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MULTISCALE PROBABILISTIC APPROACHES AND STRATEGIES FOR THE MODELLING OF CONCRETE CRACKING

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Abstract. Since the 90s, Ifsttar has developed probabilistic models for a reliable description of the cracking processes in concrete structures. These models take into consideration the inherent heterogeneity of the material and the consecutive scale effects on its mechanical properties via appropriate, and experimentally validated, size effect laws and random distributions. After a brief presentation of the models, the paper delivers some considerations on the fact that the efficiency and the relevance of the approach, for a given scale of modelling, directly depends on the judicious choice of the couple of *type of local behaviour model* - *type of numerical support*. It is also shown that this choice has an important and obvious impact on the richness of the local information obtained for cracking processes (micro and macro cracking).

1 INTRODUCTION

The numerical modelling of concrete cracking is a challenging issue in Civil Engineering. The development of models providing information on the characteristics of concrete cracks, for a given environment, loading and limit conditions set, is actually very popular. Cracks openings, lengths, orientations and spatial distributions are essential factors. Their impact on the long-term structural performance maintenance must be accounted for an accurate prediction of structures lifespan.

In the literature, a number of approaches for describing the nucleation/evolution of cracks can be found: however, results are rarely fully predictive [4], especially when crack openings and spacings are concerned. The experimental analyses held at IFSTTAR(former LCPC) since more than 20 years [5], [10], [7] have led to the observation that these phenomena can be correctly described by explicitly taking into account concrete heterogeneity (which is at the origin of volume effects) in the frame of a probabilistic approach. An original discrete model based on these concepts was first proposed by Rossi [9] in the 90' and has been more recently resumed in [11]. Since the model is based on a local probabilistic elastic-brittle behaviour of the concrete and uses contact elements for the description of the kinematic discontinuities of the displacement field when cracks appear, the approach can provide local information on both micro and macro cracks and it is well suited for modelling crack patterns. But the increase of the node number (due to the use of contact elements) can rapidly have a detrimental effect on the size of the problem and lead to prohibitive computation costs when dealing with real structures. Therefore, a macroscopic probabilistic approach has been developed to overcome this difficulty and to be efficient at the scale of a real concrete structure. The main principles of this latter are given in [12] [13]. These approaches, developed in the frame of the FEM, considers the finite element as a given volume of heterogeneous material exhibiting therefore, in local uniaxial tension, random mechanical properties (tensile strength, Young modulus, local dissipated energy, ...) directly depending on its size. Size effect laws are obtained by coupling experimental results [10] and inverse analysis.

The paper proposes a discussion on the consistency between the behaviour model and the numerical support, and its consequence on the efficiency and the relevance of the approach to describe crack creation and to approximate their propagation. After a brief description of the cracking processes in concrete and the main bases of the original (discrete elastic-brittle) probabilistic approach and these of the macroscopic approach as well, the paper focuses on the effect of the choice of the type of contact element by presenting a comparison between experimental results and numerical results obtained with these two approaches.

2 CRACKING PROCESSES IN CONCRETE

Concrete is, by nature, a heterogeneous material, whose heterogeneity is essentially due (mechanically speaking) to the difference in properties of the cement paste and of the granular skeleton. The cement past also contains inner defects such as pores and cracks, consequences of hydration of the cement paste and restrained shrinkages at early ages. Initial conditions are therefore responsible of internal autobalanced stresses (even under no external load), which also can determine the long-term concrete behaviour [1].

When an external load is applied, a new crack state is observed. The cracks may be located across several hydrates of the cement paste. They are either newly created or due to the propagation of existing cracks. A crack is created or does propagate when a critical value of the local tensile stress is reached. The spatial distribution of cracks is the direct consequence of the spatial distribution of stresses and strengths, these latter being also the consequence of the heterogeneity of the material. In addition, the creation or the propagation of a crack leads to a local spatial redistribution of stresses in its vicinity. Therefore, cracks newly formed or propagated constitute an induced heterogeneity. When the load is increased, new cracks are again formed or propagated, so that the process evolves in two ways:

– the crack density increases,
– cracks appear increasingly long and open.

