VIII International Conference on Fracture Mechanics of Concrete and Concrete Structures
FraMCoS-8
J.G.M. Van Mier, G. Ruiz, C. Andrade, R.C. Yu and X.X. Zhang (Eds)

# EFFECT OF FIBER REINFORCEMENT ON THE SHEAR BEHAVIOR OF REINFORCED CONCRETE BEAMS

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**Key words:** steel fibers, crack spacing, shear strength, fiber reinforced concrete

**Abstract:** Results from an experimental program aimed at evaluating the shear behavior of steel fiber reinforced concrete (SFRC) beams without stirrup reinforcement are presented. A total of 28 simply supported beams with a test shear span-to-effective depth ratio of approximately 3.5 were tested under a monotonically increased concentrated force up to failure. Among the 28 beams tested, four beams were constructed with regular concrete, three without stirrups and one with stirrups satisfying the minimum stirrup reinforcement requirements in the 2008 ACI Building Code. Three types of steel fibers were evaluated, all with hooks at their ends. Test variables were: 1) beam depth (455 or 685 mm); 2) fiber length-to-diameter ratio (55 or 80); 3) fiber tensile strength (1100 or 2300 MPa); 4) fiber volume fraction (0.75, 1.0 or 1.5%); and 5) longitudinal tension reinforcement ratio (1.6, 2.0 or 2.7%). All beams were designed so that a shear failure would ultimately develop, either prior to or after flexural yielding initiated. Regular strength concrete was used in all beams, with cylinder strengths at test day ranging between 29 and 51 MPa.

Test results showed that the use of hooked steel fibers in a volume fraction greater than or equal to 0.75% led to multiple diagonal cracks and a substantial increase in shear strength compared to regular concrete beams without stirrup reinforcement. All SFRC beams failed at a shear stress greater than or equal to $0.33\sqrt{f'_c}$, MPa, where $f'_c$ is the concrete cylinder strength. Also, the fiber reinforced concrete beams exhibited higher shear strength and better diagonal crack distribution compared to the beam with minimum amount of stirrups. No significant difference in average shear stress at shear failure was observed with an increase in beam depth from 455 to 685 mm. Although changes in fiber length did not lead to appreciable changes in beam overall behavior, maximum diagonal crack width prior to shear failure was found to be on the order of 5% of the fiber length. A simple model to predict the shear strength of SFRC beams is proposed based on the assumption that shear is resisted by fibers crossing diagonal cracks and shear carried in the concrete compression zone. Reasonable agreement between experimental and predicted strengths was found when applying this model to SFRC test beams reported in the literature.

## 1 INTRODUCTION

The use of discontinuous fibers as shear reinforcement has been extensively studied in the past forty years (for example, see References [1-8]. Fibers increase shear strength by bridging diagonal cracks, and by reducing crack spacing and width, which leads to increased aggregate interlock. However, most experimental research on the shear behavior of fiber reinforced concrete beams has focused on small-scale beams. In a test database published by Parra-Montesinos [9] of steel fiber reinforced concrete (SFRC) beams, only 14 beams out of a total of 147 fiber reinforced concrete beams were deeper than 610 mm.

Given the decrease in shear strength with increased depth exhibited by reinforced concrete beams without stirrups [10], experimental data on the shear behavior of relatively large fiber reinforced concrete beams are needed. This need led to the experimental investigation reported herein, in which 24 SFRC beams and 4 companion reinforced concrete beams with overall depth of either 455 mm or 685 mm and shear span-to-effective depth ratio of approximately 3.5 were tested under monotonically increased shear to failure.

## 2 EXPERIMENTAL PROGRAM

The experimental program consisted of the testing of 28 large-scale beams, 16 beams with overall depth of 455 mm (denoted B18 for the overall depth in inches) and 12 beams with an overall depth of 685 mm (B27 beams). All beams had a rectangular cross section, with a width of 152 mm for the 455 mm deep beams and 203 mm for the 685 mm deep beams. Four regular concrete beams were tested for comparison purposes, three of those without stirrup reinforcement and one satisfying the minimum stirrup reinforcement requirements in the 2008 ACI Building Code [11]. The main properties of each specimen are listed in Table 1, while the specimen reinforcement details are shown in Fig. 1.

As shown in Fig. 1, all test beams were simply supported and a single concentrated force was applied to the top of the beams. No stirrup reinforcement was used within the test shear span (longer span), except for Beam B27-8, which contained minimum stirrup reinforcement, while the other shear span was reinforced with sufficient stirrup reinforcement to ensure negligible shear-related damage during the test.

