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FRACTURE ENERGY OF CONCRETE AT VERY EARLY AGES BY INVERSE ANALYSIS

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Abstract. In this paper, the fracture characteristics of concrete at very early ages are investigated. Experimentally, three point bending tests are performed on notched beams at very early-ages (before 24 h) and the Load-CMOD curves are obtained. In order to find the trend of the evolution of the fracture energy, an inverse analysis procedure is developed. A damage model based on an energetic regularization is combined with a non linear inverse analysis algorithm. As the stress-strain curve (and the stress-crack opening one) given by the damage model is non linear, the Levenberg-Marquardt algorithm is chosen. Results obtained from numerical simulations show that fracture energy of concrete increases with age.

1 INTRODUCTION

The fracture energy is one of the most important fracture characteristics of concrete. The determination of the evolution of the fracture energy at early-ages has been already investigated by some authors [1–4]. It was found that the fracture energy increases with age. Based on the three-point bending test or the wedge splitting test, the fracture energy could be evaluated directly from the load-displacement curves [5–7] according to the RILEM recommendation, or indirectly, by numerical inverse analysis.
When using inverse analysis, experimental results are approximated by repeated numerical simulations of the tests. Thus, a function is needed to describe concrete softening. Discrete models (Fictitious crack models [8]) or smeared crack models based on the crack band theory [9] are often used for modelling crack initiation and propagation in concrete. However these models require a stress-crack opening relationship. The area under the tensile softening curve, described by this relationship, determines the specific fracture energy G_f.

As defined by the RILEM technical committee, the specific fracture energy measurement is based on the load-displacement $(p - \delta)$ curve and expressed as :

$$G_f = \frac{1}{B(W-a)} \int p \partial\delta \qquad (1)$$

Where B is the specimen thickness, W its width in the crack direction and a the initial notch or crack length. The RILEM energy definition is based on Hillerborg's fictitious crack model. So, the motivation of the RILEM was to provide (from load and load-displacements curves of concrete) a constant parameter G_f representing the area under the tensile softening curve which is supposed unique. However, as it is widely agreed, size effect on the RILEM G_f exists [10–12]. G_f depends not only on the specimen size W but also on the crack length a even if W is constant. There is a size/ligament effect on G_f [13, 14]. Eq 1 gives only an average value where the fracture energy is assumed to be uniform over the crack length. Definition of G_f from a complete $p - \delta$ curve includes the boundary region which explains why the RILEM G_f is ligament dependent.

Inverse analysis obtained without considering boundary effect implies a constant $\sigma - dw$ (stress-crack opening) relation. If the inverse analysis is performed with a model that does not take into account boundaries effects, the solution is to fit only a part of the $P - \delta$ (or $p - CMOD$) curve where the distribution of the local fracture energy is uniform. In this region (the inner zone), a good estimation of the size independent fracture energy (G_F) could be obtained.

The objective of the present study is to contribute to the identification of the fracture energy at very early ages using numerical inverse analysis. Experimentally, three point bending tests are performed on rectangular notched specimens at ages between 12 hours and 24 hours [15]. The Load-CMOD curves are obtained.

In order to estimate the fracture energy a damage based model is used to represent the softening behaviour of concrete. A fracture energy regularization is applied to this model in order to avoid spurious mesh sensitivity. Therefore, the energy dissipation due to the fracture per unit length (or unit width) described by the stress-strain curves is constant. As the softening behaviour is non linear, the Levenberg-Marquardt [17, 18] algorithm (LMA) is used to perform inverse analysis. Only a part of the $P - CMOD$ curves is fitted in order to estimate the size independent fracture energy.

2 Experimental program

2.1 Materials and Methods

2.1.1 Concrete formulation

The materials constituting the CEOS concrete [19] are: semi-crushed gravel of class 4/20 mm, alluvial sand of class 0/4 mm and Cement CEMI 52.2 N (EN 197-1 and EN 197-2). A small amount of super-plasticizer (Optima 206) was adjusted manually for workability. The formulation used is given in table 2 . The water to cement ratio (W/C) is 0.42 and the gravel to sand ration (G / S) is 1.25.