As long as, the induced heterogeneity, due to microcrack creation and/or propagation in the material, does not generate strain localization, the concrete structure may accept loadings of greater magnitude. Otherwise, it leads to strain localization (local increase of the microcrack density) in the structure that enters in an unstable fracture process. This is known as macro-crack coalescence and propagation into the structure.

Therefore, heterogeneity, inner defects, applied loads and boundary conditions, provoke stress gradients in concrete and, of course, cracking. Thus, the global mechanical response of the structure is a consequence of the combined effect of these stress gradients and localized cracks propagating during the loading. And as the material is by nature heterogeneous, its behaviour obviously exhibits some randomness.

3 MODELLING CRACKING PROCESSES IN CONCRETE VIA PROBABILISTIC APPROACHES

The general numerical framwork of these developments is the one of the finite element method. As already exposed in the introduction, the underlying and basic ideas of the original model developed by Rossi [9] is to consider a finite element volume as a volume of heterogeneous material and to assume that physical mechanisms influencing the cracking processes remain the same whatever the scale of observation. At the scale of the finite element, mechanical properties are then functions of its own volume. To describe the heterogeneity of the material, these mechanical properties have to be randomly distributed over the finite element mesh. That is the reason why the model is probabilistic. Caracteristics of random distribution functions (Weibull for example for the tensile strength) are directly depending on particular indicators of (1) the heterogeneity of the material (the ratio between the volume of the material and the volume of the coarsest agregate) and (2) of the quality of the cement past, as the location of inner defects, (for example the mean compressive strength).

Note that only cracking in mode I is considered here.

3.1 Discrete approach

This model aims to explicitly represent localized (micro and macro) crack patterns in concrete taking into account volume effects.

From a numerical point of view, interface elements are used to describe the displacement discontinuities consecutive to the crack creation. As already mentionned, the mechanical properties of these elements (essentially the tensile strength) are considered as randomly distributed variables. The volume of the massive elements, that are adjacent to the considered interface element, acts as the reference volume (of heterogeneous material). The scale effect laws cited in introduction have been extrapolated [11], by inverse analysis (see also [3]), from the scale of experimental test results [10] to the scale of this reference volume and used as input data in the numerical modelling.

Volumes of concrete associated to interface elements follow a local elastic-purely brittle behaviour while massive elements, representing the uncracked concrete, remain purely elastic. Contact elements verify some Rankin (in tension) and Tresca (in shear) criteria. In that sense, it is assumed that the no-consideration of the local irreversible energy dissipation little influences the global behaviour of the structure. In other words, the local energy dissipation is assumed to be very small compared to the elastic energy stored in the structure. This assumption is acceptable when the phenomenon is located in a very small scale with respect to the scale of the structure.

3.2 Macroscopic approach

The main limit of the use of the discrete approach is related to the high number of nodes that it induces, increasing then the computational cost, which is considerably perceptible for fine meshes. In that sense, it may be interesting to use a more efficient approach that better suits the simultation of behaviours at the scale of real concrete structures.

The macroscopic model aims to represent the global behaviour of concrete structures with localized macro crack patterns, of course taking into account scale effects.

The basic idea of this model is again to consider the finite element as a given volume of heterogeneous material. The tensile strength is here also considered as a random variable and is still determined in the same way as in the discrete approach. But at the scale of the element, the model considers that the cracking processes induce some energy dissipation. ”Cracking processes” means here creation and propagation of cracks in the element. When the whole amount of energy, that the element can consume, is reached this latter is considered as failed. That means that its contribution can be removed from the numerical model.

