VIII International Conference on Fracture Mechanics of Concrete and Concrete Structures
FraMCoS-8
J.G.M. Van Mier, G. Ruiz, C. Andrade, R.C. Yu and X.X. Zhang (Eds)

HIGH PERFORMANCE FIBRE REINFORCED CONCRETE: FUNDAMENTAL BEHAVIOUR AND MODELLING

STEPHEN J. FOSTER[*], TREVOR HTUT[†] AND TIAN SING NG[*]

[*] The University of New South Wales, School of Civil and Environmental Engineering
Centre for Infrastructure Engineering and Safety, Sydney, Australia
e-mail: s.foster@unsw.edu.au - Web page: http://www2.civeng.unsw.edu.au/staffinfo/stephen_foster/

[†] Sinclair Knight Merz, Adelaide, South Australia
e-mail: thtut@globalskm.com

Key words: Steel fibre, tension, dispersion, distribution, X-ray, imaging, concrete, modelling.

Abstract: This paper reports on the results of X-ray imaging of steel fibre reinforced concrete in tension. The observations on fundamental behaviour and influences in the development of models for structural design are discussed. It is observed that crack paths find ways of least resistance through a section and divert around fibre ends, where possible. In large scale structural members where crack locations are not fixed by either geometry or load state, variability in fibre dispersion should be considered at the model level, rather than incorporated in materials or member safety factors. Based on image analysis, the fibre dispersion influence factor is quantified for the case where multiple cracking paths may occur.

1 INTRODUCTION

By 3000 BC the Egyptians had used a combination of lime and gypsum mortar as binding agents in their construction of the Pyramids. In latter period of the Roman Republic, the Romans of southern Italy mixed a volcanic sand, pozzuolana, found near Pozzuoli on the Bay of Naples to form a hydraulic-setting cement paste. This mixed with aggregate gave rise to Roman concrete with good compressive strength properties, which was used together with complex structural shapes such as domes and arches. One of the first uses of hydraulic-setting cement was in 27 BC for the dome of the Pantheon. At around the 2nd century BC, the Chinese used cementitious materials in the Great Wall of China. Thus, while early concrete technologies were available, this type of concrete was not used elsewhere and stone and brick masonry remained the dominant construction materials for many centuries.

The modern use of concrete can be traced to John Smeaton who, born in 1724, is famous for his work on the Eddystone Lighthouse in Cornwall, England, which he had been commissioned to rebuild in 1756. Smeaton used interlocking courses of masonry bound with a pozzolanic mortar. In his later work, he added aggregate to the mix and built Ramsgate Harbour, Perth, and Coldstream Bridges and the Forth and Clyde Ship Canals. In the early 1850's, the use of reinforcing steel was introduced by Jean-Louis Lambot in his boats. In 1889, the first concrete reinforced bridge was built, the Alvord Lake Bridge, USA.

Basic cement tests were standardised in the early 1900's and in 1904, with the construction of the Ingalls building USA, the first concrete skyscraper was built. In 1916, the Portland Cement Association was founded and, a year later, the US Bureau of Standards and the

American Society for Testing Materials (ASTM) established a standard formula for Portland cement. Concrete construction was set to revolutionise the building and infrastructure construction industries.

Interest in advanced cement-based materials sawed, not only because of their increased strength but also due to their generally high-performance characteristics. The earliest use of high performance concrete (HPC) can be traced to the 1950’s and in the time since there has been numerous projects that have used HPC in their construction. In 1973, Water Tower Place reached 260 metres with concrete strengths as high as 60 MPa and in the following two to three decades, high performance concrete became widely used in bridge and high rise building construction.

In the early 1960's, pioneering research into fibre reinforced concrete was undertaken by Romualdi and Batson [1] where it was demonstrated that the tensile strength and crack resistance of concrete can be improved by providing suitably arranged, closely spaced, wire reinforcement. After 50 years of research in the development and placement of fibres in reinforced concrete, the concept has matured to the stage where it is finding increasing use in practice. The materials used for fibres have also seen significant advancements including stainless steels and complex polymers. As early as 1994, Banthia and Trottier [2] recorded that fibres are used as a form of shear reinforcement in reinforced concrete (RC) structural elements, for blast resistance in structures, as shotcrete in tunnel linings, for use in slope stabilisation works and to limit early age shrinkage cracking in large concrete pavements.

By adding fibres to a concrete mix the objective is to bridge discrete cracks providing for some control to the fracture process and increase the fracture energy. Since the early work, the pullout mechanism of discontinuous fibres embedded in a variety of cementitious materials has been studied by a number of researchers [3-13].

