A thirty-year survey of Microplane Models

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Origin of microplane models

Slip theory of plasticity [Taylor (1938) and Batdorf and Budianski (1949)] and subsequent works (e.g. Lin and Ito 1965, 1966, Kroner 1961, Budianski and Wu 1962, Hill, 1965, 1966, Rice, 1970).

It was used in arguments about the physical origin of strain hardening, and was shown to allow easy modeling of anisotropy as well as the vertex effects for loading increment to the side of a radial path in stress space. In all the formulations only the inelastic shear strains (slips) were considered to occur (no inelastic normal strain).

The theory was also adapted to anisotropic rocks under the name ***multilaminate model*** (Zienkiewicz and Pande 1977, Pande and Sharma 1981, 1982; Pande and Xiong 1982).

Origin of microplane models

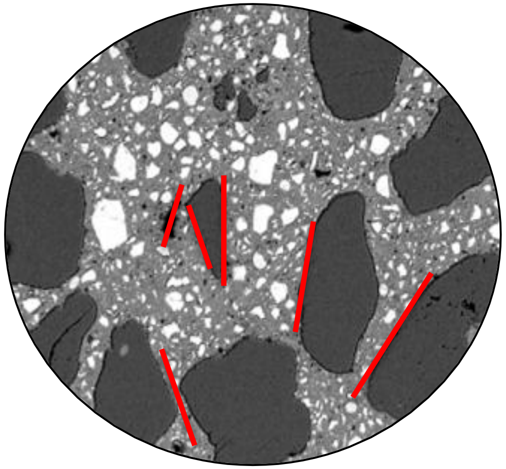
The first microplane model was presented at a 1983 Tucson (AZ) conference (Bažant 1984).

Microplane models differ conceptually from the slip theory of plasticity:

- the kinematic constrain is utilized instead of the static one to prevent instability in post-peak softening damage;
- a variational principle (such as the principle of virtual work) relates the stresses on the microplanes to the macro-continuum stress tensor;
- the previous models considered only plastic slips, microplane model introduced normal inelastic strains on the microplanes.

Origin of microplane models

The neutral term ''***microplane***'' (Bažant 1984) was coined to reflect the fact that the model is not restricted to plastic slip and it does not imply actual simulation of the microstructure geometry.



Microplane formulation

Macroscopic Strain
ε_{ij}

Classical (tensorial) models

Macroscopic Stress
σ_{ij}

Microplane formulation

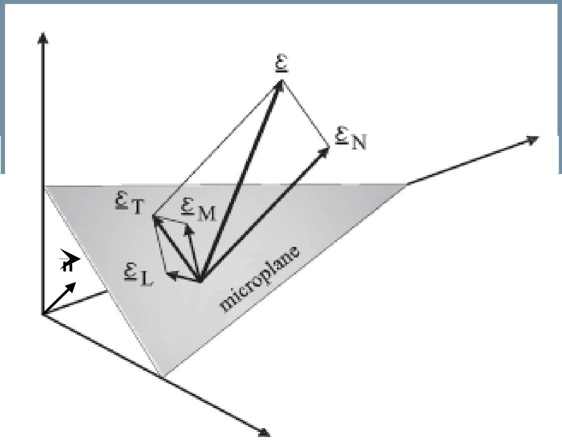
Macroscopic Strain
ε_{ij}

Classical (tensorial) models

- plasticity
- damage mechanics
- their combination

Macroscopic Stress
σ_{ij}

Microplane formulation



Macroscopic Strain
ε_{ij}

Kinematic constraint

Strains on each microplane
ε_V, ε_D, ε_N, ε_T

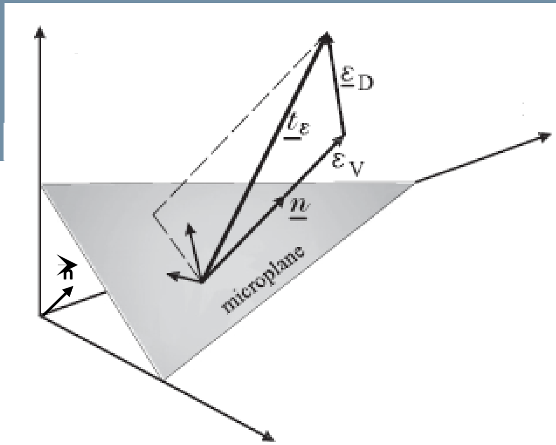
Classical (tensorial) models

Macroscopic Stress
σ_{ij}

The microplane model with Normal–Tangential split (N–T split) (Bažant et al., 1983; Bažant and Gambarova, 1984; Bažant and Oh, 1985)

The microplane model with Volumetric–Deviatoric–Tangential split (V–D–T split), i.e. the microplane normal strain is further split into the volumetric and deviatoric components (Bažant and Prat, 1988; Carol et al., 1991, 1992, 2001, 2004; Bažant et al., 1996, 2000; Ožbolt et al., 2001; Kuhl et al., 2001; Di Luzio 2007, 2009; Di Luzio and Cusatis 2013, Caner and Bažant 2013).

Microplane formulation



Macroscopic Strain
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Classical (tensorial) models

Macroscopic Stress
σ_{ij}

The microplane model with Volumetric-Deviatoric split (V–D split) which can be viewed as a special case of the more general V–D–T split based formulation was proposed (Leukart and Ramm, 2002, 2003, 2006). In this split methodology, the macroscopic strain is decomposed into its bulk and deviatoric components which are then projected onto the specific microplane to obtain the microscopic stress components.

Microplane formulation

Macroscopic Strain
ε_{ij}

Kinematic constraint

Strains on each microplane
ε_V, ε_D, ε_N, ε_T

Microplane constitutive laws

Stresses on each microplane
σ_V, σ_D, σ_N, σ_T

Classical (tensorial) models

Macroscopic Stress
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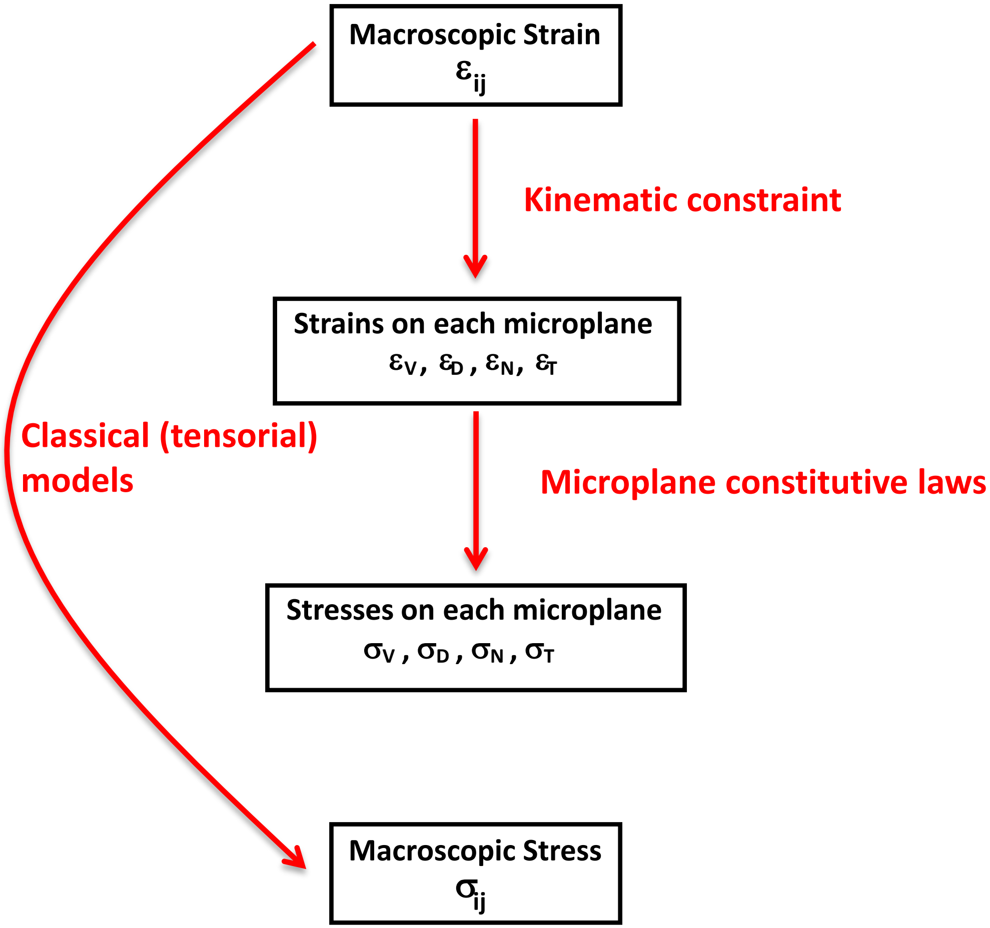
Classical (tensorial) models

