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ANALYTICAL PREDICTION OF CRACK WIDTH OF FRC/RC BEAMS UNDER SHORT AND LONG TERM BENDING CONDITION

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Abstract: It is well known that fibers are effective in modifying the cracking pattern development of concrete structural element, causing an higher number of cracks and, consequently, lower crack spacing values and narrower crack widths compared to the matrix alone. This effect could be exploited to improve durability of Reinforced Concrete (RC) structures, especially of those exposed to aggressive environments.

The analytical prediction of crack width and spacing in Fiber Reinforced Concrete (FRC) structural elements in bending is still an open issue. A crack width relationship for RC elements with fibers similar to those developed for classical RC structural members would be desirable for designers. The recent development of important technical design codes, such as RILEM TC 162 TDF and the new MC2010, embrace this idea. However further validation of these models by experimental results are still needed. On the other hand, the study of the influence of a sustained load on crack width in presence of the fiber reinforcement is a topic almost unexplored and important at the same time.

In the present work, the cracking behaviour of full-scale concrete beams reinforced with both traditional steel bars and short fibers has been analyzed under short and long term flexural loading. A theoretical prediction of crack width and crack spacing was carried out according to different international design provisions. The analytical results are discussed and compared in order to highlight the differences between the models and to check the reliability of the theoretical predictions on the basis of the experimental data.

1 INTRODUCTION

The presence of cracks in concrete may significantly influence the aesthetic of the structure as well as its durability. In this context, the ability of fibers in restraining crack width of concrete structural elements can be conveniently exploited to improve the durability of building and infrastructures and therefore the constructions sustainability.

The effectiveness of the fibers in reducing cracking in concrete structural elements depends on several factors, such as the type of fibers, their geometry, their amount as well as on parameters that normally influence the cracking phenomena in reinforced concrete elements, namely the reinforcement ratio, the concrete cover, the presence of stirrups, the bars diameter, the bars spacing, etc. The analytical prediction of crack width and spacing in FRC/RC elements in bending is still an open problem. In fact, to date there are not widely accepted relationships able to predict crack widths in presence of short fibers; this lack is still an obstacle to a widespread application of short fiber as crack-controlling reinforcement. According to Borosnyói and Balázs [1] the cracking process and the influence of fibers on the cracking development may be analysed at different levels of accuracy, related basically to four main approaches, namely analytical, semi-analytical, empirical and numerical. For practical uses and design purpose, the second or third approach are normally considered; in fact available Codes, as the Model Code [2] and the Eurocode 2 (EC2) [3], join this kind of approaches. As the crack width prediction of FRC/RC (Reinforced Concrete) is concerned, it would be highly desirable to provide design equations formally similar to those used for plain concrete, making the design approach much easier. The RILEM TC 162 TDF [4] and the new MC2010 [5] strongly embrace this idea. The RILEM TC 162 TDF provisions are based on the European pre-standard ENV 1992-1-1 [6] while the crack spacing expression is modified in order to take into account the presence of fibers.

The aforementioned design models need to be further validated by experimental studies. In fact, to date, there are few works in the literature which quantitatively relate the cracking behaviour (crack width and spacing) to FRC properties [7, 8 , 9, 10, 11]. Moreover, the influence of a sustained load on crack width in presence of the fiber reinforcement remains a topic almost unexplored in literature.

In the present study, the crack width and crack spacing relationships of RILEM TC 162 TDF and MC2010, together with EC2 [3] provisions, have been adopted for RC/FRC full scale beams, reinforced with steel or polyester fibers. The beams were tested under short and long term bending condition and their cracking pattern, namely crack width and crack spacing, was accurately registered during the tests at regular intervals. The theoretical predictions of crack width and spacing, carried out according to the available codes, were compared each other in order to evidence the main differences as well as the influence of empirical coefficients introduced by the different codes. They were also compared with experimental results in order to validate or propose modification to the available formulae.

2 EXPERIMENTAL PROGRAM

Two sets of beams, S1 and S2, were designed and poured: S1 beams were used for long term bending test while S2 beams were tested under monotonic load (short term bending test). One year after casting, the S1 beams were positioned under two loading steel frames; five beams per frame were piled up and loaded by means of a screw jack (Figure 1).

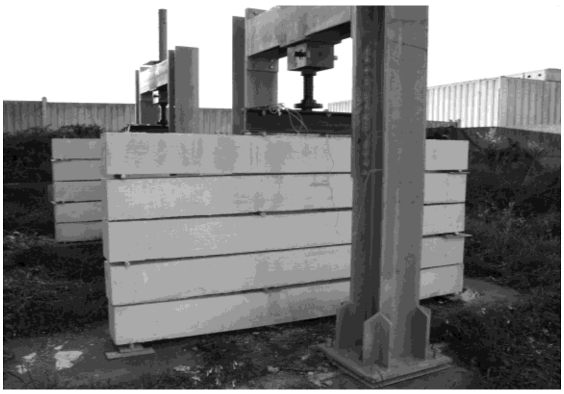


Figure 1: S1 beams under sustained loading

In order to simulate service conditions, a sustained load equal to 50 kN, about 50% of designed ultimate load, was applied. Two loading cells were placed under the screw jacks to monitor the applied load periodically. The beams of frame 1 were unloaded after seventeen months for laboratory testing, while the beams of frame 2 are still under load. During the seventeen months of loading, measurements on the cracking pattern, namely crack width, crack length and crack position were carried out, periodically. Up to 9 months, crack widths were measured by means of an optical scale loupe, with a precision of 0.05 mm. Afterwards an handheld digital microscope with 200x magnification was used. The crack widths were measured along each beams between the loading points, at the bottom of their tension side.

