

## SIMULATION OF CRACKING IN MASONRY ARCH BRIDGES

STEPHANIE A. FRANCK<sup>\*</sup>, NICK BRETSCHEIDER<sup>\*</sup>, MARC GEBHARDT<sup>\*</sup>,  
VOLKER SLOWIK<sup>\*</sup> AND STEFFEN MARX<sup>†</sup>

<sup>\*</sup> Leipzig University of Applied Sciences  
PF 301166, 04251 Leipzig, Germany  
e-mail: stephanie.franck@fb.htwk-leipzig.de

<sup>†</sup> Leibniz Universität Hannover  
PF 6009, 30060 Hannover, Germany  
e-mail: marx@ifma.uni-hannover.de

**Key words:** Arch Bridge, Masonry, Safety Evaluation, Smeared Cracking

**Abstract:** The concept of smeared cracking has been adopted for the safety evaluation of masonry arch bridges. In order to realistically simulate the specific load-carrying behavior of such structures nonlinear analyses are required. The formulation of appropriate failure criteria and the safety concept are discussed. Parametric studies show the influence of the assumed material parameters on the calculated safety level. Based on the numerical results and on the experience with the analysis of real masonry arch bridges, recommendations for sufficiently accurate and computationally efficient nonlinear finite element analyses of these structures are formulated.

### 1 INTRODUCTION

The preservation of historic masonry arch bridges under the conditions of increasing traffic loads may require the re-analysis of such structures. Other possible reasons for a renewed safety evaluation are observed damage patterns or retrofitting measures.

The mechanical behavior of masonry arch bridges is characterized by structural changes due to inelastic strain, softening and cracking. These effects result under loading in significant stress redistributions which may not be captured by linear-elastic analyses. Only by considering the physical nonlinearities the load-carrying behavior of these structures may be realistically reproduced by the adopted mechanical model. It has also to be considered that not only the masonry arch itself but to a large extent supporting structural members like the spandrel walls and the backfill are contributing to the resistance against external loads. In order to take this into

account at least 2D models, preferably 3D models of the structure are required. By using simplified structural models, for instance framework models, or simplifying assumptions concerning the material behavior, the ultimate load level of masonry arch bridges is usually underestimated. This explains why for this particular type of structure a comparatively large discrepancy between the predicted load-carrying capacity and the real one may be observed. The objective of the presented work is to use a nonlinear analysis method for evaluating the safety of historic masonry arch bridges in order to preserve them.

Except for framework-type analysis models which are no longer recommendable, there are basically two numerical methods for simulating the mechanical behavior of masonry arch bridges. One of them is the Discrete Element Method (DEM) which has been successfully used for simulating the failure of such structures [1, 2]. With this

method, the opening of the “weak” masonry joints is appropriately reproduced resulting in realistic damage patterns. The authors preferred, however, the Finite Element Method (FEM), mainly because of the shorter computing times and the better availability of software tools. The physical nonlinearities have to be considered by assigning appropriate material laws to the elements. It is possible to apply a plasticity model in order to limit the stresses in the masonry and to account for inelastic strains. Schlegel [1] used an orthotropic material law with a failure surface for masonry which was originally proposed by Ganz [3]. In this way, the characteristic stress redistributions could be reproduced in the numerical model and regions of nonlinear material behavior were identified. As far as the behavior under tension is concerned, damage occurs as yielding in a plastic zone which is not exactly reproducing the real damage process characterized by individual cracks formed in masonry under tension. For this reason, in the present investigation the smeared crack approach, which is widely used for describing the behavior of concrete, has been adopted. It allows to reproduce crack patterns in the numerical simulations which may directly be compared to those observed at the real structures. Simplifying assumptions

are the homogeneity of the material and the isotropy of the material properties. The last mentioned simplification will be discussed in Section 3. Analyses of masonry arch bridges by using concepts of nonlinear fracture mechanics were successfully conducted before. Chandra Kishen et al. [4] and Audenaert et al. [5] simulated the crack propagation in such structures on the basis of the smeared crack approach.