The current understanding of the behaviour of fibre-matrix interfacial mechanics is based on a number of pullout studies using single or multiple fibres where steel fibres are embedded within a cementitious matrix. The experimental parameters investigated include the rate of loading [14-16], curing and environmental temperatures [2, 17, 18], the quantity and quality of the matrix [19-21], addition of adhesive agents [22] and fibre type and fibre orientation [2, 23, 24]. In spite of a belief sometimes expressed [2] that no correlation exists between the behaviour of a single fibre pullout test and the behaviour of bulk fibres in a real composite matrix, the effectiveness of a fibres as a medium of stress transfer is often assessed using fibre pullout tests where slip between the fibres and the matrix is monitored as a function of the applied load.

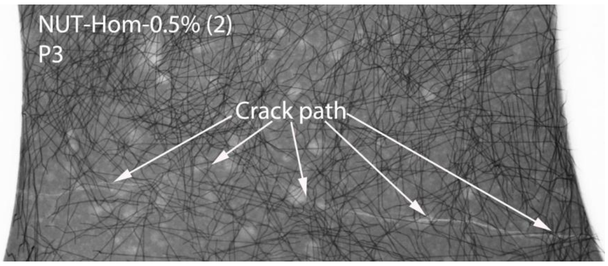
2 OBSERVATIONS FROM X-RAY IMAGING

Htut [25] conducted X-ray imaging on seven dog-bone shaped specimens subjected to uniaxial tension. It was observed that the crack initiation is likely to commence at the point of minimum resistance and, consequently, cracks initiate from the location at which least number of fibres are located; that is, cracks initiate from areas with poor fibre dispersion rather than at the narrowest part of the dog-bone specimen. Thus, fibre dispersion plays a significant role in crack initiation and, consequently, on the tensile strength.

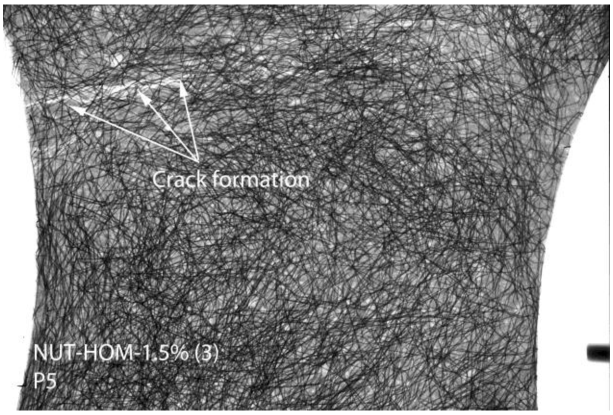
As was observed by Markovic et al. [26], the crack path follows the easiest propagation route and is often near the end of fibres or around them (Figures 1 and 2). Consequently, many of the end-hooked fibres fail to engage and do not deform during the fracture process.

3. FIBRE DISPERSION

The dispersion of fibres in the matrix and deformation of the end-hooks significantly influences the tensile behaviour of a SFRC composite. In the development of material models for design, this observation needs consideration. By digitalising X-ray images of specimens we may learn more on fibre dispersion and distribution. Eleven 30 mm thick dog-bone shaped specimens with randomly distributed 25 mm long by 0.3 mm diameter end hooked fibres and volume percentages of between 0.5% and 2% were cast and X-ray imaged prior to tensile testing. The images were then analysed for fibre concentration over various regions (Figure 3); to avoid the wall effect in the analyses, the region near the side walls was excluded.



(a)



(b)

Figure 1: X-ray images showing crack formation during a uniaxial tension test: (a) 0.5% fibres; (b) 1.5% fibres.

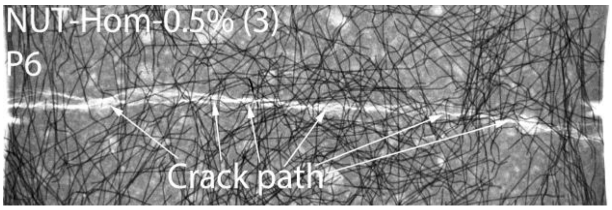
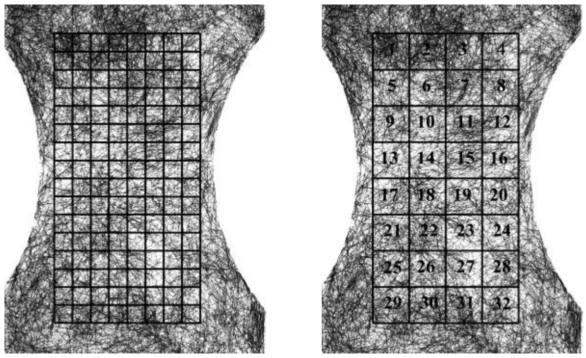


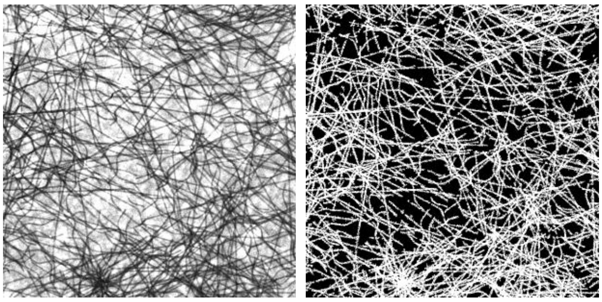
Figure 2: Crack propagation during a uniaxial tension test: (a) 0.5% fibres.

