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A PROBABILISTIC FATIGUE MODEL BASED ON THE INITIAL DISTRIBUTION TO CONSIDER FREQUENCY EFFECT IN PLAIN AND FIBER REINFORCED CONCRETE

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Abstract. The objective of this work is twofold. First, we aim to develop a new fatigue model valid for quasi-brittle materials like concrete, which properties have considerably larger standard deviation than metals. Having this in mind, we fit the measured strength data with a three-parameter Weibull cumulative distribution function and in turn take it as the initial distribution for an asymptotic fatigue model in concrete. Second, we endeavor to take into account the observed influence of frequency and stress ratio on the fatigue life in concrete, both plain and reinforced with fibers. The developed model is validated against fatigue tests in compression on cubic specimens for different stress ratios and loading frequencies. All the parameters have found physical meaning in the extensive experimental tests performed for two plain high strength concretes and two concretes reinforced with fibers. The secondary strain rate is found to be correlational with the number of cycles to failure. Finally, a reduced test procedure is proposed for fatigue strength characterization.

1 INTRODUCTION

Interest in the fatigue of concrete began more than a hundred years ago with the development of reinforced concrete bridges. Since then, numerous experiments have been conducted to study the influence of different fatigue parameters, for instance, see [3, 4, 5, 6, 7, 10, 13, 15, 16, 17, 19] and the references within. These parameters are either set by the fatigue test conditions, such as the minimum stress σ_{min}, the maximum stress σ_{max} and the loading frequency f, or determined by the material properties, for example the static material strength σ_c, which can be the compressive strength f_c or the tensile strength f_t, or any other critical stress defined accordingly. Other parameters include the stress ratio R, defined as $\sigma_{min}/\sigma_{max}$, the stress amplitude or stress range $\triangle\sigma$, calculated as $\sigma_{max} - \sigma_{min}$, or the stress level S, defined as σ_{max}/σ_c.

For metals, the stress amplitude $\triangle\sigma/2$ plays an important role, and the fatigue life (the number of cycles N resisted before failure) is often described by the Wöhler curve. For concrete, however, the influence of the stress ratio, loading frequency and stress level has been observed to be important [2, 9, 13, 19]. The fatigue equation has evolved accordingly to illustrate the role of those parameters. For instance, Aas-Jakobsen [2] proposed to include the effect of the stress ratio R as follows

$$\frac{\sigma_{max}}{\sigma_c} = 1 - (1-R)\beta \ln N \tag{1}$$

where β is a material parameter. The same relation was confirmed by Tepfers and Kutti [6] and Tepfers [7] for fatigue strength of concrete in compression and in tension for splitting tests of cubes. Even though the influence of loading frequency (or time) has been observed as early as 1960s by Rusch [1] and confirmed by Awad and Hilsdorf [3], Sparks and Menzies [4] and Holmen [8] in 1970s, it was not included in the fatigue equation until Hsu [9], Furtak [10] improved Eq. 1 by including the loading time and frequency respectively. Zhang et al [15] further improved the equation of Furtak by redefining the stress ratio R when there is stress reversal.

However, none of them considered the marked dispersion of the fatigue strength σ_c in concrete. The first consideration of the statistical distribution of concrete strength properties for fatigue tests was by Zhao *et al.* [17], who considered a normal distribution as suggested in the design codes. Recently, Przybilla, Fernández-Canteli and Castillo [18] considered the statistical feature of the characterized strength for brittle materials and derived the primary three-parameter Weibull cumulative distribution function (CDF) of fracture stress from three- and four-point bending tests. Weibull distribution was also used to fit the fatigue life of concrete at various stress levels by Oh [13] and to fit the flexural fatigue life of concrete containing nano-particles by Li et al. [16].

Castillo and his coworkers [20, 21] also postulated a general probabilistic model to predict the fatigue behavior for any stress level and range based on laboratory tests for ductile materials like steel. The nine parameters involved are defined through the physical and compatibility considerations of the Weibull model. However, their model does not consider the observed influence of loading frequency in concrete.

In the current work, we first consider the entire statistical distribution given by the characterization tests and build our fatigue model from this initial distribution. Second we take into account the loading frequency based on the dynamic-response description given by the Model Code [22]. The range of application of the proposed model is below 10 Hz according to the experimental tests realized between 1/16 and 4 Hz [19]. Even though the model itself does not limit to a given range of frequency, its application beyond 10 Hz needs further experimental collaboration.

The rest of the paper is organized as follows. A fatigue model based on an initial distribution is postulated in Section 2. The experimental program and validation of the model are given in Section 3. A reduced test procedure is proposed based on the developed model in Section 4. Finally we summarize the current work in Section 5.

2 THE FATIGUE MODEL WITH INITIAL DISTRIBUTION

As mentioned above, we aim to develop a fatigue model for concrete, taking into account the statistical distribution of the characterized strength data and the influence of loading frequency and stress ratio, the following hypotheses are assumed.

- The characterized (or experimentally measured) material property of concrete, such as the compressive or tensile strength, follows a Weibull distribution. In the current work, we focus on the compressive strength measured from cubic specimens.