Macroscopic Stress
σ_{ij}

Linear elastic behavior

$$\begin{cases} \sigma_V = E_V \epsilon_V \\ \sigma_D = E_D \epsilon_D \\ \sigma_L = E_T \epsilon_L \\ \sigma_M = E_T \epsilon_M \end{cases}$$

Microplane formulation



Inelastic behavior:
based on the scalar damage theory (Bažant and Prat 1988; Bažant and Ožbolt 1990; Ožbolt and Bazant 1992; Ožbolt et al 2001; Leukart and Ramm, 2002, 2003, 2006), e.g. Ožbolt et al 2001:

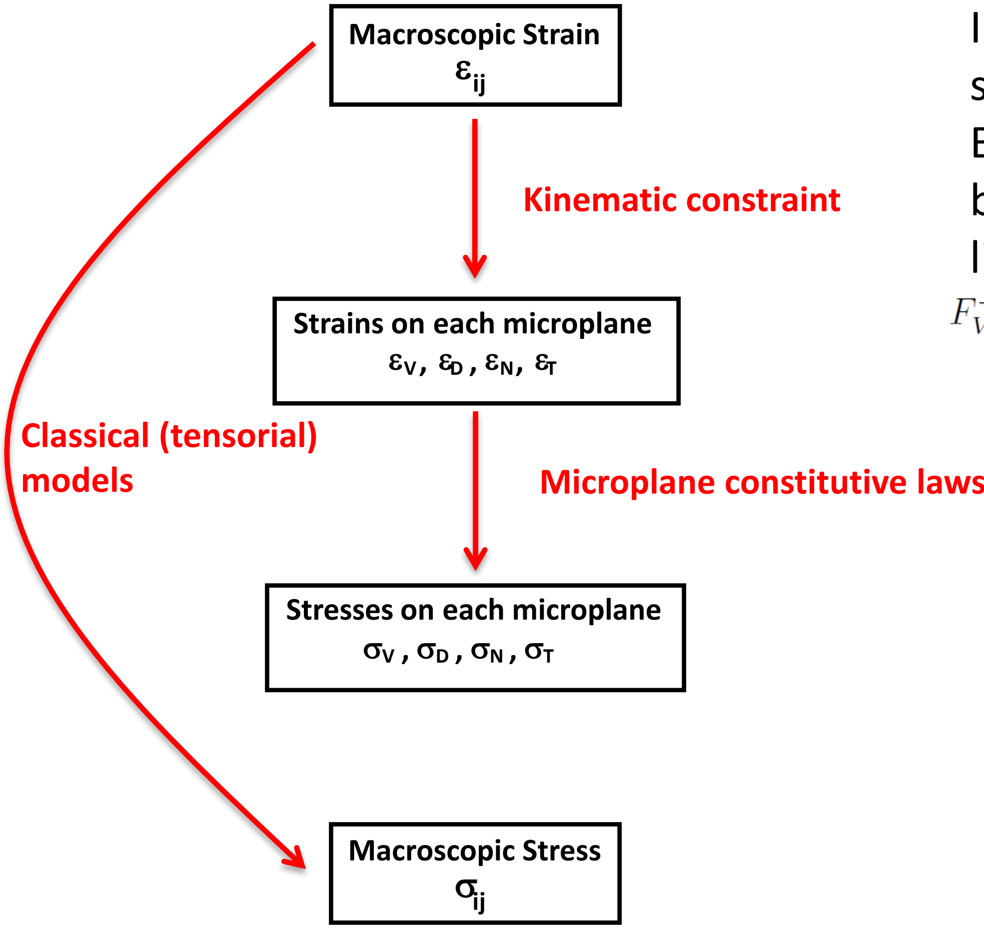
$$\sigma_V = C_V \varepsilon_V, \quad \sigma_D = C_D \varepsilon_D, \quad \sigma_T = C_T \varepsilon_T$$

$$C_V = E_{V,0}(1 - \omega_V), \qquad C_D = E_{D,0}(1 - \omega_D),$$

$$C_T = E_{T,0}(1 - \omega_T),$$

$$C_V = E_{V,0}\left[\left(1 + \left|\frac{\varepsilon_V}{a}\right|\right)^{-p} + \left|\frac{\varepsilon_V}{b}\right|^{q}\right],$$