Three types of fibers were evaluated, all made of steel with hooks at their ends and manufactured by Bekaert Corp. For convenience, these fibers are referred to as Type 1, Type 2 or Type 3 fibers. Fiber parameters studied were fiber length, aspect ratio, and tensile strength. Fiber Type 1 (length: 30 mm, diameter: 0.55 mm, minimum tensile strength: 1100 MPa) was evaluated in volume fractions of 0.75, 1.0 and 1.5%. Fiber Type 2 (length: 60 mm, diameter: 0.75 mm, minimum tensile strength: 1050 MPa), on the other hand, was evaluated at volume fractions of 0.75 and 1.0%. Fiber Type 3 (length: 30 mm, diameter: 0.38 mm, minimum tensile strength: 2300 MPa) was evaluated only at a volume fraction of 0.75% given its substantially higher tensile strength, combined with a small diameter and large aspect ratio.

Another variable evaluated was tensile longitudinal reinforcement ratio $\rho$, where $\rho = A_s/(bd)$ and $A_s$ is the area of longitudinal tension reinforcement. In the test beams, $\rho$ was either 1.6, 2.0 or 2.7%. This allowed the evaluation of the behavior of SFRC beams failing in shear either prior to or after flexural yielding. For beams with $\rho$ = 1.6%, flexural yielding was expected prior to the development of a shear failure, while for beams with $\rho$ = 2.0 or 2.7%, whether shear failure occurred prior to or after flexural yielding depended on the type and amount of fibers used.

## 3. MATERIAL PROPERTIES

Concrete was either provided by a local ready-mix concrete supplier or mixed in the

**Table 1:** Beam properties and main test results

| Beam | $d$ (mm) | $a/d$ | $\rho$ (%) | Fiber type | $V_f$ (%) | $f'_c$ MPa | $P_u$ (kN) | $v_u$ (MPa) | $\frac{v_u}{\sqrt{f'_c}}$ | $n_d$ | Failure mode |
|---|---|---|---|---|---|---|---|---|---|---|---|
| B18-0a | 381 | 3.43 | 2.7 | - | - | 42.8 | 168 | 1.1 | 0.17 | 1 | DT |
| B18-0b | 381 | 3.43 | 2.7 | - | - | 42.8 | 162 | 1.1 | 0.17 | 1 | DT |
| B18-1a | 381 | 3.43 | 2.0 | 1 | 0.75 | 44.8 | 441 | 2.9 | 0.44 | 5 | SC+ST** |
| B18-1b | 381 | 3.43 | 2.0 | 1 | 0.75 | 44.8 | 413 | 2.8 | 0.41 | 4 | ST+DT** |
| B18-2a | 381 | 3.50 | 2.0 | 1 | 1.0 | 38.1 | 437 | 3.0 | 0.49 | 5 | ST+DT** |
| B18-2b | 381 | 3.50 | 2.0 | 1 | 1.0 | 38.1 | 445 | 3.1 | 0.50 | 5 | ST+DT** |
| B18-2c | 381 | 3.50 | 2.7 | 1 | 1.0 | 38.1 | 503 | 3.5 | 0.57 | - | N.A.** |
| B18-2d | 381 | 3.50 | 2.7 | 1 | 1.0 | 38.1 | 367 | 2.6 | 0.41 | - | N.A.* |
| B18-3a | 381 | 3.43 | 2.7 | 1 | 1.5 | 31.0 | 384 | 2.6 | 0.46 | 2 | ST+DT* |
| B18-3b | 381 | 3.43 | 2.7 | 1 | 1.5 | 31.0 | 507 | 3.4 | 0.61 | 5 | SC+ST |
| B18-3c | 381 | 3.43 | 2.7 | 1 | 1.5 | 44.9 | 494 | 3.3 | 0.49 | 3 | ST+DT |
| B18-3d | 381 | 3.43 | 2.7 | 1 | 1.5 | 44.9 | 490 | 3.3 | 0.49 | 4 | ST+DT |
| B18-5a | 610 | 3.43 | 2.7 | 2 | 1.0 | 49.2 | 445 | 3.0 | 0.43 | 3 | DT |
| B18-5b | 610 | 3.43 | 2.7 | 2 | 1.0 | 49.2 | 565 | 3.8 | 0.54 | 5 | ST+DT |
| B18-7a | 610 | 3.43 | 2.0 | 3 | 0.75 | 43.3 | 498 | 3.3 | 0.50 | 7 | ST+DT** |
| B18-7b | 610 | 3.43 | 2.0 | 3 | 0.75 | 43.3 | 490 | 3.3 | 0.50 | 7 | ST+DT** |
| B27-1a | 610 | 3.50 | 2.0 | 1 | 0.75 | 50.8 | 908 | 2.9 | 0.41 | 4 | ST+DT |
| B27-1b | 610 | 3.50 | 2.0 | 1 | 0.75 | 50.8 | 837 | 2.7 | 0.38 | 6 | DT |
| B27-2a | 610 | 3.50 | 2.0 | 2 | 0.75 | 28.7 | 872 | 2.8 | 0.53 | 4 | SC+ST |
| B27-2b | 610 | 3.50 | 2.0 | 2 | 0.75 | 28.7 | 854 | 2.8 | 0.52 | 5 | DT |
| B27-3a | 610 | 3.50 | 1.6 | 1 | 0.75 | 42.3 | 846 | 2.7 | 0.42 | 7 | F** |
| B27-3b | 610 | 3.50 | 1.6 | 1 | 0.75 | 42.3 | 863 | 2.8 | 0.43 | 7 | SC+ST** |
| B27-4a | 610 | 3.50 | 1.6 | 2 | 0.75 | 29.6 | 663 | 2.1 | 0.40 | 4 | ST+DT* |
| B27-4b | 610 | 3.50 | 1.6 | 2 | 0.75 | 29.6 | 556 | 1.8 | 0.33 | 4 | ST+DT* |
| B27-5 | 610 | 3.50 | 2.1 | 1 | 1.5 | 44.4 | 1081 | 3.5 | 0.53 | 6 | SC+ST** |
| B27-6 | 610 | 3.50 | 2.1 | 2 | 1.5 | 42.8 | 1046 | 3.4 | 0.52 | 6 | ST+DT** |
| B27-7 | 610 | 3.50 | 1.6 | - | - | 37.0 | 402 | 1.3 | 0.21 | 1 | DT |
| B27-8 (†) | 610 | 3.50 | 1.6 | - | - (†) | 37.0 | 570 | 1.8 | 0.30 | 3 | DT |

