ON THE CHARACTERIZATION OF STRAIN HARDENING CEMENT-BASED MATERIALS BY INVERSE ANALYSIS OF BENDING TESTS

N. BRETSCHNEIDER, B. VILLMANN AND V. SLOWIK

Leipzig University of Applied Sciences
PF 301166, 04251 Leipzig, Germany
e-mail: nick.bretschneider@fb.htwk-leipzig.de, www.htwk-leipzig.de

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Abstract: An inverse analyses procedure for determining the mechanical material properties of strain hardening cement-based materials is presented. It allows to determine the uniaxial stress-strain curve up to the material's localization point on the basis of the results of four-point bending tests. In these tests, the beam curvature is measured and serves as input value for the inverse analysis. An evolutionary optimization algorithm is used. It is demonstrated that the beginning of damage localization in a bending test does not necessarily lead to a decrease of the external load. Results of direct tension tests were used for validating the inverse analysis procedure.

1 INTRODUCTION

Fibre reinforced cementitious materials may exhibit a strain hardening behaviour under uniaxial tension. This means that after crack initiation the stress will increase or at least remain constant under increasing strain. In this hardening stage, multiple parallel cracks are being formed. Eventually, the traction in one of these cracks will start to decrease resulting in a closing of the neighbouring cracks. The uniaxial stress-strain curve up to this point of damage localization is the subject of the present investigation.

Whereas the softening behaviour of cementitious materials may be determined by testing notched specimens, preferably in wedge splitting or bending tests, the investigation of the hardening behaviour requires tests of unnotched specimens. Otherwise, the characteristic multiple cracking may not be activated.

An accepted experimental method for determining the material-specific stress-strain curve of hardening materials are uniaxial tension tests. Since such tests are time-consuming and expensive, easy to perform bending tests are an alterative for practical material testing. However, these tests require inverse analyses because the uniaxial stress-strain curve can not be obtained directly from the test results. The inverse analyses include numerical simulations of the fracture process taking place in the specimen and an iterative fitting of the numerical results to the experimental ones. In this way, the best fitting set of material parameters is identified.

In the following, an inverse analysis procedure for determining the strain hardening behaviour of cementitious materials is described and discussed. The underlying mechanical model and the optimization method were already presented at a previous FraMCoS conference [1]. In the present paper, an improved experimental setup is proposed and for the validation of the method new experimental as well as numerical results are presented.

Inverse analysis methods have been used successfully in concrete fracture mechanics for decades, in most cases for identifying softening properties [2-5]. For strain hardening materials, Qian et al. [6] developed a simplified inverse analysis procedure to be applied in quality control. Tensile strength and strain capacity are derived from the results of bending tests by using previously determined master curves linking the flexural behaviour to the uniaxial one. Østergaard et al. [7] modified their hinge model and the corresponding inverse analysis method in order to determine both the hardening and the softening behaviour of cementitious materials.

The inverse analysis procedure described in the present paper allows to identify only the hardening behaviour. It has to be considered, however, that for most structural applications of cementitious strain hardening materials only the stress-strain curve up to the localization point is of technical interest. If, in spite of that, the softening curve of strain hardening cementitious materials needs to be determined, the authors refer to a previously proposed two-step method involving bending tests of notched specimens in addition to those of unnotched specimens [1, 10-11].

The objective of the authors' work on the inverse analysis of fracture experiments was, firstly, to adopt mechanical models which would not require the assumption of model parameters in addition to the material properties to be identified and, secondly, to use a suitable optimization method for the fitting process [1, 4, 12-13]. As far as the optimization method is concerned, an evolutionary algorithm including local neighbourhood attraction for convergence improvement has proved to be an appropriate method for determining different material parameters of cement-based materials. This algorithm was used in inverse analyses of fracture tests for determining softening parameters [4] as well as in inverse analyses of drying experiments for the determination of the moisture dependent diffusion coefficient [8] and of the infinitesimal shrinkage strain [9]. Evolutionary algorithms are biologically motivated iterative stochastic optimization methods. They originate from Darwin's model of evolution by natural selection of individuals, i.e., of possible solutions in computational applications, and incorporate genetic operations like combination of individuals (heredity) or mutations [14-16].

