STATISTICAL EVALUTION OF FATIGUE TESTS OF PLAIN C30/37 AND C45/55 CLASS CONCRETE SPECIMENS

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Abstract: The aim of this paper is to present and compare basic fatigue parameter values obtained for plain C30/37 and C45/55 class concrete specimens during dynamic tests. Selected approximation curves of mechanical-fracture parameter values over time – compressive cube strength, modulus of elasticity, effective fracture toughness and specific fracture energy – are used to determine the most accurate fatigue parameter values corresponding to the age of specimens when dynamic tests were performed. The power law and Weibull model developed by Castillo et al. are used for standard description of the $S-N$ curve. Both of the methods of fatigue test evaluation will be compared and the suitability of their application within numerical models will be discussed.

1 INTRODUCTION

Many structures are often subjected to repetitive cyclic loads of high stress amplitude. Examples of such cyclic loads include automobile and train traffic, machine vibration and wind action. The phenomenon known as material fatigue, a process in which progressive and permanent internal damage occurs in materials subjected to repeated loading, is a serious problem for concrete structures such as bridges, tunnels, airport/highway pavements, railway sleepers, etc. Concrete is a highly heterogeneous material and the processes occurring within its structure and leading to its degradation under cyclic loading are more complicated in comparison to those affecting metals (see [1]). This is one reason why the understanding of fatigue failure in cementitious composites is still lacking in comparison to that of ferrous materials, even though concrete is a widely used construction material.

Fatigue tests of concrete materials and structures are expensive, and for this reason numerical modelling [2] can represent a powerful approach for the prediction of the damage process and fatigue life of such materials under different service conditions. For the effective and correct use of a numerical (material) model it is often necessary to tune its parameters using data obtained during experiments. The correct evaluation of such data is becoming a prerequisite for the correct use of numerical models in practice.

Various approaches have been used to assess the fatigue life of structural members in recent years. The generally accepted approach in engineering practice is based on empirically derived *S*–*N* diagrams known as Wöhler curves (stress *S* vs. the number of cycles to failure *N*). Another possibility is to use the statistical evaluation of dynamic tests, e. g. the Weibull non-linear regression model developed by Castillo et al. [3, 4].

Both of the above-mentioned methods of fatigue test evaluation will be compared in this paper and the suitability of their application within numerical models will be discussed.

2 EXPERIMENT DETAILS

An extensive laboratory experiment was conducted on a set of specimens of plain C30/37 and C45/55 class concrete. The specimens were used to determine the values of fundamental fracture characteristics and related fatigue parameters using static fracture and dynamic experiments.

2.1 Static experiments

The experimental data (values for the modulus of elasticity, effective fracture toughness and specific fracture energy) was obtained from three point bending fracture tests of beam specimens with a central edge notch. The nominal dimensions of the beams were 100×100×400 mm; span length 300 mm. Cubes with edge lengths of 150 mm were used for determination of the compressive strength values. Because of the gradually increasing age of the concrete samples during the dynamic tests, the above-mentioned specimens were tested at the age of 28, 98 (91 for C45/55 class concrete) and 159 days.

Analytical expressions were determined for regression curves as approximations of the above-mentioned fracture mechanics parameter values over time. Power, logarithmic and polynomial functions were used (EXCEL software). The coefficient of determination R^2 was obtained for each type of regression curve.

Coefficients of analytical expressions for approximation curves for C30/37 and C45/55 class concrete are collected in Tab. 1 and Tab. 2., respectively.

In the equations, x indicates time in days and y indicates the dimensionless relative values of the appropriate parameter.

Table 1: Coefficients of analytical expressions for simple approximation curves for C30/37 class concrete

Power regression curve $y = a \cdot x^b$

Relative parameter	*a*	*b*	R^2
Compressive strength	0.6367	0.1377	0.9396
Modulus of elasticity	0.6673	0.1113	0.2676
Fracture toughness	0.4909	0.2007	0.5347
Fracture energy	0.2877	0.3639	0.7960

Logarithmic regression curve $y = a \cdot \ln(x) + b$

Relative parameter	*a*	*b*	R^2
Compressive strength	0.1542	0.4939	0.9404
Modulus of elasticity	0.1185	0.5786	0.2906
Fracture toughness	0.2380	0.1664	0.5255
Fracture energy	0.4882	-0.6575	0.7889