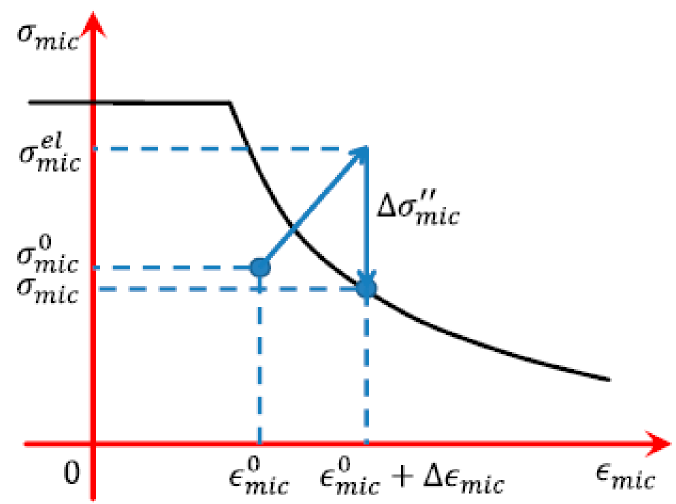
Microplane formulation



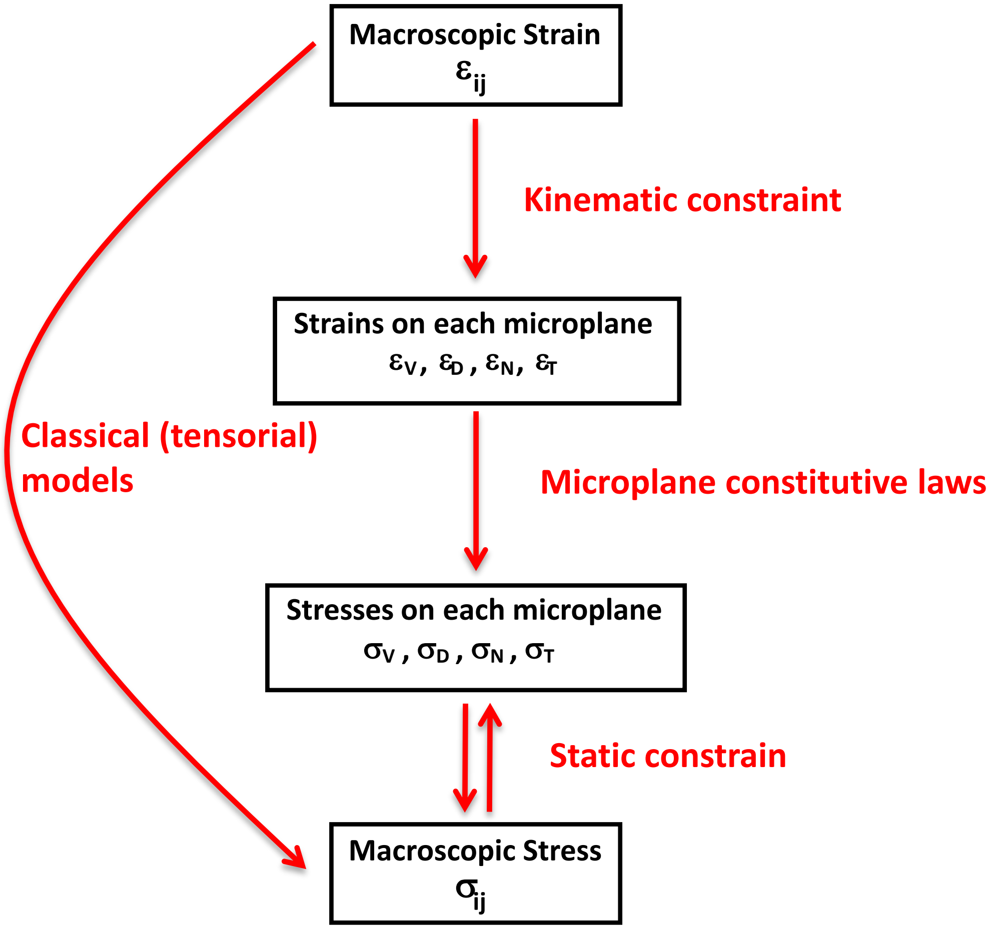
Inelastic behavior:
stress-strain boundaries (Bažant et al. 1996; Bažant et al., 2000). These boundaries may be regarded as strain-dependent yield limits.:

$$F_V^-(\epsilon_V) \leq \sigma_V, \quad F_D^-(\epsilon_D) \leq \sigma_D \leq F_D^+(\epsilon_D), \quad \sigma_N \leq F_N(\epsilon_N),$$

$$|\sigma_T| \leq F_T(\sigma_N, \epsilon_V)$$



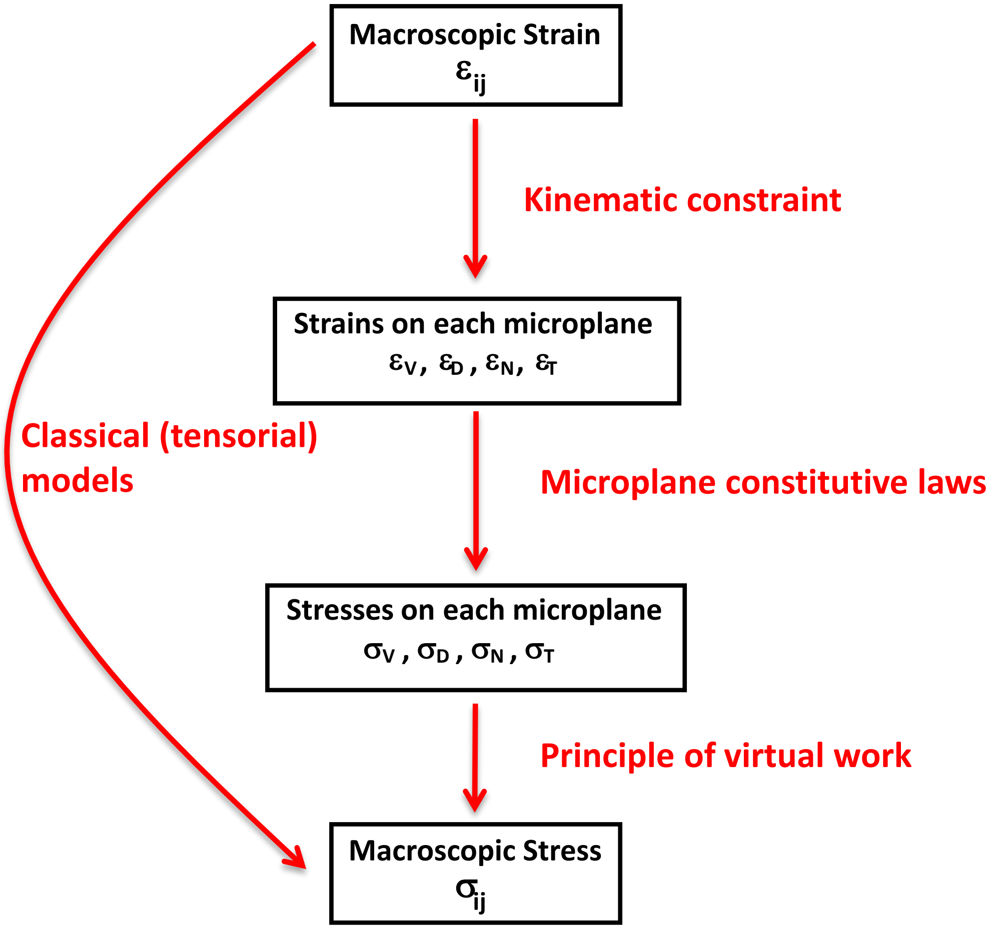
Microplane formulation



A micro–macro homogenization:

- micro–macro force equilibrium relation which means double kinematic-static constraint. Unfortunately, this double constraint does not hold for the general microscopic material laws (Carol and Bažant, 1997).