Width of beams in Series B18 and B27 = 152 mm and 203 mm, respectively.
(†) This beam contained minimum shear reinforcement (see Fig. 1 for reinforcement details)
* Significant bond degradation near support; **Flexural reinforcement yielded
***d***: beam effective depth; ***a***: shear span; ***ρ***: tension reinforcement ratio; ***$V_f$***: fiber volume fraction; ***f'c***: concrete; cylinder strength; ***$P_u$***; peak load; ***$v_u$***: peak average shear stress; $n_d$: number of diagonal cracks
DT: Diagonal Tension; SC: Shear Compression: ST: Shear Tension; F: Flexure

Structural Engineering Laboratory at the University of Michigan. Course aggregate consisted of crushed limestone with a maximum size of 10 mm. Fibers were added last to the concrete, and in the case of ready-mix concrete, fibers were added to the concrete truck on site.

Concrete compressive strength was evaluated through tests of 100x200 mm cylinders. Concrete cylinder compressive strength values are listed in Table 1. Flexural performance was evaluated through ASTM 1609 [12] four-point bending tests. Each ASTM beam had a 152 mm square cross section and a span length of 455 mm. As required in ASTM 1609, the beams were tested under displacement control up to a mid-span deflection of 3 mm.

In general, the ASTM beams with longer Type 2 fibers (60 mm in length) exhibited a more ductile response compared to the shorter (30 mm long) Type 1 and Type 3 fibers. This

