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MODELING FATIGUE CRACK PROPAGATION IN CONCRETE USING DISSIPATION POTENTIAL

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Abstract. Majority of concrete structures are subjected to fatigue loading. It is important to mathematically model fatigue in order to predict the remaining life of these structures. Fatigue crack propagation is a complex and irreversible process, hence an energy approach is adopted in this work by using thermodynamics as the framework. In thermodynamics, a dissipation potential is used to describe the evolution of internal variables of a dissipative phenomenon. An analytical expression for dissipation potential defined for fatigue crack propagation in concrete is derived using the concepts of dimensional analysis and self-similarity. A fatigue crack propagation model is obtained using this potential as a guideline and is validated using available experimental results. An attempt is made to impart physical meaning to this potential and also to the various dimensionless products involved. It is shown that the proposed expression for dissipation potential captures size effect in concrete and therefore resulting in more objective results.

1 INTRODUCTION

Fatigue is a progressive, localized, permanent damage that occurs in a structure, when subjected to cyclic loading. It is important to understand and mathematically model the rate of fatigue crack propagation in a structure in order to predict its residual life. Fatigue crack propagation in concrete structures is a very complex process, involving irreversible changes at micro, meso and macro levels. Initially, irreversible changes occur at micro level; and with increasing number of cycles, the damage accumulates and takes the form of a macrocrack. This crack propagates further leading to the ultimate failure of the structure. The presence of a fracture process zone ahead of crack tip in quasi-brittle materials like concrete further complicates the process. The theory of fracture mechanics is one of the several approaches through which the fatigue phenomenon in concrete is studied. This involves the determination of a fatigue crack propagation law, from which the number of cycles to failure can be predicted. It could be stress based approach, revolving around stress intensity factors, more appropriate in linear elastic fracture mechanics. Alternatively, it could be energy based, dealing with the energy release rate and is suitable for both linear as well as non-linear fracture mechanics. The energy based approach is a more viable one as it gives a global response of the structure and, since we assert to understand and model a complex process through an energy approach, it seems reasonable to use the concepts of thermodynamics. A model based on fundamental physical principles such as those of thermodynamics is more desirable than a model based on analogy. Fatigue is an irreversible process and hence the theory of irreversible (non-equilibrium) thermodynamics is suitable to describe it.

The thermodynamic formalism is based on the assumption of existence of two kinds of potentials, thermodynamic potential and a dissipation potential. Any material behavior can be expressed as a mathematical model, if the second law of thermodynamics is satisfied and a proper choice of state variables, analytical expressions of the state potential and dissipative potential are made. The choice of variables depends upon the purpose of modeling, the phenomena to be modeled, the conditions under which it occurs, and the predictions that are to be made from the model. Generally, the free energy is chosen as the thermodynamic potential and the state laws are derived from it. To describe a dissipative process, a dissipation potential is required [1]. The choice of dissipation potential is essentially an assumption of the constitutive model and is usually not derived from thermodynamic foundations. Such a potential has no apriori physical meaning [2]. The available expressions for dissipation potential are mostly applicable to metals and are derived empirically by fitting experimental data. They are mostly developed using the framework of continuum damage mechanics [1,3,4]. The existing expressions for dissipation potential contain parameters with no physical meaning. In this work, an analytical expression for dissipation potential is developed in fracture mechanics framework to model fatigue crack propagation in concrete. This is done using the concepts of dimensional analysis and intermediate asymptotics. The crack length is chosen as the internal variable and hence its evolution is obtained from this expression. In other words, the fatigue crack propagation model is just an outcome of this exercise, but the dissipation potential is used as a guideline. Also, an attempt is made to impart physical meaning to this potential.

2 THERMODYNAMIC POTENTIALS

Thermodynamic potentials are scalar functions of a set of independent state variables used to represent the thermodynamic state of a system from which all the characteristics of the system can be deduced. Associated with this set of independent state variables is a set of dependent state variables called the thermodynamic properties or associated variables or dual variables. These play a duality type role in that each state variable has a thermodynamic property and it is occasionally desirable to reverse these roles. For example, if Π is the thermodynamic potential which is a function of a set of state variables say, $\Pi = \hat{\Pi}(\chi_0, \chi_1, ...)$, then from chain rule

$$d\Pi = \frac{\partial \Pi}{\partial \chi_0} + \frac{\partial \Pi}{\partial \chi_1} + ... \quad (1)$$

The derivatives $\frac{\partial \Pi}{\partial \chi_j}$ are the thermodynamic properties, also called thermodynamic forces associated with each independent variable χ_j [5].