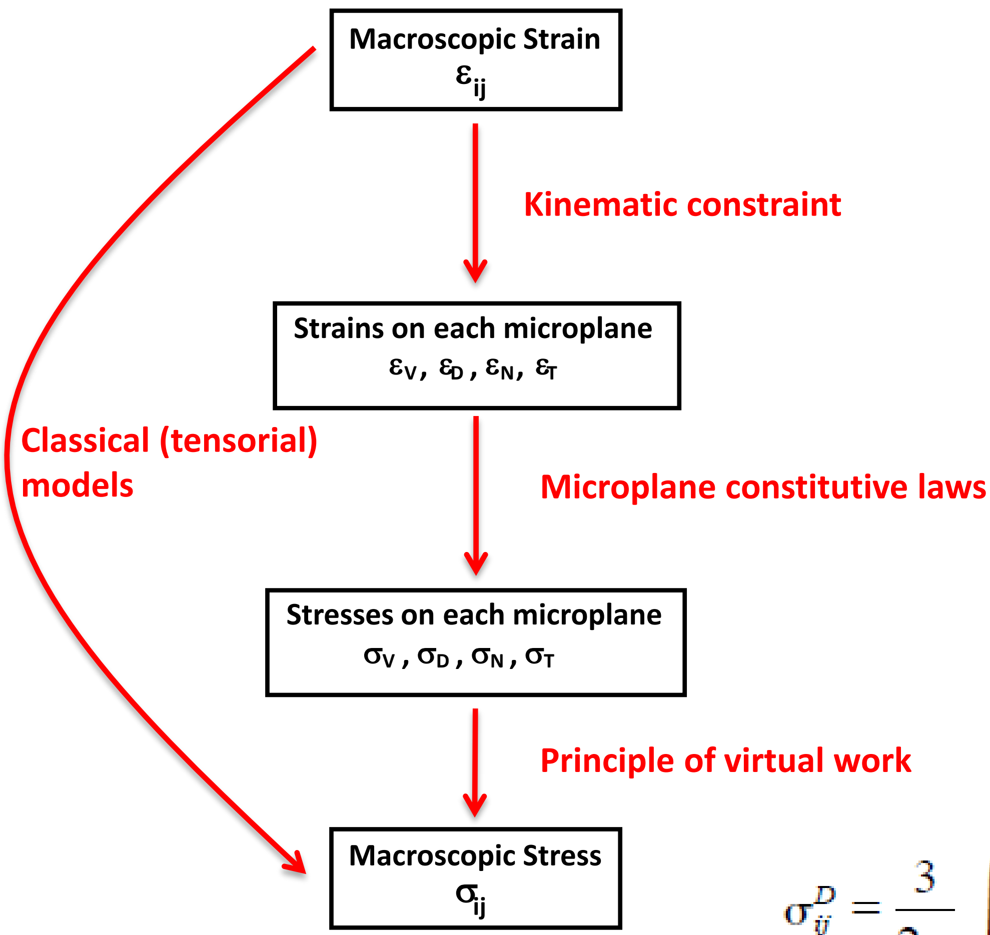
Microplane formulation



A micro–macro homogenization:

- principle of virtual work (PVW) based micro–macro homogenization which imposed the equilibrium in weak sense (Bažant, 1984).

Microplane formulation

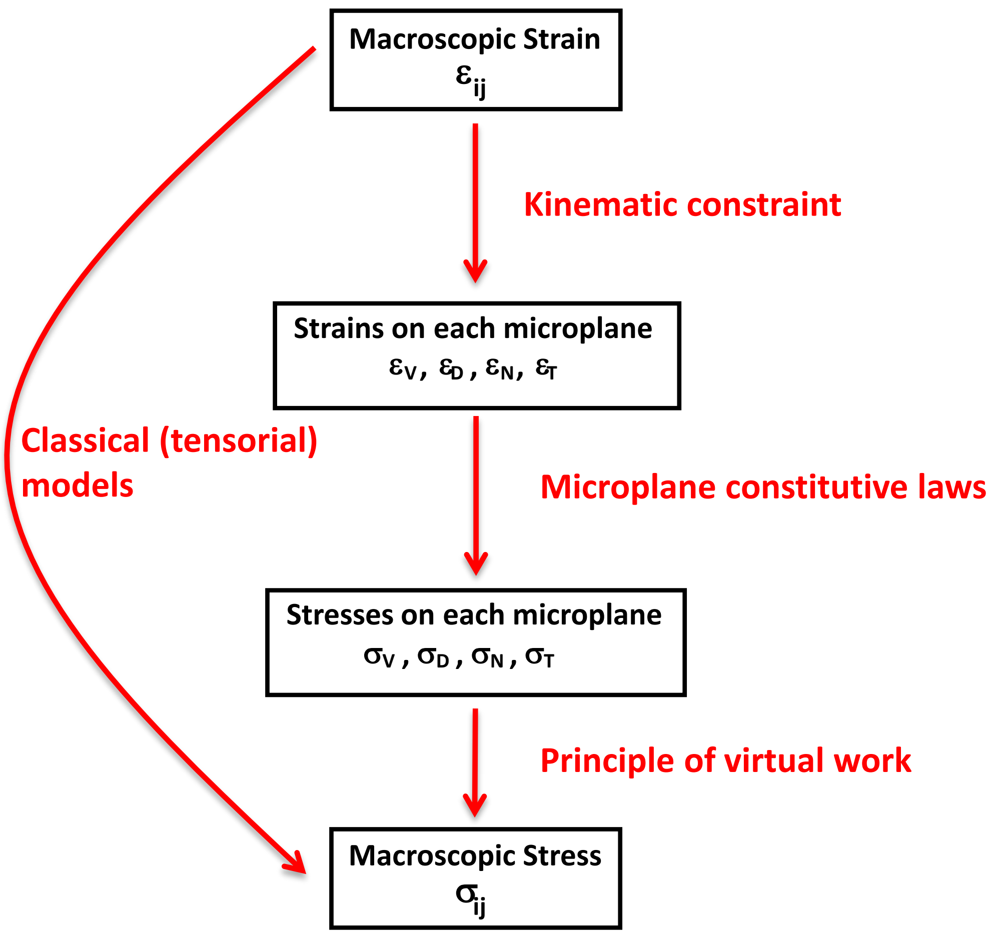


The PVW based formulation was adopted for the microplane model with N–T split and then for the case with V–D–T split. However, Carol et al. (2001) pointed out that such derived microplane model with V–D–T split violates the second principle of thermodynamics due to the non-symmetric material behavior of deviatoric stresses, leading to spurious energy deficiency (Bažant et al., 2000; Carol et al., 2004; Bažant and Caner, 2005; Leukart and Ramm, 2002; Di Luzio 2007). The macro-stress tensor is

$$\sigma_{ij} = \sigma_V \delta_{ij} + \sigma_{ij}^D$$

$$\sigma_{ij}^D = \frac{3}{2\pi} \int_\Omega \left[\sigma_D \left(N_{ij} - \frac{\delta_{ij}}{3} \right) + \sigma_L L_{ij} + \sigma_M M_{ij} \right] d\Omega$$

Microplane formulation



Numerical integration over the unit sphere (half-sphere).

The numerical method must consider:

- the accuracy (degree of the formula);
- the induced anisotropy;
- the symmetry.

(Krylov 1962; Stroud 1971; Bažant and Oh 1985, 1986; Neutsch 1996; Fliege and Maier 1999; Hannay et al 2004; Ehret et al 2009)

Microplane formulation

Macroscopic Strain
ε_{ij}

Kinematic constraint

Strains on each microplane
$\varepsilon_V, \varepsilon_D, \varepsilon_N, \varepsilon_T$

Microplane constitutive laws

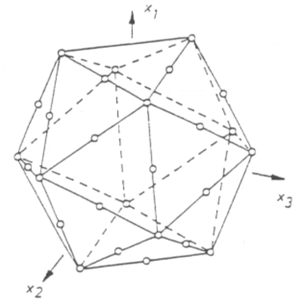
Stresses on each microplane
$\sigma_V, \sigma_D, \sigma_N, \sigma_T$

Principle of virtual work

Macroscopic Stress
σ_{ij}

Classical (tensorial) models

The most efficient formula that still yields acceptable accuracy involves 42 (21 with the half-sphere) microplanes and is of 9th degree with full symmetry (Bažant and Oh 1986).



M1 (Bažant and Oh 1983, 1985; Bažant 1984; Bažant and Gambarova 1984)
Model M1 was focused only on tensile failure. It simulated smeared multidirectional tensile cracking and postpeak tensile softening of concrete. In tandem with the crack band concept, model M1 is capable of representing well all the fracture tests of notched concrete specimens, including the size effect, and was also extended to fit well the basic data on dilatant shear on preexisting cracks in concrete.

