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A MULTISCALE ORIENTED CONCEPT FOR THE ANALYSES OF STEEL FIBER REINFORCED CONCRETE MATERIALS AND STRUCTURES

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Abstract: A multiscale oriented approach to the analyses of steel fiber reinforced concrete materials and structures is proposed. At the micro level, the pullout behavior of a steel fiber embedded in concrete matrix, either straight or with hooked end, with or without inclination to the loading direction, is investigated by means of analytical or numerical models which take into account the interfacial slip, the plastic deformation of fiber and the localized damage of concrete. A representative volume element (RVE) containing a number of fibers is used to describe the composite behavior at the mesoscale. For a certain opening crack, the bridging effect is obtained by the integration of the individual pullout response of all fibers crossing the crack; the mechanical properties of the RVE is analyzed using a modified fracture-micromechanics model. At the macroscopic level, the Finite Element Method is used, applying the Embedded Crack approach, for which the cohesive behavior along macro-cracks and the material properties of the continuous parts are obtained from the homogenization of fiber-crack interactions and the corresponding RVE, respectively.

1 INTRODUCTION

1.1 Background

The development of fiber reinforced concrete (FRC) can be traced back to the 1960's, when the significance of adding steel fibers to enhance the ductility of plain concrete was recognized. The expansion of the research has brought us FRC composites featuring various types of fiber reinforcement using different materials, sizes, shapes, surfaces, contents, etc., recently leading to high performance materials such as engineered cementitious composites (ECC) [1].

In addition to the experimental investigation, large efforts have been devoted to the modeling of FRC on the material as well as on the structural level. Typically, a phenomenological approach is adopted, where the enhanced ductility of FRC is taken into account by means of a modification of the softening law in plain concrete models (e.g. [2]). More recently, advanced Finite Element technologies are applied, for example the Partition of Unity approach, by representing the slip behavior of individual fibers at a macroscopic level [3].

1.2 Multiscale oriented modeling concept

For the numerical analyses of FRC composite materials and structures, a multiscale oriented approach enables us to formulate the behavior of every constituent (fiber and matrix in our case) and the interactions between them at different length scales [4]. Aiming at the development of a simulation and optimization platform for structures (e.g. segmental tunnel linings) made of FRC and hybrid FRC composite in conjunction with conventional reinforced concrete, a multiscale modeling concept is proposed in the present work (Figure 1):

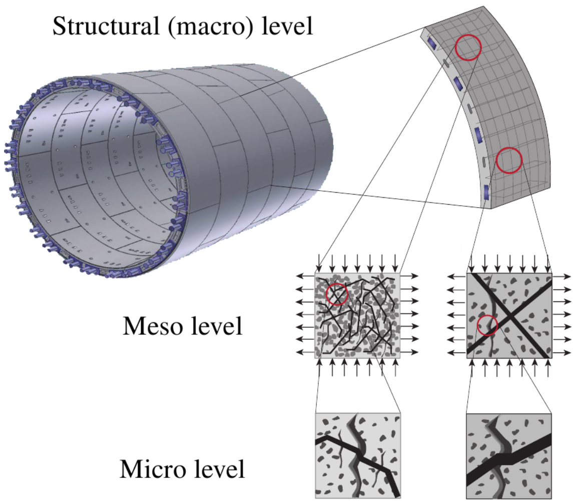

Figure 1: Multiscale oriented approach to the analyses of structures made of steel fiber reinforced concrete (in conjunction with conventional reinforced concrete).

- At the structural (macroscopic) level, the Finite Element Method is used, making use of a model which is suitable to capture propagating cracks. The interface behavior along the macro-cracks and the material properties of the intact parts are obtained from the analysis of the corresponding representative volume elements (RVE).
- An RVE containing a number of distributed fibers is used to describe the composite material behavior at the meso-scale. For a certain growing crack within the RVE, the bridging effect is obtained from the summation of pullout responses of all the fibers intercepting the crack. The effective mechanical properties of the RVE is generated by means of micromechanical homogenization.
- At the micro-level, analytical or numerical models are developed in order to describe the pullout behavior of single fiber embedded in the matrix with arbitrary inclination w.r.t. the crack plane.