The digital image analysis was undertaken using IMAQ™ Vision Builder software. Each sample image was filtered to distinguish the fibres from the background image (Figure 4). A particle analysis was then undertaken to determine the area of fibres in the image (white area in Figure 4b) with the fibre dispersion/distribution factor (F_{fd}) defined as the ratio of white area to the total sample area. Only a limited improvement in result of the analysis (approximately 2%) was observed when the sampling size was reduced much below the length of the fibres and a sampling size of 25 mm × 25 mm was adopted for further investigation. The dispersion results are presented in Figure 5.



(a) (b)

Figure 3: Sampling locations for fibre dispersion analysis: (a) 12.5 mm × 12.5 mm; (b) 25 mm × 25 mm.



(a) (b)

Figure 4: Example of 50 mm square sample; (a) original image before filtering and colour inversion, and (b) digital image after filtering.

For a sample of statistical significance, the median value of F_{fd} for samples taken within one dog-bone specimen represents the average fibre volume fraction, ρ_f. This data is plotted in Figure 6 for the dog-bone shaped specimens for the different, known, fibre volumetric ratios (ρ_f from 0.5% to 2.0%). With the relationship F_{fd} and ρ_f established, for the given fibre type and specimen thickness, (Figure 6), the volume fraction as a function of F_{fd} is determined as:

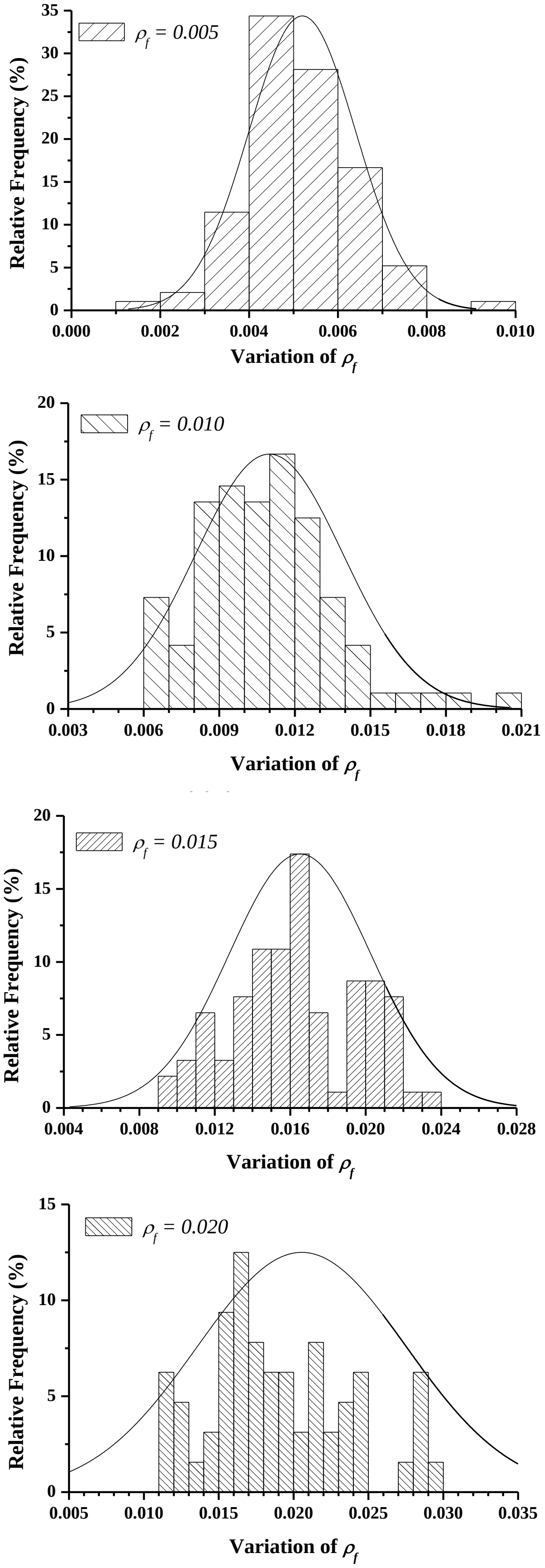


Figure 5: Distribution of fibres for fibre volume fractions (ρ_f) of 0.005, 0.010, 0.015 and 0.020.