M2 (Bažant and Prat 1988; Bažant and Ozbolt 1990, 1992; Carol et al. 1992; Hasegawa and Bažant 1993; Ožbolt et al 2001)

Model M2 was able to model simultaneously tensile and compressive failures by introducing a volumetric-deviatoric split of the normal strains and stresses on the microplanes. While the tensile failure is basically a uniaxial behavior, describable simply by a scalar relation between the normal stress and strain, compression failure is a triaxial phenomenon, in which failure is triggered by lateral expansion and by slip on inclined planes.

A nonlocal generalization of model M2 was developed to prevent spurious excessive localization of damage in structures and spurious mesh sensitivity (Bažant and Ožbolt 1990; Ožbolt and Bažant 1992, 1996).

M3 (Bažant et al. 1996a,b)
Model M3 introduced the concept of stress-strain boundaries (or softening strain-dependent yield limits) on the microplane level. Aside from simplicity and clarity, the advantage of this approach is that several independent
boundaries for different stress components can be defined as functions of different strain components. This is helpful for simultaneous modeling of tensile, compressive, and shear softening.
A softening stress-strain curve for shear employed in model M2 was replaced in model M3 by a strain-independent linear frictional-cohesive yield surface relating the normal and shear stress components on the microplane.

M4 (Bažant et al. 2000; Caner and Bažant 2000; Di Luzio 2007)
Model M4 presents a work-conjugate definition of the volumetric-deviatoric split and improved formulations of boundary surfaces, frictional yield limits and damage.

The model M4 was also extended to strain-rate sensitivity, creep and to arbitrarily large finite strain (Bažant et al 2000).
A nonlocal version of the model M4 was presented by Bažant and Di Luzio 2004, Di Luzio 2007.

However, the volumetric-deviatoric split leads to excessive lateral expansion and stress locking in far postpeak uniaxial tension and unrealistic unloading and reloading. To overcome this deficiency a modified M4 was re-formulated with a no-split of the normal component for dominant tensile failure and removing the tensile volumetric boundary (Bažant et al 2004; Di Luzio 2007).

M5-M6-M7 (Bažant and Caner 2005a, b; Caner and Bažant 2011; Caner and Bažant 2013a, b)
The volumetric-deviatoric split, however, brought about other problems—especially, excessive lateral expansion and stress locking in far postpeak uniaxial tension and unrealistic unloading and reloading.
The expansion and locking problems were mitigated in M5 by a series coupling of separate microplane models for compression and for tension.
All these problems are overcome in the model M7, in which the key idea is to abandon the volumetric-deviatoric split for the elastic part of microplane strains and for the tensile stress-strain boundary while retaining it for the compressive normal and deviatoric stress-strain boundaries.

Microplane formulation

Impact loading (Bažant et al 2000; Ožbolt et al 2006; Travaš et al 2009; Adley et al 2012; Kirane et al 2015)

Large strain extensions (Bažant et al 2000; Carol et al 2004; Caner et al 2007)

Fiber reinforced concrete (Di Luzio and Cedolin 2004; Beghini et al 2007; Caner et al 2013; Vrech et al 2016)

Shape Memory Alloys (Brocca et al 2000, Kadkhodaei et al 2007; Mehrabi et al 2014; Poorasadion et al 2015; Karamooz Ravari et al 2015)

Composites: fiber reinforced polymer composite (Ožbolt et al 2011); woven fabric composites (Salviato and Bažant 2015) or braided laminates (Caner et al 2011); composite laminates (Brocca et al 2001, Cusatis et al 2008, Beghini et al 2008)

Microplane formulation

Blood soft tissue (Caner and Carol 2006)

Geomechanical materials: rock (Prat et al 1997; Adachi and Oka 1995, Bažant and Zi 2003), jointed rock mass (Chen et al 2014), clay (Bažant and Prat 1987), sand (Chang and Sture 2006).

Metal plasticity (Brocca and Bažant 2000, Ožbolt et al. 2016)

Micropolar formulation, in the spirit of the “Cosserat media”, (Etse et al. 2003; Etse and Nieto 2004).

High order microplane model (Cusatis and Zhou 2013).

The microplane model has been introduced into various commercial programs (ATENA, OOFEM, DIANA, SBETA, ANSYS, MARS, MASA, ...).

Over-nonlocal formulation of Microplane model
(Bažant and Di Luzio 2004; Di Luzio 2007)

"Localization limiters"

Nonlocal strains: $$\bar{\varepsilon}_{ij}(\mathbf{x}) = \int_V W(\mathbf{x},\xi)\cdot\varepsilon_{ij}(\xi)\cdot dV(\xi)$$

Refined nonlocal formulation $$\hat{\varepsilon}_{ij} = m\cdot\bar{\varepsilon}_{ij} + (1-m)\cdot\varepsilon_{ij}$$ with $m > 1$

(Vermeer & Brinkgreve 1994)

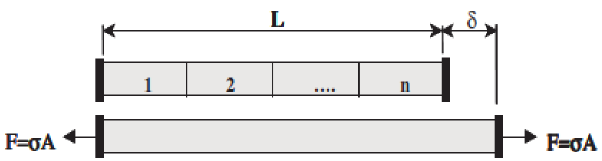
Macro-nonlocal Strain	Kinematic constraint →	Micro-nonlocal Strains
$\hat{\boldsymbol{\varepsilon}}_{ij}$		$\hat{\boldsymbol{\varepsilon}}_V, \hat{\boldsymbol{\varepsilon}}_D, \hat{\boldsymbol{\varepsilon}}_N, \hat{\boldsymbol{\varepsilon}}_T$ on each microplane

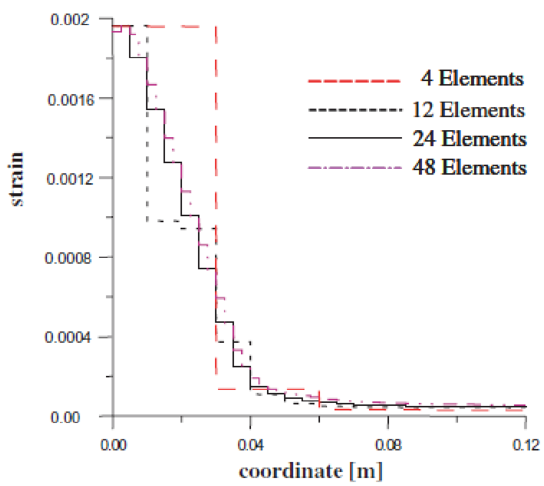
All the boundaries are function of the nonlocal microplane strains:

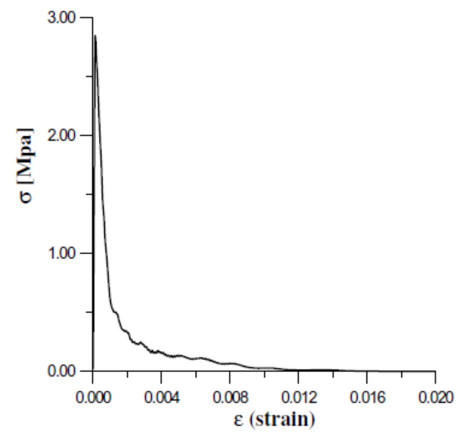
$$\sigma_N^b = F_N(\hat{\varepsilon}_N); \quad \sigma_D^{b^+} = F_D(\hat{\varepsilon}_D); \quad \sigma_D^{b^-} = F_D(\hat{\varepsilon}_D); \quad \sigma_N^0 = \sigma_N^0(\hat{\varepsilon}_V)$$