In the following sections, we first show the models for the single fiber pullout behavior which are validated for different situations. Next, making use of the pullout models and by up-scaling the information from the micro level, the fiber bridging effect on an opening crack and the constitutive response of an RVE are analyzed at the meso-scale. In the third part, the Embedded Crack model is introduced and applied for the preliminary numerical simulation of an FRC structure.

2 Micro-scale: single fiber pullout models

2.1 Straight fiber pullout without inclination

Being the most basic case, straight fiber pullout without inclination with respect to the loading direction has been well investigated by means of laboratory tests, numerical simulations as well as analytical models. It is generally accepted that the pullout procedure can be divided into three stages: Bonded state, debonding stage and pulling-out phase (Figure 2).

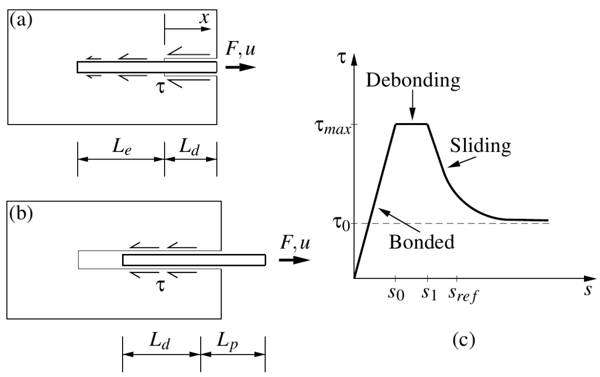

Figure 2: Straight fiber pullout without inclination: (a) debonding stage, (b) sliding phase, (c) proposed interfacial friction law.

According to the *shear-lag* concept (e.g. [5]), an interfacial friction law ($\tau - s$ relation) is proposed in the present work, as illustrated in Figure 2c, where τ denotes the interface frictional stress and s the relative displacement at a point on the fiber axis x; τ_{max} is the bonding strength of the interface, τ_0 the asymptotic value of the frictional stress and s_{ref} a parameter controlling the descending branch of the curve. Applying the proposed interfacial friction law, the load-displacement relations ($F-u$ diagrams) at the free end are obtained for the whole pullout process.

2.2 Pullout of inclined straight fiber

Compared to the situation described above, modeling the pullout behavior of an inclined fiber involves additional complexities correlated with the additional frictional stress, plastic deformation of the fiber and change of the geometrical state, caused by the lateral pressure on the interface, yielding of the steel and partial damage of the matrix, repectively (see e.g. the experimental observation in [6] and the analytical model in [7]).

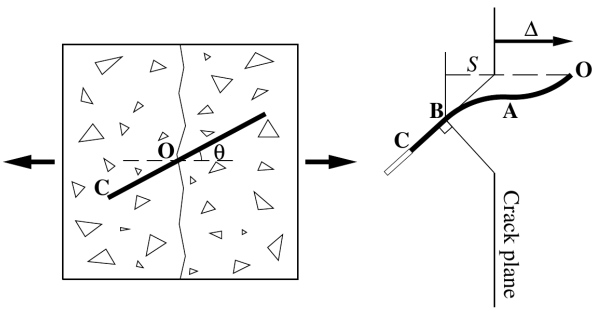

Figure 3: Illustration of the inclined straight fiber pullout problem: (a) crack initiation, (b) geometrical state of (one side of) the fiber during pullout.