To take into account this local energy dissipation, a simple isotropic damage model can be used. Many different modelling strategies are available and can be used: damage, smeared crack for the most popular. And it is well none that they are quite similar as recalled by Bazant in 83 [2]. A bilinear formulation of the stress-strain relation is chosen here in a sake of simplicity. The damage parameter is driven by the evolution of the strain in the direction of the principal major stress (in tension). In this formulation and numerical context, the total amount of energy dissipation up to failure is represented by the whole area under the stress-strain curve multiplied by the volume of the element. The model considers that this energy is entirely consumed in the creation and propagation of a macrocrack inside the element. The value of the dissipated energy density can be taken equal the one experimentally obtained by

Rossi [5]:

$$G_c \; = \; \frac{K_{Ic}^2}{E} \simeq 1.31410^{-4} MN.m/m^2 \quad (1)$$

This has two consequences:
– the energy consumed in the creation and propagation of a macrocrack can be viewed, in mean value, as a material parameter, and only depends on the type of concrete. This hypothesis seams to be reasonable with regard to the results obtained in [5] and confirmed in [8];
– due to the heterogeneity of the material (whose impact is enhanced at the scale of the finite element) the dissipated energy shows in fact some randomness. Its dispersion can be considered as directly influenced by the size of the solicitated volume, in that sense that this volume may (or not) contain barriers to crack propagation.

The dissipated energy is then ramdomly distributed over the mesh. The chosen distribution function is a log-normal law. And again, as the mean value of this distribution is chosen constant whatever the scale, the standard deviation is the only parameter to be dentermine. This determination has to be performed by inverse analysis.

4 VALIDATION EXAMPLE

4.1 A double cantilever beam test

This test is related to the study of macrocrack propagation in a very large double cantilever beam (DCB) specimen. It was performed in the framework of a PhD thesis aimed to study applicability of linear fracture mechanics to concrete structures [5]. The geometry and the dimensions of the specimen are given in Figure 1.

Prestressing forces(1230 kN) are applied to concentrate the propagation of the macrocrack in the center of the specimen and to avoid any premature bifurcation. The concrete mix design formula is given Table 1. And the mechanical properties of the material are reported in Table 2.

Table 1: Mix design formula of the concrete (kg/m^3) .

Agregate (4-12 mm)	1105
Sand (0-5 mm)	700
Cement	400
Water	190

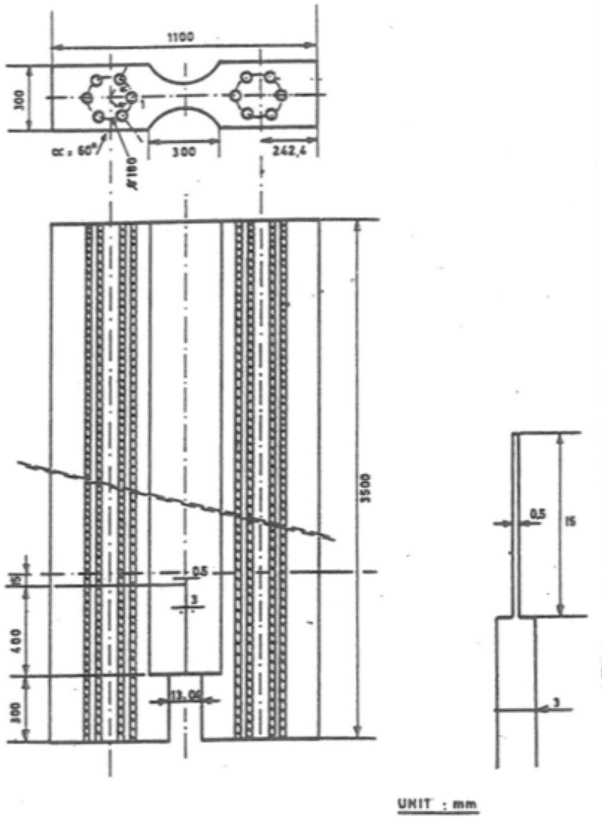

Figure 1: Geometry of the DCB specimen.

Table 2: Concrete mechanical properties

Young modulus	$\simeq$ 36000 MPa
Compressive strength	$\simeq$ 50 MPa

It is important to note the huge size of the specimen, so that the (mechanical) responses of the experimental test can be, as a consequence, representative of the material behaviour.

4.2 Results obtained with the discrete approach

The modelling is performed here in the frame of a 2D plain stress state hypothesis. The dog-bone shape of the cross section of the specimen is represented by an equivalent section, as schematized Figure 2, to fit this plane stress hypothesis. Model parameters are given Table 3.

