VIII International Conference on Fracture Mechanics of Concrete and Concrete Structures
FraMCoS-8
J.G.M. Van Mier, G. Ruiz, C. Andrade, R.C. Yu and X.X. Zhang (Eds)

ANALYTICAL ESTIMATION FOR MECHANICAL CHARACTERISTICS OF CONCRETE AS A PERMEABLE MATERIAL

HIDEKI OSHITA *

* Chuo Unversity
1-13-27 Kasuga, Bunkyo-ku, Tokyo, 112-8551 Japan
e-mail: oshita@civil.chuo-u.ac.jp

Key words: Two Phase Material, Liquid Migration, Surface Tension, Cavitation

Abstract: It is well known that the static and dynamic strength of cement based material are influenced by the water content, moreover its value is greatly dependent on the surface tension of the liquid. However, the mechanism for the effect of the surface tension of liquid on the solid strength has not been investigated yet. In the beginning, the cavitation phenomenon is incorporated to the developed model which is assumed to be a two phase porous material of solid and liquid and then the effect of the erosion of bubbles on the solid phase is estimated and compared with the experimental results.

1 INTRODUCTION

It is noted that the static or dynamic strength of the cement based materials such as the compressive, tensile and flexural strength are influenced by the water content and then it is reported that the gradual decrease of these behavior occur with the increase of water content [1][2][3][4]. Benedicks[3] and Hori[5] pointed out that the microcracks easily occur and propagate for the wetting specimens since the surface energy of solid phase decrease for the surface energy of liquid phase due to the presence of liquid. Therefore, it may be seen that the strength varies by the liquid type, namely the decrease of the strength of cement based materials occur with the increase of the surface tension of liquid [3][5][7][8]. However, the decrease of the strength simply is defined as an index such as the interface energy between the solid and liquid phase and therefore, its mechanism does not have been investigated yet.

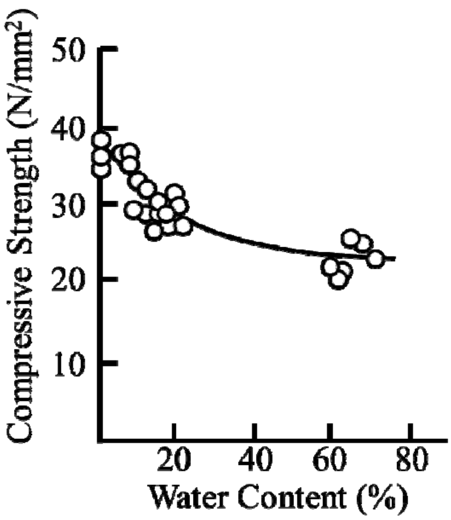
On the other hands, Oshita et al.[6] pointed out that the water migration are expected to significantly affect the mechanical characteristics and then the positive pressure such as a disjoining pressure occurring in the elastic region causes the microcracks and the negative pressure occurring tin the plastic region causes the crush between the solid phases due to the experimental and analytical procedures.

In this study, an extended mathematical model that incorporated the cavitation phenomenon in the model developed by authors[6] which is assumed to be a two phase porous material of solid and liquid is developed to investigate in microlevel the detailed mechanism such that the liquids with the different surface energy and its content influences on the decrease of the strength.

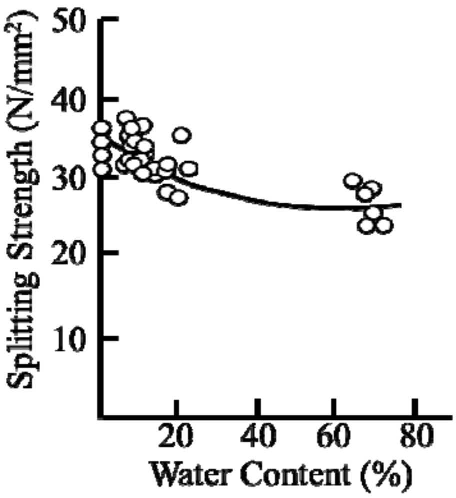
2 MECHANICAL BEHAVIOR DUE TO PRESENCE OF LIQUID

2.1 Influence of Water Content on Strength

The representative research referring the effect of water content in concrete on the various strength is shown in figure 1. Figure 1



(a) Compressive Strength



(b) Splitting Strength

Figure 1: Effect of Water Content.

shows the relationship between the strength and water content[2].