The situation of an inclined fiber embedded in the matrix is illustrated in Figure 3a, where θ indicates the inclination angle, O is the initial intersection of the fiber axis and the crack plane and C is the embedded end of the fiber. For a certain pullout state corresponding to the crack opening Δ and concrete spalling S (see Figure 3b), the additional fiber-concrete interactions is analyzed via two sub-models referred to as the *cantilever*-AB and the *beam on elastic foundation*-BC. In both submodels, the force equilibrium on the fiber sections is analyzed; the free end pullout force corresponding to the current pullout state is calculated.

The model described above is validated by means of the experimental results reported in [6]. First, the model for the pullout without inclination is calibrated, with the interfacial parameters determined; then we proceed with a numerical algorithm to generate the complete force-displacement diagram for the situation of straight fiber pullout with an arbitrary inclination angle. The model developed in the present work describes the pullout behavior with satisfactory performance (see the diagrams in [8]).

2.3 Pullout of inclined hooked end fiber

In comparison to straight fibers, steel fibers with deformed geometry usually exhibit higher ductility during the pullout process. One of the most widely applied types of deformed steel fibers is the hooked end fiber, characterized by the hook on each end. During the pullout procedure, the resistance of the hooked end to straightening often contributes, as an *anchorage effect*, to the main portion of the total pullout force, in comparison to the case of a straight fiber, where the interfacial behavior plays the main role [9].

The hooked end fiber pullout behavior is often investigated by means of laboratory tests. However, due to the highly nonlinear local behavior of the hooked end and the surrounding matrix, as well as the complicated interactions between them (see the experimental observation in e.g. [10]), only a few analytical models describing the hooked end fiber pullout without inclination to the crack plane (e.g. [11]) can be found in the literature; the inclined situation is only considered in [12].

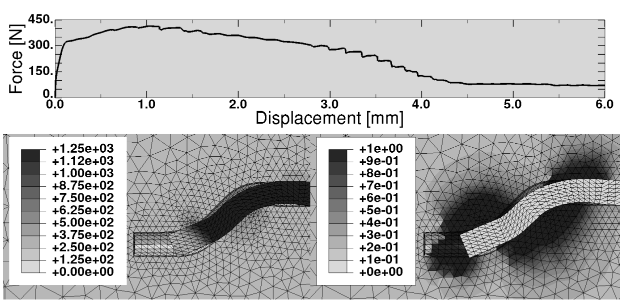

Figure 4: Numerical simulation of the hooked end fiber pullout: Load-displacement diagram (upper), contour plot of the von Mises stress (lower-left) and the compressive damage (lower-right).

To support the analytical formulation, numerical models for the single fiber pullout behavior are developed, using the Finite Element Analysis software Abaqus, by defining appropriate material models for the concrete and steel, as well as the interface properties between them. The numerical simulation provides not only the pullout load-displacement relation at the free end, but also an insight into the problem and a reference for the formulation of the analytical model (Figure 4).

Regarding the analytical model proposed in the present work, the anchorage effect of the hooked end is represented by a multi-linear load-displacement relation, capturing a sequence of key states (Figure 5).

For every key state, the force equilibrium on the segments of the hooked end is analyzed and the resulting anchorage force is calculated, taking into account the yielding of steel and the damage of concrete. This submodel is then combined with the straight fiber pullout model described in the previous section, in order to predict the load-displacement relation during the hooked end fiber pullout with an arbitrary inclination w.r.t. the loading direction.

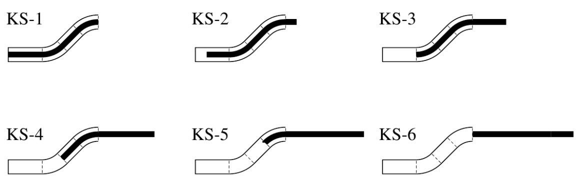

Figure 5: Key states during the hooked end fiber pullout.

The model is validated with the experimental results reported in [13]. From Figure 6 we can see that the analytical model captures successfully the major feature observed during the pullout of a hooked end steel fiber in concrete matrix for different values of the strength of concrete and steel fiber and the inclination angle.