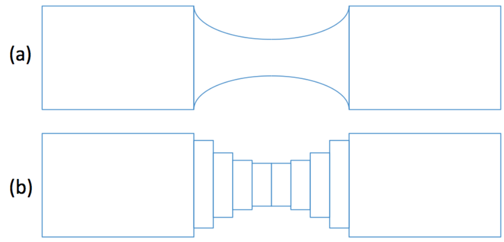

Figure 2: Real (a) and idealized (b) shapes of the cross section of the specimen.

Quadratic (contact and massive) elements are used in this numerical study. 6 different simulations have been performed. Mesh, boundary and loading conditions are given Figure 3. Results are given Figure 4 (curves load vs. COD). And Figure 5 shows an example of crack pattern.

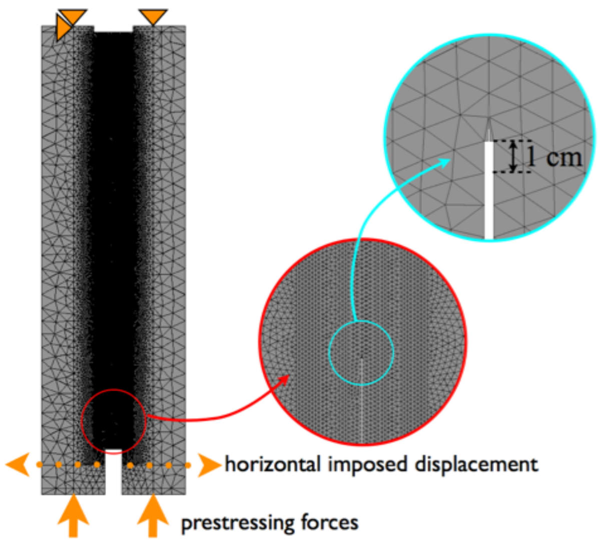

Figure 3: Discrete model – Mesh, boundary and loading conditions.

Table 3: Discrete model parameters.

Young Modulus	36000 MPa
Compressive strength	50 MPa
Agregate diameter	12 mm
Tensile strength	Mean val. 4 MPa
(at the elem. scale)	Std dev. 0.6 MPa

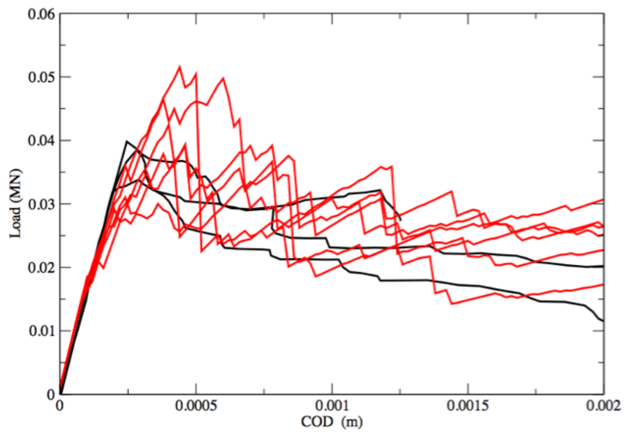

Figure 4: Discrete model – Global behaviour of the DCB specimen; comparison between experience (black) and computations using quadratic contact elements (red).

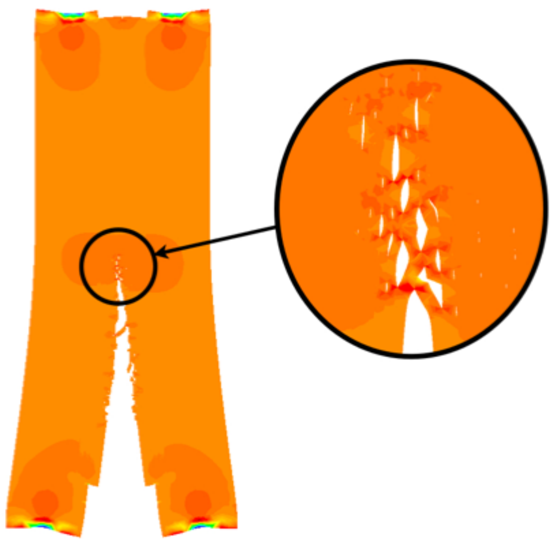

Figure 5: Example of crack pattern obtained with the discrete model.