Polynomial regression curve $y = ax^2 + bx + c$

Relative parameter	*a*	*b*	*c*	R^2
Compressive strength	-2.15E-05	0.0060	0.8494	0.9706
Modulus of elasticity	1.44E-05	-0.0010	1.0160	0.4004
Fracture toughness	5.04E-05	-0.0058	1.1224	0.7849
Fracture energy	1.52E-05	0.0040	0.8758	0.8428

Table 2: Coefficients of analytical expressions for simple approximation curves for C45/55 class concrete

Power regression curve $y = a \cdot x^b$

Relative parameter	*a*	*b*	R^2
Compressive strength	0.7173	0.0983	0.8548
Modulus of elasticity	0.8573	0.0409	0.2384
Fracture toughness	0.7554	0.0834	0.5026
Fracture energy	0.5892	0.1532	0.7944

Logarithmic regression curve $y = a \cdot \ln(x) + b$

Relative parameter	*a*	*b*	R^2
Compressive strength	0.1073	0.6313	0.8412
Modulus of elasticity	0.0429	0.8395	0.2495
Fracture toughness	0.0903	0.6969	0.4834
Fracture energy	0.1748	0.3956	0.7857

Polynomial regression curve $y = ax^2 + bx + c$

Relative parameter	*a*	*b*	*c*	R^2
Compressive strength	3.59E-06	0.0008	0.9735	0.9300
Modulus of elasticity	1.57E-05	-0.0022	1.0496	0.6550
Fracture toughness	-4.84E-06	0.0021	0.9444	0.4862
Fracture energy	9.61E-06	0.0007	0.9727	0.9032

In addition, an advanced approximation curve was used for compressive cube strength values:

$$y = a\left(1 - e^{-b(x)^c}\right) \qquad (1)$$

Coefficients of analytical expressions for advanced approximation curves for both classes of concrete are collected in Tab. 3.

In the equations, x indicates time in days and y indicates the dimensionless relative value of the compressive cube strength.

Table 3: Coefficients of analytical expressions for approximation curves for compressive cube strength

	Advanced regression curve $y = a\left(1-e^{-b(x)^c}\right)$			
Compressive strength	a	b	c	R^2
C30/37 class concrete	1.2694	-0.1920	0.6269	0.9705
C45/55 class concrete	1.2769	-0.4360	0.3470	0.8026

2.2 Dynamic experiments

Fatigue properties were obtained from three point bending tests of beam specimens with a central edge notch. The nominal dimensions of the beams were 100×100×400 mm; span length 300 mm. The initial notches were made by a diamond bladed saw. Note that the depth of the notches was 10 mm.

The fatigue experiments were carried out in a computer-controlled servo hydraulic testing machine (INOVA–U2). The controlled values for temperature and relative humidity were 22±2 °C and 50%.

Fatigue testing was conducted under load control. The stress ratio $R=P_{min}/P_{max}=0.1$, where P_{min} and P_{max} refer to the minimum and maximum load of a sinusoidal wave in each cycle. The load frequency used for all repeated-load tests was 10 Hz. The number of cycles before failure was recorded for each specimen.

Concrete specimens were loaded in the range of high-cycle fatigue; therefore, the upper limit to the number of cycles to be applied was selected as 2 million cycles. The test finished when the failure of the specimen occurred or the upper limit of loading cycles was reached, whichever occurred first.

3 RESULTS OF THE FATIGUE TESTS

The results of the fatigue tests under a varying maximum bending stress level are summarized in Fig. 1 and Fig. 2 for C30/37 and C45/55 class concrete, respectively, where the logarithm of the maximum bending stress (S) used in the fatigue experiments is plotted against the logarithm of the number of cycles to failure (N).

The fatigue experiments lasted for a long time, which is problematic from the point of view of the ageing of the specimen material. Because of this, the data obtained from the fatigue tests were standardized to a specimen age of 28 days. Selected approximation curves (Tabs. 1−3) obtained from fracture mechanics parameter values over time were used for this purpose.

The measured data and examples of the corrected fatigue data (using the power regression curve of fracture toughness for C30/37 class concrete and the polynomial regression curve of fracture energy for C45/55 class concrete) are shown in Figs. 1 and 2.