The state variables include the observable variables and the internal variables. The observable variables are the usual field quantities like the total strain (ϵ) and temperature (T). For a reversible (elastic) phenomenon, the state depends uniquely on the observable variables. But, for dissipative phenomena, the current state also depends on the internal variables which describe the internal structure of the material and also are able to capture the past history effect. The choice of the internal variables is dictated by the phenomenon under study and its application. The plastic strain ϵ^p, the damage variable D or the crack length a are few internal variables depending on whether the phenomenon under study is plasticity, damage or fracture. The thermodynamic potential allows one to write relations between observable variables and its associated variables. However, for internal variables it allows only the definition of their associated variables. Whereas, a dissipation potential allows one to get the relationship between the internal variables and its associated variables. In order to describe the dissipation process or the evolution of the internal variables, a dissipation potential is needed [1].

2.1 Dissipation potential ϕ

The laws of elasticity are derived from the thermodynamic potential, whereas the constitutive equations for dissipative phenomena like plasticity or damage are derived from dissipation potential. Let V_k be the internal variables and A_k, their corresponding associated variables. Dissipation is defined as the sum of product of the thermodynamic force (associated variable) A_k and the respective flux variable ($\dot{V}_k$).

$$D = \Sigma A_k \dot{V}_k \tag{2}$$

Dissipation potential ϕ is a function of the flux variables, the gradient of which will give the thermodynamic force causing it.

$$A_k = \frac{\partial \phi}{\partial \dot{V}_k} \tag{3}$$

According to the second law of thermodynamics, dissipation must be positive. The dissipation potential must essentially be a positive, convex scalar valued function possessing a value zero when $V_k = 0$ to ensure automatic satisfaction of second law of thermodynamics [1]. It is more easy to express the complementary laws in the form of evolution laws of flux variables as functions of dual variables. The Legendre-Fenchel transform enables us to define the corresponding potential $\phi^*(A_k)$, the dual of ϕ with respect to the variables $\dot{V}_k$.

2.2 Dual of Dissipation Potential ϕ^*

Legendre transformation is an operation that transforms one real-valued function of a real variable into another. The Legendre transform of a convex function f is the function f^* defined by

$$f^*(f'(x)) = \sup_x (x\ f'(x) - f(x)) \tag{4}$$

Generally, a function expresses a relation between two parameters; an independent variable or control parameter (x) and a dependent value or function (f). This information is encoded in the functional form of $f(x)$. In some circumstances, it is useful to encode the information contained in $f(x)$ in a different way. Given a function $f(x)$, the Legendre Transform provides a more convenient way of encoding the information in the function when it is strictly convex and is smooth, and it is easier to measure, control, or think about the derivative of f with respect to x than it is to measure or think about x itself [6]. The Legendre-Fenchel transform enables us to define the corresponding potential $\phi^*(A_k)$, the dual of ϕ with respect to the variables $\dot{V}_k$. If the function ϕ^* is differentiable, the normality property is preserved for the variables $\dot{V}_k$. The complementary laws of evolution can be written as [1]

$$\dot{V}_k = \frac{\partial \phi^*}{\partial A_k} \tag{5}$$

The whole problem of modeling a phenomenon lies in the determination of the analytical expressions for the thermodynamic potential and for the dissipation potential ϕ or its dual ϕ^* and their identification in characteristic experiments. Analytical expression can be derived using dimensional analysis.