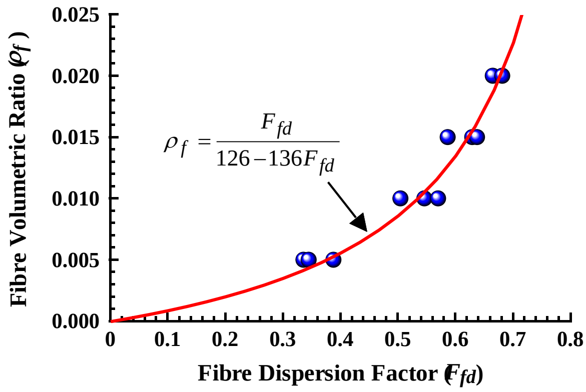


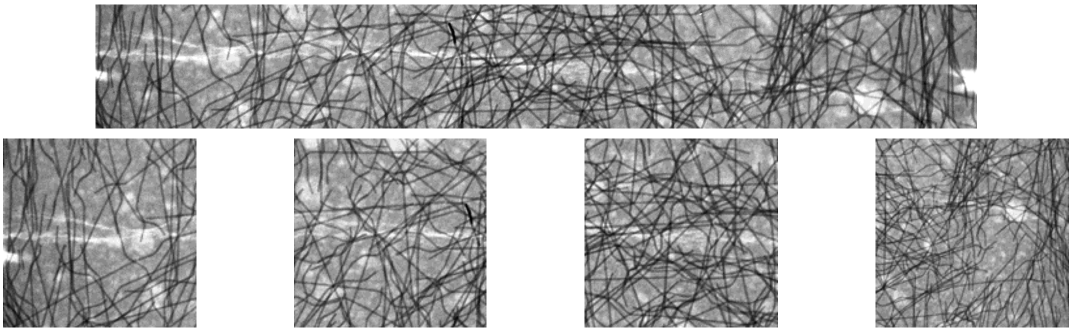
Figure 6: Median fibre dispersion factor versus fibre volume concentration for the 30 mm thick specimens.

$$\rho_f = F_{fd} / \left(126 - 136 F_{fd}\right) \qquad (1)$$

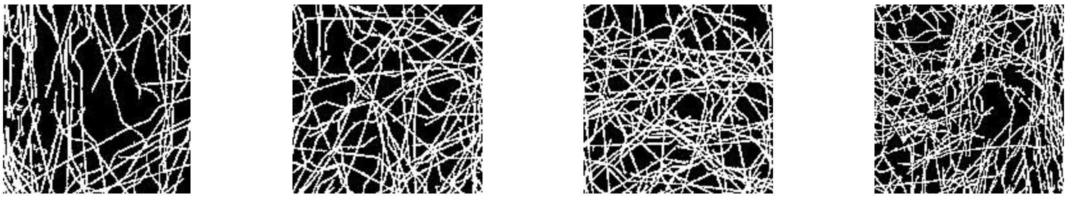
With the fitting Eq. 1 established, variations in the fibre volume fraction within one specimen can then be determined. Note that the best predictions from Eq. 1 are when the results are interpolated, that is between fibre volume fractions of between 0.005 and 0.02 (fibre dispersion factors of approximately 0.375 to 0.677). Beyond the limits of the data, extrapolation is needed and the results are less certain.

From the fibre dispersion data, the standard deviation for the test series was $\sigma = 0.27\rho_f$ and, considering the fibres to be normally distributed, the 75th and 90th percentiles are $\rho'_{75} = 0.82\rho_f$ and $\rho'_{90} = 0.65\rho_f$, respectively.

It is important to note that, in this study, the specimen thickness was 30 mm or 100 times the diameter of a single fibre. Fibres are more likely to overlap one another in the digital image analysis as the specimen thickness increases. Consequently, in the digital image analysis, specimen thickness is a significant component in determining of the fibre dispersion factor. For the hypothetical case of a fibre diameter thick specimen, there is no possibility of fibre being overlapped and the fibre dispersion factor is linearly proportional to the fibre volume concentration. Probability of fibre overlapping is a function of the specimen thickness, and, consequently, a non-linear relationship is observed.



(a)



(b)

Figure 7: Digital X-ray image along the crack path: (a) before the image analysis and (b) after the image analysis.

4. INFLUENCE OF FIBRE DISPERSION ON FRACTURE PROCESS

The digitised X-ray images taken during the tension test of the dog-bone specimens show the importance of fibre dispersion on the crack formation/initiation and propagation processes. To validate the findings presented in Section 3, further digital image analysis was undertaken to determine the fibre dispersion along the crack path of the specimens containing fibre volume concentrations of 0.5%, 1.0% and 1.5%. A typical X-ray image around the crack path is shown in Figure 7a. Digital image analysis was undertaken on a sample size of 12.5 mm square (Figure 7b) and the fibre volume ratio versus fibre dispersion ratio is plotted in Figure 8.

The results show that the cracks are likely to form or propagate along the path of least resistance. The fibre volume concentration along the crack path was found to average through the 75th percentile characteristic value. This confirms the conclusion that fibre dispersion contributes significantly to the fracture process in uniaxial tension and this observation needs to be taken into consideration during the development of behavioural models.