- This distribution is influenced by the dynamic condition through the loading frequency. In addition, the relation given by the Model Code [22] to describe the dynamic properties of concrete is extendable to consider the influence of loading frequency according to the experimental data of Ruiz *et al.* [19].

- There exists a minimum stress which is the asymptote given by the zero probability of failure.

Given sufficient number of characterization tests carried out at a certain reference loading rate $\dot{\sigma_0}$, which is considered static, where the dot $\dot{}$ represents derivation with respect to time, denominating the measured strength data or the failure stress at one cycle as σ_{f_0} ($_f$ for failure, $_0$ for static loading), the probability of failure (PF) corresponding to each stress level can be fitted by a three-parameter Weibull CDF as follows

$$PF(\sigma_{f_0}) = 1 - \exp\left[-\left(\frac{\sigma_{f_0} - \sigma_{min_0}}{\lambda}\right)^k\right] \quad (2)$$

where λ and k are the scale and shape parameter respectively, whereas σ_{min_0} is the location parameter or the threshold stress below which no failure will occur, it plays the role of endurance limit. Note that, through Eq. 2, the concept of absolute failure or damage is replaced by the the probability of failure, which ranges from 0 to 1. We define the distribution in Eq. 2 as the initial distribution D_i, which is a property of the material, and is determined necessarily through experimental characterization.

2.1 The influence of loading frequency

In order to relate the dynamic failure strength under compression f_{cd} with its static counterpart f_{c_0}, we start with the empirical expression provided by the Model Code [22], which is written as follows

$$\frac{f_{cd}}{f_{c_0}} = \left(\frac{\dot{\sigma}_d}{\dot{\sigma}_0}\right)^{\alpha} \quad (3)$$

where $\dot{\sigma}_d$ and $\dot{\sigma}_0$ are the loading rate of the compressive fatigue test and that of the compressive characterization test respectively. The exponent α is fitted as 0.014 in the Model Code [22], where the effect of loading frequency is not taken into account.

The loading rate $\dot{\sigma}_d$ in Eq. 3 in each cycle can be roughly related to the loading frequency f and the stress range $\Delta\sigma$ through

$$\dot{\sigma}_d = 2f\Delta\sigma \quad (4)$$

Meanwhile an expression for the exponent α that takes into consideration the influence of loading frequency is obtained by fitting experimental data of Ruiz *et al.* [19] for frequencies below 10 Hz, the function is

$$\alpha = 0.014\exp[\gamma f] \quad (5)$$

where the parameter γ needs to be determined by fitting experimental data for different loading frequencies. The coefficient 0.014 for static loading conditions is recovered for a zero frequency. As a result, the influence of frequency in a fatigue test is manifest on both the loading rate through Eq. 4 and the exponent α through Eq. 5. In addition, Eq. 3 allows us to shift the initial distribution D_i for f_{c_0} to the distribution of f_{cd} in dynamic conditions.

3 FAILURE CURVES OF ISO-PROBABILITY

In this Section, we explore all the conditions of the failure curves in order to obtain the specific expression $\sigma_f(f, \sigma_{max}, \sigma_{min_0}, \sigma_{f_0}, R, N)$. On the one hand, each curve represents one probability of failure and intercepts the σ_f axis at σ_{f_0}, the PF (probability of failure) of which is determined by the distribution D_i defined by Eqn. 2, see Fig. 1. On the other hand, there are three limit conditions that all the failure curves of iso-probability should comply with.

$$\lim_{N\to\infty} \sigma_f = \sigma_{min_0} \quad (6)$$

$$\lim_{R\to 1} \sigma_f = \sigma_{f_0} \quad (7)$$

$$\lim_{N\to 1} \sigma_f = \sigma_{f_0} \quad (8)$$

It needs to be emphasized again that σ_{min_0} is the threshold stress below which no fatigue failure will occur, whereas the σ_{f_0} is the static strength (when stress ratio equals one or when the failure occurs after one cycle).

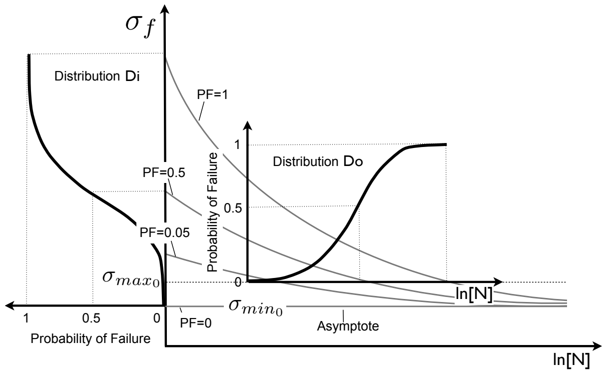

Figure 1: Failure curves of iso-probability, where D_i is the initial CDF, determined from characterization tests and fitted according to Eq. 2, whereas D_o is the output CDF fitted by the fatigues tests.