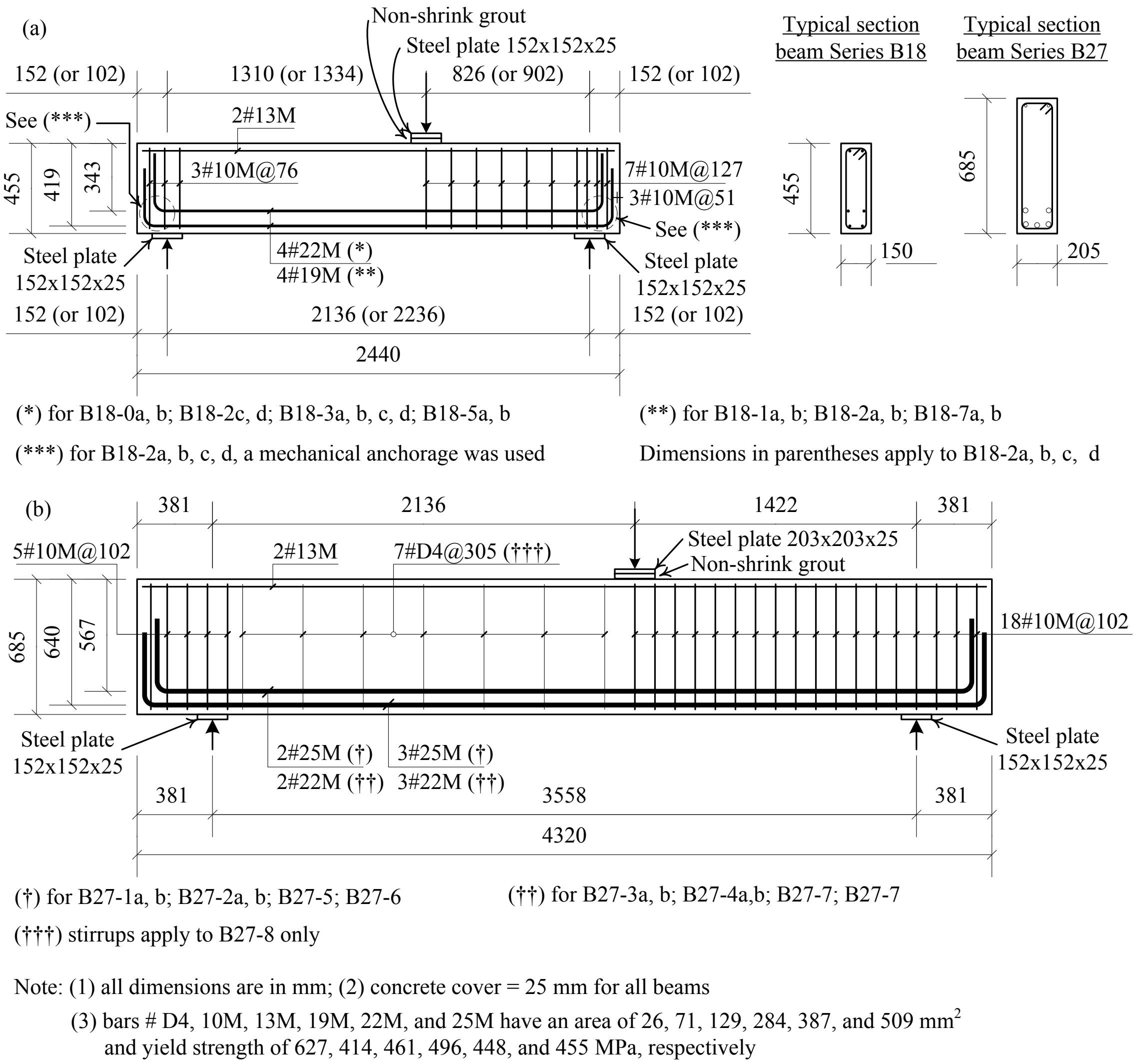


**Figure 1:** Reinforcement details in test beams

was because of the better ability of the longer fibers to bridge wide cracks (on the order of 2.5 to 5 mm at the end of the test). The shorter high-strength fibers (Type 3) were very effective soon after first flexural cracking, leading to a pronounced, but short hardening response. Type 1 fibers, on the other hand, exhibited a deflection softening response when used at a 0.75% volume fraction, but a slight hardening response, after a small drop in load immediately after fist cracking, when used in a 1.5% volume fraction. Figure 2 shows typical equivalent flexural stress versus deflection response for various steel fiber reinforced concrete mixtures with either 0.75 or 1.5% fiber volume fraction. Detailed information about these ASTM 1609 tests can be found elsewhere [13].

Stress-strain response of the reinforcing bar steel was obtained through direct tension tests. Yield and ultimate strength values, as well as the strain at initiation of strain hardening, are listed in Table 2.

## 4 EXPERIMENTAL RESULTS

In the following, a summary of the test results in terms of cracking pattern, failure mode, and load versus deflection response are provided.

## 4.1 Cracking pattern and failure mode

Typical crack patterns exhibited by the SFRC beams prior to failure are shown in Fig. 3, while the number of diagonal cracks crossing the beam mid-depth level at an angle less than 80º with respect to the beam longitudinal axis is reported in Table 1.

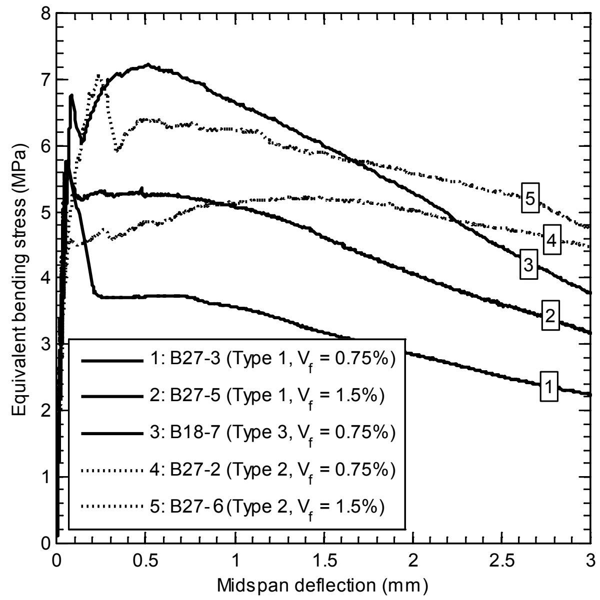


**Figure 2:** Representative equivalent bending stress versus deflection response from ASTM 1609 tests for various SFRCs

**Table 2:** Reinforcing steel properties

| Bar size | $f_y$ MPa | $\varepsilon_{sh}$ | $f_{su}$ MPa |
|---|---|---|---|
| D4(†) | 627* | * | 661 |
| 10M | 414 | ** | 579 |
| 13M | 461 | 0.0080 | 689 |
| 19M | 496 | 0.0090 | 751 |
| 22M | 448 | 0.0080 | 675 |
| 25M | 455 | 0.0080 | 689 |