3 DIMENSIONAL ANALYSIS

Dimensional analysis is a tool to find relationship between quantities occurring in a phenomenon by comparing the dimensions of the quantities involved. This is done by obtaining dimensionless products which helps us to reduce the number of variables involved in the problem. When properly formed, these dimensionless products have clear physical interpretation and thus offer physical understanding of the phenomenon under study. Let there exist a relationship between a quantity a, the dependent variable (governed parameter), and a set of quantities that are independent (governing parameters), which can be written as

$$a = f(a_1, ..., a_k, b_1, ...b_m) \tag{6}$$

where, the parameters $a_1, ..., a_k$ are those with independent dimensions and are chosen to be those governing parameters which are definitely significant for the phenomenon. Having independent physical dimensions means none of these quantities have a dimension that can be represented in terms of a product of powers of dimensions of the remaining quantities. The parameters (b_1, ..., b_m) can be expressed as the product of powers of the dimensions of the parameters (a_1, ..., a_k). Applying Buckingham's Π theorem to Equation 6, we get

$$\Pi = \Phi(\Pi_1, .., \Pi_i, ..., \Pi_m) \quad (7)$$

where, Π terms are the dimensionless parameters defined using the expressions for dimensions of $a, b_1, ..., b_m$ through the powers of the dimensions of $a_1, ..., a_k$ and are given by

$$\Pi = \frac{a}{a_1^p ... a_k^r}, \Pi_i = \frac{b_i}{a_1^{p_i} ... a_k^{r_i}}, ... \; i = 1, ...m \quad (8)$$

Φ is a function of dimensionless parameters Π_i. Dimensional analysis thus transforms f, a function of $k + m$ variables to Φ, a function of m variables only.

$$f(a_1, ..., a_k, b_1, ...b_m) = a_1^p ... a_k^r \Phi\left(\frac{b_1}{a_1^{p_1} ... a_k^{r_1}}, .. \frac{b_m}{a_1^{p_m} ... a_k^{r_m}}\right) \quad (9)$$

Although dimensional analysis is considered as a universal tool, however, there are physical problems that cannot be solved by dimensional analysis in principle. For example, the problem that involves information about the initial and boundary conditions, the system behavior in the initial times, the details of process generation, its behavior near the system boundaries, decay via equilibration, energy dispersion or dissipation during the process evolution. The present problem come under this category as it deals with energy dissipated during fatigue crack propagation. Consequently, more sophisticated tools must be employed to cope successfully with these problems. The theory of intermediate asymptotics, which can be considered as a generic extension of dimensional analysis can be adopted [7].

3.1 Intermediate Asymptotics

The intermediate asymptotic is a timespace dependent solution of an evolution equation that has already forgotten its initial conditions, but still does not feel the limitations imposed by the system boundary. It is an approximate solution to a complex problem, valid in a certain range. It can be represented by the self-similar solution, which is the exact solution to a simplified problem, valid in the whole range. The consideration of self-similar solutions as intermediate asymptotics allows us to understand the role of dimensional analysis in establishing self-similarity and determining self-similar variables [8]. Two kinds of self-similar solutions exist which are discussed below.

3.2 Self-similar solutions of first and second kind

Considering Equation 7, b_i is said to be essential if the corresponding dimensionless parameter Π_i is not too large and not too small. On the other hand, if the dimensionless parameter Π_i corresponding to dimensional parameter b_i is either very small or very large compared to unity and the function Φ has a finite limit then the parameter may be considered to be non essential.

If there exists a finite limit of the function Φ when the parameters $\Pi_{l+1}, ..., \Pi_m$ all go to zero or infinity, while other similarity parameters $\Pi_1, ...\Pi_l$ remain constant, then the function Φ can be replaced by smaller number of arguments as

$$\Pi = \Phi_1(\Pi_1, ...\Pi_l) \quad (10)$$

This is the case of the phenomenon to be self similar of the first kind or complete similarity in parameters $\Pi_{l+1}, ..., \Pi_m$. On the other hand when the parameters $\Pi_{l+1}, ..., \Pi_m$ tends to zero or infinity, if Φ also tends to zero or infinity, then the quantities $\Pi_{l+1}, ..., \Pi_m$ become essential, no matter how large or small it becomes. However, in some cases, the limit of the function Φ tends to zero or infinity, but the function Φ has power type asymptotic representation which can be written as,

$$\Phi = \Pi_{l+1}^{\alpha_{l+1}} ... \Pi_m^{\alpha_m} \Phi_1 \left(\Pi_1, ..., \Pi_l \right) \qquad (11)$$

This is the case of incomplete self similarity or self similarity of second kind [8]. Here, $\alpha_{l+1}, ..., \alpha_m$ are constants and cannot be obtained from dimensional analysis. These constants can be obtained either from a best fitting procedure on experimental results or through numerical simulations.