S2 beams were cast in laboratory and tested under a four point bending scheme up to failure after about two months from casting. The cracking pattern of each beam was accurately analyzed at five load steps: Step 1: 20kN; Step 2: 30kN; Step 3: 50kN; Step 4: 80kN; Step 5: 100kN. At each load step, the number of cracks, the crack widths and the crack lengths were registered. As for S1 beams, the crack widths were measured at the bottom of their tension side by means of a digital microscope with a magnification of 200x.

2.1 Materials

Three different concrete mixes were prepared for the S1 and S2 beams: a control mix without fiber reinforcement (TQ), a concrete mix embedding steel fibers with a 0.6% volume dosage (ST) and a concrete mix embedding polyester fibers with a 0.9% volume dosage (POL). The geometrical and mechanical properties of the fibers are summarized in Table 1 where L is the length of the fibers while D is their diameter.

Table 1: Geometrical and mechanical characteristics of the fibers

	L	L/D	Tensile Strength	Elastic Modulus
ST	30 mm	50	> 1150 N/mm2	210 x103 N/mm2
POL	30 mm	66	400-800 N/mm2	11.3 x103 N/mm2

All the mixes had a water/cement ratio equal to 0.65, a cement type 32.5R II-A/LL and a workability class S5. Four cubes (150 mm side) for each mix (TQ, ST and POL) were cast for quality control. In Table 2, the values of the compressive strength obtained after 28 days from casting are reported.

In Table 3, the mechanical properties of longitudinal bars and stirrups employed in the beams, determined following UNI EN ISO 15630-1 [12] are reported. The nomenclature of the S1 and S2 beams is given in Table 4.

Table 2: Cube (150 mm) compressive strength

	Beam	Cube strength (MPa)	COV (%)
S1	TQ	25.8	4
	ST	21.4	8
	POL	23.2	8
S2	TQ	22.70	6
	ST	19.84	6
	POL	22.65	5

Table-3:-Mechanical properties of steel bars

	Diam.	Yield strength	Ultimate strength	Max Elong.
Longit.bars	14mm	520 MPa	614 MPa	12.2 %
Stirrups	8mm	567 MPa	600 MPa	4.8 %

Table-4:-Specimen labels

Series	Beam typology	Beam code
S1	Exposed beams – Frame 1	TQ1_E, ST1_E, ST2_E, POL1_E, POL2_E
S1	Exposed beams – Frame 2	TQ2_E, ST3_E, ST4_E, POL3_E, POL4_E
S2	Not exposed beams	TQ1, TQ2, ST1, ST2, POL1, POL2

2.2 Beams details

Rectangular reinforced concrete beams were designed according to Italian Code and EC2 [3, 13], by adopting the loading scheme shown in Figure 2: it is a four point bending scheme with a 280 cm span length and a 90 cm distance between the two loading points. The amount of steel reinforcement was calculated in order to have a ductile bending failure of the beam, with concrete crushing after steel yielding. Vertical stirrups were also provided to prevent premature shear failure, in accordance with the design code. Figure 2 shows the geometry of the beams and bars details. Three 14 mm diameter bars were placed at the tension region and two 14 mm diameter bars were placed at the upper compression region of the beam as longitudinal reinforcement, while 8 mm diameter stirrups were placed at 14 cm over the entire length of the beam, except near the supports where the spacing was 7 cm.

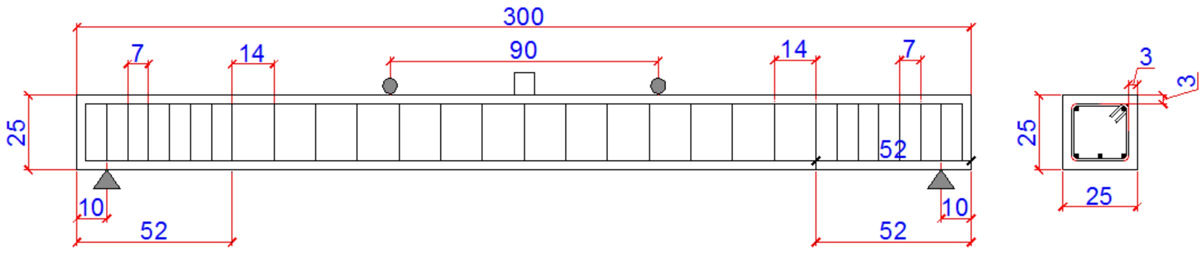


Figure 2: Loading scheme

3 EXPERIMENTAL RESUTS

3.1 Long term bending test

Position, width and length of the cracks of each S1 beam were registered during the loading period. The results of the measurements are extensively reported and commented in [14]. In Figures 3 and 4, the average crack width measured between the two loading points of each beam under frames 1 and 2, respectively, are reported versus time. The crack width values measured on FRC beams were lower than those measured on plain concrete beams (TQ). This effect increased with the loading time. In fact, the crack width in FRC beams (ST and POL) seems to be stabilized after ten months of exposure, while that in TQ beams continued to grow until the last measurement.