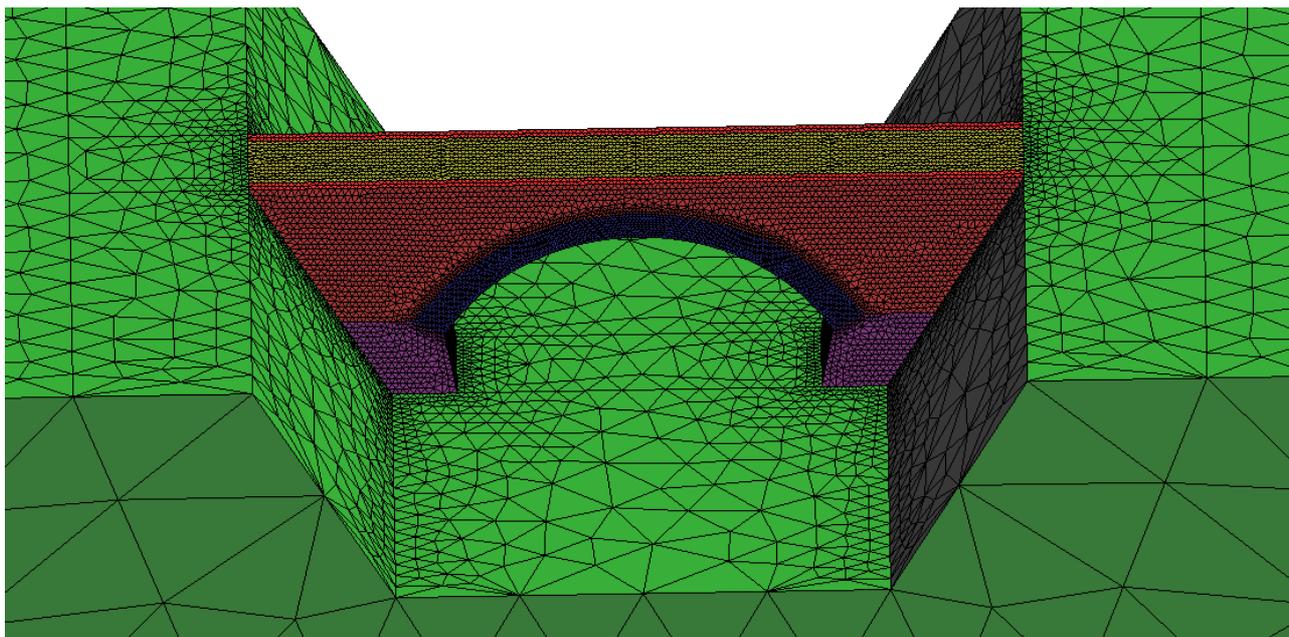
## 2 ANALYSIS CONCEPT

The main characteristics of the proposed analysis procedure are:

- Exact geometrical modeling of the masonry arch bridge with all structural members including columns, spandrel walls and backfill, preferably in 3D.
- Incorporation of the adjacent soil into the geometrical and finite element model.
- Consideration of the nonlinear material behavior of both masonry and soil.

Except for the material behavior of the masonry, which is subject of the next section, the aforementioned characteristics will be discussed in the following.

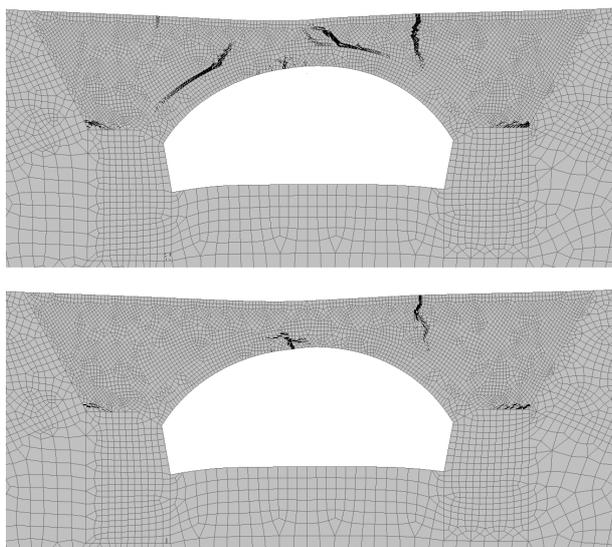
The spandrel walls considerably contribute into the load-carrying capacity of a masonry arch bridge. According to results by Weber [6]



**Figure 1:** Finite element model of a masonry arch bridge (complete model of the surrounding soil is not shown).

this contribution amounts to 24 %, the one of the backfill to 17 %. Whereas historic analysis procedures oftentimes discarded this contribution, the spandrel walls may easily be included in finite element models. In 2D models, to the parts of the structure which correspond to the spandrel walls with the backfill between them effective material parameters should be assigned which depend on the properties of both components.