It can be seen that the compressive and tensile strength gradually decreases with the increase of the water content and therefore its value with drying is the most high. Okajima[2] expressed the dependency of water content on the tensile strength by modified the Griffith theorem taken into the reduction of the surface energy of solid by the adsorption of vapors account, as shown in following equation. However, its model does not show an agreement with the experiments since the stress concentration in the tip of cracks and microcracks are taken the Griffith theorem itself into account.

$$\sigma_t = \sqrt{\frac{2E(\gamma_S - \Delta\gamma)}{\pi C}} \qquad (1)$$

where σ_t is the tensile strength, E is a young's modulus, C is half length of crack, γ_S is a surface energy of solid and $\Delta\gamma$ is a interfacial energy between the solid and liquid which is a function of the relative humidity shown as Gibbs equation.

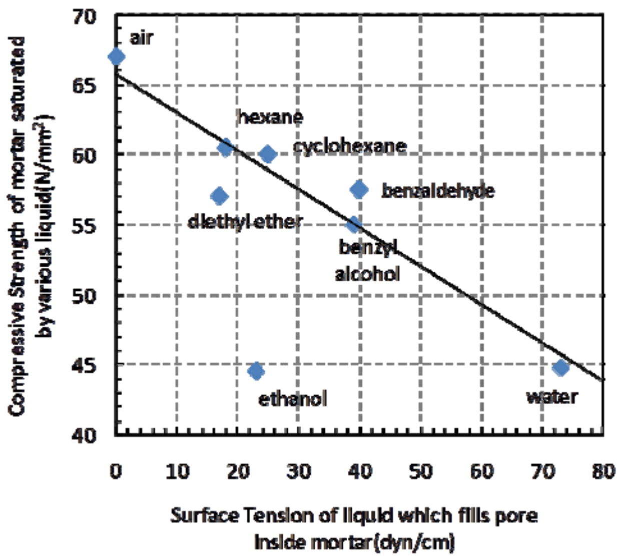
2.2 Influence of Surface Tension of Liquid on Strength

The representative research referring the strength characteristics of the mortar in which the pore are completely saturated the various liquids with the different surface tension is shown in figure 2. Figure 2 shows the relationship between the strength and the surface tension of the liquid.

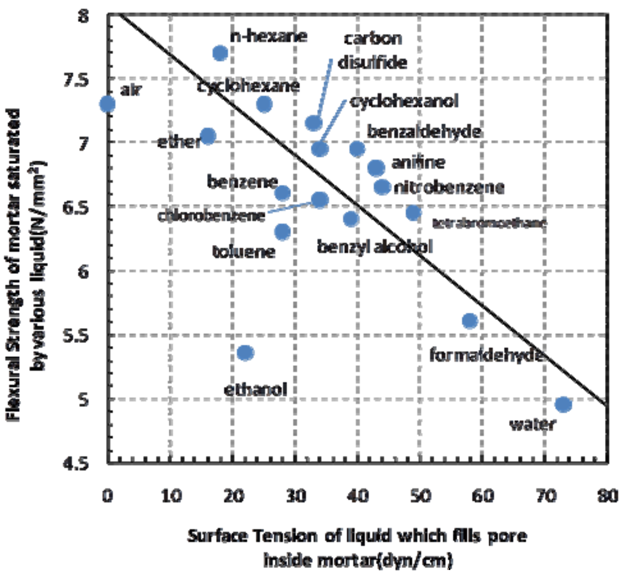
It can be seen that the decrease of each strength linearly occur with the increase of the surface tension except for the ethanol and methanol. It may be said that such strength behavior dependent on the surface tension of the liquid are caused by that the microcracks easily occur and propagate due to the reduction of the surface energy of the solid for the surface energy of liquid [5]. Namely, the strength seems to be proportional to the interface energy between the solid and liquid as shown in the following equations.

$$\gamma_{sl} = \gamma_s - \gamma_l \;\rightarrow\; \sigma \propto \gamma_s - \gamma_l \qquad (2)$$

where γ_{sl}, γ_s and γ_l are interface energy between solid and liquid, surface energy of solid and liquid, respectively. σ is the strength. However, Matsushita[8] pointed out that the effect of the surface tension of the liquid on the strength behavior is different due to the water cement ratio (W/C) of the mortar and concrete material and then its effect becomes higher with the increase of W/C. This fact is that the strength behavior is dependent on the surface tension of the liquid, but not directly the interface energy between the solid and liquid. Therefore, the mechanism related the surface tension of the liquid to the occurrence



(a) Compressive Strength



(b) Flexural Strength

Figure 2: Effect of Surface Tension of Liquid.

and progress of cracks and the reduction of the strength are remained to be not clear. There is no doubt that the strength characteristics of the porous material such as the mortar and concrete are influenced by the surface tension of the liquid.