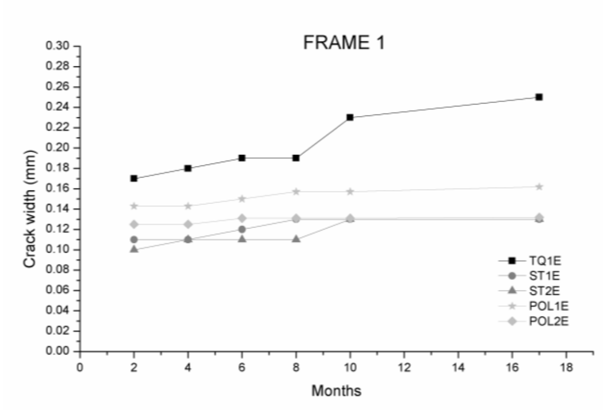


Figure 3: Average crack width vs time

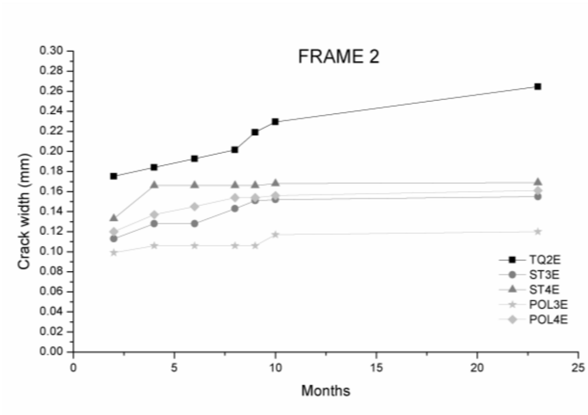


Figure 4: Average crack width vs time

The number of cracks did not change during the period in which regular monitoring was performed. Figures 5 and 6 show the average crack spacing calculated between the loading points of each beam of frame 1 and 2, respectively.

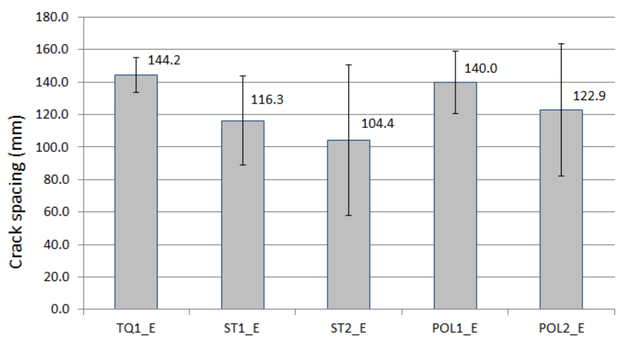


Figure 5: Average crack width after 17 months of loading (S1 beams in frame 1)

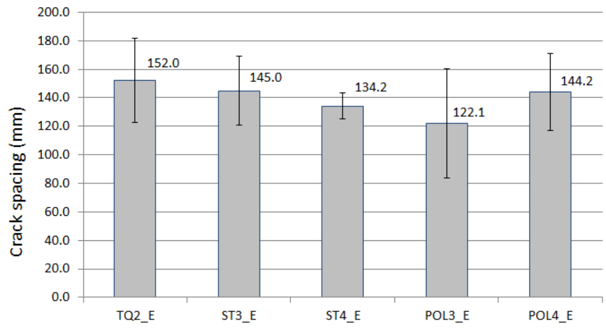


Figure 6: Average crack width after 17 months of loading (S1 beams in frame2)

From a statistical analysis of variance (ANOVA) it results that, at 5% level of significance, the presence of the adopted fibers did not influence the crack spacing that seems mainly depend on the stirrup spacing, equal to 140 mm. Similar results were found by Tan et al. [8].

3.1 Short term bending test

The results of tests carried out on S2 beams in bending are extensively described and commented in [15]. Crack width measurements were made on S2 beams during the laboratory tests up to failure. In order to compare results of short and long term loading conditions, the average crack widths of S2 beams under the service load (50kN) are considered in Figure 7.

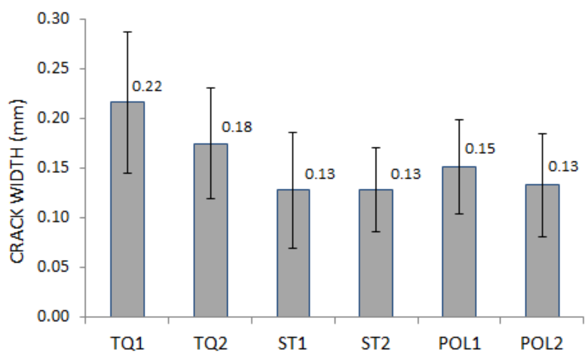


Figure 7: Average crack width of S2 beams at 50kN

Comparing the long and short term crack width values, it can be noticed that the mean value of crack opening of TQ beams change from 0.20 mm in short term loading to 0.24 mm in long term loading, while the FRC crack width change slightly with the loading condition. Thus, the presence of fibers seems to reduce the crack growth with age respect to the behavior observed in plain concrete beams.