4 EXPRESSION FOR DUAL OF DISSIPATION POTENTIAL

In this section, an expression for dissipation potential is derived using the theory of intermediate asymptotics, dimensional analysis and self similarity. The system is a cracked concrete beam under three-point bending; the thermodynamic process is the propagation of crack with increasing number of load cycles. This process being irreversible, energy is dissipated. To describe a dissipative process a dissipation potential is needed. Crack length is the internal variable, the evolution of the flux of this variable, i.e. the rate of crack propagation is the quantity of interest. The conjugate of this variable, that is the thermodynamic force causing it is the strain energy release rate G. The dissipation potential is in terms of the rate of crack propagation $\dot{a}$; when differentiated with respect crack rate, it gives the energy release rate G. But it is easier to calculate G rather than rate of crack propagation $\dot{a}$. The Legendre-Fenchel transformation as elucidated in the previous section, enables us to use the dual of dissipation potential, the differentiation of which with respect to G will give the rate of crack propagation $\dot{a}$. Firstly, the relevant variables on which the dual of the dissipation potential is likely to depend on is listed. Then dimensional analysis is used to obtain dimensionless products. Proceeding further, the existence of self similarity is explored and the model is worked out. The model contains unknown constants, which are obtained from experimental results.

4.1 Dimensional analysis - Parameters and dimensionless products

Dissipation potential in the present context is defined as the energy dissipated per unit volume, hence the dimensions are FL^{-2}. The list of variables on which the dissipation potential depends on must include a loading parameter, displacement parameter,geometric parameter and material parameters. It may also include the state variables itself. Each of these influencing parameters are discussed below.

1. The loading parameter considered here is the strain energy release rate which is the energy required for unit crack propagation. It depends on the loading, crack length and specimen geometry and the material. During fatigue loading, for each cycle, part of the strain energy is used for crack propagation and the remaining is dissipated. Hence, this is one of the most important parameters that will affect dissipation potential. Since load is applied in cycles, varying between a minimum and maximum amplitude, the load range is considered in terms of the increment in the energy release rate range ΔG_I. The subscript I represents crack propagation in mode I.

2. In cracked specimens, generally, the crack mouth opening displacement ($CMOD$) is more widely used as the displacement parameter. Under fatigue loading, the area in between the unloading curve and the reloading curve of a typical load displacement plot gives the energy dissipated. In practical situations, $CMOD$ may be considered as the crack width and is assumed to have some effect on the dissipation potential. The notation used for $CMOD$ is w.

3. It is well known that a quasi-brittle material such as concrete exhibits strong size effect. Hence the size parameter in terms of the depth of the beam specimen, D is

also included in order to obtain a size independent expression for dissipation potential.

4. The material parameters included are the fracture energy G_f, and the tensile strength of the material f_t.

5. Sometimes the variable, in the evolution of which we are interested can itself become a parameter [1]. In this problem, the crack length, a is also considered as a parameter on which the dissipation potential depends.

Considering all the above parameters, the dual of the dissipation potential Φ^* can be written as

$$\Phi^* = f(\Delta G_I, w, a, G_f, f_t, D) \qquad (12)$$

Table 1: Variables on which the dual of dissipation potential depends and their dimensions

Variable	Definition	Dimension
Φ^*	Dual of dissipation potential	FL^{-2}
ΔG_I	Energy release rate range	FL^{-1}
w	$CMOD$	L
a	Crack length	L
G_f	Fracture energy	FL^{-1}
f_t	Tensile strength	FL^{-2}
D	Structural size	L