3.1 *S*−*N* curves

The first formula for fitting the experimentally obtained data used in this paper is based on empirically derived S–N diagrams known as Wöhler curves:

$$S = a \times N^{b} \tag{2}$$

where a, b are the material parameters.

In an ideal, theoretical case, all specimens at a certain stress level would fail after the same number of cycles. However, the fatigue behaviour of a heterogeneous material like concrete is far from being ideal, so the results are usually highly scattered. Accordingly, it is necessary to determine not only the analytical expression of the relevant S–N curve but also a measure of the scatter, such as the coefficient of determination R^2.

According to (2), the power function and the coefficient of determination for C30/37 class concrete are as follows:

$$S = 3.7256 \times N^{-0.0230} \text{ and } R^2 = 0.3806 \tag{3}$$

and for C45/55 class concrete:

$$S = 5.9227 \times N^{-0,0345} \text{ and } R^2 = 0.8248 \tag{4}$$

The coefficients of analytical expressions (2) for S–N curves corrected by approximation curves and coefficients of determination are summarized in Tab. 4 and Tab. 5 for C30/37 and C45/55 class concrete, respectively.

Table 4: Coefficients of *S–N* curves for C30/37 class concrete, and coefficients of determination

$S = a \times N^b$	using power regression curve		
Relative parameter	a	b	R^2
Compressive strength	3.2782	-0.0261	0.7400
Modulus of elasticity	3.4954	-0.0255	0.6769
Fracture toughness	3.2618	-0.0276	0.8248
Fracture energy	2.8009	-0.0313	0.6379
$S = a \times N^b$	using logarithmic regression curve		
Relative parameter	a	b	R^2
Compressive strength	3.2734	-0.0261	0.7334
Modulus of elasticity	3.4684	-0.0255	0.6734
Fracture toughness	3.2221	-0.0278	0.8212
Fracture energy	2.7500	-0.0315	0.6415
$S = a \times N^b$	using polynomial regression curve		
Relative parameter	a	b	R^2
Compressive strength	3.2617	-0.0263	0.7661
Modulus of elasticity	3.5079	-0.0252	0.6307
Fracture toughness	3.4140	-0.0273	0.6754
Fracture energy	2.8468	-0.0310	0.6320
$S = a \times N^b$	using advanced regression curve		
Relative parameter	a	b	R^2
Compressive strength	3.2561	-0.0262	0.7028

Table 5: Coefficients of *S–N* curves for C45/55 class concrete, and coefficients of determination

$S = a \times N^b$	using power regression curve		
Relative parameter	a	b	R^2
Compressive strength	5.3889	-0.0335	0.8698
Modulus of elasticity	5.7726	-0.0341	0.7916
Fracture toughness	5.4358	-0.0336	0.8514
Fracture energy	5.1290	-0.0329	0.9194
$S = a \times N^b$	using logarithmic regression curve		
Relative parameter	a	b	R^2
Compressive strength	5.3778	-0.0335	0.8691
Modulus of elasticity	5.7630	-0.0341	0.7929
Fracture toughness	5.4207	-0.0336	0.8507
Fracture energy	5.1051	-0.0329	0.9181
$S = a \times N^b$	using polynomial regression curve		
Relative parameter	a	b	R^2
Compressive strength	5.4907	-0.0335	0.8810
Modulus of elasticity	5.9137	-0.0342	0.7965
Fracture toughness	5.4524	-0.0335	0.8643
Fracture energy	5.2676	-0.0329	0.9315
$S = a \times N^b$	using advanced regression curve		
Relative parameter	a	b	R^2
Compressive strength	5.3947	-0.0334	0.8833

3.2 *Castillo et al.'s model*

The second formula for fitting experimentally obtained data uses the non-linear Weibull regression model proposed by Castillo et al. [3, 4] in the following form:

$$(\log N - B)(\log S - C) = \lambda + \delta\left(-\frac{L_0}{L_i}\log(1-P)\right)^{1/\beta} \quad (5)$$

where N is the fatigue life measured in cycles, S is the stress, P is the probability of failure, L_0 is the reference length, L_i is the specimen length (in the case of this study $L_0/L_i = 1$) and β, B, C, δ and λ are the model parameters to be estimated, with the following meaning: B is the threshold value for N or the limit number of cycles, C is the threshold value for S or the endurance limit, β and δ are the shape and scale parameters of the Weibull distribution, and λ is the parameter fixing the position of the zero probability curve.