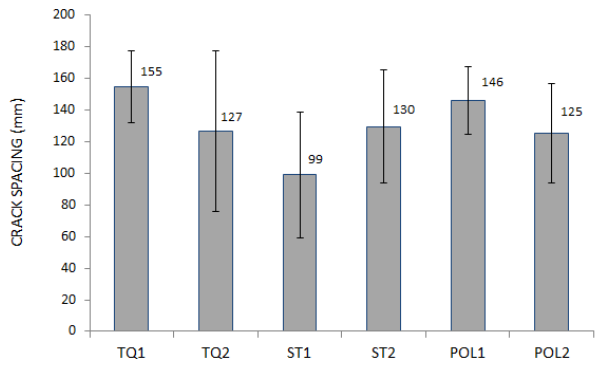


Figure 8: Average crack spacing of S2 beams at 50kN

In Figure 8, the average crack spacing calculated between the loading points of each S2 beam is reported. As for S1 beams, an high scatter of results can be observed: from a statistical analysis of variance (ANOVA) it results that, at 5% level of significance, the presence of fibers did not influence the crack spacing values. As in the case of long term loading, the crack spacing is mainly affected by the presence of stirrups.

4 ANALYTICAL PREDICTION OF CRACK WIDTHS

An analytical prediction of crack width for S1 and S2 beams was performed on the basis of the recommendations available in Eurocode 2 [3], New MC 2010 [5] and RILEM TC 162-TDF [4]. While the Eurocode2 provisions refer only to plain concrete beams, the MC 2010 and RILEM TC 162-TDF account for the presence of short fibers and consider a specific formulation for FRC structural members. Eurocode2 has been extended herein to the case of FRC, taking into account the contribution of fibers in tension when evaluating the stress distribution within the cracked cross section. Specifically, a constant stress (f_{Fts}) distribution over the tension part of the cross-section is adopted. Following Italian CNR DT-204-2006 guidelines [16], the value of f_{Fts} is given by:

$$f_{Fts} = 0.45 f_{eq1} \tag{1}$$

where f_{eq1} is the post-cracking strength obtained by tests performed on notched beams in four point bending condition according to UNI 11039 [17]. In Table 5, the values of f_{Fts} calculated from the test results on ST and POL small-size notched beams are reported [14].

Table-5:-Specimen labels

	ST beams	POL beams
f_{Fts}	1.1 MPa	0.54 MPa

4.1 EUROCODE 2

According to EC2, the maximum crack width should be calculated as follows:

$$w_{max} = s_{r,max} (\varepsilon_{sm} - \varepsilon_{cm}) \quad (2)$$

where $s_{r,max}$ is the maximum crack spacing, and ε_{sm} and ε_{cm} are the average strains of the steel bars and the concrete in tension, respectively, over the length $s_{r,max}$. The maximum value of crack width is related to the average value (w_m) by the expression:

$$w_{max} = \beta\, w_m \quad (3)$$

where β is a statistical coefficient equal to 1.7 [1, 6].

The difference between steel and concrete strains (ε_{sm}-ε_{cm}) in Eq.2 is given by:

$$\varepsilon_{sm} - \varepsilon_{cm} = \sigma_s / E_s - k_t\, f_{ctm} / (E_s\, \rho_{s,eff})(1 + \rho_{s,eff}\, \alpha_e) \quad (4)$$

where α_e is the ratio between E_s (= steel modulus of elasticity) and E_c (= concrete modulus of elasticity); $\rho_{s,eff}$ is the ratio between A_s, that is the whole area of the longitudinal reinforcement, and A_{ce}, that is the effective area of the concrete in tension.

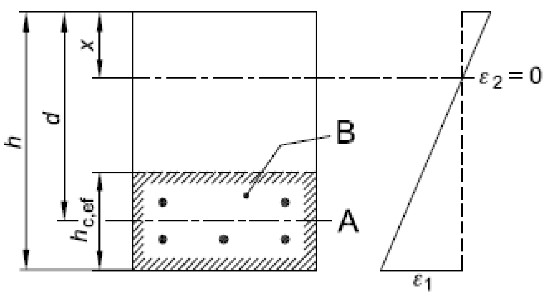


Figure 9: Ace for RC beams in bending (EC2)

The value of A_{ce} is obtained multiplying the width of the section (B) for $h_{c,eff}$, equal to the minimum value between 2.5 (h -d), (h -x)/3 and h /2 (Figure 9).

The coefficient k_t is set equal to 0.6 for short-term loading condition and 0.4 for long term or cyclic loading; σ_s is the stress in the tensile reinforcement calculated in a cracked section under the applied external load. In this work, the value of σ_s for FRC beams has been calculated considering the contribution of the short fibers in tension, as mentioned above.

The crack spacing ($s_{r,max}$) has the following semi-empirical formulation:

$$s_{r,max} = k_3\, c + k_1 k_2 k_4 \phi_s / \rho_{s,e} \quad (5)$$

where c is the concrete cover thickness (mm) and ϕ_s is the bar diameter (mm). The EC2 suggests to set k_3 equal to 3.4 and k_4 to 0.425; k_1 is a coefficient which accounts for the bond properties of steel bars (= 0.8 for corrugated bars and = 1.6 for smooth bars); k_2 is a coefficient which takes account of the form of strain distribution along the cross section (= 0.5 for bending and = 1 for pure tension).

In the case of long term loading, the effect of shrinkage (ε_{cs}) must be taken into account when evaluating the concrete strain (Eq. 2).

