theta_s+\theta_w)}{A_c/\sin\theta_s} + 0.31\sqrt{f_c'}\left(\frac{\varepsilon_{cr}}{\varepsilon_1}\right)^{0.40} \tag{15}$$

where A_s and A_{sw} are the total areas of longitudinal and web reinforcement respectively; f_y and f_{yw} are the yield strengths of longitudinal and web reinforcement; θ_w is the inclined angle of web reinforcement with respect to the horizontal (Fig. 5); ε_{cr} is the concrete strain at cracking, taken as 0.00008; ε_1 is the principal tensile strain of concrete strut calculated from $\varepsilon_1 = \varepsilon_s + (\varepsilon_s + \varepsilon_2)\cot^2\theta_s$, where ε_s and ε_2 are the tensile strain of longitudinal steel and peak compressive strain of concrete strut at crushing respectively. The value of ε_2 is taken as 0.002 for the normal strength concrete. The term ν is the efficiency factor and taken as the product of ξ and ζ, where ξ accounts for the effect of strut geometry and ζ for the effect of boundary condition influenced by web reinforcement. These parameters are expressed as

$$\xi = 0.8 + \frac{0.4}{\sqrt{1+(l-s)/50}}$$
$$\zeta = 0.5 + \sqrt{\frac{kd_s}{l_s}} \le 1.2 \tag{16}$$

where l and s are the strut length and width, respectively. The term d_s is the diameter of web steel bar; when the web steel is not provided, d_s is taken as the minimum diameter of the bottom longitudinal steel bars and the material factor incorporating steel bar yield strength f_y and concrete tensile strength f_{ct} is given as

$$k = \frac{1}{2}\sqrt{\frac{\pi f_y}{f_{ct}}} \quad (17)$$

When web steel is not provided, it is taken as half of the above value. The term, l_s is the maximum spacing of web steel intercepted by inclined strut.

Being the latest size effect model, the above Zhang & Tan's model is taken for comparison with the proposed model.

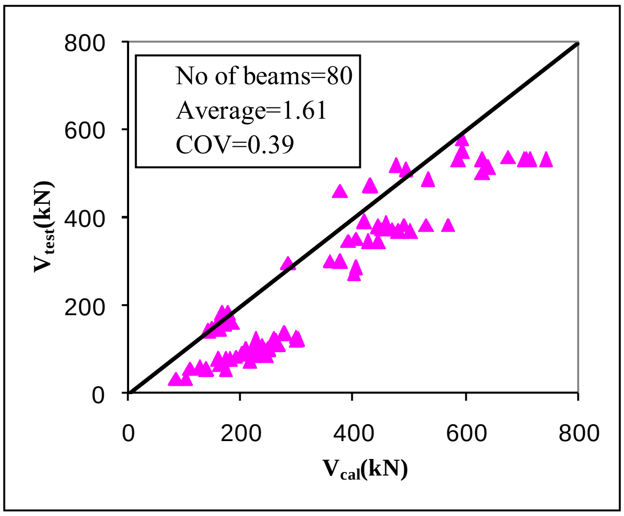

Figure 6. Comparison of experimental with the calculated data by using Zhang & Tan's size effect model.

Zhang and Tan's size effect model is found to predict the size dependent shear strength of deep and short beams. However, the procedure is iterative requiring a computer to solve the problems accurately.

From Figures 4 & 6, it is found that the proposed equation predicts the size dependent shear strength of RC deep beams having $a/d \le 1$ with lesser variation from the mean.

7 CONCLUSION

With a limited available experimental data, the size dependent shear strength of reinforced concrete deep beams is developed. This simple expression is in better agreement with the available test data. From this model the existence of size effect in deep beams with web reinforcement is well confirmed.

REFERENCES

Bazant, Z. P., (1984), "Size effect in blunt fracture: concrete, rock, metal," *Journal of Engineering Mechanics, ASCE*, 110(EM4): 518-535

de Paiva, H. A. R., & Siess, C. P., (1965), "Strength and behavior of deep beams in shear," *Journal of the Structural Division, ASCE*, 91(ST5): 19-41

Kani, G. N. J., (1967), "How safe are our large reinforced concrete beams?" *ACI JOURNAL, Proceedings* 64(3): 128-141

Kong, F. K., Robbins, P. J., & Cole, D. F.,(1970), "Web reinforcement effects on deep beams," *ACI JOURNAL, Proceedings* 67(12), 1010-1017

Kim, J. K., & Eo, S. H., (1990), "Size effect in concrete specimens with dissimilar initial cracks," *Magazine of Concrete Research (London)*, 42(153): 233-238

Kim, J. K., & Park, Y. D., (1996), "Prediction of shear strength of reinforced concrete beams without web reinforcement," *ACI Materials Journal*, 93(3): 213-222

Oh, J. K., & Shin, S. W., (2001), "Shear strength of reinforced high-strength concrete deep beams," *ACI Structural Journal*, 98(2): 164-173

Smith, K. H., & Vantsiotis, A. S., (1982), "Shear strength of deep beams," *ACI JOURNAL, Proceedings,* 79(3): 201-213

Schlaich, J., Schafer, K., & Jennewein, M., (1987), "Towards a consistent design of structural concrete," *PCI Journal*, 32(3): 74-150

Shioya, T., Iguro, M., Nojiri, Y., Akiayma, H., & Okada, T., (1989), "Shear Strength of Large Reinforced Concrete Beams," *Fracture Mechanics: Application to Concrete, SP-118, American Concrete Institute, Detroit*, 259-279

Siao, W., B., (1993), "Strut-and-tie model for shear behavior in deep beams and pile caps failing in diagonal splitting," *ACI Structural Journal*, 90(4): 356-363

Tan, K. H., Kong, F. K., Teng, S., & Guan, L., (1995), "High-strength concrete deep beams with effective and shear span variations," *ACI Structural Journal*, 92(4): 395-405

Tan, K. H., Tong, K., & Teng, S., (2001), "Direct strut-and-tie model for prestressed deep beams," *Journal of the Structural Engineering*, 127(9): 1076-1084

Tan, K. H., Tang, C. Y., & Tong, K., (2003a), "A direct method for deep beams with web reinforcement," *Magazine of concrete research,* 55(1): 53-63

Tan, K. H., Tong, K., & Tang, C. Y., (2003b), "Consistent strut-and-tie modeling of deep beams with web openings," *Magazine of concrete research*, 55(1): 65-75

Tan, K. H. & Cheng, G. H., (2006), "Size effect on shear strength of deep beams: investigating with strut-and-tie model," *Journal of Structural Engineering, ASCE*, 132(5): 673-685

Zhang, N. & Tan, K. H., (2007), "Size effect in RC deeps: Experimental investigation and STM verification," *Engineering Structures*, 29: 3241-3254