The evolutionary algorithm used by the authors allows within a moderate computing time for very good fits which could not be achieved by manual fitting. In its current version, the associated software tool allows to optimize models with up to 12 parameters. In the case of the hardening behaviour, these 12 parameters may describe a multi-linear stress-strain curve with seven segments. The comparatively high number of parameters which may be optimized is an advantage of evolutionary algorithms. The optimization problem discussed here, i.e., the identification of the stress-strain curve, requires indeed a high number of parameters. This may be attributed to the fact that large variations in the hardening function cause only comparatively small differences in the numerically obtained load-displacement curves. Consequently, only quite exact approximations provide reliable inverse analysis results, i.e., sufficiently accurate hardening functions. This entails the formulation of an adequate error measure. In the work presented here a mixed error criterion based on vertical and horizontal deviations between the experimentally and numerically obtained curves was used [4].

Because of the stochastic nature of evolutionary algorithms the risk of getting caught by local minima in the error function is reduced. This is another advantage of this type of optimization algorithms and of particular importance for the parameter identification in concrete fracture mechanics since the corresponding error functions are usually quite jagged [4].

Finally, it should be mentioned that in evolutionary algorithms the underlying physical model is completely separated from the optimization method. There are no restrictions concerning type and mathematical formulation of the model. This allows for a high variability in the process of finding an appropriate material law.

2 PROPOSED METHOD

2.1 Experiments

As explained before, the investigation of the hardening behaviour requires bending tests of unnotched specimens. In order to completely activate the high strain capacity of the respective material, four-point loading is applied, see Figure 1, resulting in a comparatively large highly strained volume. The specimen dimensions for the tests described in Section 3.1 are given in Table 1. Of course, the inverse analysis procedure may also be applied to bending tests with other specimen dimensions.

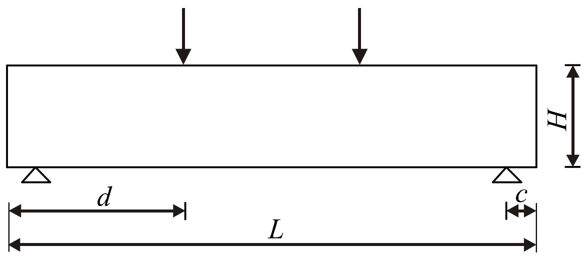

Figure 1: Geometry of the tested beams.

Table 1: Specimen dimensions.

H	80 mm
L	400 mm
c	25 mm
d	125 mm
Thickness	40 mm
Length of region with constant moment	150 mm

Figure 2 shows the experimental setup for the four-point bending tests. In view of the inverse analysis, it appeared to be useful to measure the curvature of the beams in order to have an additional or alternative measurand to be compared to the analysis results. For that, LVDTs are installed in axial direction at the side faces of the respective beam, on each side one at the top and one at the bottom, see Figure 2. On the basis of the measured displacement differences, the average curvature in the centre part of the beam may be calculated. The LVDTs used for the curvature measurement are connected to the beam's surface by hinges, see Figure 3. The vertical displacements of the loading points and of the supports are also measured by LVDTs.

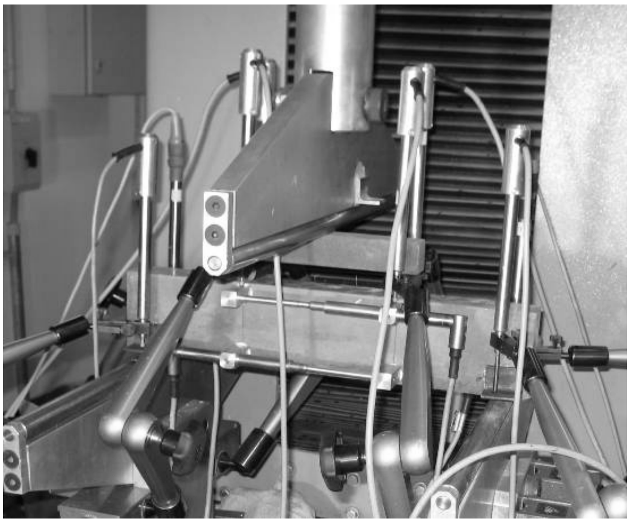

Figure 2: Experimental setup for the bending tests.

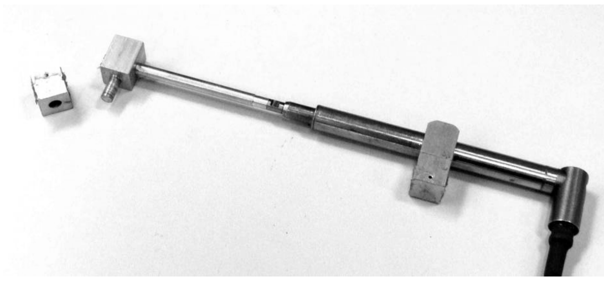

Figure 3: LVDT with hinge connectors for the curvature measurement.