3 COUPLED ANALYTICAL MODEL WITH LIQUID AND SOLID PHASE

3.1 Modelling of Concrete by Saturated Porous Two Phase Material

In the analysis which is a coupled model In the analysis which is a coupled model with liquid and solid phase of concrete structure (called as Analysis 1) developed by authors[7], concrete is regarded as a porous material which is composed of aggregate, cement paste and water. Aggregate is considered as perfectly elastic material while cement paste is assumed to behave as an elasto-plastic permeable material.

(a)Formulation of equilibrium equation

With the presence of pore water pressure p, the relation of effective stress $\{\sigma'\}$ and total stress $\{\sigma\}$ will become

$$\{\sigma\} = \{\sigma'\} - \{m\}p \qquad (3)$$

where the sign of tensile stress is taken as positive and $\{m\} = \{1\ 1\ 1\ 0\ 0\ 0\}^T$.

The incremental effective stress-strain relation for concrete can be written as

$$d\{\sigma'\} = [D_S^{ep}]\left(d\{\varepsilon^t\} - d\{\varepsilon^{pr}\} - d\{\varepsilon^h\} - d\{\varepsilon^{cr}\}\right) \qquad (4)$$

where $[D_T^{ep}]$ is the elasto-plastic stiffness matrix of concrete where voids are not saturated with water, and $d\{\varepsilon^{pr}\}$ is the incremental strain of solid phase resulted from the incremental pore water pressure $d\{p\}$. Elasto-plastic stiffness of concrete can be written with the use of average elasto-plastic stiffness matrix of solid phase $[D_S^{ep}]$ and porosity ξ as follows.

$$[D_T^{ep}] = (1-\xi)[D_S^{ep}] \qquad (5)$$

$$d\{\varepsilon^{pr}\} = -D[D_S^{ep}]^{-1}\{m\} \qquad (6)$$

Then by using the principle of virtual work and appropriate shape function, the equilibrium equation can be written in differential form as

$$K_T\frac{d\{\bar{u}\}}{dt} - L\frac{d\{\bar{p}\}}{dt} - \frac{d\{\bar{f}\}}{dt} = 0 \qquad (7)$$

where matrices K_T and L are the tangential stiffness and effect of both pore water pressure and volume change of solid phase, respectively. The vector $\{f\}$ denotes the effect of external force on displacement. These notations can be defined as follows.

$$K_T = (1-\xi)\int_\Omega B^T D_s^{ep} B\, d\Omega \quad , \quad L = \xi\int_\Omega B^T \{m\} \overline{N}\, d\Omega$$
$$\{\bar{f}\} = \int_\Omega N^T \{b\} d\Omega + \int_\Gamma N^T \{t\} d\Gamma \qquad (8)$$

where N and $\overline{N}$ are the shape functions for displacement, pore water pressure ,while B is the strain-displacement matrix.

(b) Formulation of Flow Continuity Equation

Water head h can be written as

$$h = z + \frac{p}{\gamma} \qquad (9)$$

where z is the vertical coordinate of the point which is positive for upward direction, γ is the specific gravity of fluid. From the mass conservation law, the volume change in a unit volume ΔV is equal to the difference of the amount of inflow q and outflow from this volume, which can be written as

$$\Delta V = q - \nabla^T \{v\} \qquad (10)$$

where v is the flow velocity, which is considered to follow Darcy's law. The factors which contribute to the volume change can be summarized as follows.