The stiffness of the spandrel walls influences the crack patterns formed under external loads. If these sidewalls are weak, i.e., if their thickness is small, tangential cracks separating them from the arch, see Figure 2 (top), may be formed. In the case of thicker spandrel walls, these cracks are less likely to occur and, instead, localized fracture at midspan may be observed, see Figure 2 (bottom). By the non-uniform stiffening effect of the spandrel walls, a “weak spot” of the arch is formed at its crown.



**Figure 2:** Crack patterns formed in a masonry arch bridge with different thickness of the spandrel walls (top: 25 % of the arch width, bottom: 50 %; 2D model; unsymmetric loading on the left side; soil model is significantly larger than the shown detail).

The modeling in 3D allows to directly reproduce the geometry of the spandrel walls. Other advantages of 3D models are the consideration of failure modes involving longitudinal cracks, the possibility of modeling curved bridges, and the consideration of load

eccentricities as well as lateral loads, for instance wind or centrifugal forces. Furthermore, the soil-structure interaction may be reproduced more realistically in 3D. In certain cases, it appears to be inevitable to use a 3D model [7]. If the nonlinear material behavior is considered in the analysis and if, for the sake of a comparison, out-of-plane loading is excluded, a 3D model is likely to reveal an additional load-carrying capacity with respect to a corresponding 2D model [7]. This may mainly be attributed to out-of-plane stress redistributions.

It is recommended to incorporate the soil in the finite element models of masonry arch bridges, see Figure 1. Rigid supports at the abutments may lead to a significant overestimation of the load-carrying capacity. An alternative to the finite element model of the soil is the definition of longitudinal and rotational spring supports. The specification of the respective spring stiffness, however, would require simplifications or additional numerical analyses. The major advantages of a direct modeling of the soil are the geometrically correct representation of the foundation and the possibility to directly assign material properties to the soil, including those describing nonlinear deformations. Furthermore, the material properties may spatially vary within the model.

If linear-elastic soil behavior is assumed, unrealistic tensile stresses may be built up in the model and, as a result, the load-carrying capacity of the bridge would be overestimated. In certain cases, the material between the bridge supports may act like an internal tie constraining the arch. Hence, it is necessary to limit the tensile stresses being built up in the soil surrounding masonry arch bridges. For this purpose, the Drucker-Prager plasticity model has proved to be suitable. The influence of the material parameters required for this material law is comparatively small if technically feasible values are chosen. The main purpose of applying this model is to limit the tensile stresses in the soil.

Masonry arch bridges are characterized by a high self-weight. Hence, the portion of the internal forces resulting from the self-weight is

comparatively large. For this reason, the sequence of the load application during the construction process should be considered in numerical simulations of the load carrying behavior. Some structural members, for instance the arches, are added to the structure after other structural members, for instance the piers, have already undergone deformations. These deformations will then not affect the subsequently added structural members, in this case the arches. If the construction sequence is neglected in the simulations, the load-carrying capacity of the masonry arch bridge might be underestimated. A simple way of considering the construction sequence is to start the simulation with a negligible stiffness of the members to be added later, for instance the arches. In a subsequent load step, the real stiffness of these members is activated and their self-weight is imposed on the structure.

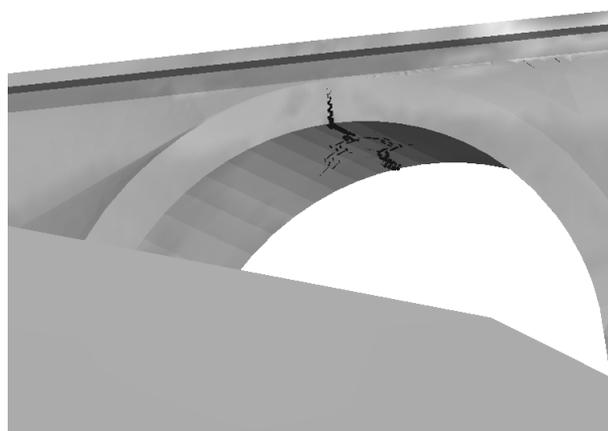
### 3 MATERIAL BEHAVIOR OF THE MASONRY

As already stated in the Introduction, the intention of the authors is to reproduce the cracking of masonry arch bridges in appropriate numerical models. Since such models account for the characteristic stress redistributions taking place in the structures, the analysis results are expected to allow for a realistic safety evaluation. Furthermore, the direct modeling of the crack propagation allows to formulate criteria for the limit state of strength on the basis of the crack pattern, see Section 4.