4.2 RILEM TC 162- TDF

The RILEM TC 162- TDF proposes a crack width formulation which takes into account the presence of steel fibers. Starting from [6], the crack width is calculated according to:

$$w_k = \beta\, s_{r,m}\, \varepsilon_{sm} \quad (6)$$

in which β is a coefficient (equal to 1.7 for load induced cracking) relating the average crack width to the maximum crack width; $s_{r,m}$ is the average crack spacing, which has a semi-empirical formulation:

$$s_{r,m} = (50 + 0.25\, k_1\, k_2\, \phi_s / \rho_{s,eff})\, (k\phi / L) \quad (7)$$

and ε_{sm} is the average steel strain, which takes into account the additional contribute of concrete in tension (tension stiffening) and is expressed as:

$$\varepsilon_{sm} = \sigma_s / E_s\, [1 - \beta_1\, \beta_2\, (\sigma_{sr} / \sigma_s)2]\, (k\phi / L) \quad (8)$$

where k is set equal to 50, k_1 and k_2 have the same values already specified in Eq. (4); β_1 is a coefficient which takes account the bond properties of the bars (1 for ribbed bars and 0.5 for smooth bars); β_2 is a coefficient which takes account of the duration of the loading or of repeated loading (1 for single short term loading, 0.5 for sustained load or for many cycles of repeated loading); ϕs is the bar diameter (mm); $\rho_{s,eff}$ is the ratio between A_s, and A_{ce}. The calculation of A_{ce} is that reported in EC2, assuming $h_{c,eff}$ equal to 2.5 (h-d) (Figure 9). σ_s is the stress in the tensile reinforcement calculated at the cracked section; σ_{sr} is the stress in the tensile reinforcement calculated at a cracked section under loading conditions causing the first cracking; L is the length of steel fiber (mm) and ϕ is its diameter (mm). For FRC, σ_s and σ_{sr} are determined taking into account the post cracking tensile strength of fiber reinforced concrete in the hypothesis of a constant stress (f_{Fts}) distribution over the tension part of the cross-section (Eq.1). In the case of long term loading, the contribute of shrinkage has to be taken into account for the evaluation of ε_{sm} (Equation. 8).

4.3 MODEL CODE 2010

The new MC 2010 suggests two distinct formulations for plain and fiber reinforced concrete structural members. In both cases the maximum crack width can be calculated as:

$$w_k = 2l_{smax} (\varepsilon_{sm} - \varepsilon_{cm} - \varepsilon_{cs})(k\phi/L) \qquad (9)$$

where l_{smax} is the length (mm) over which slip between concrete and steel occurs. ε_{sm} and ε_{cm} are the average strains of steel bars and concrete, respectively, over the length l_{smax}. ε_{cs} is the strain of the concrete due to free shrinkage. The average crack width can be calculated by dividing the maximum crack width (Eq. 9) for 1.5 [1].

l_{smax} has two different expression for plain and fiber reinforced concrete (Eqs. 10 and 11, respectively):

$$l_{smax} = k\, c + f_{ctm}\, \phi_s /(4\tau_{bm}\, \rho_{s,eff}) \qquad (10)$$

$$l_{smax} = k\, c + (f_{ctm} - f_{tsm})\, \phi_s /(4\tau_{bm}\, \rho_{s,eff}) \qquad (11)$$

where f_{tsm} (eq.1) is the residual tensile strength of FRC equal to 0.45 f_{R1} (similar to f_{Fts} of CNR DT 204-2006 that is equal to $0.45f_{eq(0\text{-}0.6)}$ [18]). The relative mean strain in Eq. (9) follows from:

$$\varepsilon_{sm} - \varepsilon_{cm} - \varepsilon_{cs} = (\sigma_s - \beta\, \sigma_{sr})/Es + \eta_r \varepsilon_s h \qquad (12)$$

where σ_s is the stress in the steel rebars at a cracked section, in which the effect of fibers needs to be taken into account; σ_{sr} is the maximum steel stress in a cracked section at the crack formation stage, which is:

$$\sigma_{sr} = f_{ctm}\, (1+\rho_{s,eff}\, \alpha_e) / \rho_{s,eff} \qquad (13)$$

for plain concrete and:

$$\sigma_{sr} = (f_{ctm} - f_{tsm})\, (1+\rho_{s,eff}\, \alpha_e) / \rho_{s,eff} \qquad (14)$$

for FRC.

The values of τ_{bm} is equal to 1.8 f_{ctm} for stabilized cracking in both short and long term loading. β is equal to 0.6 and 0.4, for short and long term loading, respectively. η_r is equal to 0 or 1, for short and long term loading, respectively. $\rho_{s,eff}$ and α_e are those reported in EC2 (4.1). When comparing to Model Code 1990 [2], the following parameter has been introduced:

$$(h-x)/(d-x) \qquad (15)$$

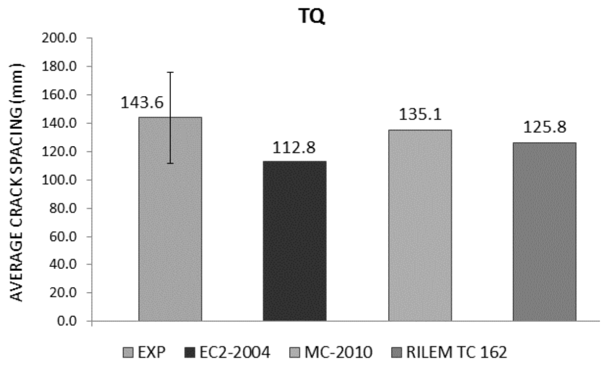
That, multiplying the w_k value of Eq (9), allows to calculate the crack width at the bottom of the tension side of the beams.