Due to total strain change

Due to the change of volume of particles caused by changes of hydrostatic pressure

Due to the change of fluid volume

Due to the change of particle size by effective stresses

Substituting these factor into Eq.(10) and applying the Galerkin's method, Eq.(10) becomes in differential form as

$$H\{\bar{p}\} + S\frac{d\{\bar{p}\}}{dt} + L^T \frac{d\{\bar{u}\}}{dt} - \{f_p\} = 0 \qquad (11)$$

where

$$H = \int_\Omega \left(\nabla \overline{N}\right)^T k' \nabla \overline{N} d\Omega$$
$$S = \int_\Omega \overline{N}^T s \overline{N} d\Omega$$
$$L^T = \int_\Omega \overline{N}^T \{m\}^T B d\Omega$$

$$\{f_p\} = \int_\Omega \overline{N}^T q d\Omega - \int_\Omega \left(\nabla \overline{N}\right)^T k' \nabla \gamma z d\Omega$$
$$s = \frac{\xi}{k_f}, \quad k' = \frac{k}{\gamma} \qquad (12)$$

(c) Formulation of Coupled Equations

The coupled equations which satisfy the force equilibrium and continuity condition can be written in the next matrix form.

$$\begin{bmatrix} [0] & [0] \\ [0] & -[H] \end{bmatrix} \begin{Bmatrix} \{\bar{u}\} \\ \{\bar{p}\} \end{Bmatrix} + \begin{bmatrix} [K_T] & -[L] \\ -[L]^T & -[S] \end{bmatrix} \begin{Bmatrix} \frac{d\{\bar{u}\}}{dt} \\ \frac{d\{\bar{p}\}}{dt} \end{Bmatrix} = \begin{Bmatrix} \frac{d\{f\}}{dt} \\ -\{f_p\} \end{Bmatrix} \qquad (13)$$

In Eq.(13), if the initial conditions are known, it can be solved for both the displacement and the pore water pressure. The same equation can be transformed to the following form

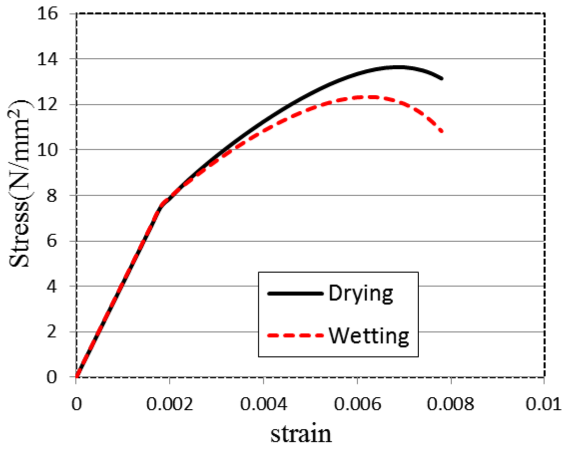
$$\begin{bmatrix} [K_T] & -[L] \\ -[L]^T & -\Delta t[H] - [S] \end{bmatrix} \begin{Bmatrix} \{\Delta \bar{u}\} \\ \{\Delta \bar{p}\} \end{Bmatrix} = \Delta t \left[\begin{Bmatrix} \frac{d\{f\}}{dt} \\ -\{f_p\} \end{Bmatrix} - \begin{bmatrix} [0] & [0] \\ [0] & -[H] \end{bmatrix} \begin{Bmatrix} \{\Delta \bar{u}\}_i \\ \{\Delta \bar{p}\}_i \end{Bmatrix} \right] \qquad (14)$$

where $\{\Delta \bar{u}\}_i$, $\{\Delta \bar{p}\}_i$ are the nodal displacement and the nodal pore water pressure at the previous step.

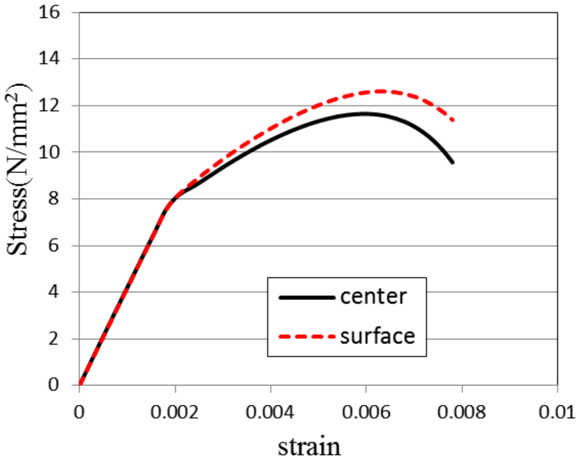
Since, in the analysis, incompressibility of solid by pore pressure is assumed, porosity ξ and the incremental strain of solid phase resulted from the incremental pore $d\{\varepsilon^{pr}\}$ are ignored in the above equations.