3 Meso-scale: FRC composite properties

With the pullout behavior of single steel fiber embedded in concrete matrix investigated, the scope is now to analyze the behavior of fiber reinforced concrete at the meso level, considering an RVE of the composite material under tensile loading. Initially all the fibers are well bonded to the matrix and both constituents are linear elastic. From the micromechanical point of view, the increasing tension on the boundary of the RVE will lead to the initiation of microcracks at the inherent flaws within the RVE [14]. As the load increases, those microcracks tend to open and propagate, however, unlike in the situation of plain concrete, their opening is constrained by the bridging effect of the individual fibers intersecting the cracks. To this end, it is essential to investigate the bridging stress vs. crack opening relationship.

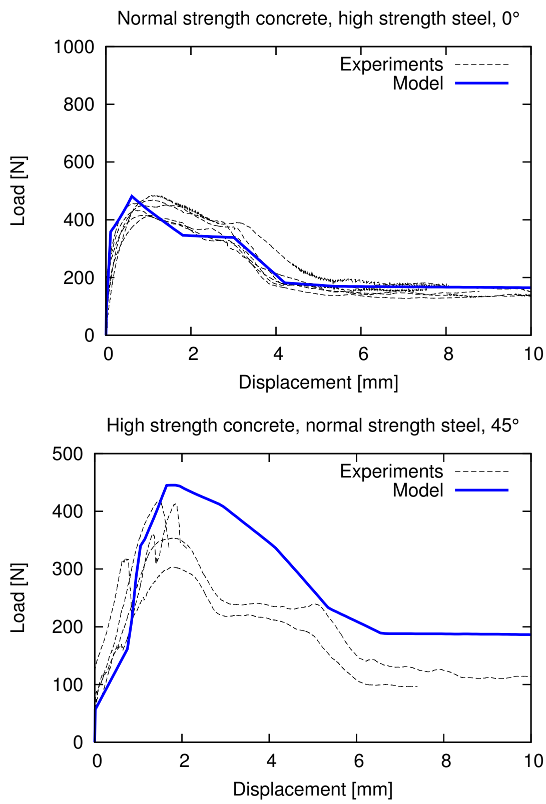

Figure 6: Inclined hooked end fiber pullout model validation: Comparison between the results of analytical model and experiments in different cases.

3.1 Crack bridging effect

In [15], for a homogeneous (in terms of position) and isotropic (w.r.t. orientation) distribution of fibers in the composite, the integration of the pullout forces of all fibers crossing a crack divided by the cross sectional area of the RVE provides the bridging law, i.e. the relation between the fiber bridging stress σ^f and the crack opening displacement Δ. While in [15], explicit analytical formulations of pullout force-displacement relation are used, the pullout response in the present work is obtained numerically, and the integration over different positions and orientations gives the fiber bridging effect:

$$\sigma^f(\Delta) = \frac{1}{A^{cr}} \sum_z \sum_\theta F(z, \theta, \Delta). \qquad (1)$$

A^{cr} denotes the cross sectional area of the RVE; z and θ represent the position and orientation of individual fibers, respectively; F is the single fiber pullout force calculated by means of the models described in the previous section.

Figure 7 shows some results of the crack bridging relation, considering different situations of the orientation distribution of fibers, which is represented by a spheroid: Z-axis coincides with the crack normal direction; X- and Y-axes are parellel to the crack plane; the semi-axes a and c represent the anisotropy (e.g. $a/c = 1$ indicates the isotropic case).