5 COMPARISON BETWEEN THE ANALYTICAL AND EXPERIMENTAL RESULTS

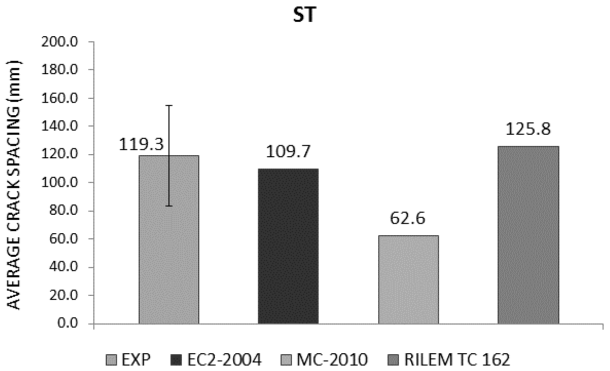
5.1 CRACK SPACING

In Figure 10, the values of the average crack spacing experimentally obtained and the theoretical values calculated according to EC-2, MC 2010 and RILEM TC 162 design codes are reported. From the experiments it was observed that the crack spacing did not change with time (3.1); thus, the average experimental value used in the comparison was calculated as the mean value of the average crack spacing of S1 and S2 beams.

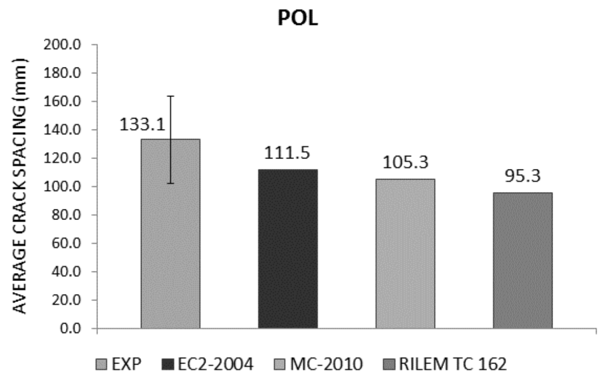
The crack spacing prediction obtained by RILEM TC 162 and MC 2010 relationships was found to be in good accordance with experimental results for TQ beams (Figure 10a), while the EC2 formulation underestimates (21%) the experimental values. The main difference between RILEM TC 162 and EC2 lies in the $h_{c,eff}$ value that results lower in the EC2 (see Sections 4.1 and 4.2).



(a)



(b)



(c)

Figure 10: Comparison between experimental and theoretical average crack spacing (a: TQ beams; b: ST beams; c: POL beams)

A worse comparison with experimental results is given by MC2010 in ST beams. In fact, the code provision strongly under-estimates (48%) the experimental crack spacing. MC 2010 differs from the other codes as it considers the residual tensile strength of FRC (f_{tsm}). This effect causes a lower value of the average crack spacing compared to the other formulations. According to the Authors' opinion, the poor accordance with the experimental results may be related to the presence of stirrups, which strongly influence the crack spacing of the tested ST beams. For this reason, a modification of the crack spacing formula is proposed (Equation 16) to take into account of the presence of stirrups in the case of steel fiber reinforced beams:

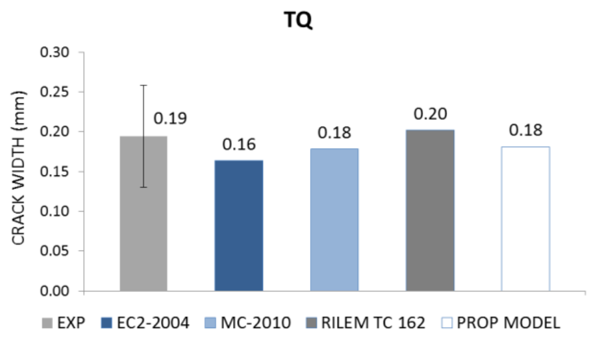
$$s_{pm} = (2l_{smax}/1.5 + s_{st})/2 \qquad (16)$$

where s_{pm} is the proposed mean value of crack spacing; l_{smax} refers to Eq. (11), 2 l_{smax} is the maximum crack spacing and 2 l_{smax} /1.5 is the average crack spacing according to MC 2010; s_{st} is the spacing between stirrups. By applying Eq. 16 to the tested beams, a value of 101.3 mm is obtained as cracks spacing, that is narrower to the experimental results (119.3 mm). Equation (16) can be applied also to TQ and POL beams, obtaining a crack spacing of 137.6 mm and 122.7 mm, respectively; both values are found to be in accordance with the experimental ones (143.6 mm and 133.1 mm for TQ and POL beams, respectively).