3.2 Analytical Estimation of Compressive Strength

In this section, the effect of the pore water pressure occurring in pore structure on the uniaxial compressive strength of concrete is discussed analytically. The analysis are performed for the perfectly drying state and perfectly saturated state with water. Figure 3 shows the relationship between the concrete stress and strain, figure(a) and (b) shows the



(a) Average Stress



(b) Inside Stress

Figure 3: Stress Characteristice.

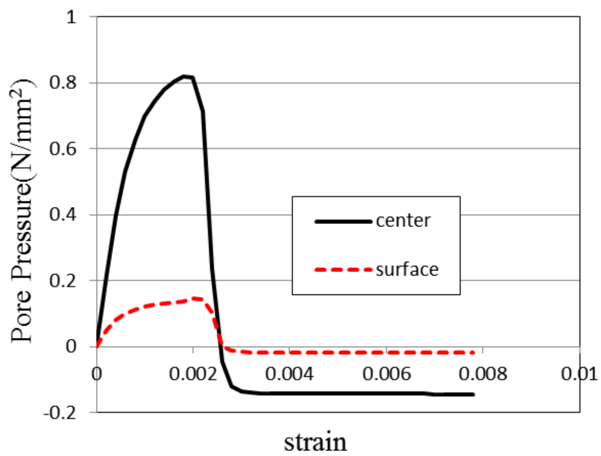


Figure 4: Pore Pressure Characteristice.

average stress and the inside of concrete stress, respectively. In figure(a), the result of the drying concrete and saturated concrete are shown with solid line and dotted line, respectively. In figure(b), the stress at the center and surface neighborhood of the concrete are shown with solid and dotted line. Figure 4 shows the relationship between the pore water pressure and strain. In this figure, the result of the saturated concrete at the center and surface neighborhood of concrete are shown with solid and dotted line, respectively. Young's modulus of the concrete in the elastic region used in the analysis is 1700MPa. In the plastic region, the initial cohesion is taken as a half of compressive strength and the final internal friction is taken as 27° for the failure surface. The permeability of liquid is 1.67×10^{-8} cm / s. The boundary conditions of the flow analysis are that the pore pressure at the surface of the concrete is equal to the atmospheric pressure.

As shown in figure 3(a), it may be seen that the compressive strength of the drying state is higher around 20% than that of saturated state. In the elastic region, there seems no difference of concrete stress between the both, but in the plastic region, the concrete stress of the drying state becomes gradually higher than that of the saturated state. On the other hands, as shown in figure 4, the positive pore pressure occurs and becomes the maximum value at the elastic limit of concrete. After yielding, the negative incremental pore pressure occurs due to the plastic expansion of the concrete and then the sudden decrease occur due to the progress of microcracks, finally becomes the slightly negative value. Concerning with the difference of the pore pressure behavior by the position, the maximum and minimum value at the center of the concrete is higher than that of surface, because the water migration cannot easily occur inside the concrete. Therefore, the effect of the pore pressure on the concrete stress becomes greater as the inside of concrete and then the value of the concrete stress becomes smaller. Namely, the positive pore pressure distributing a part of the external load is released and then the released stress transfer to the solid phase. As the result, the concrete stress reduces due to the occurrence and progress of microcracks caused by the increase of the compressive stress among the solid phase. These phenomenon gradually progress with the increase of the load and finally the concrete stress reaches the maximum value due to the occurrence of the negative pore pressure and then the failure occurs.

4 EXPANSION OF CAVITATION FLOW MODEL

4.1 Surface Tension and Cavitation

Once the pressure of the liquid becomes below the saturated vapor pressure, the bubbles as the nucleus such as an air, particles etc. occur in the liquid due to the evaporation caused by the boiling phenomenon, namely the cavitation phenomenon occurs. The pressure inside the bubbles have the value that only a value based on the Young-Laplace equation as follows has a bigger than the pressure of the liquid and the Young-Laplace equation shows that the difference between the pressure inside the bubble and of the liquid is dependent on the surface tension of the liquid and the diameter of the bubble. Therefore, the pressure inside the bubble becomes larger with the decrease of the diameter and the increase of the surface tension of the liquid. On the other hand, when the evaporation of the liquid gradually occurs toward the bubbles, the pressure inside the bubble becomes smaller with the gradual increase of the diameter of bubble. Finally, the bubbles collide with and cling to the wall surface of the solid by the liquid migration and after the surrounding liquid collapses the bubbles, the microcracks occur on the solid surface due to the shock wave, namely the erosion occurs.

$$p_G - p_L = \frac{2\gamma}{r_G} \tag{15}$$

where p_G, p_L are the pressure of bubble and liquid, respectively, $r_G$is a diameter of a bubble, γ is a surface tension of the liquid.