In Fig. 1, we give a schematic plot for the failure curves of three iso-probabilities, 0.05, 0.5 and 1.0. On the left of the σ_f-axis is the initial distribution D_i determined from experimental characterization, on the right is the output distribution D_o, which represents the fatigue equation we are looking for and is going to be defined later on. Having in mind the limiting conditions given in Eqs. 6-8, the following expression for each failure curve is proposed

$$\sigma_f = \sigma_{min_0} + (\sigma_{f_0} - \sigma_{min_0}) N^{-a(1-R)} \tag{9}$$

where the parameter a is related to the loading frequency as follows

$$a = b + c \ln(1+f) \tag{10}$$

where b and c are the only parameters which need to be fitted with the experimental data of fatigue tests. The logarithmic function in Eq. 10 has been inspired by earlier work of Furtak [10] and it is the function that best fits the experimental results, which we will see in the following Section.

For a fatigue test, σ_{max}, R and $\ln N$ are known parameters, meanwhile, the static counterpart of σ_{max}, denominated as σ_{max_0}, corresponds to the value given in the characterization tests. According to Eq. 3, they are related through the following dynamic equation

$$\sigma_{max_0} = \sigma_{max} \left(\frac{\dot{\sigma}_0}{\dot{\sigma}_d} \right)^{\alpha} = \sigma_{max} \left(\frac{\dot{\sigma}_0}{2f\Delta\sigma} \right)^{\alpha} \tag{11}$$

By plugging in the value of σ_{max_0} for σ_f to Eq. 9, we easily obtain σ_{f_0} as follows

$$\sigma_{f_0} = \sigma_{min_0} + (\sigma_{max_0} - \sigma_{min_0}) N^{a(1-R)} \tag{12}$$

Introducing the value of σ_{f_0} into Eq. 2, we arrive at the general expression for the cumulative probability of failure for any point of the fatigue test

$$PF(N; \sigma_{max}, f, R) = 1 - \exp\left\{ - \left[\frac{\sigma_{max_0} - \sigma_{min_0}}{\lambda N^{-a(1-R)}} \right]^k \right\} \tag{13}$$

where N is considered as the main variable, whereas the rest are secondary ones, which are parameters set by a given fatigue test. Notice that, due to the shifting of the CDF from Eq. 2 to Eq. 13, the shape parameter k is preserved, but the scale parameter is now related to loading frequency, stress ratio and the number of cycles suffered. Insert Eq. 11 and Eq. 10 to Eq. 13, the following explicit CDF is obtained

$$PF(N; \sigma_{max}, f, R) = 1 - \exp\left\{ - \left[\frac{\sigma_{max} \left(\frac{\dot{\sigma}_0}{2f\Delta\sigma} \right)^{\alpha} - \sigma_{min_0}}{\lambda N^{-[b + c\ln(1+f)](1-R)}} \right]^k \right\} \tag{14}$$

This is the output distribution D_o shown in Fig. 1. It can be observed that, in Fig. 1, for a fatigue test performed at a given level of σ_{max_0}, the probability of failure increases with the number of cycles endured. The number of cycles N resisted for a given PF under given loading conditions is also easily derived, see Eq. 15. In addition, by writing the stress range in terms of the stress ratio and the maximum stress as $\Delta\sigma = (1-R)\sigma_{max}$ in Eq. 11, we can also predict the maximum stress for a given PF and a designed fatigue life at a given stress ratio and loading frequency, see Eq. 16.

$$N(PF; \sigma_{max}, R, f) = \left[\frac{\lambda \sqrt[k]{-\ln(1-PF)}}{\sigma_{max_0} - \sigma_{min_0}}\right]^{\frac{1}{a(1-R)}} \tag{15}$$

$$\sigma_{max}(PF, N; R, f) = \left[\sigma_{min_0} + \frac{\lambda \sqrt[k]{-\ln(1-PF)}}{N^{a(1-R)}}\right]^{\frac{1}{1-\alpha}} \left[\frac{2f(1-R)}{\dot{\sigma}_0}\right]^{\frac{\alpha}{1-\alpha}} \tag{16}$$

where α is given by Eq. 5.

4 Validation against experimental data

The experimental program was designed to look into the influence of the stress ratio and loading frequency on the fatigue strength of concrete. Two groups of tests on two different types of concrete C1 and C2 were carried out for cubic specimens. Twenty compressive tests on cubes (80 mm in edge length) for C1, loaded at a rate $\dot{\sigma}_0$ of 0.25 MPa/s, and six compressive tests for cubes (100 mm in edge length) for C2, loaded at 0.2 MPa/s, were carried out for characterization purpose. The fitted Weibull distribution D_i for both materials are plotted in Fig. 2, the corresponding parameters are listed in Tab. 1. Note that for both type of concretes C1 and C2, even though the scale and shape parameters λ and k are quite different, the minimum stress σ_{min0} is 3.1 MPa, which plays the role of the endurance limit. Next, the model is validated against concrete reinforced with steel fibers CF1 and polypropylene fibers CF2. Finally, after establishing the one-to-one relation between the secondary strain rate and the fatigue life from observed experimental trend, the fatigue Eq. 14 is also expressed in terms of the secondary strain rate.

Table 1: Fitting parameters for the initial distribution D_i given in Fig. 2 for concrete C1 and C2.