(†) Area = 25.8 mm$^2$
* No clear yield point (calculated based on 0.2% strain offset)
** Strain hardening initiated as soon as steel yielded

At least three diagonal cracks were observed in all beams containing fiber reinforcement, except for Beam B18-3a, in which 2 diagonal cracks formed prior to failure. Only a single diagonal crack formed in the RC beams without stirrup reinforcement. The use of minimum stirrup reinforcement in Beam B27-8, on the other hand, led to a minor improvement in the cracking pattern compared to the RC beams without stirrups.

Cracking spacing was evaluated by determining the average horizontal crack spacing at beam mid-depth. Horizontal rather than perpendicular spacing between cracks was measured due to the fact that inclined cracks are generally not parallel to each other and tend to change direction as they propagate to the beam compression zone. Horizontal crack spacing, however, can be easily converted into a perpendicular crack spacing by assuming an average crack angle. Although multiple diagonal cracks occurred in all SFRC beams, there seemed to be a dependency of diagonal crack spacing on beam depth. In general, it was found that the average horizontal spacing between diagonal cracks could be reasonably approximated as 0.4***d***, regardless of the beam depth. Also, the width of the critical diagonal crack prior to shear failure was found to be dependent on fiber length and on the order of 5% of the fiber length. This indicates that performance criteria based on results from ASTM 1609 beams should be tied either directly or indirectly to crack width dependent on fiber length, as discussed in Section 5.1.

All test beams ultimately failed in shear, except for Beam B27-3a, which exhibited a flexural failure characterized by crushing of the beam compression zone near the load point after substantial yielding of the longitudinal tension reinforcement (Fig. 3c).

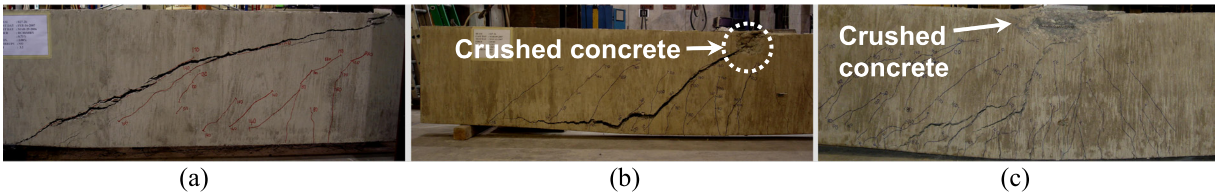


**Figure 3:** Cracking pattern in SFRC beams failing in (a) diagonal tension (Beam B27-2b); (b) combination of shear compression and shear tension (Beam B27-3b); and (c) flexure (Beam B27-3a)

Although three types of shear failures were observed, 1) diagonal tension failure (Fig. 3a); 2) a combination of diagonal tension and shear-tension failure; and 3) a combination of shear-compression and shear-tension failure (Fig. 3b), failure modes within each pair of nominally identical beams were not consistent, as can be seen in Table 1. It should also be mentioned that failure in four of the test beams seemed to have been associated with substantial bond degradation between the longitudinal reinforcement and the surrounding concrete near the support. This is believed to be the consequence of lumping of fibers along the top layer of tension steel reinforcement, or air voids during concrete casting, or a combination of both.

## 4.2 Load versus deflection response

All beams constructed with regular concrete exhibited a linear behavior up to shear failure, which even for the beam with minimum stirrup reinforcement was brittle. The load versus displacement response for the SFRC beams, on the other hand, was influenced by the amount of flexural reinforcement, fiber type and fiber content, which determined whether shear failure occurred prior to or after flexural yielding. The beams that exhibited flexural yielding prior to shear failure are identified in Table 1. For SFRC beams that failed in shear prior to flexural yielding, the response was approximately linear up to failure. However, the presence of fibers allowed the development of multiple diagonal cracks and the widening of at least one of them prior to a shear failure, which provided some warning about the imminence of failure. Ultimate failure in these beams, however, was rather sudden.