Measuring the curvature rather than the deflection has already been recommended by the Japan Concrete Institute [17], although a completely different analysis procedure was proposed in this recommendation. The investigations presented here have confirmed that the inverse analysis results are more accurate when the beam curvature is measured and used as an input parameter for the iterative curve fitting. This may be attributed to the fact that only the deformation in the inner part of the beam length is captured by the curvature measurement whereas the mechanical behaviour outside this part does not influence the measurement results. Since these influences are excluded, a better accordance between experiment and mechanical model may be achieved. This is an improvement with respect to the method proposed in [1].

2.2 Mechanical model

Taking advantage of symmetry, only half of the beam is modelled, see Figure 4. The inner region of the 2D model, i.e., the region with constant maximum moment, consists of non-linear axial members and the side region of a linear-elastic finite element model. Hence, the mechanical model does not require a plane section assumption, except in the plane of symmetry. Cracking outside the inner region of maximum moment is neglected. For practical reasons, the linear-elastic part, i.e., the side region, is pre-analyzed by finite elements and replaced by a linear-elastic macro-element in the actual inverse analyses. It has to be pointed out that evolutionary optimization algorithms require a high number of individual simulations. By the linear-elastic pre-analysis, the number of degrees of freedom and, consequently, the computing time are significantly reduced.

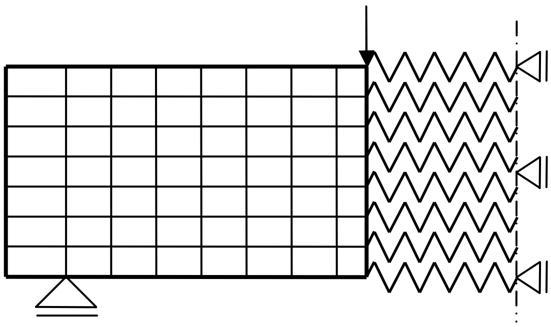

Figure 4: Model for the numerical simulations.

The mechanical behaviour of the nonlinear axial members is described by a multi-linear stress-strain curve with strain hardening in the tensile range, see Figure 5. In order to obtain correct simulation results, it appeared to be necessary to consider a nonlinear stress-strain behaviour also in the compressive range. In this range, a bilinear curve with a linear-elastic branch and a constant post-peak stress was adopted.

With the mechanical model shown in Figure 4 and the material law shown in Figure 5, the numerical simulations required for the inverse analysis are carried out, however only up to the localization point which is characterized by the maximum tensile stress. Thereafter, further damage would take place in a single fracture process zone and should be described by a softening curve [1, 11]. A continuing simulation with the model shown in Figure 5 would not be physically correct since it can not correctly reproduce the strain localization. Hence, the hardening curves obtained in the inverse analyses of the bending tests are truncated at the point of maximum stress.

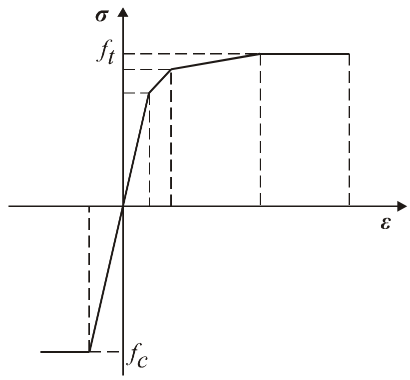

Figure 5: Stress-strain curve assumed for the nonlinear axial members.

The specimen self-weight is taken into account in the simulations, although parametric studies have shown that its influence is negligible for practical specimen sizes.

2.3 Inverse analysis of numerical experiments

The proposed analysis procedure has been tested on the basis of simulated bending tests, i.e., on the basis of artificial “experimental” results. Initially, the four-point bending tests described in Section 2.1 were numerically simulated by using a 2D finite element model and under the assumption of uniform material properties, i.e., assuming a homogeneous material. Thereby, the material law shown in Figure 5 was assigned to the elements. The mechanical behaviour of the models turned out to be too ductile when compared to real experiments. The uniform stress distribution led to the opening and softening of numerous parallel cracks the spacing of which was strongly mesh dependent rather than a material characteristic. The obtained crack patterns and force-curvature curves were not realistic.