Material	λ [MPa]	k	σ_{min_0} [MPa]
C1	94.7	12.4	3.1
C2	76.1	19.8	3.1

4.1 Tests on Concrete C1 for two different stress ratios

For the purpose of studying the influence of stress ratio, two series of compressive fatigue tests on cubic specimens with an edge length of 80 mm, fifteen each, were carried out at a loading frequency of 4 Hz, a maximum stress σ_{max} of 90 MPa. Fitting the experimental data with Eq. 13, the values for a and γ shown in Tab. 2, give an error of 2.36% and 4.56% respectively. The agreement, as can be seen from Fig. 3, is remarkable. After inserting the obtained parameters a and γ to Eq. 2, we arrive at the fatigue equation shown in Eq. 17, which predicts the probability of failure for any compressive fatigue test carried out at 4 Hz, with a stress ratio R and maximum stress σ_{max} after N cycles.

$$PF(N; \sigma_{max}, R) = 1 - \exp\left\{-\left[\frac{\sigma_{max}^{0.9634}}{107.4(1-R)^{0.0366}} - \frac{1}{30.5}\right]^{12.40} N^{0.75(1-R)}\right\} \tag{17}$$

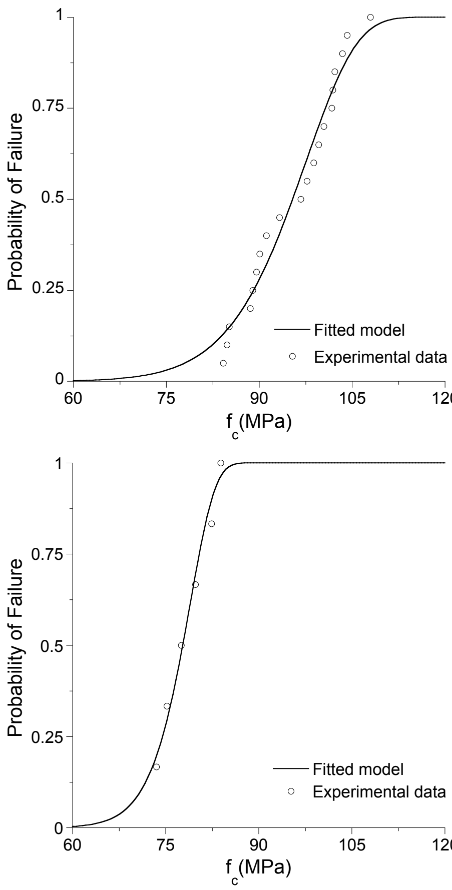

Figure 2: Initial distribution of the compressive strength fitted with the experimental measurements on cubes for concrete C1 and C2.

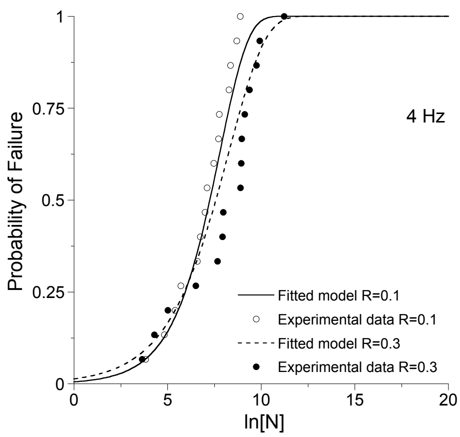

Figure 3: Distribution D_o given by Eqn. (13), fitted through the fatigue tests with R = 0.1 and R = 0.3 for concrete C1.

4.2 Tests on concrete C2 under four different loading frequencies

In order to pinpoint the influence of loading frequency, four series of compressive fatigue tests, on cubic specimens with an edge length of 100 mm, were carried out at four different frequencies for concrete C2. Fitting the data curves with Eq. 2, the adjusting parameters b, c and γ listed in Tab. 2, give an error of 3.52%, 3.10%, 1.76% and 3.19% for the frequency of 0.0625, 0.25, 1 and 4 Hz respectively, see Fig. 4. Again the agreement is noteworthy. As a result, all the parameters in Eqs. 14-16 are determined, in turn, the fatigue life or probability of failure for concrete C2 is characterized for any given maximum stress, stress ratio or any loading frequency (below 10 Hz). Notice that the parameter γ for the two concretes studied remains the same.

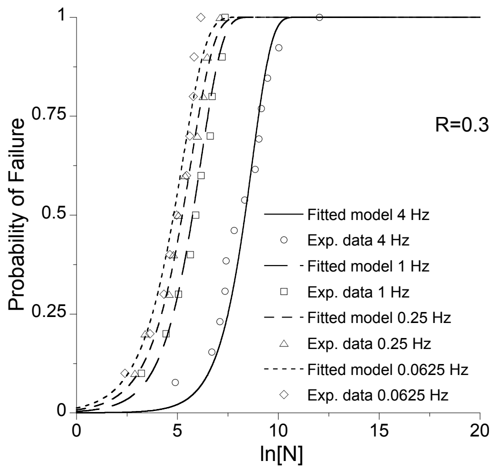

Figure 4: Model and experimental data for concrete C2 for fatigue loading at four different frequencies.