For cases in which flexural yielding preceded a shear failure, the load (or average shear stress) versus displacement response exhibited a well-defined yield plateau. However, because the shear force demand associated with flexural yielding was close to the expected beam shear capacity if the beam were to behave linearly elastic, the degree of yielding often varied, even within the same pair of nominally identical beams. Fig. 4 shows representative responses for the regular concrete beams (with and without stirrups), as well as for the SFRC beams that failed prior to or after flexural yielding.

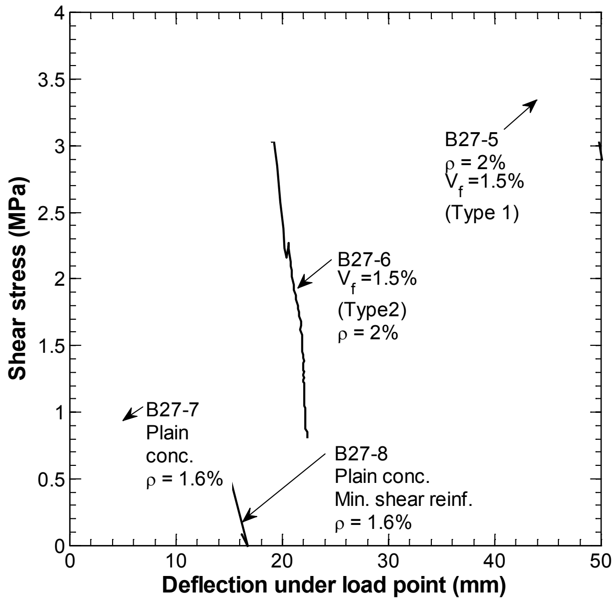


**Figure 4:** Shear stress versus deflection behavior of selected beam specimens

Peak average shear stress for each test beam is listed in Table 1. Average shear stress $v = V/(bd)$, where $V$ is the shear force, and $b$ and $d$ are the beam width and effective depth, respectively. All SFRC beams exhibited a shear strength between $0.33\sqrt{f_c'}$ and $0.61\sqrt{f_c'}$ (MPa), while the shear strength of the regular concrete beams without stirrups was $0.17\sqrt{f_c'}$ and $0.21\sqrt{f_c'}$ (MPa) for the beams with 455 and 685 mm of depth, respectively. The beam with stirrup reinforcement satisfying the minimum requirements in the 2008 ACI Code exhibited a nearly 50% increase in shear strength ($0.30\sqrt{f_c'}$, MPa) compared to the regular concrete beam without stirrups.

## 5 PREDICTION OF SHEAR STRENGTH

From test observations, a simple model has been proposed [14] for estimating the shear strength of beams with deformed steel fibers. The model is based on the assumption that at failure, beam shear strength can be calculated as the summation of shear due to diagonal tension resisted by fibers and shear carried in the compression zone. Fiber contribution through diagonal tension is not independent of aggregate interlock. For simplicity, however, the entire contribution from stresses at a crack is assumed to come from diagonal tension resisted by the fiber reinforcement.

Fig. 5 shows the assumed shear failure mode for SFRC beams. A critical diagonal crack that leads to shear failure, with a horizontal projection at the level of the tension reinforcement of ($d$-$c$) (i.e., $\alpha$ = 45 degrees in Fig. 5), where $c$ represents the neutral axis depth, is assumed at this stage. Consistent with the measured crack widths at failure, the width of the critical crack at the level of the longitudinal tension steel is taken equal to $0.05L_f$, where $L_f$ is the fiber length. The beam shear strength is then calculated as the summation of the vertical component of the resultant tension force across the critical diagonal crack, $V_{FRC} = T_f\ cos(\alpha)$, and the shear carried by the beam compression zone, $V_{cc}$. The determination of $V_{FRC}$ and $V_{cc}$ is discussed next.

### 5.1 Contribution of fiber reinforcement to beam shear strength

The fiber contribution to beam shear strength requires the estimation of the tensile stresses across the critical diagonal crack. For this purpose, an analogy is made between the critical diagonal crack in an actual beam, which is assumed to increase in width as the distance from the neutral axis increases, and the flexural crack that forms in the middle third of an ASTM 1609 beam under four-point bending. Assuming that the average tensile stress perpendicular to these two cracks is equal at a given maximum crack width, the problem can be simplified by looking at the average tensile stress in the ASTM 1609 beam at a crack width corresponding to the assumed crack width that leads to failure in the actual, i.e., $0.05L_f$. It is worth mentioning that if more than one flexural crack develops in the ASTM beam, the procedure outlined next is not applicable. Such a case, however, is rare in SFRC materials with fiber contents not exceeding 1% by volume.

For design purposes, the specification of a mid-span deflection at which the average tensile stress in an ASTM 1609 beam is to be calculated is more attractive than the use of a target crack width. In [15], it was shown that $L_f/24$ is a reasonable estimation of the deflection associated with a crack width of $0.05L_f$.