For all analyses presented in this paper, the program ATENA by Cervenka Consulting (Prague, Czech Republic) has been used as a numerical tool. Cracking is simulated by using a smeared crack model whereby a homogeneous material is assumed. The latter behaves linear-elastic until the maximum principal stress reaches the tensile strength. Then, a crack is formed and with increasing local crack opening the stress will decrease according to an exponential softening curve. The applied crack model is a rotating one. This assumption is justified since the direction of the principal strains is normally not subject to

significant changes during the loading of masonry arch bridges. Figure 3 shows a crack pattern obtained in a fracture simulation.

The fracture properties are considered to be independent on the orientation. Although the orientation of the masonry joints causes orthotropic material properties [1, 3], the simplifying assumption of isotropy is justified here by the fact that the maximum principal normal strains as well as the crack opening displacements will normally occur perpendicular to the radial joints in the masonry of the arch. The assumed isotropic properties are considered to describe the material resistance in this direction, i.e., in the direction of the tangent to the arch. These properties are controlling the fracture process in the respective masonry arch.



**Figure 3:** Cracks at the bottom side of a masonry arch bridge.

Under compression, a nonlinear stress-strain curve is assumed for the pre-peak range. For the corresponding post-peak range, a linear softening curve is used.

### 4 INVESTIGATION INTO CRITERIA FOR THE LIMIT STATE OF STRENGTH

The safety evaluation of masonry arch bridges by nonlinear analyses is not yet regulated by technical codes. The present paper is intended to contribute into the ongoing discussion of this subject.

Since the work of the authors was focused on railway bridges, the criteria for the limit state of strength proposed in the following

were derived from a code issued by the International Union of Railways (UIC). It has to be pointed out, however, that the criteria specified in this code were originally not formulated for the usage in nonlinear analyses.

The authors propose the following four failure criteria for nonlinear finite element analyses of masonry arch bridges. The first one, referred to as criterion 1 in the following, is based on the UIC Code 778-3 [8] and met when the maximum tensile stress reaches a certain value. Although in [8] a fixed value of  $0.5 \text{ N/mm}^2$  is specified, in the following parametric study this value will be varied. Criterion 1 corresponds to the event of crack initiation in the arch. Cracks in other parts of the bridge, for example in the spandrel walls, are not of relevance for criterion 1 since these cracks do not significantly affect the structural safety. The value of  $0.5 \text{ N/mm}^2$  specified in the UIC Code 778-3 [8] is not meant to be a masonry tensile strength. The intention is to allow for a small tensile zone in linear-elastic analyses and, thereby, to consider indirectly the partial cracking of the masonry arch.

The second criterion for the limit state of strength adopted here for nonlinear analyses is based on the allowable crack length in the masonry arch. According to UIC Code 778-3 [8], in linear-elastic analyses a criterion is met when the distance of the resultant compressive force to the compressed face is equal to or less than  $1/12$  of the arch thickness. In case of a failing tensile zone and a linear stress distribution in the compressive zone, this means that the minimum length of the compressive zone is equal to  $1/4$  of the arch thickness. From this consideration, criterion 2 for nonlinear analyses has been derived. It is met when the crack length in the arch reaches  $3/4$  of the arch thickness.

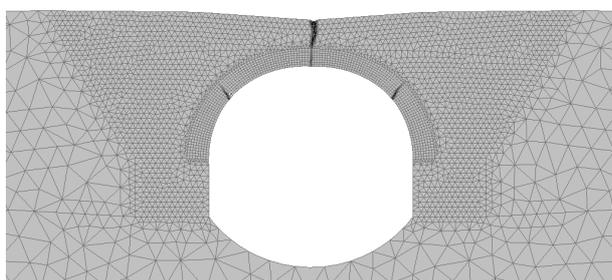
The two criteria described above were not derived from the physical behavior of the material. They may be considered to be empirical rules. When these criteria are applied, it is not advisable to consider partial safety factors on the resistance side which would reduce the individual material properties used in the nonlinear analysis. Because of the complexity of the material

models, changes to individual model parameters might lead to a different material behavior, for instance to an unrealistic material brittleness. It is necessary, however, to check the maximum compressive stresses occurring in the structure when the aforementioned criteria are met. These compressive stresses should not exceed the compressive strength reduced by a material factor. In this way, an adequate safety margin with respect to the ultimate load level is assured. After criteria 1 and 2 have been met, it is normally possible to significantly increase the live loads in the nonlinear finite element simulation. States of equilibrium are still being found until finally compressive failure occurs or the system becomes unstable. The ultimate load level may be considerably higher than the one reached when criterion 2 is met. However, the concept of limiting crack lengths in masonry members has been part of technical regulations for a long time. It limits the influence of the material inhomogeneity on the global structural behavior. Durability considerations also lead to a limitation of crack lengths in masonry arches.