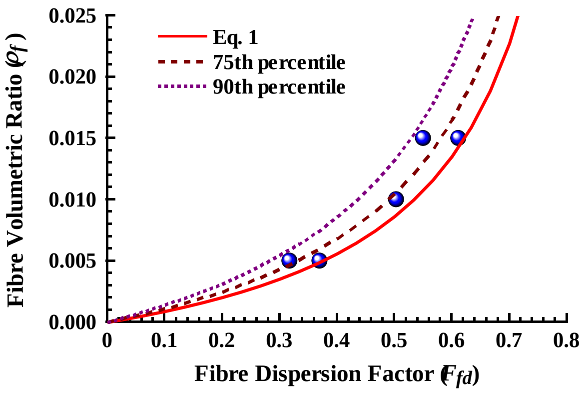


Figure 8: Fibre volume concentration ratio versus average fibre dispersion along the crack path.

5. INFLUENCE OF FIBRE DISPERSION ON MODELLING

Despite numerous publications on fibre reinforced concrete behaviour, limited research has been undertaken on developing general design models for fibre reinforced composites in tension. Visalvanich and Naaman [27] derived a semi-empirical model for the tension-softening curve in discontinuous randomly distributed steel fibre-reinforced mortar by assuming a purely frictional fibre-matrix interface and complete fibre pullout. With the same assumptions and taking into account an additional frictional effect called the snubbing effect, Li [13] derived an analytical model named the fibre pullout model (FPM) that predicts the bridging stress-COD relationship for fibre reinforced brittle-matrix composite.

A micro-mechanical model known as the fibre pullout and rupture model (FPRM) was developed by Maalej et al. [28]. In their model, the FPM model of Li [13] was extended to account for the possibility of fibre rupture in the composite. The model is able to predict the composite bridging stress-COD relationship, account for fibre pullout, fibre rupture and the local frictional effect or snubbing.

Using the "fibre-matrix misfit" theory of Timoshenko [29], Naaman et al. [30] proposed an analytical model for straight, undeformed, circular steel fibres aligned perpendicularly to the cracking plane. The model was shown to capture the pullout-slip relationship between steel fibres and concrete when compared to the experimental data as published by the authors. However, this model is limited for use in directionally orientated plain fibre composites.

Gilles [31] developed a micro-mechanical model taking into account the different phenomena observed during pullout of a deformed fibre, including the interfacial adhesion between the fibre and the matrix, friction and fibre deformation. The model can be used for predicting pullout behaviour of fibres having various geometries but is limited to the case where the fibres are aligned perpendicular to the cracking plane.

Marti et al. [32] developed a simple parabolic model to describe the stress-COD relationship of randomly orientated fibre reinforced composites where it was assumed that, after cracking of the matrix, there is zero contribution to tensile strength from the matrix, the fibre pulls out from the shorter embedded length and that the bond-shear stress is constant along this length.

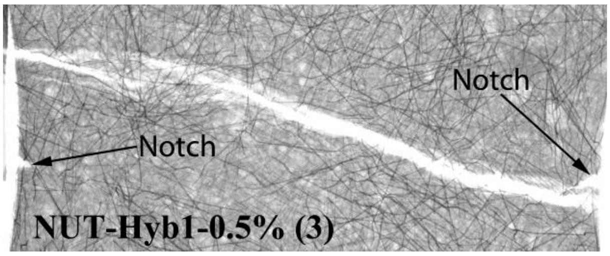
Many other models have been proposed [1, 33-39] but these models are generally limited in their use as tools for structural designers due to limitations of the models or due to the complexity of the models.

In 2003/2004, the first form of the Variable Engagement Model (VEM) was published by Voo and Foster [40-41]. The model has since been used as the basis for the design of RPC and SFRC members for shear [42-45], for punching shear in SFRC slab-column connections [46] and as a constitutive model for FEM [47].

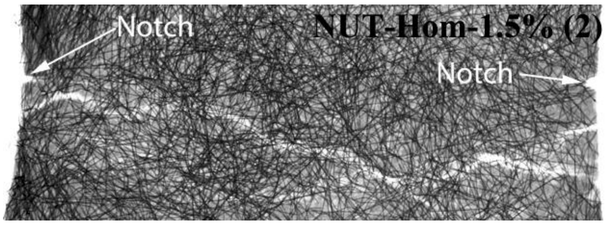
Since 2003, considerable progress has been made in development of generic models for SFRC, including that of Lee et al [48] and the Unified Variable Engagement Model (UVEM) of Ng et al [49].