Table 1 gives the dimensions of each of these quantities. Choosing G_f and f_t as having independent dimensions and using them to non-dimensionalize the remaining quantities, we obtain,

$$\frac{\Phi^*}{f_t} = f\left(\frac{\Delta G_I}{G_f}, \frac{f_t}{G_f}w, \frac{f_t}{G_f}a, \frac{f_t}{G_f}D\right) \qquad (13)$$

We obtain Φ^* as a function of dimensionless products

$$\Phi^* = f_t\, f(\Pi_1, \Pi_2, \Pi_3, \Pi_4) \qquad (14)$$

where, $\Pi_1 = \frac{\Delta G_I}{G_f}$, $\Pi_2 = \frac{f_t}{G_f}w$, $\Pi_3 = \frac{f_t}{G_f}a$, $\Pi_4 = \frac{f_t}{G_f}D$.

Now, we explore the possible existence of any self similar behavior in these four dimensionless products.

1. First consider, Π_1, it is a function of the strain energy release rate range. If we assume complete self similarity, it means that Φ^* must be independent of ΔG_I, since complete similarity will render the quantity non essential. But ΔG_I is the most important parameter as it is the loading parameter and Φ^* will invariably depend on load. Hence we assume that incomplete self similarity exists with respect to Π_1.

2. Π_2: The softening portion of the stress (σ)-CMOD(w) curve is typically used for concrete. If we non-dimensionalize these parameters, we get $\frac{\sigma}{f_t}$ vs $\frac{f_t w}{G_f}$. The problem of obtaining an analytical expression for the softening curve lies in assuming reasonably a linear, bilinear or exponential relationship between these two dimensionless quantities. Thus Π_2 is an important quantity responsible for determining the shape of the softening curve. The value of Π_2 typically varies between 0.1 to 12. Thus we see that if Π_2 tends to 0, the energy dissipated is less since there is no softening tail. If on the other hand, Π_2 value is very high, it indicates a longer softening tail, indicating more energy is dissipated.

3. Π_3: It is the non dimensional crack length. Its value typically ranges from 100 to 10^6 or higher depending on specimen size. At failure, a lower value indicates a ductile specimen and higher value indicates brittle. Generally, the parameter Π_3 shows dependence of fatigue behavior on initial notch length and on a characteristic length scale that describes the ductility of the material. By introducing a characteristic length parameter which is defined as

$$l_{ch} = \frac{EG_f}{f_t^2} \tag{15}$$

the dimensionless parameter Π_3 is dependent on $\frac{a}{l_{ch}}$, where l_{ch} characterizes the length of the fracture process zone. The smaller the value of l_{ch}, the more brittle is the material. Hence, this dimensionless parameter governs the transition between ductile and brittle behavior when $\Pi_3 \to 0$ and $\Pi_3 \to \infty$, respectively [9]. Experimental results have shown the dependence of crack growth rate on crack length (a). Therefore, we can assume the existence of incomplete self similarity in this quantity.

4. Π_4: Its value typically ranges from 10^3 to 10^6 and more as size of structure increases. This parameter captures the size effect, since it is based on the size of the specimen. Large sized specimens dissipate less energy than small sized specimens in a normalized sense.

Thus for Π_2, Π_3 and Π_4 also we assume the existence of incomplete self similarity. The substantiation of this assumption is done and it is indeed found that this assumption is valid. Thus, the dual of the dissipation potential can be written as

$$\Phi^* = f_t \ \Pi_1^{\gamma_1} \ \Pi_2^{\gamma_2} \ \Pi_3^{\gamma_3} \ \Pi_4^{\gamma_4} \tag{16}$$

$$or \quad \Phi^* = f_t \left(\frac{\Delta G_I}{G_f}\right)^{\gamma_1} \left(\frac{f_t}{G_f} w\right)^{\gamma_2} \left(\frac{f_t}{G_f} a\right)^{\gamma_3} \left(\frac{f_t}{G_f} D\right)^{\gamma_4} \tag{17}$$

$$or \quad \Phi^* = \Delta G_I^{\gamma_1} G_f^{(-\gamma_1-\gamma_2-\gamma_3-\gamma_4)} f_t^{(1+\gamma_2+\gamma_3+\gamma_4)} w^{\gamma_2} a^{\gamma_3} D^{\gamma_4} \tag{18}$$