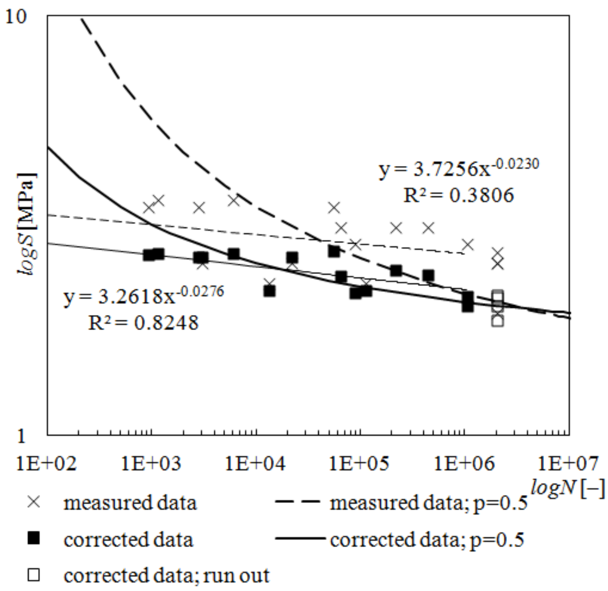

Figure 1: The power function and Castillo et al.'s model for C30/37 class concrete: measured and corrected data.

The analytical expression of the *S–N* field given by:

$$S = \exp\left(\frac{\left(\log\left(\frac{1}{1-P}\right)\right)^{1/\beta}\delta + \lambda}{\log N - B} + C\right) \quad (6)$$

allows probabilistic prediction of the max constant amplitude loading for the required quantity of cycles.

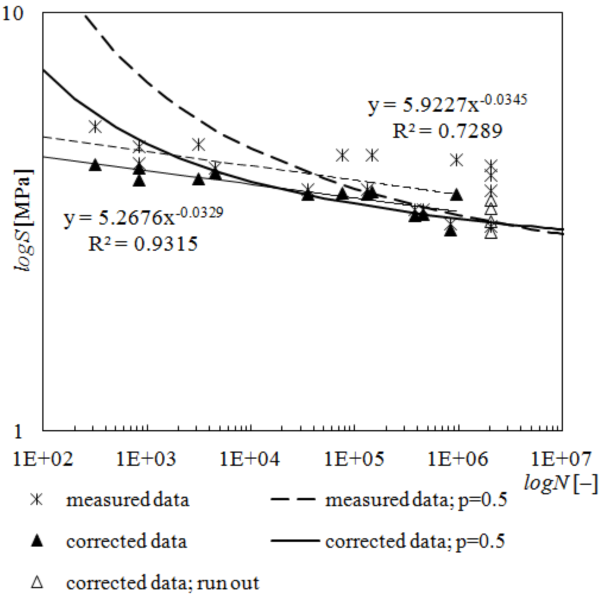

Figure 2: The power function and Castillo et al.'s model for C45/55 class concrete: measured and corrected data.

The following analytical expression for C30/37 class concrete was obtained according to (6) using Castillo et al.'s model:

$$S=\exp\left(\frac{\left(\log\left(\frac{1}{1-0.5}\right)\right)^{1/1.20}5.71+8.92}{\log N-0.00}-0.17\right) \qquad (7)$$

and for C45/55 class concrete as follows:

$$S=\exp\left(\frac{\left(\log\left(\frac{1}{1-0.5}\right)\right)^{1/1.02}4.67+6.82}{\log N-0.00}+0.46\right) \qquad (8)$$

Eqs. 7, 8 and Tabs. 6, 7 are referred to as the quantile curve P = 0.5.

Tabs. 6 and 7 show the parameters estimated using Castillo et al.'s model for corrected data by approximation curves of relative values of mechanical-fracture parameters over time for C30/37 and C45/55 class concrete.

In Fig. 3 corrected data are introduced using advanced approximation curves of relative compressive strength. S–N curves and curves obtained by Castillo et al.'s model are plotted corresponding to the quantile curve P = 0.5 and the 0.90 broad confidence interval range.