When the compressive strength is reached in the masonry of a structural member, criterion 3 is met. According to this criterion, softening under compression is not allowed. It is recommended to consider not only the compressive stresses being built up in the arch and in the abutments, but also those in the spandrel walls. As stated before, it is not advisable to assign safety factors to the individual material properties used in the nonlinear analysis. Instead, a global safety factor should be used on the resistance side in case criterion 3 is the critical one.

Finally, an arch may fail when it becomes globally unstable due to the formation of multiple hinges. This is referred to as criterion 4 in the present investigation. Figure 4 shows an example of an arch under symmetric loading with three marked zones (shaded areas) of high compressive principal strain. The bridge model is supported by a finite element soil model larger than the cut-out shown in Figure 4. A state of equilibrium could no longer be found when this load level was reached. The nonlinear

finite element solution did not converge. It should be pointed out, however, that a not converging solution does not necessarily mean that there is no possible state of equilibrium. Convergence problems may also have numerical reasons. Therefore, the application of criterion 4 for the analysis of real structures appears to be problematic. On the other hand, before criterion 4 is met, normally the compressive strength (criterion 3) has already been reached. As for criterion 3, a global safety factor should be applied on the resistance side when criterion 4 is used.

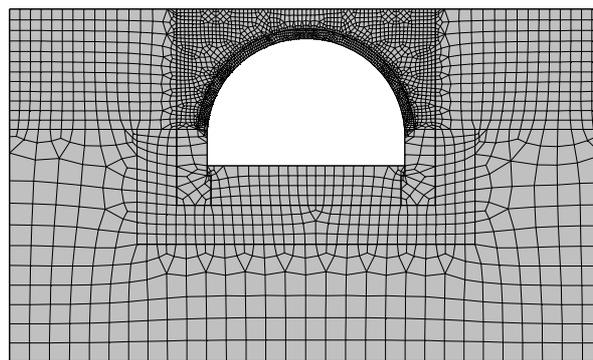


**Figure 4:** Zones of high compressive strain in a masonry arch bridge (dark areas).

It is recommended to express the level of structural safety by a single numerical value  $\eta$ . It represents the factor the live loads specified in the technical regulations may be multiplied with such that the level of structural safety is exactly matching the required one. Thereby, all load factors, i.e., the partial safety factors on the side of the actions, as well as the dynamic factors are considered. In case of criteria 3 and 4, these  $\eta$ -values should be divided by a global material factor as explained before. Hence, structural safety is proved under consideration of all safety and dynamic factors if  $\eta$  is equal to or larger than one. If  $\eta < 1$ , the safety level does not meet the requirements of the technical regulations. For the four failure criteria described above, normally different  $\eta$ -values are obtained. If the same tensile strength is applied, the  $\eta$ -value for criterion 2 (based on crack length) is generally larger than the one for criterion 1 (based on tensile stress). The smaller the difference, the more brittle is the fracture behavior of the arch.

In a parametric study on the basis of 2D bridge models, the sensitivity of the different failure criteria to the type of loading and to the masonry tensile strength as an important fracture parameter was investigated. Unfortunately, it is technically difficult to experimentally determine material properties for a particular masonry arch bridge. Nonlinear analyses are therefore often based on assumptions made on the basis of material properties given in technical recommendation or of those published in the literature.