The average tensile stress at the critical crack width ($\sigma_{fu}$ in Fig. 5), assuming a neutral axis depth of $0.1h$, can then be calculated based on the moment in the middle third of an ASTM 1609 beam at a mid-span deflection $\delta = L_f/24$ as follows,

$$\sigma_{fu} = \frac{2(M)_{\delta=\frac{L_f}{24}}}{0.9bh^2} \tag{1}$$

where $b$ and $h$ are the width and height of the ASTM 1609 beam ($b = h$ in ASTM 1609). In order to account for potential differences between the behavior of SFRC in ASTM 1609 beams and that in the actual beam (e.g. due to fiber distribution, member size), it is recommended that a strength reduction factor, arbitrarily selected as 0.8, be applied to the average tensile stress in Eq. (1) for use in the calculation of $V_{FRC}$.

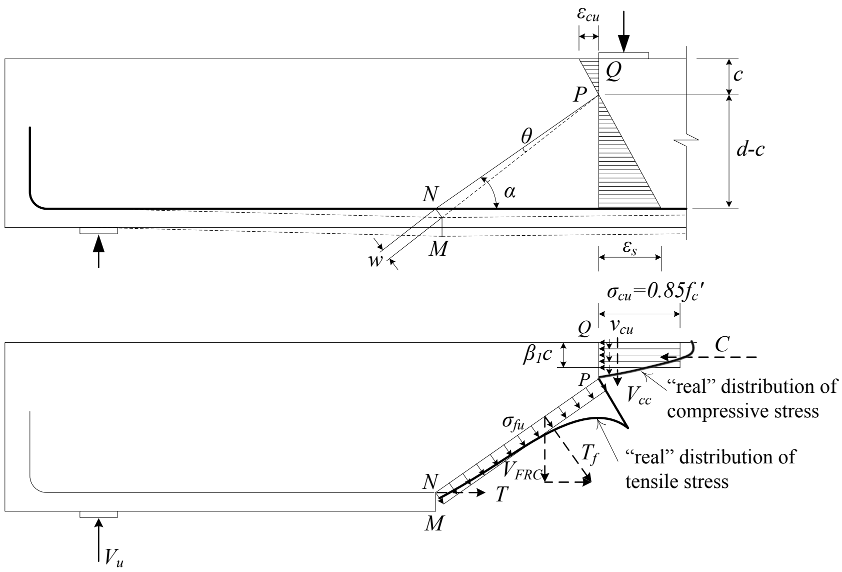


**Figure 5:** Assumed critical crack and stress distribution at shear failure

## 5.2 Contribution of concrete compression zone to beam shear strength

The shear carried by the beam compression zone is determined based on the failure criterion developed by Bresler and Pister [16] for concrete subjected to combined shear and compression stresses. This failure criterion is defined as follows,

$$\frac{v_{cu}}{f_c'} = 0.1\left[0.62 + 7.86\left(\frac{\sigma_{cu}}{f_c'}\right) - 8.46\left(\frac{\sigma_{cu}}{f_c'}\right)^2\right]^{1/2} \qquad (2)$$

where $v_{cu}$ and $\sigma_{cu}$ are the acting shear stress and normal compressive stress at failure, respectively. It has been shown [13] that rather than calculating $v_{cu}$ based on a nonlinear compressive stress distribution over the beam compression zone, the use of Whitney's stress block, as defined in the ACI Building Code [11], leads to conservative estimations of the shear carried in the compression zone. In this case, $\sigma_{cu} = 0.85f_c'$ and $v_u = 0.11f_c'$ over a depth $a = \beta_1 c$, where $\beta_1 = 0.85$ for $f_c' \leq 28$ MPa and $\beta_1 = 0.65$ for $f_c' \geq 55$ MPa. Linear variation of $\beta_1$ is assumed for 28 MPa < $f_c'$ < 55 MPa. The shear carried by the beam compression zone is thus calculated as,

$$V_{cc} = 0.11f_c'\beta_1 cb = 0.11\frac{T_s}{0.85} = 0.13A_s f_y \qquad (3)$$

where $A_s$ is the area of tension steel and $f_y$ is the yield strength of the tension reinforcement. In Eq. (3), the neutral axis depth is calculated at yielding of the tension steel, assuming the beam is under-reinforced, as required for design.

The proposed model predicted reasonably well the shear strength of test SFRC beams reported in the literature and with characteristics satisfying the following: 1) shear span-to-effective depth ratio $a/d \geq 2.5$; 2) beam depth $h$ between 230 and 685 mm; 3) longitudinal tension reinforcement ratio ρ between 1.2 and 4.5%; 4) concrete cylinder strength $f_c'$ between 20.7 and 104 MPa; 5) hooked steel fibers in volume fractions ranging between 0.5% (39 kg/m$^3$) and 2% (157 kg/m$^3$); 6) fiber tensile strength≥ 1030 MPa; and 7) fiber length-to-diameter ratio between 55 and 100.