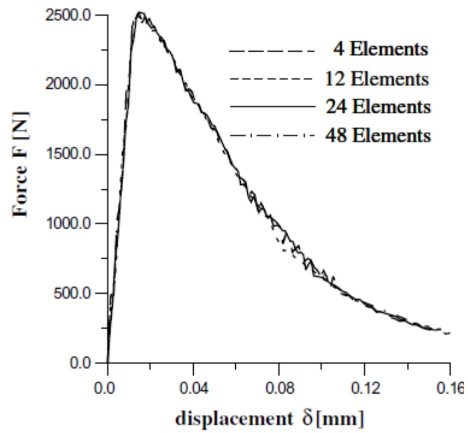
Over-nonlocal formulation of Microplane model
(Bazant and Di Luzio 2004; Di Luzio 2007)

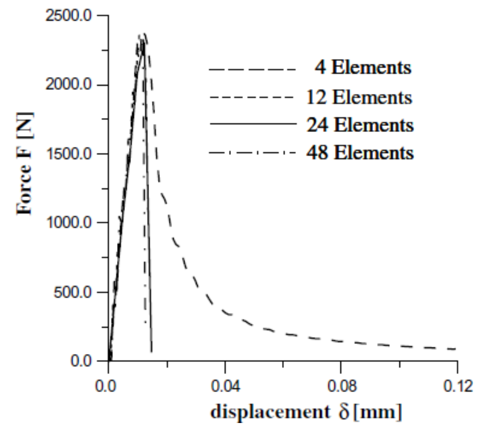
Bar subjected to uniaxial tension





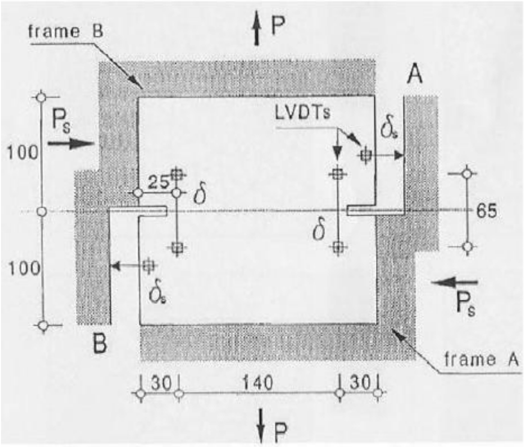


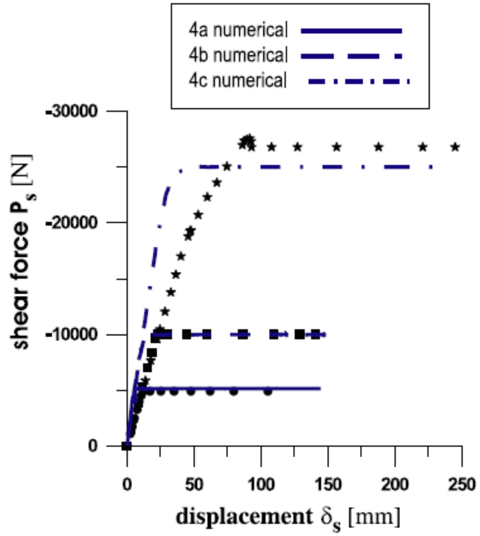


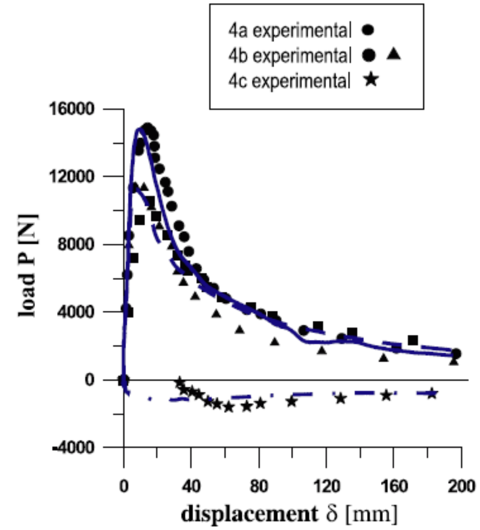


Over-nonlocal formulation of Microplane model
(Di Luzio 2007)

The double-edge-notched (DEN) specimen experimentally tested by Nooru-Mohamend (1992): loading paths 4 tensile test under constant shear force

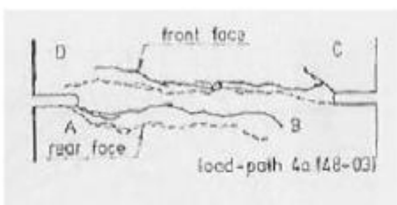


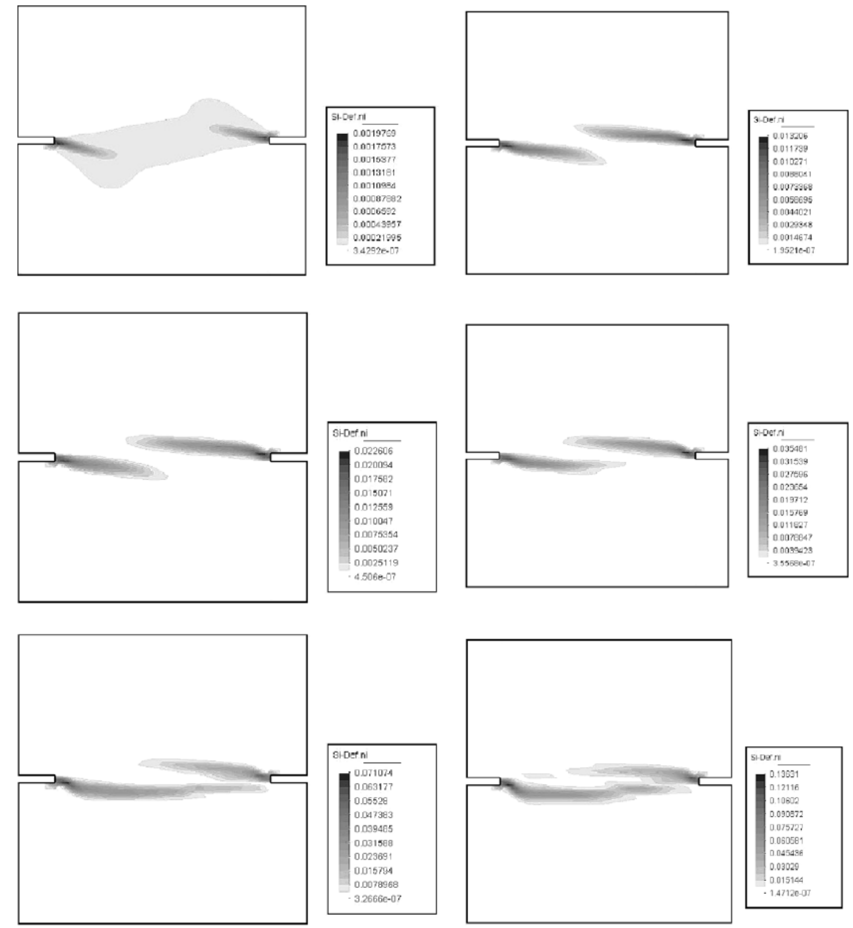




Over-nonlocal formulation of Microplane model
(Di Luzio 2007)

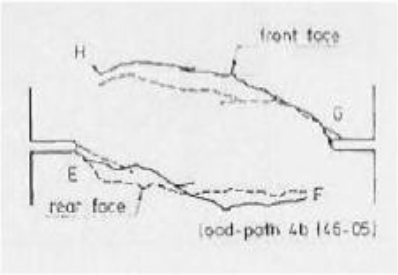
Double-edge-notched (DEN) tested by Nooru-Mohamend (1992) load-path 4a: evolution of the maximum principal strain.