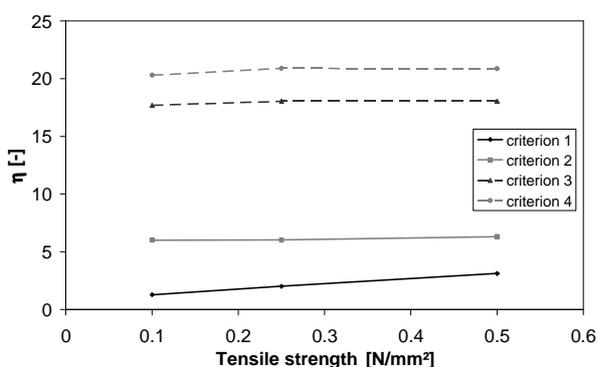
A first example of a 2D bridge model used for the parametric study is shown in Figure 5. The superstructure of this masonry bridge consists of the arch and of the spandrel walls. The latter have a total thickness of 1.6 m while the bridge has a width of 5 m. Between the spandrel walls, a filling is assumed which has a realistic density but a negligible stiffness. The foundation rests on a soil model the edges of which are supported in the normal direction. The Drucker-Prager plasticity model was used for modeling the soil. In addition to the self-weight of the superstructure, a constant line load of 156 kN/m was applied according to the UIC load model 71. For the calculation of the  $\eta$ -values, this external load was multiplied by a load factor of 1.45 and a dynamic factor of 1.2. In the case of *symmetric loading*, the line load was acting on the top edge of the model between the inner faces of the abutments and in the case of *unsymmetric loading* only on the left hand side of the bridge span.



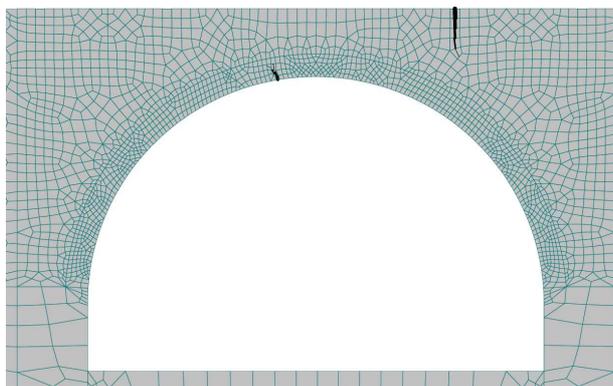
**Figure 5:** 2D finite element model used for a parametric study regarding the influence of the type of loading and of the masonry fracture properties.

In Figure 6, the influence of the masonry tensile strength on the  $\eta$ -values for the

different criteria is presented for the case of unsymmetric loading which is the critical load case. Simultaneously to the tensile strength, the fracture energy has been varied such that the critical crack opening, i.e., the crack opening at zero stress, was constant. In this way, an unrealistic increase of the brittleness with increasing tensile strength was to be avoided. A previous investigation has shown, however, that the influence of the fracture energy on the analysis results is comparatively small for masonry arch bridges when technically sound values are assigned [7]. The assumed compressive strength amounted to  $12 \text{ N/mm}^2$ .



**Figure 6:**  $\eta$ -values for the structure shown in Figure 5 under unsymmetric loading as dependent on the tensile strength of the masonry.



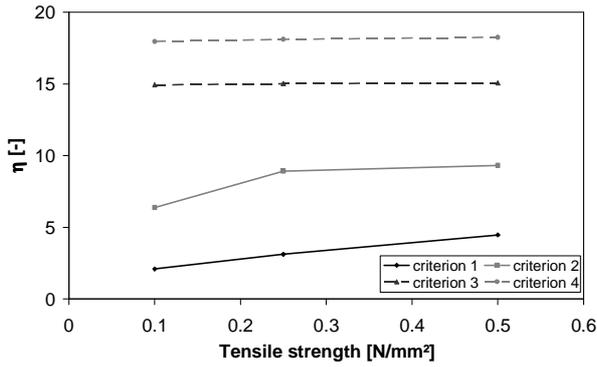
**Figure 7:** Crack pattern in a masonry arch under unsymmetric loading on the left side (figure contains detail of the finite element model shown in Figure 5).

As expected, the tensile strength has a significant influence on  $\eta_1$ , i.e., the  $\eta$ -value for criterion 1 which is based on the maximum tensile stress in the arch. The influence of the tensile strength on criterion 2 is considerably smaller. This means, the correct assumption of

the masonry tensile strength is more important for criterion 1 than for criterion 2. Still, this material property has a noticeable influence on the analysis results. When criterion 2 is met, the compressive stresses in the arch and in the spandrel walls do not reach the compressive strength here. In the arch, they amount to  $-0.641$ ,  $-0.644$ , and  $-0.482 \text{ MPa}$  for the tensile strength values of  $0.1$ ,  $0.25$ , and  $0.5 \text{ MPa}$ , respectively, and in the spandrel walls to  $-3.991$ ,  $-4.015$ , and  $-4.328 \text{ MPa}$ . Figure 7 shows a characteristic crack pattern observed when criterion 2 was met under unsymmetric loading. In contrast to the  $\eta$ -values for the other criteria, the  $\eta_2$ -value exhibits a mesh dependency which is not negligible. This may be attributed to the limited number of elements along the uncracked portion of the arch thickness and also to slight changes of the crack patterns in the spandrel walls observed when the present model was refined. Systematic convergence studies are under way.