As reported in [14], mean value and standard deviation of the predicted versus experimental strength ratios were 0.79 and 0.12, respectively. Also, the strength ratios were consistent for the range of fiber volume fractions considered.

## 6 CONCLUSIONS

Results from large-scale tests of steel fiber reinforced concrete (SFRC) beams subjected to monotonically increased shear to failure indicate that the hooked steel fibers evaluated in this investigation, when used in volume fractions greater than or equal to 0.75%, are effective in ensuring multiple diagonal cracks and substantially increasing shear strength in beams without stirrup reinforcement. The minimum shear strength of the test SFRC beams was $0.33\sqrt{f_c'}$ (MPa), which exceeded that of a beam with stirrup reinforcement satisfying the minimum requirements in the 2008 ACI Building Code ($0.30\sqrt{f_c'}$, MPa).

A simple model in which the shear strength of a SFRC beam without stirrup reinforcement is taken as the summation of shear resistance contributed by fiber tensile stresses across diagonal cracks and shear carried in the beam compression zone can be used to estimate the shear strength of SFRC beams with reasonably accuracy. Average tension stresses across a critical diagonal crack can be estimated from standard ASTM 1609 four-point bending tests, while the shear carried in the beam compression zone is determined from a failure criterion for concrete subjected to combined compressive and shear stresses.

## 7 ACKNOWLEDGMENTS

The writers would like to acknowledge the support of Bekaert Corp., which donated the steel fibers used in this investigation.

## REFERENCES

[1] Batson, G., Jenkins, E., and Spatney, R., 1972. Steel fibers as shear reinforcement in beams. *ACI J.* **6**:640-44.

[2] Swamy, R.N. and Bahia, H.M., 1985. The effectiveness of steel fibers as shear reinforcement. *Concrete Int.* **7(3)**:35-40.

[3] Mansur, M.A., Ong, K.C.G., and Paramasivam, P., 1986. Shear strength of fibrous concrete beams without stirrups. *ASCE J. Struct. Eng.* **112:**2066-79.

[4] Shantz, B.A., 1993. The effect of shear stress on full scale steel fiber reinforced concrete beams. *MS Thesis*, Dept. of Civil and Environmental Eng., Clarkson University, Potsdam, NY; 86 pp.

[5] Adebar, P., Mindess, S., St.-Pierre, D., and Olund, B., 1997. Shear tests of fiber concrete beams without stirrups. *ACI Struct. J.* **94**:68-76.

[6] Noghabai, K., 2000. Beams of fibrous concrete in shear and bending: experiment and model. *J. Struct. Eng.* **126:**243-51.

[7] Rosenbusch, J., and Teutsch, M., 2002. Trial beams in shear. *Brite/Euram Project 97-4163*, Final Report, Sub Task 4.2, Technical Univ. of Braunschweig.

[8] Kwak, Y.-K., Eberhard, M.O., Kim, W.-S., and Kim, J., 2002. Shear strength of steel fiber-reinforced concrete beams without stirrups. *ACI Struct. J.* **99**:530-38.

[9] Parra-Montesinos, G.J., 2006. Shear strength of beams with deformed steel fibers. *Concrete Int*. **28(11)**:57-66.

[10] Wight, J.K., and MacGregor, J.G., 2008. *Reinforced Concrete. Mechanics and Design*, Fifth Edition, Pearson Prentice Hall; pp. 247-48.

[11] ACI Committee 318, 2008. *Building code requirements for structural concrete and commentary (ACI 318-08)*. American Concrete Institute, Farmington Hills, Michigan.

[12] ASTM International, 2006. *C 1609/C 1609M-06, Standard test method for flexural performance of fiber-reinforced concrete (using beam with third-point loading)*; 8 pp.

[13] Dinh, H.H., 2009. Shear behavior of steel fiber reinforced concrete beams without stirrup reinforcement. *Doctoral Dissertation*, Dept. of Civil and Environmental Eng., Univ. of Michigan, Ann Arbor, MI; pp. 285.

[14] Dinh, H.H., Parra-Montesinos, G.J., and Wight, J.K., 2011. A shear strength model for fiber reinforced concrete beams without stirrup reinforcement. *J. Struct. Eng.* **137:**1039-51.

[15] Dinh, H.H., Parra-Montesinos, G.J., and Wight, J.K., 2010. Shear behavior of steel fiber reinforced concrete beams without stirrup reinforcement. *ACI Struct. J.* **107:**597-06.

[16] Bresler, B. and Pister, K.S., 1958. Strength of concrete under combined Stresses. *ACI J.* **55**:321-45.