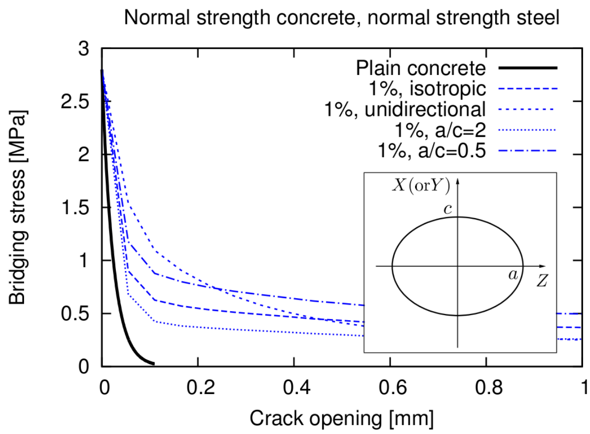

Figure 7: Crack bridging effect obtained through integration of single fiber pullout responses (in different cases of orientation distribution).

3.2 Mechanical properties of the RVE

The composite behavior under further tensile loading after the initiation of microcracks, i.e. whether those cracks grow quickly, leading to the softening and failure of the material, or the RVE shows pseudo strain-hardening response, which is typically observed in *high performance fiber reinforced cement composites materials* (HPFRCC [16, 17]), depends on the contribution of the evolving crack bridging stress σ^f. In the present work, we propose a "modified fracture mechanics-micromechanics model" based on the *combined fracture-micromechanics model* for brittle materials with microcracks in [18]. However in the proposed model for FRC the stress intensity factor, on which the Griffith fracture criterion

$$K_I = K_{Ic} \qquad (2)$$

is based, is modified by defining $K_I^* = K_I + K^f$ [19, 20], in order to take into account the influence of fiber bridging mechanisms. K^f depends on the crack size r and the interfacial friction behavior. The full model is currently in progress. As a first attempt, a simple relationship $K^f(r) = k^f\, r$ is investigated (k^f is a stiffness-like constant representing the effect of fibers). The macroscopic constitutive behavior of the composite material, obtained from using $K^f(r)$ in the upscaling procedure proposed in [18], is illustrated in Figure 8. The transition from a brittle to ductile behavior is well represented.

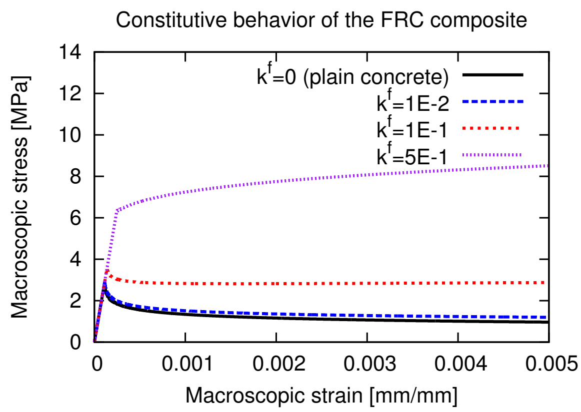

Figure 8: Preliminary results the model: Different constitutive behavior of the composite material containing different content of fiber (represented by the parameter k^f).

The meso-scale model provides the overall mechanical behavior of the composite material generated in the form of a continuum constitutive law at the level of the RVE including an ascending branch. Transition from distributed cracks to localized fracture is detected on the basis of the localization tensor related to the concrete-fiber composite material [21]. As soon as the loss of ellipcity is signalled, the behavior is governed by the opening of a macro-crack which is represented by using the Embedded Crack model.

4 Macro-scale: Embedded Crack model

The localization of deformation into macro-cracks is represented on the level of Finite Elements by adopting the *Embedded Crack* approach [22, 23]. In this model, the scale transition is accomplished by an additive decomposition of the displacement field $\mathbf{u}$ into a large scale (continuous) portion $\bar{\mathbf{u}}$ and a discontinuous portion $\hat{\mathbf{u}}$, representing the local displacement jump:

$$\mathbf{u}(\mathbf{x}) = \bar{\mathbf{u}}(\mathbf{x}) + \hat{\mathbf{u}}(\mathbf{x}). \tag{3}$$

Since the element enrichment used for $\hat{\mathbf{u}}$ is restricted to the element domain, the additional parameters connected with the displacement jump are resolved by static condensation without introducing global degrees of freedom [22, 24].