4.3 Validation against concrete reinforced with fibers

In order to further validate the fatigue equation presented in Eq. 14, we also recur to the experimental results presented by Ruiz *et al.* [19] for concrete reinforced with steel fibers, denoted as CF1, and for concrete reinforced with polypropylene fibers, denominated as CF2. The parameters for the initial distribution D_i and the fatigue equation Eq. 14 are given in Tabs. 2 and 3. We also demonstrate separately the output distributions for concrete reinforced with polypropylene fibers CF2 at four loading frequencies together with the experimental data in Fig. 5, the fitting error is below 5% for all the cases considered. The distribution curves, or the curves of failure probability, at four loading frequencies are plotted together in Fig. 6 for C2 and CF1. Notice that, one the one hand, for the same failure probability, the specimen resists more cycles at a higher frequency. This is attributed to the dynamic behavior of concrete, which results an increase of the dynamic exponent α, see Zhang *et al.* [23]. On the other hand, due to the effect of added steel fibers, the influence of the loading frequency on concrete CF1 is less pronounced as the frequency increases. This is consistent with the known fact that the fatigue behavior of steel is not influenced by loading frequency. In contrast, adding polypropylene fibers also alters the influence of loading frequency on curves of failure probability, such a tendency is not observed in Fig. 5.

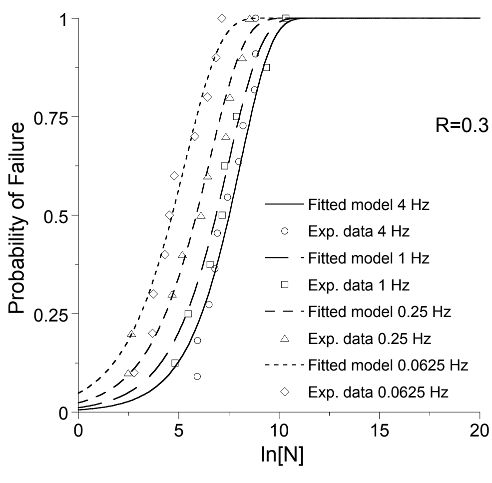

Figure 5: Model and experimental data for concrete reinforced with polypropylene fibers CF2 for fatigue loading at four different frequencies.

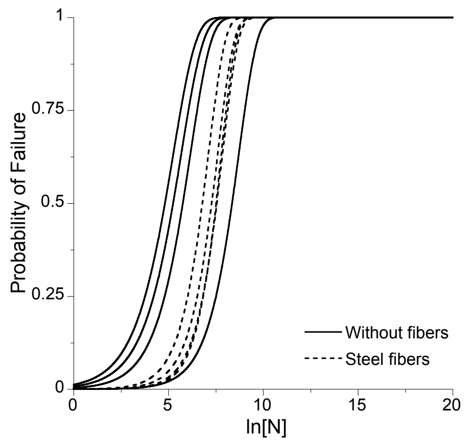

Figure 6: Influence of the frequency on concrete reinforced with steel fibers CF1 compared to that on plain concrete C2.

4.4 Secondary strain rate versus fatigue life

In a fatigue test for concrete, when the deformation at the upper stress level σ_{max} is plotted as the function of the number of cycles under testing, the resulted curve is known as the *cyclic creep curve*. This curve has a middle part, denominated as the *secondary branch*, as illustrated in Fig. 7, in which the increase of deformation per load cycle is constant. The slope of this secondary branch is called the *secondary strain rate* or the *secondary creep rate*, denominated as $\dot{\varepsilon}_{sec}$, of a specific fatigue test [11, 12]. In the current work, without causing confusion, we simply denote it as $\dot{\varepsilon}$. According to the work of Hordijk *et al.* [14] and of Cornelissen [11], there appears to be a strong relation between the secondary strain rate and the number of cycles to failure, with diminishing secondary strain rate, the fatigue life increases. In general, a prediction of the number of cycles to failure based on this secondary strain rate is more accurate than that based on stresses in an S-N diagram. In this Section, we attempt to find the relation between the secondary strain rate with the parameters in the fatigue equation Eq. 14.

The experimental results for the materials plain concrete C2 and concrete reinforced with fibers CF1, are plotted in the space of secondary strain rate and fatigue life ($\ln \dot{\varepsilon}$ and $\ln N$) in Fig. 8. Note that, there are 43 tests for C2 and

40 for CF1, performed for one stress ratio and four different loading frequencies. It can be observed that, there is a strong correlation between the secondary strain rate and the number of cycles resisted in the log-log scale. Denominating the intercept of a straight line in Fig. 8 for loading frequency f as $\ln \dot{\varepsilon}_i$, the equation for this straight line is written as

$$\ln \dot{\varepsilon} = \ln \dot{\varepsilon}_i + d \ln N \tag{18}$$

where d is the slope.

Meanwhile, the relation between $\ln \dot{\varepsilon}_i$ and the loading frequency can also be fitted as follows

$$\frac{\ln \dot{\varepsilon}_i}{\ln \dot{\varepsilon_0}} = 1 - \eta \ln \left(\frac{f}{f_0} \right) \tag{19}$$

where η is the slope, whereas $\dot{\varepsilon_0}$ is the reference secondary strain rate, corresponding to a fatigue test carried out at the reference loading frequency f_0. The frequency f_0 is an upper limit, below which, the test is considered static. This means that the following limit conditions should be satisfied for the secondary strain rate of a fatigue test.