A refinement of the mesh can also be performed to ameliorate the description of the microcracking at the crack tip (Figure 6).

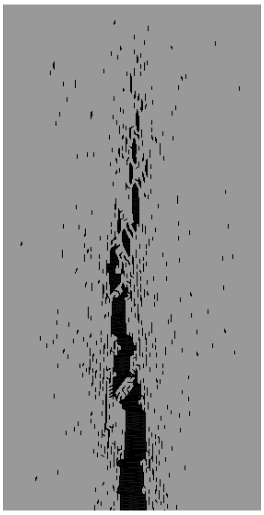

Figure 6: Example of crack pattern obtained with the discrete model on a refined mesh (zoom on the crack tip).

A previous study [14] has also shown the influence of the choice of the numerical support (i.e. the type of element) on the accuracy (and the pertinence) of the response of the model. In this study, both linear and quadratic contact elements have been (coupled to a local elasti-brittle behaviour of the material). Due to the poorness of the description of the displacement field at the scale of the element, the linear element overestimates the stresses at its centre, compared to the quadratic. The stress redistributions after failure during the nonlinear iterations until convergence are then completely different for both elements. As a consequence, the failure is premature in the case of the linear element, which concentrates much more the stresses and leads to a more localized failure. One way to counteract this phenomenon is to enrich the local behaviour by allowing the material to locally dissipate some energy. A cohesive-crack type model can therefore be used in that way. Figure 7 shows the comparison between these two results. The consequence of this enrichment lies essentially in the scale of representation of cracking processes: the dissipated energy represents the effect of the development of the microcracking at surrounding of the crack tip during the propagation of the macrocrack. As this microcracking is not explicitely described here, the representation of cracks by this model only concerns localized macrocracks.

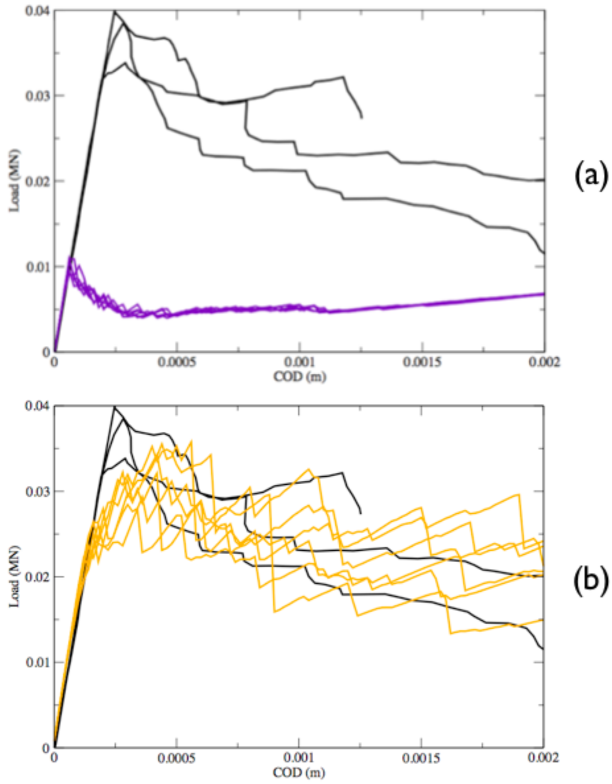

Figure 7: Results obtained with the discrete model – same mesh as in Figure 3 – and the use of linear contact elements: (a) local elastic-brittle behaviour, (b) cohesive-crack approach. Experiments are in black.

4.3 Results obtained with the macroscopic approach

The modelling is again performed in 2D (plane stress) and the same schematization of the cross section is used. The mesh for the macroscopic model is shown Figure 8. The mesh is made of (basic) linear triangles to reduce computation costs to their maximum. Note also that it is coarser than the preceding (cf. Figure 3). Considering remarks made in the preceding section of this paper, the use of linear elements in conjonction with a dissipative formulation of the cracking processes is reasonable. The coarser character of the mesh refinement (compared to the one used for the discrete model) is not a problem in the sense that the dispersion of the energy distribution directly takes into account the size of the finite element.