The representative structures in which the cavitation is a very important problem such as the fatigue fracture, the loss of the energy are the fluid machine such as the pomp, impeller, vane wheel, screw etc. In the cement based materials, it may be seen that the cavitation phenomenon easily occur under the circumstances that the pressure of the liquid becomes below the saturated vapor pressure because there are much air, micro particles such as a dust, unreacting cement etc. inside the pore which is able to becomes a nucleus and the bubbles are easy to adsorb and cling to the rough wall surface of the micro porous structure.

The analytical method to estimate the cavitation erosion is a mainly surface tracking or capturing model and a bubble flow method. The former is to estimate the bubble flow by tracing and capturing the interface between the liquid and vapor considering the phase changing for one or some bubbles. The latter is to estimate the liquid flow included the bubble by one flow model which considers one density and one velocity (called as homogeneous flow model) or two flows model which considers two densities, two velocities and constitutive equation (called as bubble flow model). In this study, the bubble flow model is introduced to Analysis 1 (called as Analysis 2). This model is a coupled procedure which satisfy the bubble motion equation such as a Rayleigh-Plesset equation, the bubble number density conservation and the bubble flow conservation for momentum, is applied to the liquid flow in micro porous pore of the cement based material.

Rayleigh-Plesset equation, which describes a volumetric motion and an erosion behavior, is expressed as following equation.

$$r_G\frac{\partial^2 r_G}{\partial t^2} + \frac{3}{2}\left(\frac{\partial r_G}{\partial t}\right)^2 = \frac{p_G - p_L}{\rho_L} + \frac{1}{4}(v_{Li} - v_{Gi})(v_{Li} - v_{Gi}) \tag{16}$$

where v_G, v_L are a velocity of the bubble and liquid, respectively, ρ_L is a density of the liquid, t is a time, a subscription i is a direction and the pressure inside a bubble p_G can be expressed as follows.

$$p_G = p_B + p_V - \frac{2\gamma}{r_G} - 4\mu\frac{1}{r_G}\frac{\partial r_G}{\partial t} \tag{17}$$

where p_B is a pressure of incondensable gas, p_V is a saturated vapor pressure and μ is a coefficient of viscosity.

Void ratio f_G can be expressed as follows by assuming a bubble to be a sphere.

$$f_G = \frac{4}{3}\pi r_G{}^3 n_G \tag{18}$$

where n_G is a bubble number density.

Then the density of the liquid included the bubbles ρ is can be written as follows by assuming the bubble density to be zero.

$$\rho = (1 - f_G)\rho_L \tag{19}$$

where ρ_L is a liquid density.

The coefficient of the viscosity is expressed as follows.

$$\mu = (1 - f_G)\mu_L + f_G\mu_G \tag{20}$$

where μ_L, μ_G are a coefficient of the liquid and bubble viscosity, respectively.

This research is located the fundamental one which estimate the effect of cavitation erosion in micro pores of the cement based materials on the strength. Therefore, in this study, the following assumptions are introduced in analysis.

Bubble is compressibility and liquid is incompressibility

All bubbles are assumed to be sphere, incorporation and separation does not occur

Bubble is saturated with the air and vapor

Phase does not change and materials does not transfer at the liquid-vapor interface

Bubble density is small to be able to ignored compared with liquid density

There is no difference between liquid and bubble velocity

Ignore the coefficient of viscosity of liquid and bubble

4.2 Compressive Strength due to Surface Tension

In this section, the effect of the cavitation erosion on the strength is discussed in detail due to the comparison between the results of Analysis 1 in section 3.1 and Analysis 2 shown in section 4.1.