The constants γ_1, γ_2, γ_3 and γ_4 cannot be obtained from dimensional analysis. These constants can be obtained from either numerical computations or experiments. Here, we obtain the values of these constants from experimental results. Φ^* is the area between the unloading and reloading curve as it represents the energy dissipated. If we can compute this energy dissipated in each cycle and sum it up and then use it to calibrate through a regression analysis, we can get the unknown constants. But the value of Φ^* is almost impossible to measure from experiments. However, the flux variables and the dual variables are quite easy to measure and it is on their values that modeling and identification are based. The complementary laws of evolution are therefore directly identified but the dissipation potential is used as guideline for writing their analytical expression. It is clear that although Φ^* is difficult to measure, the flux variable, i.e., the rate of crack propagation $\dot{a}$ and the dual variable, i.e., the energy release rate range ΔG_I are easy to measure from experiments. Hence on the basis of these values the unknown constants are obtained. If the function Φ^* is differentiable, the normality property is preserved and the complementary laws of evolution can be written as [1]

$$\dot{a} = \frac{da}{dN} = \frac{\partial \Phi^*}{\partial \Delta G_I} \tag{19}$$

where, a is the crack length and N is the number of cycles.

4.2 Fatigue crack propagation model

The fatigue crack propagation model can be obtained from Equations 18 and 19 as

$$\frac{da}{dN} = \gamma_1 \ \Delta G_I^{\gamma_1 - 1} \ G_f^{(-\gamma_1-\gamma_2-\gamma_3-\gamma_4)} \ f_t^{(1+\gamma_2+\gamma_3+\gamma_4)} \ w^{\gamma_2} \ a^{\gamma_3} \ D^{\gamma_4} \tag{20}$$

The unknown constants are determined through a calibration process using experimental results. In this study, the experimental results of Shah [10] are taken. It involves testing concrete beams of three different sizes (namely small, medium and large) with an initial notch subjected to cyclic loads under three point bending. The geometry details and material properties are given in Table 2 and the loading pattern is given in Figure 1.

Table 2: Geometry and material properties of beam specimens [10]

Specimen	Depth D (mm)	Span S (mm)	Thickness B (mm)	Notch Size a_0 (mm)	G_f (N/mm)	E (N/mm^2)	f_t (N/mm^2)
Small	76	190	50	15.2	0.07	30000	3.8
Medium	152	380	50	30.4	0.07	30000	3.8
Large	304	760	50	60.8	0.07	30000	3.8

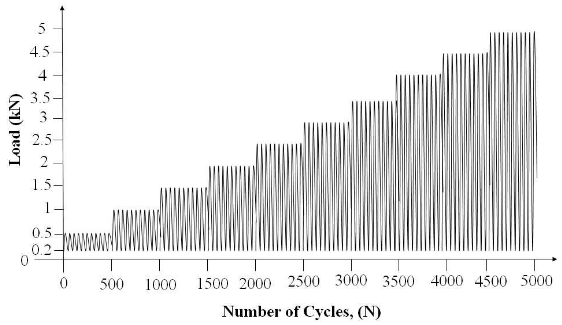

Figure 1: Loading pattern used in the experiment [10]

The load and CMOD for every load cycle is recorded during the experiment until failure. Through a finite element analysis, the CMOD compliance and crack length relation is obtained. Thus we now have the information on load, CMOD, crack length, number cycles and the geometry details. Using these one can compute ΔG_I and $\frac{da}{dN}$. All the values on the right hand side of the Equation 20 are known except for the constants and also experimental value of $\frac{da}{dN}$ is known. Through an optimization process, the constants are computed such that the error, i.e. difference between the value of $\frac{da}{dN}$ as predicted by the model and the experimental value is minimized. The data for the medium specimen is used for calibration. The value of the constants γ_1, γ_2, γ_3 and γ_4, for the best fit are $3.9544, -0.4842, -0.1685, -0.4689$ respectively. Figure 2 shows the variation of $log\left(\frac{da}{dN}\right)$ with $log\,(\Delta K_I)$ for the medium specimen that was used for calibration purpose. The model is used to predict $\frac{da}{dN}$ for other specimens. Figure 3 shows the variation of $log\left(\frac{da}{dN}\right)$ with $log\,(\Delta K_I)$ for small and large specimens. A good match between the predicted and experimental result is observed, thereby validating the model.