A simple unsystematic application of a random distribution to the local tensile strength values resulted also in a strong mesh dependence of the simulation results, especially in the post-peak range. Therefore, the spatially varying material properties were assigned to the model by using global random fields, see Figure 6. The element properties were retrieved from the respective random field by a numerical averaging over the element area. Random fields have also been used by Kabele [18] for modelling strain hardening cementitious materials.

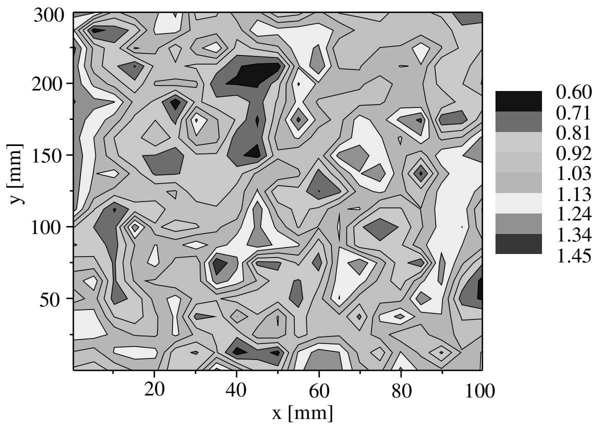

Figure 6: Random field for the material properties (dimensionless random variable).

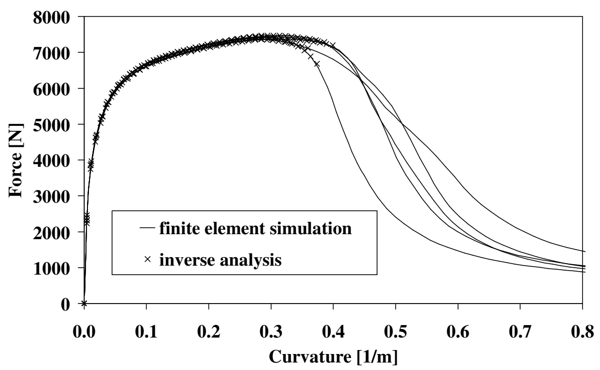

Figure 7: Simulation results of four-point bending tests.

Figure 7 shows five simulation results obtained on the basis of five different random fields. It is clearly visible that the differences between the individual curves are more pronounced in the softening range. They result from different crack patterns. Two examples of calculated crack patterns are shown in Figure 8.

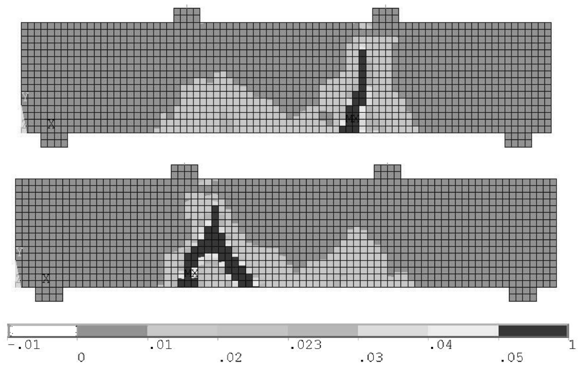

Figure 8: Simulation results of four-point bending tests, contours correspond to maximum principal strain.

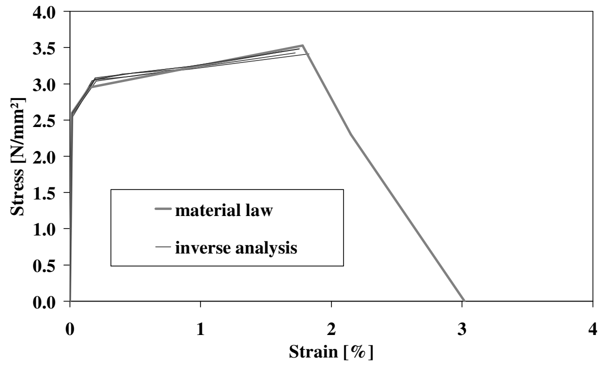

Figure 9: Hardening curves obtained by inverse analysis of the simulated four-point bending tests in comparison to the original material law.

The simulation results were used for testing the proposed inverse analysis procedure. Figure 9 shows the obtained hardening curves. As expected, these curves are in good accordance with the original uniaxial material law used for generating the different scatter fields. This indicates that the simplified mechanical model described in Section 2.2 is applicable for the inverse analyses. It yields results which are very close to those of a "full" nonlinear finite element analysis with plane elements.