Analyses are performed in the state such that the mortar are perfectly dried, in which pore are perfectly saturated with water and aniline. The surface tension and saturated

Table 1: Liquid Property

Liquid	Water	Aniline
Surface Tension (dyn/cm)	72.8	42.9
Saturated Vapor Pressure at 20℃ (kPa)	2.3	0.04

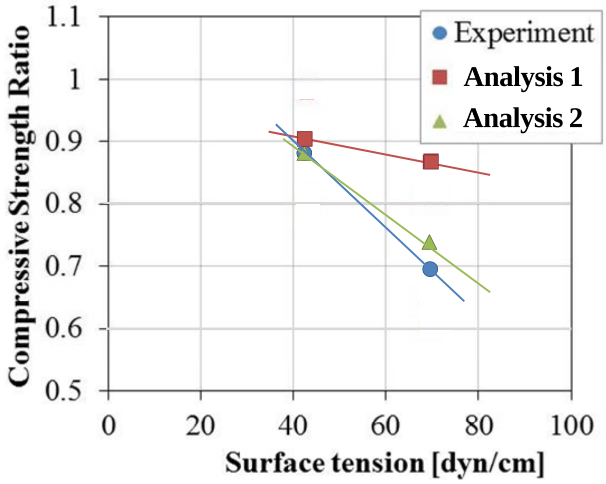


Figure 5: Compressive Strength Ratio.

vapor pressure for the water and aniline are quite different as shown in Tab.1. Young's modulus of the mortar used in analysis is 25GPa. In the plastic region, the initial cohesion is taken as a half of compressive strength and the final internal friction is taken as 27 ° for the failure surface. The permeability is 1.67 × 10^{-8} cm/s. The volumetric elastic modulus of the water and aniline are 2.2 MPa and 1.1 MPa, respectively. The boundary conditions of the flow analysis are that the pore pressure at the surface of the concrete is equal to the atmospheric pressure.

Figure 5 shows the relationship between the compressive strength ratio and the surface tension of the liquid and the compressive strength ratio is defined as the strength of the mortar immersed in the liquid normalized by that of drying mortar. In this figure, the experimental results are marked with circles and the analytical result of Analysis 1 and Analysis 2 are marked with square and triangle respectively.

As shown in figure 5, Analysis 2 shows a good agreement with the experimental results for both the liquid of water and aniline. On the other hand, Analysis 1 shows a good agreement with the experiment for the aniline having a relatively small surface tension, but for the water having a large surface tension, Analysis 1 shows a quietly different from the experiments. Namely, for the liquid with a larger surface tension, it may be seen that the strength is influenced by not only the pore pressure, but also the surface tension. This seems to be caused by that the difference between the pressure inside and outside the bubble is relatively larger.

5 CONCLUSIONS

In this study, an extended mathematical model which is introduced the cavitation phenomenon into the coupled model of solid and liquid phase is developed to investigate the detailed mechanism for the effect of the surface tension of liquid on the solid strength. It is concluded by the comparison between the experimental and analytical results that the strength is influenced by not only the pore pressure occurring in liquid phase, but also the cavitation erosion in micro pore of the porous materials.

REFERENCES

[1] Gilkey, H. J., *Eng. News Record*, Vol 119, 1937, pp.630-633.

[2] Okajima, T., *Cement and Concrete*, No.464, 1985, pp.18-26.

[3] Benedicks, M. C., *Comp, Rend.*, 1951, p.232, p.2435.

[4] Arthur, P. D., Earl, J. C. and Hodgkiess, T., Fatigue of Reinforced Concrete in Seawater, *Concrete*, 1979, pp.26-30.

[5] Hori, M., *Ceramic Soc. Japan*, Vol.70, No.7, 1962, pp.54-59.

[6] Oshita, H. and Tanabe, T., Creep Mechanism of Early Age Concrete Modelling with Two Phase Porous Material, Creep and Shrinkage of Concrete, RILEM, 1993, pp.465-470.

[7] Ogishi, S., Ono, H. and Tanahashi, A., Effect of Surface Tension of Immersed Liquid on Strength and Deformation of Cement-Mortar, *Proc.of Annual Conference of JCI*, 1981, pp.101-104.

[8] Matsushita, H. and Onoue, K., Influence of Surface Tension of Liquid Stored in Internal Void on Compressive Fatigue Strength of Concrete, *Conc. Res. and Tech.*, Vol.17, No.1, 2006, pp.19-30.