Parameters of the model are given Table 4.

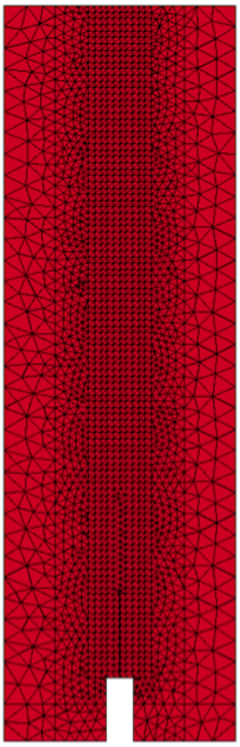

Figure 8: Macroscopic model – Mesh of the DCB specimen.

Again, global responses of the structure are given Figure 9 for six different simulations and an example of crack pattern is given Figure 10 .

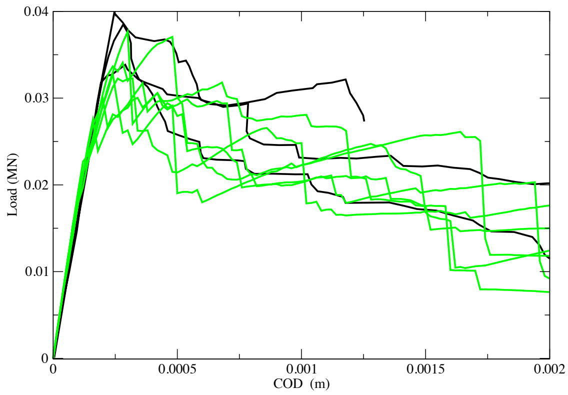

Figure 9: Macroscopic model – Global behaviour of the DCB specimen; comparison between experience (black) and computations using quadratic contact elements (green).

Table 4: Macroscopic model parameters.

Young Modulus	36000 MPa
Compressive strength	50 MPa
Agregate diameter	12 mm
Tensile strength (MPa)	Mean 4 Std dev. 0.8
Dissipated energy (MNm/m^2)	Mean 1.314 10^{-4} Std dev. 6.57 10^{-4}

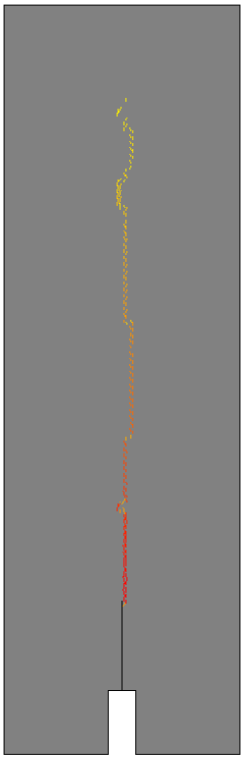

Figure 10: Macroscopic model – Example of macrocrack pattern obtained with the macroscopic model.

5 DISCUSSION AND CONCLUSION

The paper presents two different numerical strategies for the modelling of cracking processes in concrete structures. They can be essentially distinguished by the way they represent cracking processes and more precisely by the modelling scale where they act to describe these processes: a discrete and a macroscopic approach are then proposed. They are probabilistic as they take into account the inherent heterogeneity of the material via random distributions of mechanical properties.

Both of these approaches only deal with crack creation. The discrete approach represents it via a local elastic-brittle behaviour of the material, and it is here recalled that this is acceptable only if the phenomenon is considered as located in a very small scale with respect to the scale of the structure. In addition to taking into account the randomness of the tensile strength, the macroscopic model allows a local dissipation of energy at the scale of the element. This dissipation aims to macroscopically represent the local energy consumption by cracking processes (creation and propagation) considered as diffused in the finite element until its failure. As the propagation is highly influenced by the material heterogeneity, the model proposes to consider this energy as a random variable, whose mean value is a material parameter (equal to the experimental value determined in [5]) but whose standard deviation is scale sensitive.