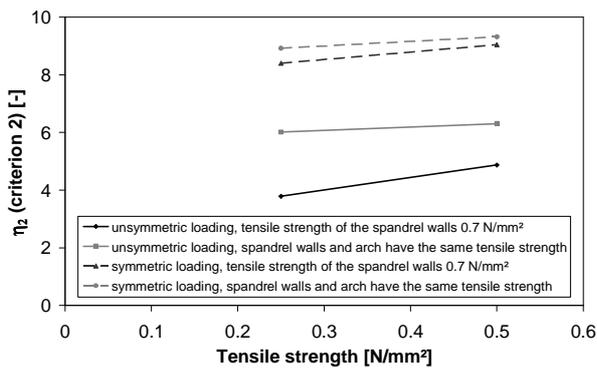
In Figure 6, the difference between  $\eta_2$  and  $\eta_3$  is comparatively large. It has to be considered, however, that for criteria 3 and 4 a global safety factor should be applied on the resistance side. This was not done in the present parametric study. Assuming a factor of 2, the reduced  $\eta_3$ -values would still be higher than the corresponding  $\eta_2$ -values. Hence, criterion 2 seems to be critical for this particular bridge and the applied load model. It has to be pointed out that the maximum compressive stress occurred in the spandrel walls. In the arch, the compressive strength was never reached for the given configuration. When the solution did no longer converge, criterion 4 was considered to be met. The difference between  $\eta_3$  and  $\eta_4$  is rather small.

The results for the case of symmetric loading are shown in Figure 8. Here, the  $\eta$ -values are higher since this load case is not the critical one. The difference between  $\eta_2$  and  $\eta_3$  is smaller than in the case of unsymmetric loading. This may be attributed to the fact that criterion 3 which is based on the compressive strength tends to become critical when the thrust line for the given loading approximates the shape of the arch.



**Figure 8:**  $\eta$ -values for the structure shown in Figure 5 under symmetric loading as dependent on the tensile strength of the masonry.

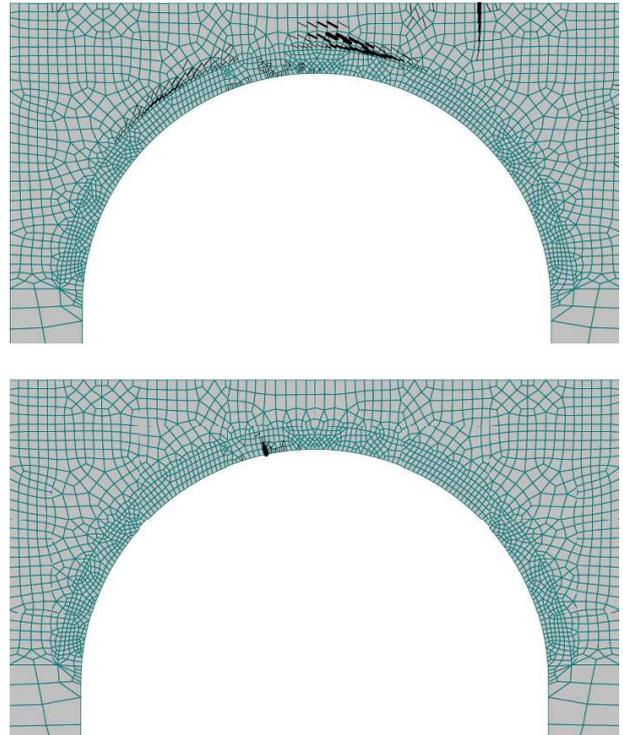
At the example of the bridge model shown in Figure 4, it may be seen that under certain conditions the failure of the bridge will occur due to compressive failure. Criterion 2 which is based on the crack length will not be met before the compressive strength is reached (criterion 3). In the case of the bridge shown in Figure 4 this is the case for symmetric loading as well as for unsymmetric loading. The arch has a comparatively large thickness. Consequently, the thrust line tends to have a large distance from the edges of the arch and the bending stresses in the arch will be small when compared to those resulting from the compressive force.