$$\lim_{R \to 1} \ln \dot{\varepsilon} = \ln \dot{\varepsilon_0} \tag{20}$$

$$\lim_{N \to 1} \ln \dot{\varepsilon} = \ln \dot{\varepsilon_0} \tag{21}$$

In analog to the limiting conditions of the failure stress in Eqs. 6-8, taking into account the fact that $\ln \dot{\varepsilon}$ also depends on the stress ratio R and the frequency f through Eqs. 9-10, we approximate the slope d as follows

$$d = d_1 + [b + c \ln(1 + f)](1 - R) \tag{22}$$

Fitting Eq. 18 with the experimental results shown in Fig. 8, d_1 is obtained as 1. Inserting Eq. 19 and Eq. 22 to Eq. 18, calculating the reference secondary strain rate according to linear elasticity, we have the equation below,

$$\ln \dot{\varepsilon} = \ln \frac{\dot{\sigma_0}}{E} \left[1 - \eta \ln \left(\frac{f}{f_0} \right) \right] - \{1 + [b + c \ln(1 + f)](1 - R)\} \ln N \tag{23}$$

The solid lines in Fig. 8 are predictions according to Eq. 23. The agreement with the experimental data is extraordinary. The significance of Eq. 23 lies in the fact that, for a specific fatigue test, once the secondary strain rate is known, the number of cycles to failure ceases to be probabilistic, since there is a one-to-one correspondence between $\dot{\varepsilon}$ and the fatigue life$N$.

Moreover, we slightly modify Eq. 23 to obtain the number of cycles to failure in terms of the secondary strain rate as a function $N(\dot{\varepsilon})$, and insert it to Eq. 14. The probability of failure related to the secondary strain rate is given by the equation 24.

$$PF(\dot{\varepsilon}; \sigma_{max}, f, R) = 1 - \exp \left\{ \left[\frac{\sigma_{max} \left(\frac{\dot{\sigma_0}}{2f\Delta\sigma} \right)^{\alpha} - \sigma_{min_0}}{\lambda N(\dot{\varepsilon})} \right]^k \right\} \tag{24}$$

Comparing Eq. 14 and Eq. 24, we can conclude that for a fatigue test carried out at a giving load condition -maximum stress level, stress ratio and loading frequency-, both the number of the cycles to failure and the secondary strain rate are probabilistic. It cannot be overemphasized that, Eq. 23 provides the possibility to determine the fatigue life N without the need of actually exhausting all the cycles.

Finally, we summarize all the parameters and the elastic modulus for the four materials in Tab. 3. It needs to be mentioned that, since fatigue tests for C1 were performed at one loading frequency of 4 Hz, it is not possible to obtain the reference loading frequency f_0, neither is it feasible for a separate fitting of parameters b and

c. Instead, a unique value for the parameter a of 0.06 is obtained. In addition, the constant γ in the expression for the dynamic exponent α, which is extended to include the influence of loading frequency, is fitted as 0.24 for the two plain concretes, zero for the concrete reinforced with steel fibers and a value in between for the concrete reinforced with polypropylene fibers.

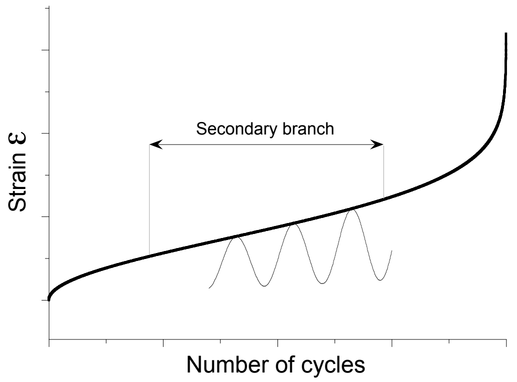

Figure 7: The concept of secondary strain rate or secondary creep rate in a cyclic creep curve for a fatigue test.

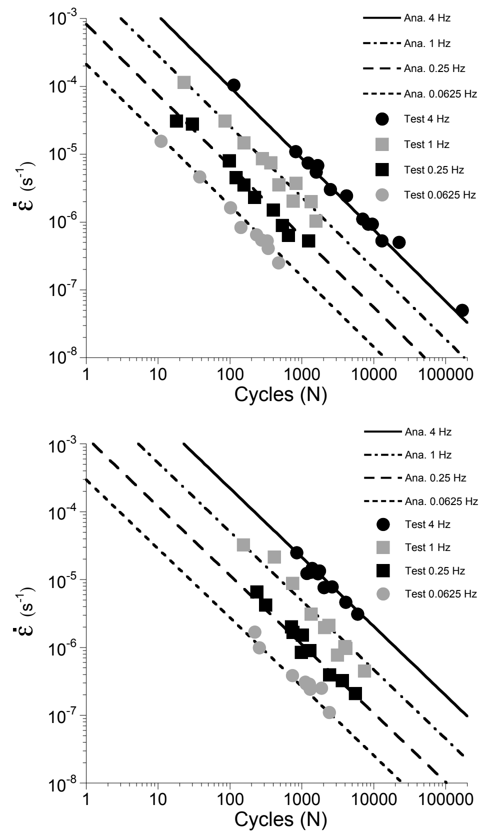

Figure 8: Secondary strain rate plotted against the number of cycles resisted for materials C2 and CF1 at four loading frequencies.