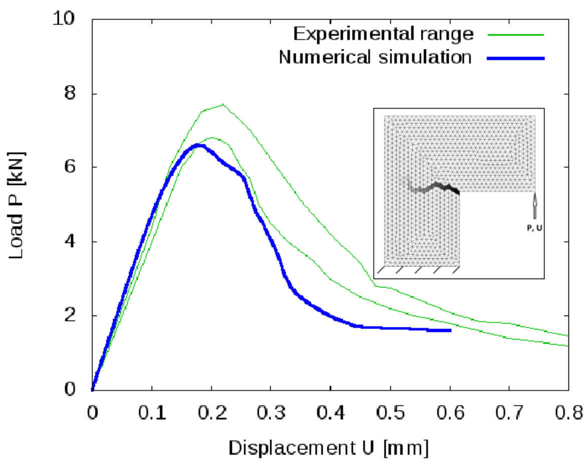

Figure 9: Numerical simulation of the L-shape experiment [25]: The computed load-displacement relation compared with the experimental results and the plot of crack pattern.

By adopting the Embedded Crack formulation proposed in [22] and defining appropriate traction-sepration laws, the behavior of structures made of plain concrete can be simulated (see Figure 9).

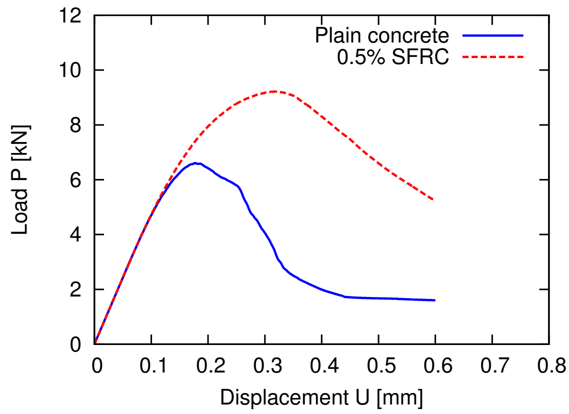

Figure 10: Numerical simulation of the L-shape experiment: Comparison of the results of plain concrete and FRC with low content of fiber.

Furthermore, by replacing the cohesive law on the crack interface with that obtained according to the traction-separation law from the meso-scale analysis of the composite material, the behavior of structures made of fiber reinforced concrete can be simulated (Figure 10).

5 Conclusions

In this paper, the essential components of a multi-scale oriented modeling framework for the finite element analyses of steel fiber reinforced concrete materials and structures have been presented. At the lowest level, the pullout behavior in various cases of single steel fiber embedded in concrete matrix is described by analytical or numerical models. The models have been successfully validated by means of representative experimental results, capturing the major mechanisms involved in the pullout of single fibers. Then, the crack bridging law at the level of an RVE of the composite material has been obtained by means of integration over the response of individual fibers distributed across the crack. A modified fracture-micromechanics model for the constitutive response of the RVE is proposed; a first testing example reveals the potential of this model. The crack bridging law is used as the basis for a macroscopic representation of cracks on the level of Finite Elements using the Embedded Crack approach. A preliminary structural simulation shows the performance of the multiscale model.

The work presented so far is not yet the full picture of the multiscale oriented scheme for the modeling, simulation and optimization of FRC and hybrid RC-FRC materials to be used for large structures, e.g. tunnel linings. The steps in progress and future work include

- the completion of the modified fracture-micromechanics model for the FRC composite behavior on the meso-scale, particularly the analysis of the strain-hardening phenomenon with correct modification on the stress intensity factor and the transition from strain hardening to strain softening (localization) behavior in connection with the Embedded Crack model,
- the implementation of the Embedded Crack model in the 3-dimensional configuration
- and the application of the complete multiscale framework to the analyses at the structural scale.

6 Acknowledgement

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