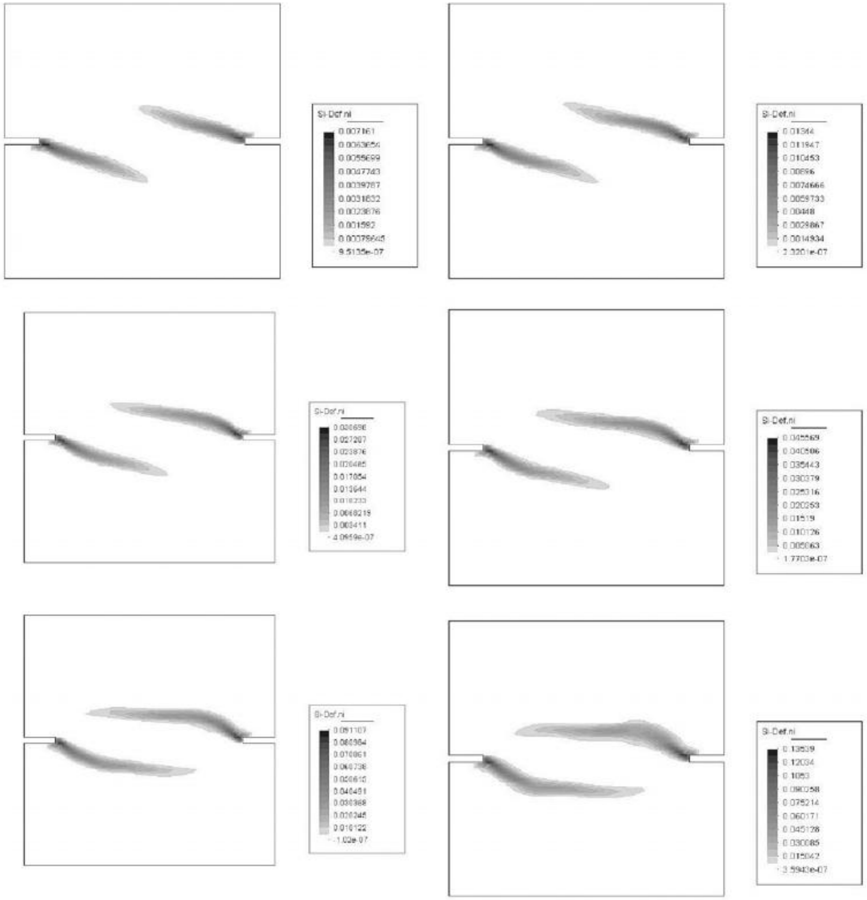




Over-nonlocal formulation of Microplane model
(Di Luzio 2007)

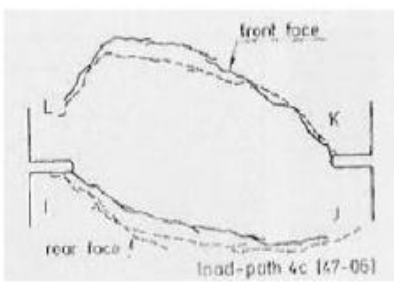
Double-edge-notched (DEN) tested by Nooru-Mohamend (1992) load-path 4b: evolution of the maximum principal strain.

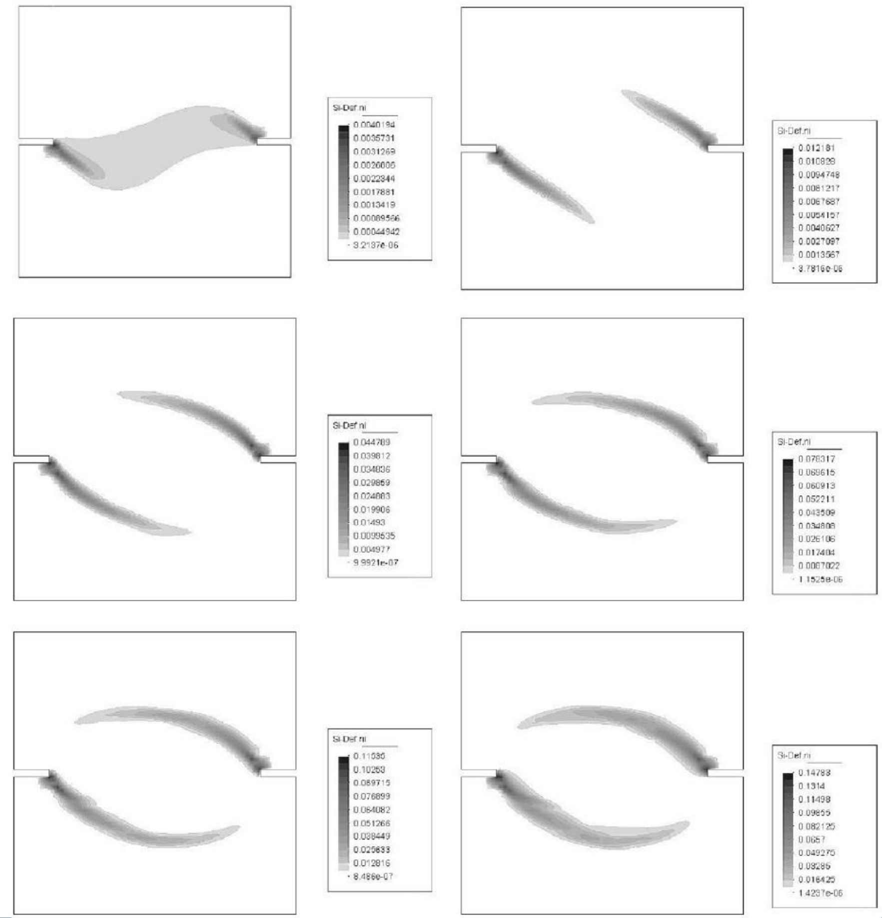




Over-nonlocal formulation of Microplane model
(Di Luzio 2007)

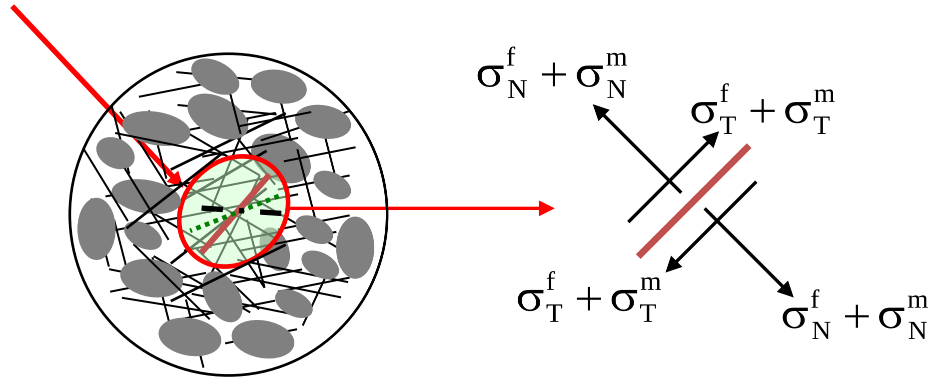
Double-edge-notched (DEN) tested by Nooru-Mohamend (1992) load-path 4c: evolution of the maximum principal strain.