Table 2: Summary of the model parameters for the four materials considered.

	b	c	η	$f_0[Hz]$	γ
C1	-	-	-	-	0.240
C2	0.061	0.0105	0.081	0.0016	0.240
CF1	0.052	0.0035	0.086	0.0019	0.086
CF2	0.049	0.0066	0.089	0.0015	0

Table 3: Summary of the model parameters for the four materials considered.

	λ [MPa]	k	σ_{min_0} [MPa]	E [GPa]
C1	94.7	12.4	3.1	35
C2	76.1	19.8	3.1	34
CF1	68.0	14.0	4.8	41
CF2	76.1	31.0	12.0	38

5 Applications to design of reduced fatigue tests

Since there is a direct correlation between the probability of failure (or the number of cycles resisted) and the conditions of loading in a fatigue test, such as stress ratio R, loading frequency f, and the fitting parameters of the corresponding characterization test, λ, k and σ_{min_0} through Eqs. 11-15, the probability of failure can be calculated for several stages of loading with different loading frequencies and stress ratios.

An example of seven stages of loading is given in Tab. 4 and Fig. 9. The test is designed to start with a null probability of failure (step 1), the probability of failure at the end of the step1 is calculated according to Eqn. 13. Next the loading condition is changed to step 2. Evaluating Eq. 15 (considering the final value of PF in the step 1), we get the equivalent number of cycles for the beginning of the step 2, denominated as N_1. Considering the loading cycles ΔN_2 at step 2, we evaluate the Eq. 13 with $N_2 = N_1 + \Delta N_2$ to get the final value of PF on this step. Repeating this procedure for all the steps of fatigue loading in Tab. 4, fatigue failure is achieved with a reduced number of cycles (less than 20000 cycles), see Fig. 9.

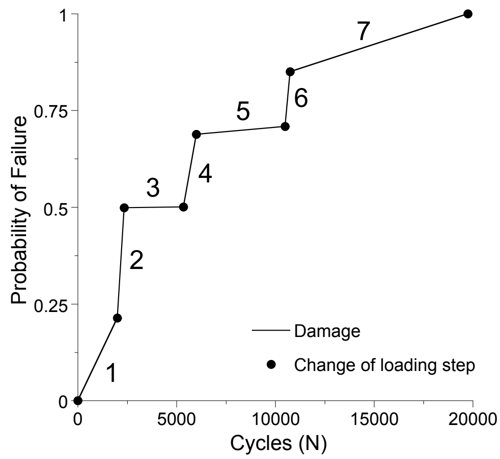

Figure 9: Damage accumulation for the different steps of loading: a reduced procedure.

Table 4: An example of a reduced procedure for fatigue characterization with seven stages of loading.

Step	σ_{max}	σ_{min}	f	$\dot{\sigma}_d$	ΔN	PF_i	PF_f
	[MPa]	[MPa]	[Hz]	[MPa/s]			
1	65	10	1	110	2000	0	0.2137
2	70	10	0.001	0.12	350	0.2137	0.4988
3	110	5	9	1890	3000	0.4988	0.5008
4	80	15	2	260	500	0.5008	0.6886
5	100	25	7	1050	4500	0.6886	0.7086
6	85	20	0.6	78	250	0.7086	0.8499
7	90	5	5	850	9000	0.8499	1

6 Conclusions

By taking into consideration the dynamic properties of concrete, fitting the results of material characterization tests with a Weibull distribution and assuming it as the initial distribution, which can be shifted along the failure axis, we have developed a fatigue model which is capable of dealing with different frequencies and stress ratios for two plain concretes and two concretes reinforced with fibers. Since only two adjusting parameters are needed, the rest are related with material properties and test conditions, reduced fatigue test procedure can be designed for fatigue life prediction.

The model is validated against a total of 153 fatigue tests for two plain high strength concrete and two concretes reinforced with steel or polypropylene fibers, performed at two different stress ratios and four different loading frequencies. In addition, we have shown the failure is probabilistic in terms of the number of cycles N or the secondary strain rate $\dot{\varepsilon}$, but there is a one-to-one relation between N and $\dot{\varepsilon}$. In practical terms, this provides the possibility of determining the fatigue life N without actually exhausting all the cycles.

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Nomenclature

- σ_{max}: Maximum stress.
- σ_{min}: Minimum stress.
- $\triangle\sigma$: Stress range σ_{max}-σ_{min}.
- R: Stress ratio defined as $\sigma_{min}/\sigma_{max}$.
- σ_{max_0}: Static equivalence of σ_{max}.
- σ_f: Failure stress.
- σ_{f_0}: Intercept of the $IPFC$ with the σ_f-axis.
- f_{c_d}: Dynamic compressive failure stress or the dynamic strength under compression.
- f_{c_0}: Static failure stress or static strength under compression .
- σ_{min_0}: Horizontal asymptote which determines the lower stress value.
- $\dot{\sigma}_0$: Loading rate of the compressive test.
- $\dot{\sigma}_d$: Loading rate of the fatigue test.
- σ_c: Critical stress, can be the compressive strength f_c or the tensile strength f_t.
- N: Number of cycles to failure.
- λ, k: : Scale and shape parameter of the Weibull distribution.
- PF: Probability of failure in any point of the domain $\sigma_f - \ln N$.
- f: Loading frequency of a fatigue test.
- f_0: Reference loading frequency.
- a, b and c: Parameter that adjusts the relation between $\ln N$ and f, R.
- $\dot{\varepsilon}$: Secondary strain rate in a fatigue test.
- $\dot{\varepsilon}_0$: Reference secondary strain rate calculated as $\dot{\sigma}_0/E$.
- $\dot{\varepsilon}_i$: Intercept of the $\ln[\dot{\varepsilon}]-\ln[f]$ curve with the $\dot{\varepsilon}$-axis.
- α: Exponent that measures the amplification of dynamic strength.
- γ: Coefficient that takes into consideration of loading frequency for α.
- η: Slope of the $\ln\dot{\varepsilon}_i$-$\ln f$ curve.
- β: Material parameter for the fatigue equation proposed in [2].