Model development can be placed in one of three categories: (i) those based on observations from direct tensile tests; (ii) those based on observations from indirect tests, such as that of di Prisco et al. [50]; (iii) physical-mechanical models built from lower order observations such as discrete fibre pull-out testing. In each case these models require validation against a wide range of direct tensile test data for different fibre types and volumes and varying matrix compositions. This is problematic as there is yet to be an agreed direct tensile test of standard configuration and boundary conditions.

For models based on direct tensile testing of unnotched specimens of reasonable size, the influence of fibre dispersion is directly considered. That is the dominant crack forms where the local fibre concentration is at its lowest across a section and where near the end of a fibre, deflects around it. This was clearly evident in the X-ray images of Htut [25]. When specimens have a dominant notch, however, the crack path forms at the notch and the impact of fibre dispersion is negated [26]. Consequently, fibres have a higher possibility of being fully deformed in the notched section tests and, thus, higher tensile strengths and ductility are observed. On the other hand, if the notch is not dominant, failure may occur away from the notch at a section where low fibre numbers, through dispersion variations, occur. As a part of his testing programme, Htut [25] undertook tension tests on three 120 mm wide by 30 mm thick dog-bone shaped specimen that had a pair of 5 mm notches at the critical section, reducing the cross-sectional area by 9%. In each case, the dominant crack formed well away from the notched section. The X-ray images of two of these tests are shown in Figure 9. Specimen NUT-Hyb1-0.5% (3) contained 0.25% by volume of 25 mm long by 0.3 mm diameter end-hooked 2300 MPa fibres and 0.25% by volume of 13 mm long by 0.2 mm diameter straight 1800 MPa fibres; Specimen NUT-Hom-1.5% (2) contained 1.5% by volume of 25 mm long fibres. From the observations with regards to the influence of fibre dispersion of the previous section, it may be postulated that a cross-sectional area reduction of the order of 20% would be needed for a reasonable probability that cracking would occur through the notched section.



(a)



(b)

Figure 9: X-Ray image of dog-bone shaped specimens with notches: Specimen (a) NUT-Hyb1-0.5% (3) and NUT-Hom-1.5% (2).

For the case of models based on indirect tests (i.e. prism bending tests), less favoured by those working in the field of fracture but more favoured by industry, the influence of fibre dispersion is unclear. In this case crack initiation is dominated by the tensile stresses at, or near, the extreme tensile fibre in the high moment region. The results will be influenced by the type of test, 3- or 4-point bending and by whether the specimen is unnotched or notched. For models based on this approach and applied to structural design, it is suggested that the influences of fibre dispersion be treated as for direct tension tests with a dominant notch.

In the final case of physical-mechanical models built from single fibre pull-out observations, case (iii), fibre dispersion needs some consideration when applied to design. In the case of punching shear, for example, where the crack location is defined largely by loading constraints and where the slabs are relatively thin, the fibre dispersion factor may play only a minor role. On the other hand, for the case of one-way shear in beams, where sections are larger and many failure paths are possible, the influence of variations of fibre dispersion cannot be ignored in the development of reliable design models, such as those of [43] and [46].

6 CONCLUSIONS

While the tensile and fracture behaviour of steel fibre reinforced concrete have been researched for nearly five decades, their use in structures has been limited by a lack of design models and standardisation. With design rules for SFRC introduced in some national concrete structures standards, and also in the *fib* Model Code 2010 [51], it could be expected that more use will be made of this higher performance material in building and bridge structures for the carrying of tensile stresses. Ideally, design models should be based on a physical understanding of fracture processes and physical-mechanical models adopted as the model basis, rather than empirically based modelling approaches. Through X-ray imaging of tensile tests, pre-loading and under load, the effect of fibre distribution and dispersion on the fracture processes is quantified. Crack diversion around fibre ends is also observed and needs consideration in the development of rationally based, physical, design models.

REFERENCES

[1] Romualdi, J.P. and Batson. G.B., 1963. Behaviour of reinforced concrete beams with closely spaced reinforcement. *Proc., ACI Journal*. **60**(6): 775-789.

[2] Banthia, N. and Trottier, J.F., 1994. Concrete reinforced with deformed steel fibres, Part I: Bond-slip mechanisms. *ACI Materials Journal*. **91**(5): 435-446.

[3] Gray, R.J., 1984. Analysis of effect of embedded fibre length on fibre debonding and pull-out from an elastic matrix; Part 1, review of theories. *Journal of Materials Science*. **19**(3): 861-870.

[4] Gray, R.J., 1984. Analysis of effect of embedded fibre length on fibre debonding and pull-out from an elastic matrix; Part 2, application to steel fibre-cementitious matrix composite system. *Journal of Materials Science*. **19**(5): 1680-1691.

[5] Gopalaratnam, V.S. and Shah, S.P., 1987. Tensile failure of steel fibre-reinforced mortar. *Journal of Engineering Mechanics*. **113**(5): 635-652.

[6] Mandel, J. A., Wei, S. and Said, S., 1987. Studies of the properties of the fibre-matrix interfaced in steel fibre reinforced mortar. *ACI Materials Journal*. **84**(2): 101-109.