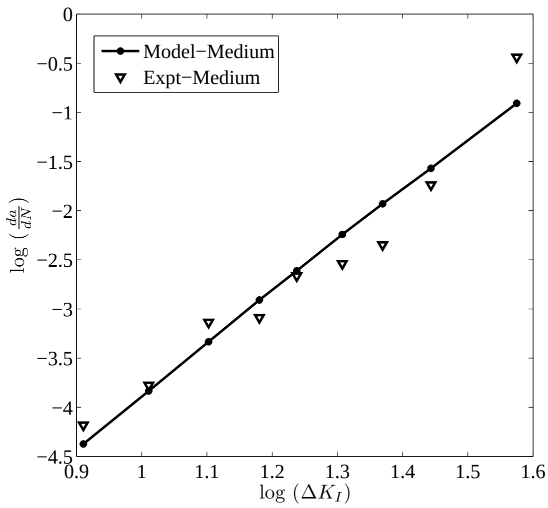

Figure 2: Calibration of the model using data of medium specimen [10]

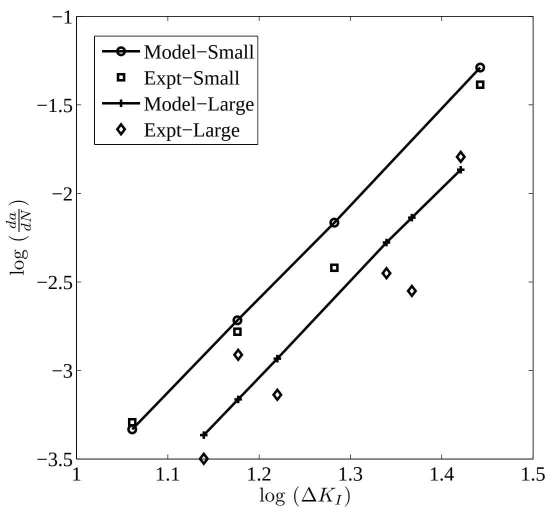

Figure 3: Comparison of the fatigue crack propagation rate using the proposed model with experimental results [10]

5 RESULTS AND DISCUSSION

Experimental data of [10] for all the three specimens; small, medium and large, is taken to present Φ^* as a function of the number of cycles, N. Figure 4 shows the variation of Φ^* with respect to number of cycles N using the proposed expression. It is observed that initially the Φ^* value is small, with increasing number of load cycles, its value increases. The same trend is observed in all the three specimens. Figure 5 shows the variation of Φ^* normalized with the initial value that is the energy dissipated after one cycle (Φ_1^*) versus the number of cycles, N for all the three specimens. It is seen that the value $\frac{\Phi^*}{\Phi_1^*}$ is constant for all the specimens as the number of cycles increases. For clarity purpose the same plot is repeated on a logarithmic scale as shown in Figure 6. The smallest specimen fails at 3256 cycles, the medium at 5537 cycles and the large one at 8027 cycles. Just before failure the energy dissipated becomes unbounded for all the three specimens as noted by the sudden rise in the graph at the time of failure. The expression for Φ^* proposed is thus able to capture the size effect in concrete leading to objective results.

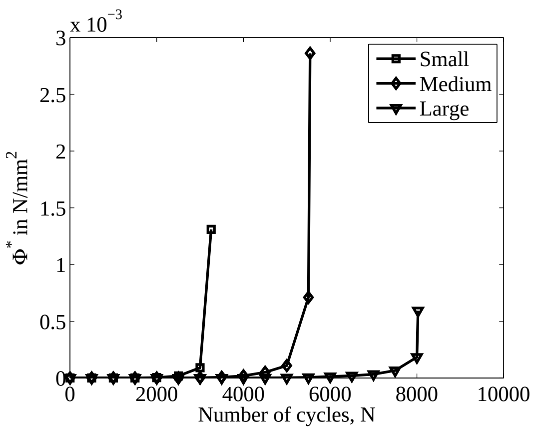

Figure 4: Variation of dissipation potential with number of cycles for different specimens [10]