**Figure 9:**  $\eta_2$ -values for the structure shown in Figure 5 for different tensile strengths of the arch and of the spandrel walls.

In another study based on the bridge model shown in Figure 5, different tensile strengths were assigned to the arch and to the spandrel walls. It can be argued that these strength

values should be different because of the crack orientation with respect to the masonry joints in these members. If the tensile strength of the spandrel walls is set to  $0.7 \text{ N/mm}^2$  which is higher than the value for the arch, the  $\eta_2$ -values will decrease, see Figure 9, whereas the other  $\eta$ -values are almost unaffected. The reason for this phenomenon is the stiffening effect of the uncracked spandrel walls. This leads to a localization of the cracking at the arch crown, see Figure 10. Consequently, the crack length associated with criterion 2 is reached at a lower load level.



**Figure 10:** Stiffening effect of the uncracked spandrel walls (figure contains detail of the finite element model shown in Figure 5; top: spandrel walls and arch have the same tensile strength, bottom: spandrel walls have a higher tensile strength; unsymmetric loading on the left side).

## 5 CONCLUDING REMARKS

The load-carrying behavior of a masonry arch bridge depends on a variety of geometrical and material parameters. Some influences have been discussed in the present paper. Each individual structure requires a separate consideration of possible failure

modes and adequate criteria for the limit state of strength. Nonlinear finite element simulations allow to reproduce the complex mechanical behavior of these structures. In future nonlinear analyses, pre-existing damage patterns should be taken into account.

It has to be considered that nonlinear analyses usually need to be based on assumptions concerning the material behavior. The validity of the analysis results may be enhanced, however, if the simulation is supported by in situ measurements at the respective structure. A method to prove the robustness of the analysis model for a particular bridge and to identify the most influential material parameters are multi-parameter sensitivity analyses [9].

## REFERENCES

- [1] Schlegel, R., 2004. Numerische Berechnung von Mauerwerksstrukturen in homogenen und diskreten Modellierungsstrategien (Numerical analysis of masonry structures using homogeneous and discrete modeling strategies). PhD thesis. Bauhaus-Universität Weimar, Weimar, Germany.
- [2] Rafiee, A., 2008. Contribution à l'étude de la stabilité des massifs rocheux fracturés: caractérisation de la fracturation in situ, géostatistique et mécanique des milieux discrets. PhD thesis. Université Montpellier II, Montpellier, France.
- [3] Ganz, H. R., 1985. Mauerwerkscheiben unter Normalkraft und Schub. PhD thesis. ETH Zürich, Birkhäuser Verlag Basel, Switzerland.
- [4] Chandra Kishen, J. M., Ramaswamy, A., 2010. Cracking analysis of brick masonry arch bridge. B. H. Oh et al. (eds.), Proceedings of the 7<sup>th</sup> International Conference on Fracture Mechanics of Concrete and Concrete Structures (FraMCoS-7). Jeju, Korea, May 23-28, 2010, Vol. 3, pp. 1872-79.
- [5] Audenaert, A., Fanning, P., Sobczak, L. and Peremans, H., 2008. 2-D analysis of arch bridges using an elasto-plastic material model. *Engineering Structures* **30**:845-55.
- [6] Weber, W. K., 1999. Die gewölbte Eisenbahnbrücke mit einer Öffnung. PhD thesis. Technische Universität München, München, Germany.
- [7] Franck, S. A., Slowik, V., Marx, S., 2012. Safety evaluation of masonry arch bridges by nonlinear finite element analyses. A. Strauss, D. M. Frangopol, K. Bergmeister (eds.), Life-Cycle and Sustainability of Civil Infrastructure Systems, Proceedings of the Third International Symposium on Life-Cycle Civil Engineering (IALCCE 2012). Vienna, Austria, October 3-6, 2012, Taylor & Francis Group, London, 2013, 2063-70.
- [8] UIC Code 778-3. 1995. Recommendations for the assessment of the load carrying capacity of existing masonry and mass-concrete arch bridges. Paris: International Union of Railways (UIC).
- [9] Schlegel, R., Will, J., 2012. Sensitivitätsanalyse, Parameter- und Systemidentifikation an historischen Mauerwerkstrukturen mithilfe moderner Optimierungsverfahren. Mauerwerks-Kalender 2012. Ernst & Sohn Verlag Berlin, pp. 626-33.