[7] Namur, G.G., Naaman, A.E. and Clark, S.K., 1987. Analytical prediction of pullout behaviour of steel fibers in cementitious matrices. *Symposium Proceedings, Materials Research Society*. **114**: 217-224.

[8] Naaman, A.E., Namur, G., Najm, H.S. and Alwan, J., 1989. *Bond Mechanisms in Fiber Reinforced Cement-Based Composites*. Report No. UMCE-89-9, University of Michigan: 253 pp.

[9] Namur, G.G. and Naaman, A.E., 1989. A bond stress model for fiber reinforced concrete based on bond stress slip relationship. *ACI Materials Journal*. **86**(1): 45-57.

[10] Wang, Y., 1989. *Mechanics of Fiber Reinforced Cementitious Composites*. PhD Thesis, Department of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, USA: 306 pp.

[11] Wang, Y., Li, C.V. and Backer, S., 1990. Tensile properties of synthetic fiber reinforced mortar. *Cement and Concrete Composites*, **12**: 29-40.

[12] Wang, Y., Li, C.V. and Backer, S., 1990. Experimental determination of tensile behavior of fiber reinforced concrete. *ACI Materials Journal*. **87**(5): 461-468.

[13] Li, V.C., 1992. Post-cracking scaling relations for fibre reinforced cementitious composites. *ASCE, Journal of Materials in Civil Engineering*. **4**(1): 41-57.

[14] Hughes, B.P. and Fattuhi, N.I., 1975. Fiber bond strength in cement and concrete. *Magazine of Concrete Research*. **27**(92): 161-166.

[15] Maage, M., 1977. Interaction between steel fibers and cement based matrixes. *Materiaux Et Constructions*. **10**(59): 297-301.

[16] Banthia, N. and Trottier, J.F., 1991. Deformed steel fiber-cementitious matrix bond under impact. *Cement and Concrete Research*. **21**: 158-161.

[17] Banthia, N. and Trottier, J.F., 1989. Effects of curing temperature and early freezing on the pullout behaviour of steel fibres. *Cement and Concrete Research*. **19**(3): 400-410.

[18] Banthia, N. and Trottier, J.F., 1992. Micromechanics of steel fiber pullout, rate sensitivity at very low temperatures. *Cement and Concrete Composites*. **14**: 119-130.

[19] Gray, R.J. and Johnston, C.D., 1984. Effect of matrix composition on fibre/matrix interfacial bond shear strength in fibre-reinforced mortar. *Cement and Concrete Research*. **14**(2): 285-296.

[20] Banthia, N. and Trottier, J.F., 1989. Effects of curing temperature and early freezing on pull-out resistance of steel fibers from a cementitious matrix. *Cement and Concrete Research*. **19**(5): 727-736.

[21] Gopalaratnam, V.S. and Abu-Mathkour, H.J., 1987. "Investigation of the pullout characteristics of steel fibres from mortar matrices". *International Symposium on Fibre Reinforced Concrete*: 201-211.

[22] Guerrero, P. and Naaman, A.E., 2000. Effect of mortar fineness and adhesive agents on pullout response of steel fibres. *ACI Materials Journal*. **97**(1): 12-20.

[23] Naaman, A.E. and Shah, S.P., 1976. Pullout mechanism in steel fibre-reinforced concrete. *ASCE, Journal of the Structurel Division*. **102**(8): 1537-1548.

[24] Soroushian, P. and Bayasi, Z., 1991. Fiber-type effects on the performance of steel fiber reinforced concrete. *ACI Materials Journal*. **88**(2): 129-145.

[25] Htut, T.N.S., 2010. *Fracture Processes in Steel Fibre Reinforced Concrete*. PhD Thesis, School of Civil and Environmental Engineeering, The University of New South Wales: 389pp.

[26] Markovic, I., Walraven, J.C. and Van Mier, J.G.M., 2004. Tensile behaviour of high performance hybrid fibre concrete. *Proc. of the 5th Inter. Conf. on Fract. Mech. of Conc. & Conc. Struct (FraMCoS-5)*, April 12-16, 2006, Vale, Colorado, USA: 1113-1120.

[27] Visalvanich, K. and Naaman, A.E., 1983. Fracture model for fibre reinforced concrete. *ACI Journal Proceedings*. **80**(2): 128-138.

[28] Maalej, M., Li., C.V. and Hashida, T., 1995. Effect of fiber rupture on tensile properties of short fiber composites. *ASCE, Journal of Engineering Mechanics*. **121**(8): 903-913.

[29] Timoshenko, S., 1941. *Strength of Materials*, MacMillan, London, U.K.

[30] Naaman, A.E., Namur, G.G., Alwan, J.M. and Najm, H.S. (1991). Fibre pullout and bond slip. I: analytical study. *ASCE, Journal of Structural Engineering*. **117**(9): 2769-2790.