Table 1: Characteristics of used aggregates

	Sand 0/4	Gravel 4/20
Water content %	3.5	2.1
Absorption %	0.7	1.39
Density (Kg/l)	2.6	2.57

Table 2: Concrete formulation

	Dosage (Kg/m^3)
Gravel 4/20	980
Sand 0/4	785
CEMI 52.2N	400
Super plasticizer	1.8
Added water	154.1
Effective Water	165.9

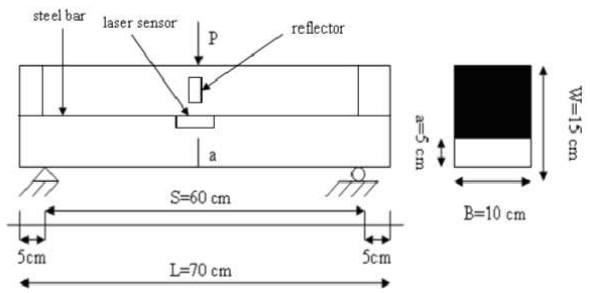

Figure 1: Schematic representation of the three-point bending test setup

Figure 2: Experimental device of the three-point bending test

2.1.2 Experimental procedure

Concrete was poured into moulds of dimensions $10 * 15 * 70cm^3$. A Teflon plate with a thickness of $3mm$ was placed in the mould before casting for the notch position. It was removed at the time of the test. To better interpret the repeatability tests, the slump test of the Abrams cone was carried out . Concrete was poured into moulds in two layers and vibrated with a plate vibrator. Specimens are then covered with an impermeable plastic sheet and kept in an air conditioned room at a temperature of 20 C and a relative humidity of 50%. The formwork was removed with care one hour before the test to glue the plates (which were used to hold the CMOD gauge) and the reflector for the measurement of the deflection.
The loading frame consists of an Instron make closed loop universal testing machine of 160 kN capacity. The load was applied with a circular jack to ensure a point load. A rubber pad was placed between the load jack and the beam to take care of the surface unevenness and to avoid damage under the load. The beam was simply supported on two circular supports. The notch mouth opening displacement was measured with a CMOD gauge. The two blades of the CMOD gauge, each 10 mm in length, were attached to two metallic plates 10 mm apart, one on either side of the notch at the bottom face of the beam. These metal plates were firmly attached to other plates (which were inserted into the mould before casting concrete) with high strength glue that ensures perfect stability of plates. All the tests were performed under crack mouth opening displacement control with a slow rate of 0.5 μm/sec. This allowed to reach the peak load in about one minute. Through these tests, the load was obtained as a function of time, CMOD and deflection. However, the load-deflection curve is not always reliable compared to the load-CMOD which is in most case good. The data were recorded every one second. The deflection was measured by a laser sensor (Figure 2) which allows to calculate the optic deflection of the reflector (corresponding to that of the beam) placed in the middle part of the beam on the neutral axis of the UN-cracked part.

2.2 Experimental results

Figure 3 shows the development of the Young's modulus with age. The Young's modulus increase is due to the formation of hydration products. The products actually continue to form and the amount of water continues to drop which makes the cohesion between the aggregates and the cement paste stronger and contributes to the increase of concrete strength. Thus, the stiffness increases over time.

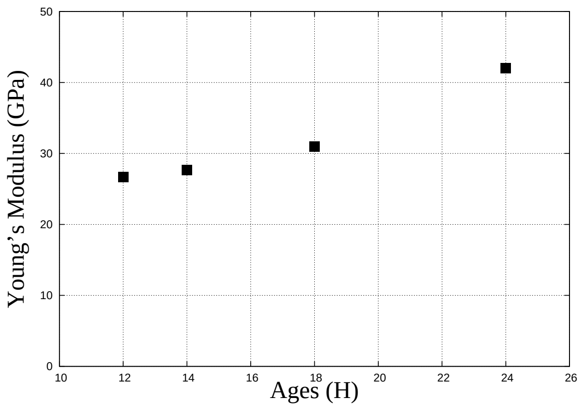

Figure 3: Young's Modulus evolution

Figure 4 shows the Load-CMOD curves at the age of 12 h-14 h-18 h-24 h ages. The curves show that the failure load and the slope of the elastic part increases with age while the post-peak part decreases rapidly. Increase in the elastic slope can be attributed to the evolution of the Young's modulus with time. The rapid fall of the curve after the peak shows that at certain ages concrete become more brittle.