3 VALIDATION OF THE METHOD BY EXPERIMENTS

3.1 Test materials and experiments

For validating the proposed procedure of conducting four-point bending tests and inverse analyses of the same, direct (uniaxial) tension tests were carried out. The results of these tests served as a reference for the hardening behaviour to be determined in the inverse analyses. Two different materials belonging to the group of High Performance Fibre Reinforced Cementitious Composites (HPFRCC) were investigated. The first one was a Strain Hardening Cement-based Composites (SHCC) with polyvinyl alcohol (PVA) fibre reinforcement. The fibres of type Kuraray REC15 had a length of 8 mm. Table 2 shows the material composition.

Table 2: Composition of the tested SHCC.

Cement [kg]	1.0
Water [l]	1.1
Fly ash [kg]	2.3
Silica flour [kg]	1.0
Silica sand [kg]	0.4
Plasticizer [kg]	0.04
Stabilizer [kg]	0.015
Fibre content [% by vol.]	2.2

The second test material was a ready mixed steel fibre reinforced mortar of type Refortec GF3 produced by Technochem Italiana Spa. This material was prepared according to the manufacturer recommendation. It is characterized by high strength in tension and compressive as well as by a high strain capacity. In contrast to that, the tensile strength of SHCC is usually as low as the one of unreinforced cement-based mortar. Hence, two very different hardening materials were used for validating the proposed inverse analysis procedure.

After casting, the specimens were left in their metal moulds and covered for three days. Thereafter, they were stored at 65 % relative air humidity and 20°C. The compressive strength after 28 days amounted to 25.5 N/mm² for the SHCC and to 163.5 N/mm² for the steel fibre reinforced mortar.

The four-point bending tests were conducted as explained in Section 2.1. For the direct tension tests, dog-bone shaped specimens with a cross-section of 40 mm by 40 mm over a length of 100 mm have been prepared. These specimens were glued between stiff platens and tested under displacement control as explained in [1].

3.2 Determination of the hardening curves by inverse analysis of the bending tests

Figure 10 shows the force-curvature curves for the SHCC. The solid lines are the experimental results of seven four-point bending tests and the dots represent the curves obtained by inverse analysis of the individual experiments. It may be seen that the evolutionary algorithm yields very good fits.

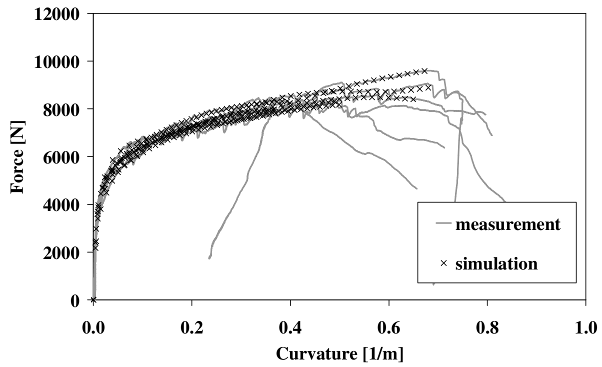

Figure 10: Comparison of measured and simulated force-curvature curves for the SHCC.

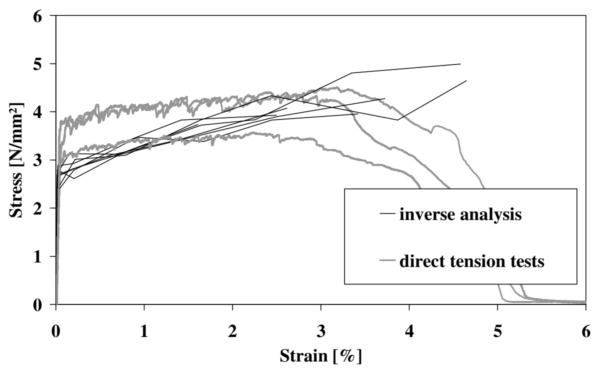

Figure 11: Obtained hardening curves for the SHCC.

The seven hardening curves obtained by the inverse analyses are presented in Figure 11 in comparison to the results of three direct tension tests conducted with the same material. It turns out that in most cases the stress at crack initiation is underestimated by the inverse analysis. The stress level at the localization point, however, is in good accordance with the reference results of the direct tension tests. The calculated strain at the localization point is in the same order of magnitude as the one in the direct tension tests, although two inverse analysis results overestimate this strain.