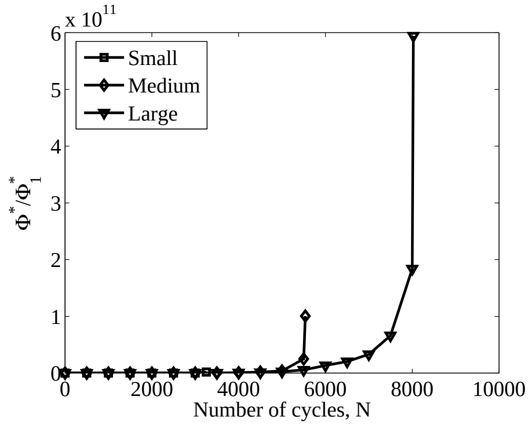

Figure 5: Dissipation potential normalized with respect to its initial value as a function of number of cycles for different specimens [10]

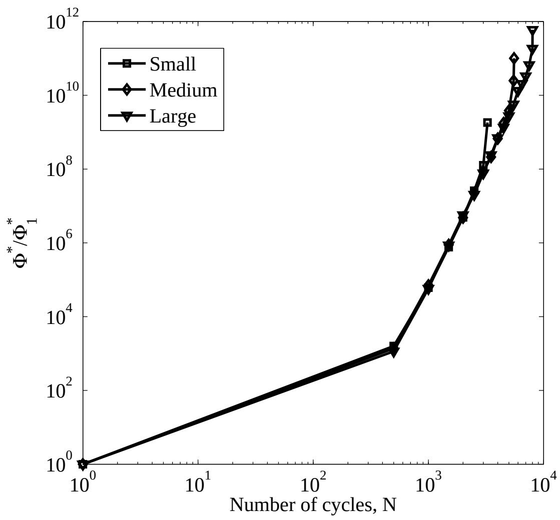

Figure 6: Dissipation potential normalized with respect to its initial value as a function of number of cycles for different specimens on a log-log plot [10]

The fatigue crack propagation model is used to predict the rate of crack propagation for specimens from other experimental sources [11]. The results are shown in Figure 7 for two different stress ratios. A good agreement in the model and experimental values is observed in Figure 7 and also shown previously in Figure 3. Thus the fatigue crack propagation model obtained from dissipation potential can be considered as robust enough and also most importantly it is derived from first principles.

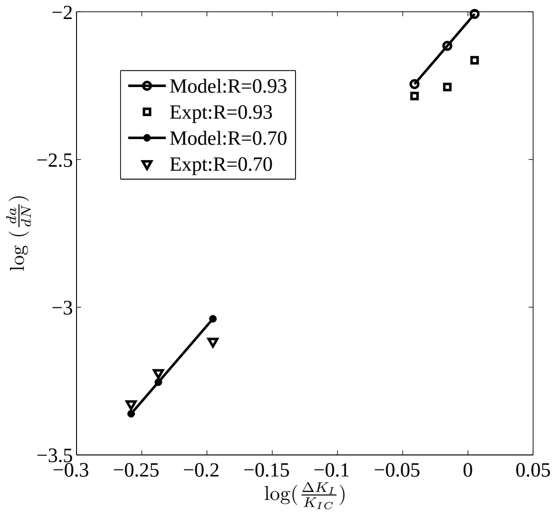

Figure 7: Comparison of the fatigue crack propagation rate using the proposed model with experimental results for different stress ratios [11]

6 CONCLUSIONS

An analytical expression for the dual of dissipation potential in the context of fatigue, as applicable to concrete structures in a fracture mechanics framework is derived using the concepts of dimensional analysis, intermediate asymptotics and self-similarity. A fatigue crack propagation model is proposed as an outcome of this exercise. The model is found to predict well the fatigue crack propagation rate in different specimens as is shown in the validation study. A physical meaning is imparted to the potential as being the energy dissipated per unit volume. The expression for the dual of the dissipation potential is found to capture size effect in concrete leading to objective results.

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