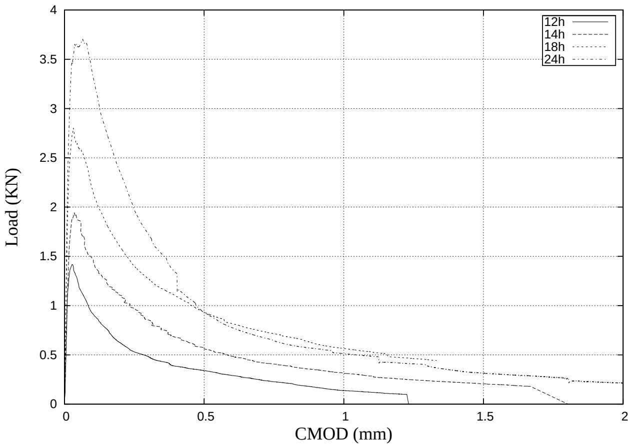

Figure 4: Load-CMOD curves for different ages

3 Modelling

3.1 Stress-strain softening curve

In order to perform inverse analysis, the stress-crack opening curve ($\sigma - dw$) is needed. A damage based model is chosen to represent the softening behaviour of concrete. Therefore, the $\sigma - dw$ softening curve is implicitly introduced. The stress-strain relationship is given by [20]

$$\sigma_{ij} = (1-d)C^0_{ijkl}\varepsilon_{kl} = (1-d)\tilde{\sigma}_{ij} \tag{2}$$

Where C^0_{ijkl} is the initial stiffness tensor, d the damage variable and $\tilde{\sigma}$ the effective stress.

The softening behaviour is represented by an exponential damage evolution law

$$d = 1 - \frac{\varepsilon_{d0}}{\varepsilon}\exp(B(\varepsilon_{d0} - \varepsilon)) \tag{3}$$

B is a parameter which commands the slope of the softening curve defined by the exponential expression and ε_{d0} is the strain threshold.

According to the Crack Band Theory, instead of treating cracks as lines as in the Fictitious crack model [8], Bazant and Oh [9] consider the fracture zone to have a certain width h over which micro-cracks are uniformly distributed. The energy dissipation due to fracture per unit length (or unit width) is therefore a constant. The fracture energy is given by :

$$G_f = \int_0^\infty \sigma\delta w \tag{4}$$

Where w is the crack opening displacement

$$\delta w = \varepsilon^f h \tag{5}$$

with ε^f the fracture strain.

To apply this to the damage model, the softening behaviour must be changed such the same energy dissipation is imposed whatever the element size [22] :

$$\frac{G_f}{h} = \int_0^\infty ((1-d)E\varepsilon)\, d\varepsilon \tag{6}$$

Where h is the characteristic length related to the element size. So, the local energy in each element g_{felem} is assumed to be uniformly smeared over an element $G_{felem} = h * g_{fleme}$. This implies that micro-cracks are assumed to be uniformly distributed over a finite element as prescribed by the crack band theory. Using the stress-strain relationship, the fracture energy is therefore given by :

$$\frac{G_f}{h} = \frac{f_t\varepsilon_{d0}}{2} + \frac{f_t}{B} \tag{7}$$

With f_t the tensile strength and E the Young's modulus $f_t = E * \varepsilon_{d0}$. An example of a stress-crack opening curve obtained by this model is given in figure 5 (more details are given in [21]. The crack opening displacement is calculated using the procedure developed by Matallah et al [21, 22]