The force-curvature curves and the corresponding inverse analysis results for the steel fibre reinforced mortar are shown in Figure 12 and Figure 13, respectively. In contrast to the results for the SHCC, the inverse analysis tends to overestimate the initial cracking strength here. The hardening stress level and the localization strain seem also slightly to be overestimated.

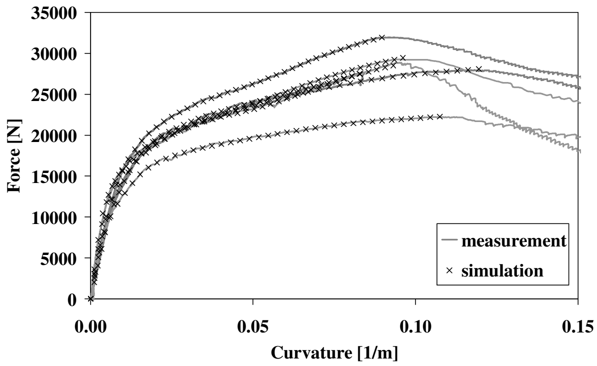

Figure 12: Comparison of measured and simulated force-curvature curves for the steel fibre reinforced mortar.

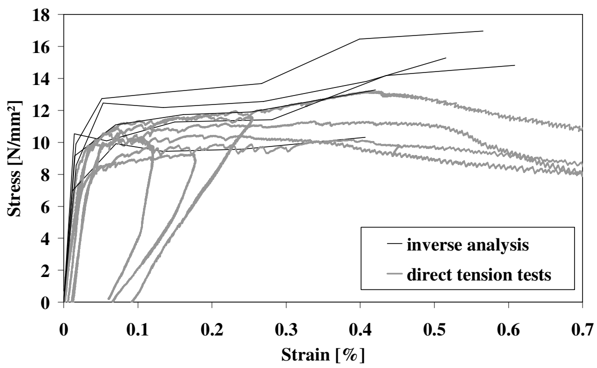

Figure 13: Obtained hardening curves for the steel fibre reinforced mortar.

A possible reason for the deviation between the inverse analysis results and the stress-strain curves obtained in direct tension tests is the simplified material law, especially as far as the nonlinear behaviour under compression is concerned. In addition, the fibres in the vicinity of the highly strained specimen boundary in the bending tests are usually oriented parallel to the surface. This results in a locally increased effective fibre content.

Considering the technical difficulties connected with direct tension tests and the scatter of the results, the inverse analysis of bending tests has proved to be a recommendable method for determining the material properties of strain hardening cementitious materials. For most practical applications, the inverse analysis yields sufficiently accurate results.

4 UNIAXIAL STRAIN HARDENING VERSUS DEFLECTION HARDENING

As stated in Section 2.2, the stress-strain curves obtained by inverse analyses of bending tests are physically correct only up to the localization point. Therefore, they are truncated at the point of maximum tensile stress although the numerical simulations may be continued beyond this point. In the bending tests, the point of local maximum tensile stress may be reached before the maximum of the external load is measured. Fracture localization in bending tests does not necessarily lead to an immediate decrease of the external load. This is demonstrated in Figure 14 which shows the results of two four-point bending tests of SHCC specimens. The dots represent the numerical results obtained by the corresponding inverse analysis. In each of the inverse analysis results, a characteristic point is marked. At these points, the maximum stress is reached at the bottom face of the respective beam, i.e., damage localization starts. In the case of beam I, the marked point coincides with the peak of the global force-curvature curve. In beam II, however, this point is reached at a load level lower that the external maximum load. The hardening occurring in the global force-curvature curve beyond the marked point is a phenomenon which may be observed only under bending. It does not indicate a continuing local strain hardening. Figure 15 shows the stress-strain curves corresponding to the results shown in Figure 14. The strain at fracture localization, also indicated by markers, is significantly smaller in beam II than in beam I. It may also be seen, that the stress decreases only slightly after fracture localization in beam II. This event does obviously not result in a sudden stress drop.