Although the objective of these strategies is to represent crack creation, validation example clearly shows that they are quite well adapted to approximate crack propagation in concrete structures, but they don't describe cracking processes at the same scale. Whereas the discrete approach approximates crack propagation via a successive creation of failure planes, the macroscopic approach represents it via a successive creation of voids.

Results clearly show, that a probabilistic discrete elastic-brittle approach (coupled to quadratic elements) can be chosen to precisely represent crack patterns, if the local information on this pattern (openings and spacings of cracks) are of primary importance for the study of the structural behaviour. But the direct consequence is a significant computational cost. On the contrary, a probabilistic damage approach (coupled to linear elements) can be efficiently used to represent the global behaviour of concrete structures with a sufficient description of the main macrocracks involved in the cracking processes. It is also interesting to note that the CPU time is significantly lower (CPU time per iteration: 100 times lower in the case of the macroscopic model with the coarse mesh – cf. Figure 8 – compared to the discrete model with the fine mesh – cf. Figure 3) in this case compared to the one of the discrete model.

Finally, when the modelling of cracking processes of a concrete structure is concerned:
– The model should take into account the heterogeneity of the material, and the cracking should be considered as a probabilistic phenomenon,
– The heterogeneity of the material depends on the scales of observation, study and modelling,
– The mechanical model must be adapted to the chosen scale of modelling, in direct relation to the type of information requested (global behaviour, crack characterization, etc.)
– The numerical support of the modelling must be adapted to the mechanical model.

REFERENCES

[1] P. Acker, C. Boulay, P. Rossi, On the importance of initial stresses in concrete and of the resulting mechanical effects, Cement and Concrete Research (17) (1987) 755764.

[2] Z.P. Bazant, B.H. Oh, Crack Band theory for fracture of concrete. Materials and Structures, 16:155-177, 1983.

[3] E. M. R. Fairbairn, N. F. F. Ebecken, C. N. M. Paz and F. -J. Ulm, Determination of probabilistic parameters of concrete: solving the inverse problem by using artificial neural networks, Computers & Structures, 78:1-3 (2000) 497-503.

[4] RFGC, Modèles de fissuration du béton, Revue Française de Génie Civil, (2003).

[5] P. Rossi, O. Coussy, C. Boulay, P. Acker, Y. Malier, Comparison between plain concrete toughness and steel fibre reinforced concrete toughness, Cem. Conc. Res. 16 (1986) 303313.

[6] P. Rossi, Fissuration du béton: du matériau à la structure Application de la mécanique linéaire de la rupture, in

French, LPC research report n 150, (1988) June, ISSN 0222-8394.

[7] P. Rossi, Influence de la présence deau libre au sein du béton sur son processus de fissuration. C. R. Acad. Sci., 310-II (1990)11551161.

[8] P. Rossi et al., Fracture properties of concrete as determined by means of wedge splitting test and tapered double cantilever beams. In *Fracture Mechanics – test Methods for Concrete*. Rilem report 5. Edtrs S. P. Shah and A. Carpinteri. Chapman and Hall, pp 87-127, 1991.

[9] P. Rossi, X. Wu, Probabilistic model for material behavior analysis and appraisement of concrete structures. Mag. Conc. Res. 44 (1992) 271280.

[10] P. Rossi, X. Wu, F. Le Maou, A. Belloc, Scale effect of concrete in tension, Materials and Structures 27 (1994) 437444.

[11] J.-L. Tailhan, J. Lombart, P. Rossi, A. Deleurence. Modelling of the volume effects related to the uniaxial behaviour of concrete, ECF-16, 2006

[12] J.-L. Tailhan, P. Rossi, S. Dal Pont, Macroscopic probabilistic modelling of cracking processes in concrete structures, WCCM-ECCOMAS, 2008.

[13] J.-L. Tailhan, S. Dal Pont, P. Rossi. From local to global probabilistic modeling of concrete cracking, Ann. Solid Struct. Mech. (2010) 1:103115

[14] J.-L. Tailhan, P. Rossi, T. S. Phan, J. Foulliaron. Probabilistic modelling of crack creation and propagation in concrete structures: some numerical and mechanical considerations, SSCS'12, Aix en Provence, May 29 - June 1, 2012.