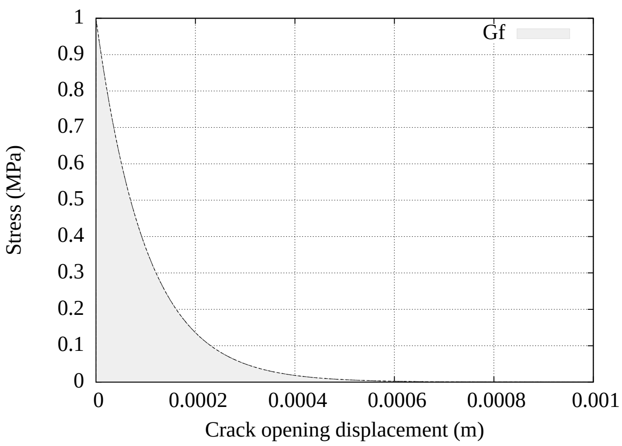

Figure 5: Stress-Crack opening curve

3.2 Inverse Analysis Algorithm

In order to find a trend of the evolution of the fracture energy with age, inverse analysis is carried out. As the stress-strain relationship describing the tensile softening curve is non linear, the Levenberg-Marquardt Algorithm (LMA) is used. Also known as the damped least-squares (DLS), it is one of the most important algorithms for solving systems of non linear equations. It is widely adopted for solving curve-fitting problems in many research area. To obtain the optimum solution of the experimental $P - CMOD$ curves, we minimise the sum of weighted squares of the errors between the measured data $F(u_i)$ and the curve-fit function $\hat{F}(u_i, p)$, where vector p is the vector of parameters. The inverse analysis procedure involves two major steps :

- formulation of the mechanical model,
- estimation of the coefficient of this model by inverse analysis.

Using the damage model exposed above, only two fracture parameters are used as input parameters p=(f_t,G_f) (the tensile strength and the fracture energy).

4 Results and Discussion

The damage model used above does not take into account boundary effects. As it has been outlined before, the solutions proposed was to fit partially the Load-CMOD curves. Hence an estimation of the size independent fracture energy is obtained. Figure 6 shows the development of the size independent fracture energy with age. Overall, The data indicate an ”upward trend” (increasing of the fracture energy with age) which is in agreement with the most experimental results.

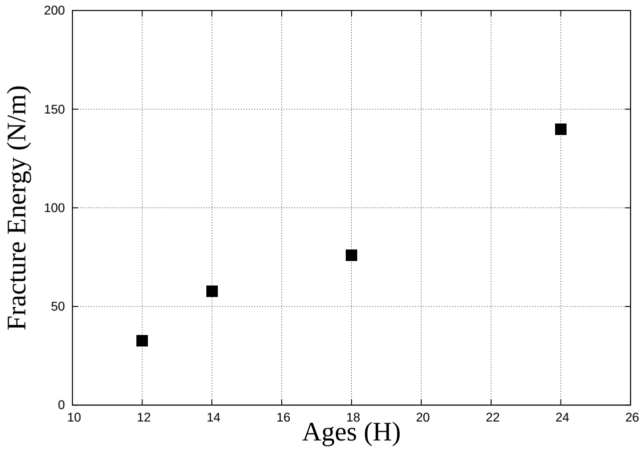

Figure 6: Development of fracture energy with age

5 Conclusion

In this study, the fracture characteristics of concrete at very early-ages were investigated. An experimental procedure was carried out to perform three point bending tests on notched beams of concrete specimens at very-early ages (before 24H). The Load-CMOD curves were established in order to determine the fracture characteristics. To this end, an inverse analysis procedure was developed. The stress-crack opening curve which is needed in such analysis is implicitly taken into account by a damage model through the stress-strain relationship. Only two physical parameters (G_F and f_t) are used as input parameters. At very-early ages, an ”upward trend” was found regarding the evolution of the fracture energy.
Summing up, the numerical inverse analysis developed in this study has shown its applicability for the identification of the fracture characteristics of concrete. The numerical results as the experimental ones confirm the basis trend already stated in the literature.

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