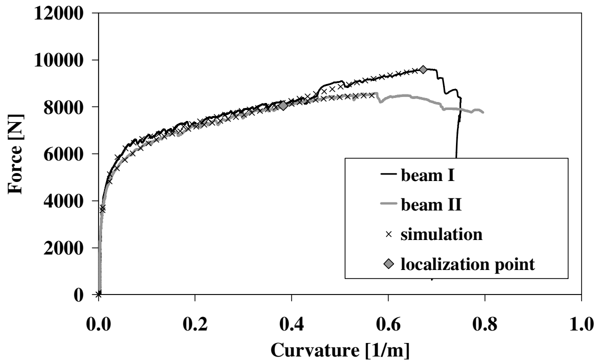

Figure 14: Comparison of measured and simulated force-curvature curves for the SHCC.

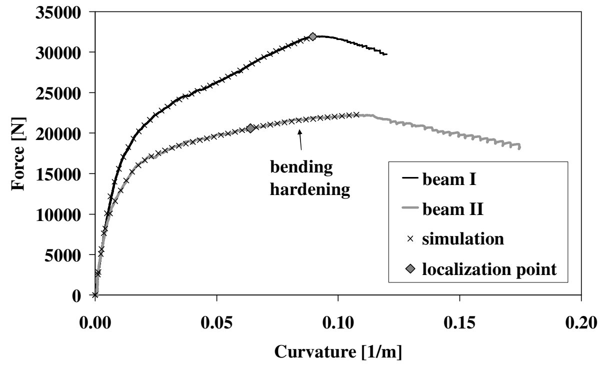

Figure 16: Comparison of measured and simulated force-curvature curves for the steel fibre reinforced mortar.

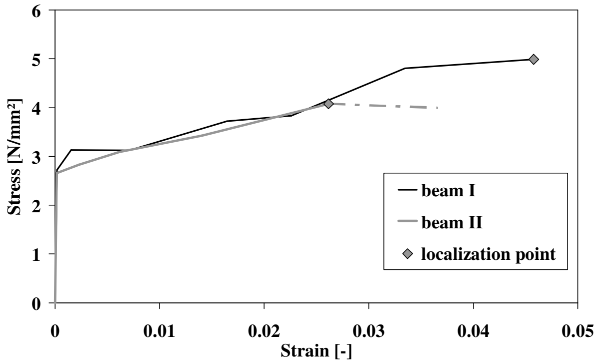

Figure 15: Obtained hardening curves for the SHCC.

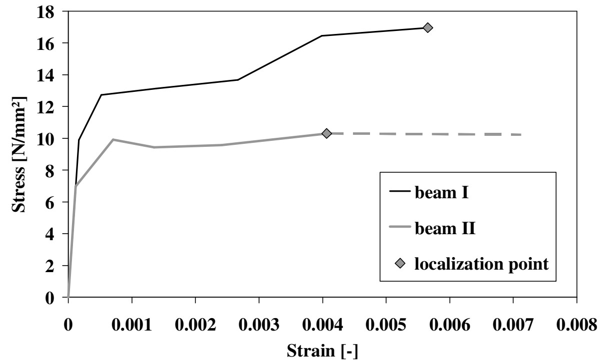

Figure 17: Obtained hardening curves for the steel fibre reinforced mortar.

Figure 16 and Figure 17 show corresponding results for the steel fibre reinforced mortar. As for the SHCC, two different four-point bending tests have been selected for demonstrating the difference between uniaxial strain hardening and deflection hardening. The results confirm that the point of local maximum stress does not necessarily coincide with the point of maximum external load in the bending test. Although the strain capacity of the steel fibre reinforced mortar is significantly lower than the one of the SHCC, this phenomenon may be observed. The results shown in Figure 16 and Figure 17 indicate that in the case of beam II deflection hardening may be observed after local softening has already started.

According to the results, it is impossible to distinguish between uniaxial strain hardening and deflection hardening purely on the basis of experimental results obtained in bending tests. An inverse analysis of these tests is required.

5 CONCLUDING REMARKS

The presented procedure for the characterization of strain hardening cement-based materials yields sufficiently accurate results for practical material testing. In four-point bending tests, the external force and the curvature are measured. For the inverse analysis of these tests, a simplified mechanical model and an evolutionary optimization method have proved to be suitable. Simplifications of the mechanical model were necessary since the high number of individual fracture simulations during the optimization requires computationally efficient analyses. Thus, within moderate computing times very good fits of the numerical results to the experimental ones are obtained.

The procedure allows to determine only the hardening behaviour, not the softening behaviour of the tested materials. It is possible, however, to identify the localization point in the bending tests and to distinguish between strain hardening and deflection hardening.

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