[31] Gilles, C., 1999. Modeling the pullout of wire-drawn steel fibers. *Cement and Concrete Research*. **29**: 1027-1037.

[32] Marti, P., Pfyl, T., Sigrit, V. and Ulaga, T., 1999. Harmonized test procedure for steel fibre-reinforced concrete. *ACI Materials Journal*. **96**(6): 676-685.

[33] Aveston, J. and Kelly, A., 1973. Theory of multiple fracture of fibrous composites. *Journal of Materials Science*. **8**(3): 352-362.

[34] Naaman, A.E., Argon, A.S. and Moavenzadeh, F., 1973. A fracture model for fiber reinforced cementitious materials. *Cement and Concrete Research*. **3**(4): 397-411.

[35] Pakotiprapha, B., Pama, R.P. and Lee, S.L., 1974. Mechanical properties of cement mortar with randomly oriented short steel wires. *Magazine of Concrete Research*. **26**(86): 3-15.

[36] Gray, R.J., 1984. Analysis of effect of embedded fibre length on fibre debonding and pull-out from an elastic matrix; Part 1, review of theories. *J. Mat. Science*. **19**(3): 861-870.

[37] Brandt, A.M., 1985. On the optimal direction of short metal fibers in brittle matrix composites. *J. Mat. Science*. **20**: 3831-41.

[38] Lim, T.Y., Paramasivam, P. and Lee, S.L., 1987. Analytical model for tensile behavior of steel-fiber concrete. *ACI Materials Journal*. **84**(4): 286-298.

[39] Easley, T.C., and Faber, K.T., 1999. Use a crack-bridging single-fiber pullout test to study steel fiber/cementitious matrix composite. *Journal of American Ceramic Society*. **82**(12): 3513-352.

[40] Voo, J.Y.L. and Foster, S.J., 2003. *Variable Engagement Model for Fibre Reinforced Concrete in Ten*sion", UNICIV Report R-420, School of Civil and Environmental Engineering, The University of New South Wales, Sydney, Australia: 86 pp.

[41] Voo, J.Y.L. and Foster, S.J., 2004. Tensile fracture of fibre reinforced concrete: variable engagement model", In Carpinteri et al (eds), *Design, Assessment and Retrofitting of RC Structures; Proc. of the 6th Inter. Conf. on Fract. Mech. of Conc. & Conc. Struct (FraMCoS-6)*, June 17-22, 2006**,** Catania, Italy: 875-884.

[42] Voo, Y.L, Foster, S.J. and Gilbert, R.I., 2006. Shear strength of fiber reinforced reactive powder concrete prestressed girders without stirrups", *Journal of Advanced Concrete Technology* **4**(1): 123-132.

[43] Foster, S.J., 2010. Design of FRC beams for shear using the VEM and the draft model code approach. In *fib* Bulletin 57, *Recent Developments on Shear and Punching Shear on RC and FRC Elements*. Chapter 12: 195-210.

[44] Voo Y.L., Poon W.K. and Foster, S.J., 2010. Shear strength of steel fiber-reinforced ultrahigh-performance concrete beams without stirrups, *ASCE, Journal of Structural Engineering*. **136**(11): 1393-1400.

[45] Spinella, N., Colajanni, P. and Recupero, A., 2010, "Simple Plastic Model for Shear Critical SFRC beams", ASCE, Journal of Structural Engineering,**136**(4): 390-400.

[46] Maya, L.F,. Fernández Ruiz, M., Muttoni, A. and Foster, S.J., 2012., Punching shear strength of steel fibre reinforced concrete slabs, *Engineering Structures*. **40**: 83–94.

[47] Foster, S.J., Voo, Y.L. and Chong K.T., 2006. Analysis of steel fiber reinforced concrete beams failing in shear: variable engagement model", In Lowes, L., and Filippou (eds). *Finite Element Analysis of Reinforced Concrete Structures*. ACI SP-237, Chapter 5.

[48] Lee, S.C., Cho, J. and Vecchio, F.J., 2011. Diverse embedment model for steel fibre reinforced concrete in tension: model development. *ACI Materials Journal*. **108**(5): 516-525.

[49] Ng, T.S., Foster, S.J. and Htut T.N.S., 2012. *Fracture of Steel Fibre Reinforced Concrete – the Unified Variable Engagement Model*. UNICIV Report R-460, School of Civil and Environmental Engineering, The University of New South Wales: 107 pp.

[50] di Prisco, M., Plizzari, G. and Vandewalle, L. 2009. Fibre reinforced concrete: new design perspectives. *Materials and Structures.* **42**: 1261-1281.

[51] International Federation for Structural Concrete (fib), *Model Code 2010 – Final Draft*. Bulletin 65: 350 pp; and Bulletin 66: 370 pp.