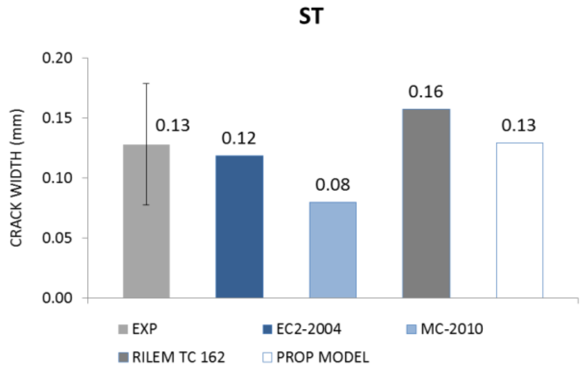
Referring to RILEM TC 162 predictions, for the analyzed ST beams the product between k and ϕ/L in Equation 7 is equal to 1, thus there are no differences in the relationship of crack spacing between ST and TQ beams; a good accordance with experimental results is still confirmed. More experimental research with other values of the fiber aspect ratio is needed to validate the RILEM TC 162 prediction, as well as the influence of stirrups. A different result has been obtained in the case of POL beams since the product between k and ϕ/L is equal to 0.75. In this case, an underestimation (28%) of the crack spacing value with respect to the experimental one can be observed (Fig. 10c). Probably, with fibers different from steel, another value of k (Eq. 7) should be considered. Furthermore, more research is needed to evaluate the influence of the presence of stirrups on crack spacing.

5.2 CRACK WIDTH UNDER SHORT TERM LOADING

In Figure 11, the experimental average crack width obtained for S2 beams and those analytically evaluated by the code's equations are reported. Furthermore, the average crack width obtained by applying the MC2010 formulation with an average crack spacing calculated according to Eq. 16 ("proposed model" in the graph) is also added.



(a)



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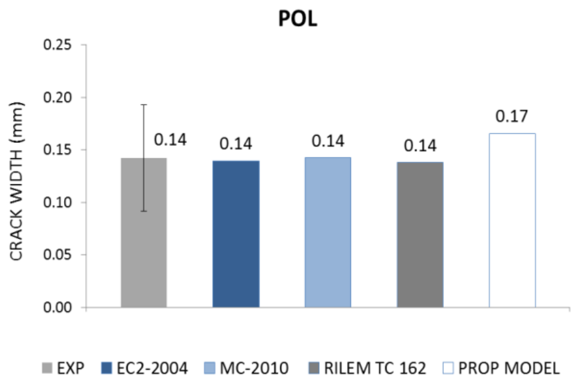


(c)

Figure 11: Comparison between experimental and theoretical average crack width of S2 beams (a: TQ beams; b: ST beams; c: POL beams).

All the reported values refers to the crack opening at the bottom of the beams; thus, for each code formulation, the crack opening, calculated at the steel bar level, has been multiplied for the factor of Eq. 15.

All the Codes investigated (included the proposed model) give a good prediction of experimental average crack width of TQ beams.

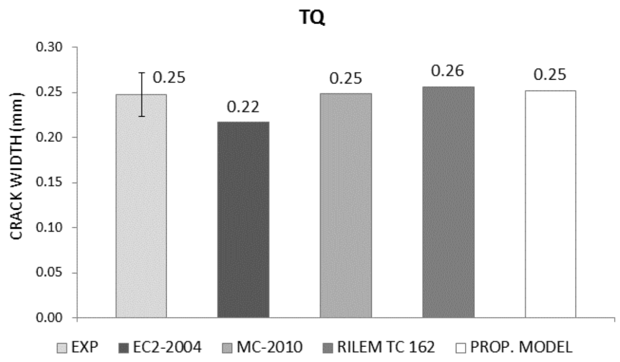
In ST beams, the EC2 prediction multiplied by the factor of Eq. 15 provides a value of crack width equal to 0.12 mm, that is in good agreement with experimental results . RILEM TC 162 TDF slightly overestimate the average crack width experimentally obtained, remaining in any case within the scatter of experimental results.

Model Code 2010 strongly underestimates (30%) the experimental crack width. This is mainly due to the low value of crack spacing obtained from Eq. 13, which does not consider the presence of stirrups. By using the crack spacing value obtained from the proposed relationship (Eq. 16), an average crack width of 0.13 mm is obtained, that is closer to the experimental results (Fig. 15b).

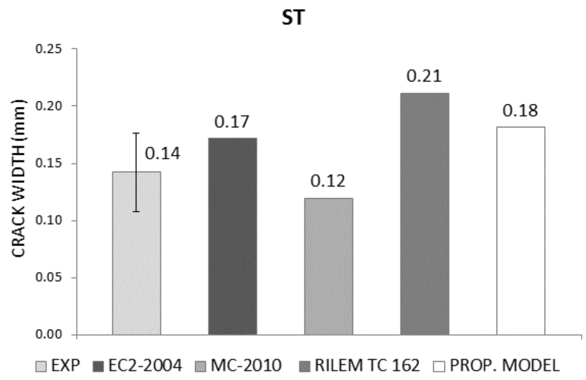
Referring to POL beams, the MC 2010, EC2 and RILEM TC 162 predictions (0.14 mm) are in good accordance with the experimental value (0.14 mm), even if the average crack spacing value of the RILEM TC 162 underestimates the experimental values (5.1). The average crack width of the proposed model slightly overestimates the experimental crack width, remaining still in the scatter range of results.