REFERENCES

[1] Rusch H., 1960. Researches toward a general flexural theory for structural concrete. *ACI Journal*, **57**(1):1-28.

[2] Aas-jakobsen K., Sept. 1970. Fatigue of concrete beams and columns. *Bulletin No. 70-1, NTH Institutt for Betongkonstruksjoner, Trondheim.*

[3] Awad M.E. and Hilsdorf H.K., Feb. 1971. Strength and deformation characteristics of plain concrete subjected to high repeated and sustained loads. *Civil Engineering Studies, Structural Research Series* No. **372**, 266 pp., Department of Civil Engineering, University of Illinois, Urbana.

[4] Sparks P.R. and Menzies J.B., Sept. 1973. The effect of rate of loading upon the static and fatigue strengths of plain concrete in compression. *Magazine of Concrete Research (London)*, **25**(83):73-80.

[5] ACI Committee 215, 1974. Considerations for design of concrete structures subjected to fatigue loading. *ACI Journal*, 97–121.

[6] Tepfers R. and Kutti T., 1979. Fatigue strength of plain, ordinary and lightweight concrete. *ACI Journal*, **76**(5):635-652.

[7] Tepfers R., 1979. Tensile fatigue strength of plain concrete. *ACI Journal*, **76**(8):919-933.

[8] Holmen J.O., 1979. Fatigue of concrete by constant and variable amplitude loading. *PhD thesis, University of Trondheim, Norway*.

[9] Hsu T.T.C., 1981. Fatigue of plain concrete. *ACI Journal*, **78**:292-304.

[10] Furtak K., 1984. Ein Verfahren zur Berech. *Cement and Concrete Research*, **14**(6):855-865.

[11] Cornelissen H.A.W., 1984. Fatigue failure of concrete in tension. *HERON*, **29**(4):1-68.

[12] CEB Comite Euro-International Du Beton., 1998. Fatigue of concrete structures: State of the art report *Bulletin D'Information*, **188**:1-300.

[13] Oh B.H., 1991. Fatigue-life distributions of concrete for various stress levels. *ACI Materials Journal*, *88*(2):122-128.

[14] Hordijk D.A., Wolsink, G.M. and de Vries J., 1995. Fracture and fatigue behavior of a high strength limestone concrete as compared to gravel concrete. *HERON*, **40**(2):125-146.

[15] Zhang B., Philips D.V. and Wu K., Dec. 1996. Effect of loading frequency and stress reversal of fatigue life of plain concrete. *Magazine of Concrete Research*, **48**(177):361-375.

[16] Li H., Zhang M.H. and Ou J.P., 2007. Flexural fatigue performance of concrete containing nano-particles for pavement. *International Journal of Fatigue*, **29**:1291-1301.

[17] Zhao D.F., Chang Q.Y., Yang J.H. and Song Y.P., 2007. A new model for fatigue life distribution of concrete. *Key Engineering Materials*, **348-349**:201-204.

[18] Przybilla C., Fernández-Cantelli A. and Castillo E., 2011. Deriving the primary cumulative distributive function of fracture stress for brittle materials from 3- and 4-point bending tests. *Journal of the European Ceramic Society*, **31**:451–460.

[19] Ruiz G., Medeiros A., Zhang, XX., 2011 Experimental study of loading frequency effect on compressive fatigue behavior of plain and fiber reinforced concretes. *Anales de Mecánica de la Fractura*, **29**, 535–540.

[20] Castillo E., Fernández-Cantelli A. and Ruiz-Ripoll M.L., 2008. A general model for fatigue damage due to any stress history. *International Journal of Fatigue*, **30**:150–164.

[21] Castillo E., Fernández Cantelli A., Koller R., Ruiz-Ripoll M.L. and García A., 2009. A statistical fatigue model covering the tension and compression Wöler fields. *Probabilistic Engineering Mechanics*, **24**:199–209.

[22] CEB-FIP., 2008. Constitutive modeling of high strength/high performance concrete. *FIB Bulletin* **42**.

[23] Zhang X.X., Ruiz G., Yu R.C., Poveda E. and Porras R., 2012. Rate effect on the mechanical properties of eight types of high-strength concrete and comparison with FIB MC2010. *Construction and Building Materials*, **30**:301–308.

[24] Yu, R.C. and Ruiz, G., 2006. Explicit finite element modeling of static crack propagation in reinforced concrete. *Int. J. Fract.* **141**:357–372.