Table 6: Parameters of Castillo et al.'s model for C30/37 class concrete

Castillo et al.'s model	using power regression curve				
Relative parameter	β	B	C	δ	λ
Compressive strength	1.40	0.00	0.20	2.64	5.36
Modulus of elasticity	1.24	0.00	0.19	2.98	5.94
Fracture toughness	1.32	0.00	0.31	1.77	4.52
Fracture energy	2.74	2.42	0.18	3.08	1.57
Castillo et al.'s model	using logarithmic regression curve				
Relative parameter	β	B	C	δ	λ
Compressive strength	1.41	0.00	0.18	2.72	5.44
Modulus of elasticity	1.25	0.00	0.18	3.02	5.99
Fracture toughness	1.53	0.00	0.30	1.86	4.41
Fracture energy	2.50	4.18	0.29	2.01	0.67
Castillo et al.'s model	using polynomial regression curve				
Relative parameter	β	B	C	δ	λ
Compressive strength	1.65	0.00	0.23	2.54	4.96
Modulus of elasticity	1.02	0.00	0.19	2.99	6.10
Fracture toughness	1.26	6.75	0.55	1.86	0.00
Fracture energy	2.01	0.00	-0.09	4.66	4.89
Castillo et al.'s model	using advanced regression curve				
Relative parameter	β	B	C	δ	λ
Compressive strength	1.52	0.00	0.11	3.12	5.80

Table 7: Parameters of Castillo et al.'s model for 45/55 class concrete

Castillo et al.'s model	using power regression curve				
Relative parameter	β	B	C	δ	λ
Compressive strength	1.02	0.00	0.64	2.82	5.15
Modulus of elasticity	1.01	0.00	0.56	3.72	6.14
Fracture toughness	1.01	0.00	0.61	3.04	5.40
Fracture energy	1.06	0.00	0.71	2.22	4.33
Castillo et al.'s model	using logarithmic regression curve				
Relative parameter	β	B	C	δ	λ
Compressive strength	1.02	0.00	0.64	2.84	5.16
Modulus of elasticity	1.01	0.00	0.56	3.71	6.12
Fracture toughness	1.03	0.00	0.61	3.03	5.42
Fracture energy	1.05	0.00	0.70	2.23	4.36
Castillo et al.'s model	using polynomial regression curve				
Relative parameter	β	B	C	δ	λ
Compressive strength	1.02	0.00	0.68	2.74	4.98
Modulus of elasticity	1.01	0.00	0.58	3.76	6.12
Fracture toughness	1.02	0.00	0.64	2.88	5.22
Fracture energy	1.42	0.00	0.76	2.21	3.96
Castillo et al.'s model	using advanced regression curve				
Relative parameter	β	B	C	δ	λ
Compressive strength	1.02	0.00	0.67	2.66	4.97

4 CONCLUSIONS

Since the asymptotic behaviour of Castillo et al.'s model in the low cycle fatigue region is not realistic, it can be concluded that Castillo et al.'s model is especially applicable for description on the middle- and high-cycle region of the Wöhler curve.

In this study, a numerical model to simulate the damage affecting concrete under tensile load has been proposed and verified. The experimental results can also be used as a valuable input to estimate the model's parameters and enhance its further development.

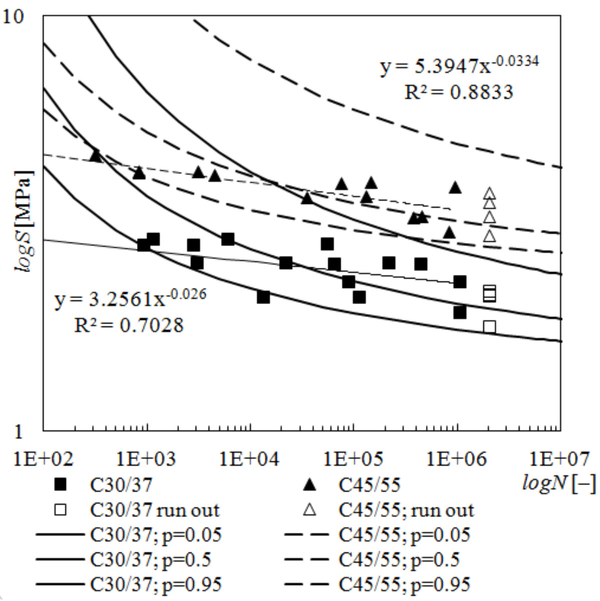

Figure 3: The power function and Castillo et al.'s model for C30/37 and C45/55 class concrete: corrected data.

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