5.3 CRACK WIDTHUNDER LONG TERM LOADING

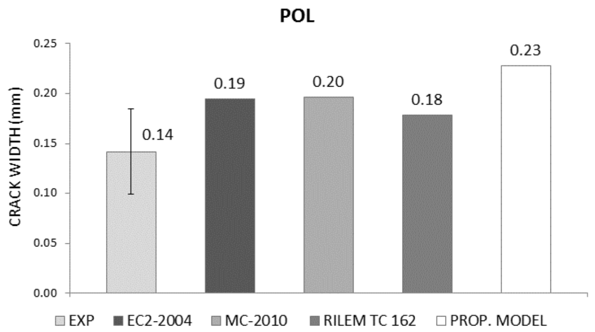
Figure 12 exhibits the average crack width obtained from long term bending test and by codes provisions. The graphs also shows the crack width determined according to MC2010 formulation by considering the average crack spacing (Eq. 16) as well as the proposed model. In the codes, the delayed concrete strains due to the effect of time can be considered as the sum of two components: a stress-dependent strain and a stress-independent strain. With temperatures ranging between 20°C and 40°C, the stress independent component considered by the codes corresponds to the free shrinkage of concrete. In order to take account the effect of loading time, a factor which multiplies concrete strain is considered, that is β in Eq. 12, β_2 in Eq. 8 and k_t in Eq. 4. In the present study, the concrete strain due to free shrinkage has been calculated according to the codes, considering a relative humidity equal to 78% and a period of drying of 868 days.



(a)



(b)



(c)

Figure 12: Comparison between experimental and theoretical average crack width of S1 beams (a: TQ beams; b: ST beams; c: POL beams)

Results from TQ beams showed that all the analytical models predict well the value of average crack width (also in the case of proposed model). In particular, the EC2 formulation slightly underestimates the average crack width obtained from experiments, as it was underlined in case of short term loading.

Referring to ST and POL beams, a little increment in experimental crack width can be observed by comparing long and short term results. On the contrary, for TQ beams the crack width increased from 0.19 mm of S2 beams (short term) to 0.25 mm for S1 beams (long term). For TQ beams the codes predict well this increment in crack width due to the effect of time, but applying the same formula to ST and POL beams, the effect of shrinkage and of loading on concrete strains appears overestimated. In Table 6, the value of average crack width predicted by the codes for short and long term loading are reported; the value of Δ_{time} evidences the increment of crack width estimated by the codes due to the effect of time.

Table–6:–Comparison between crack width with short and long term loading.

	ST BEAMS		
	LONG TERM	SHORT TERM	Δtime
EXP	0.14 mm	0.13 mm	8%
MC 2010	0.2 mm	0.08 mm	150%
EC-2	0.17mm	0.12mm	42%
RILEM TC 162	0.21mm	0.16mm	31%
	POL BEAMS		
	LONG TERM	SHORT TERM	Δtime
EXP	0.14 mm	0.14 mm	0%
MC 2010	0.2mm	0.14mm	43%
EC-2	0.19mm	0.14mm	36%
RILEM TC 162	0.18mm	0.14mm	29%

The values of Δ_{time} for all the codes are not in accordance with the experiments which evidenced a negligible increment in crack width due to the effect of time. The experimental results would evidence the significant contribution of fibers on long-term condition and, therefore, on structural durability which is not adequately considered by the present codes provisions. On the basis of this interesting results, a wider experimental research is suggested to correctly quantify the effect of fibers on the free shrinkage and long term loading in the crack width prediction.

6 CONCLUSIONS

An analytical prediction of the crack spacing and crack width of FRC/RC beams under bending loads has been carried out according to different codes, namely EC2, MC2010 and RILEM TC 162-TDF.

From the comparison between the theoretical predictions and experiments, it results that the average crack spacing given by design codes are in good accordance with experimental results referred to plain concrete beams, especially the MC 2010 provisions.

In case of FRC beams, the EC2 prediction of crack spacing is in good agreement with experimental results even if a specific formulation for fiber reinforced concrete is not considered. On the contrary, the MC 2010 prediction underestimates the experimental results especially for ST beams. It is opinion of the Authors that this is due to the presence of stirrups which are not considered by the code but strongly influences the crack spacing in the experiments. A good prediction of crack spacing is obtained by a modified relationship, proposed by the Authors, which takes into account the presence of stirrups. The RILEM TC 162 TDF prediction is in good accordance with the crack spacing obtained for ST beams for the specific analyzed aspect ratio, while it underestimates the crack spacing obtained for POL beams.

The results of short term crack width prediction of MC 2010 are in good accordance with the experiments in the case of TQ and POL beams, while for ST beams a good prediction of crack width is obtained considering the effect of stirrups (proposed model) on crack spacing.

RILEM TC 162 TDF and EC2 results are in accordance with the experiments when the increase in crack width at the bottom of the beam is considered.

Looking at crack width under long term loading, it has been found that all the codes are in good accordance with the experiments from TQ beams. However, when fibers are present (both steel and polyester), all the codes estimated an increment in crack width due to the effect of time which does not correspond to the experimental evidence.

This underlines the importance of FRC for better controlling crack development in RC beams, especially for long term loading. A smaller crack width also provides a better resistance to the penetration of aggressive agents into the beam that means an enhanced durability.

A future experimental research in the field is strongly recommended because of the little results available into the literature; in fact, the effects of FRC on the free shrinkage and long term loading of RC beams remains an important challenge for researchers.

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