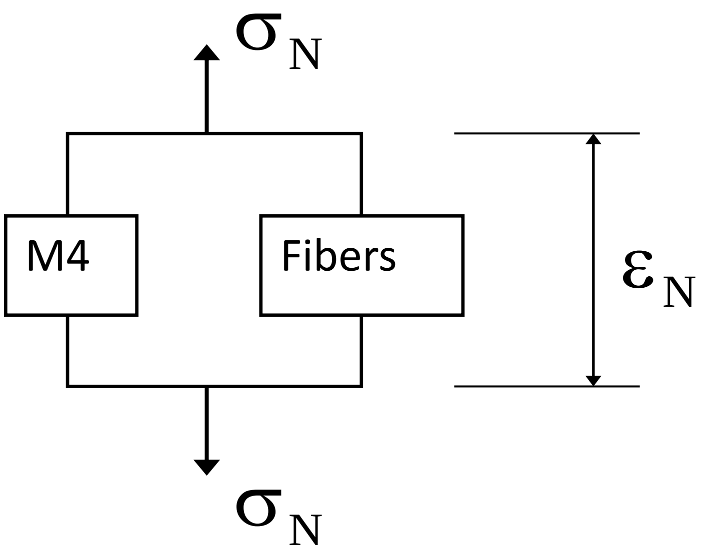


Microplane model for fiber reinforced concrete
(Di Luzio and Cedolin 2004)

Generic microplane in the material



Parallel action of fiber contribution



$$\sigma_N = \sigma_N^{Matrix} + \sigma_N^{Fiber}$$

$$\sigma_M = \sigma_M^{Matrix} + \sigma_M^{Fiber}$$

$$\sigma_L = \sigma_L^{Matrix} + \sigma_L^{Fiber}$$

+ appropriate stress-strain boundaries for fiber effect simulating the bridging effect, bending, ……..

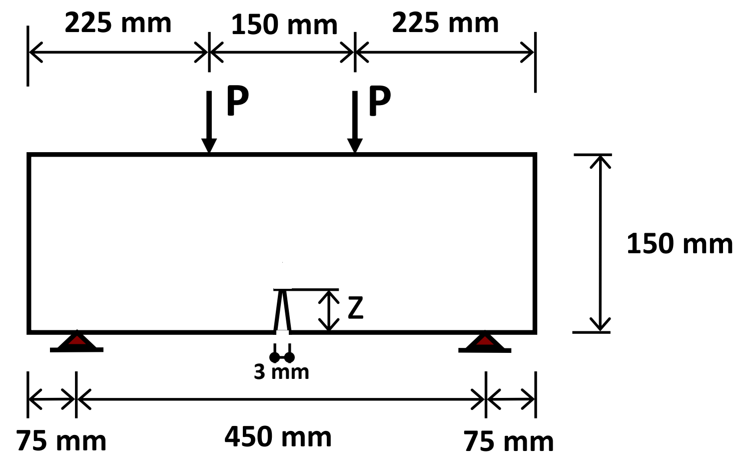
Simulations four-point-bending tests with sharp notch of different lengths: **z = 30 mm; z = 52.5 mm**; **z = 75 mm**

Steel fibers

DRAMIX ZP 30/0.5
l_f = 30 mm
l_f/d_f = 60
V_f=0.38% and V_f=0.76%

Concrete

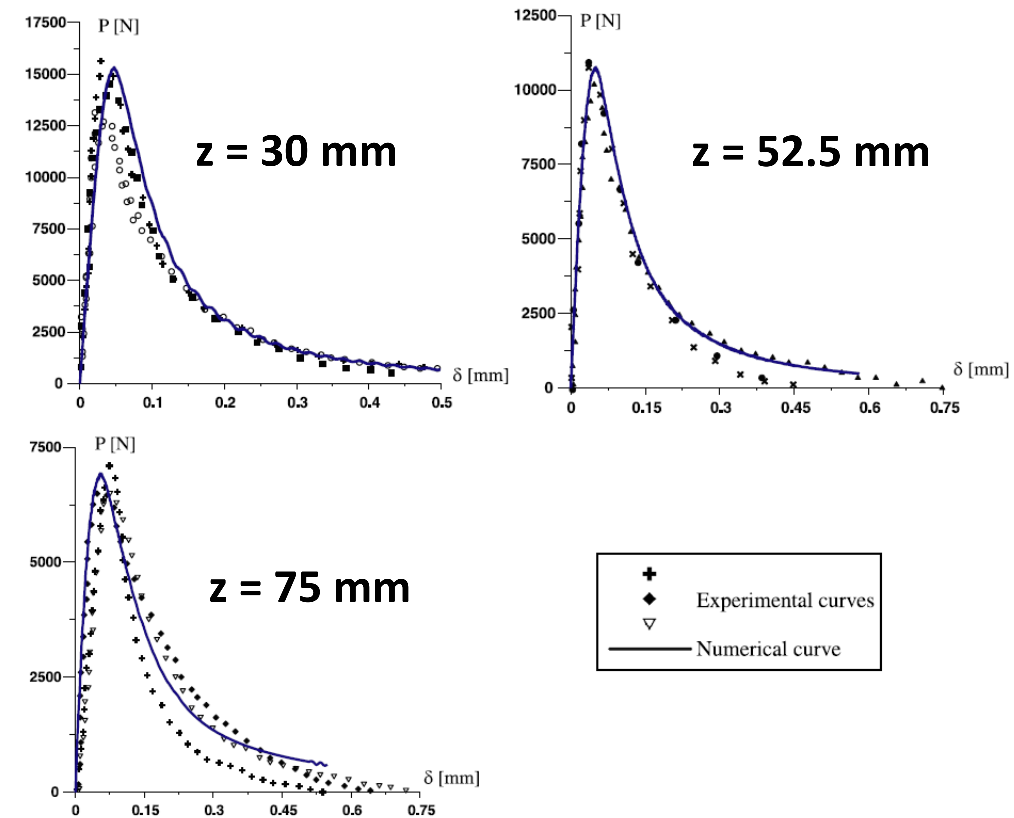
E = 37000 Mpa
f_c = 32 Mpa
$d_{a\ max}$ = 15 mm
w/c = 0.3

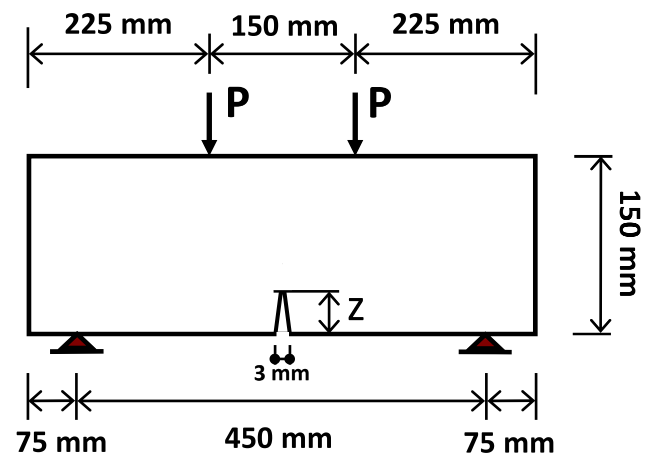


Experimental investigation of **CTG-Italcementi Group**

Microplane model for fiber reinforced concrete
(Di Luzio and Cedolin 2004)

Simulations four-point-bending tests with sharp notch of different lengths.

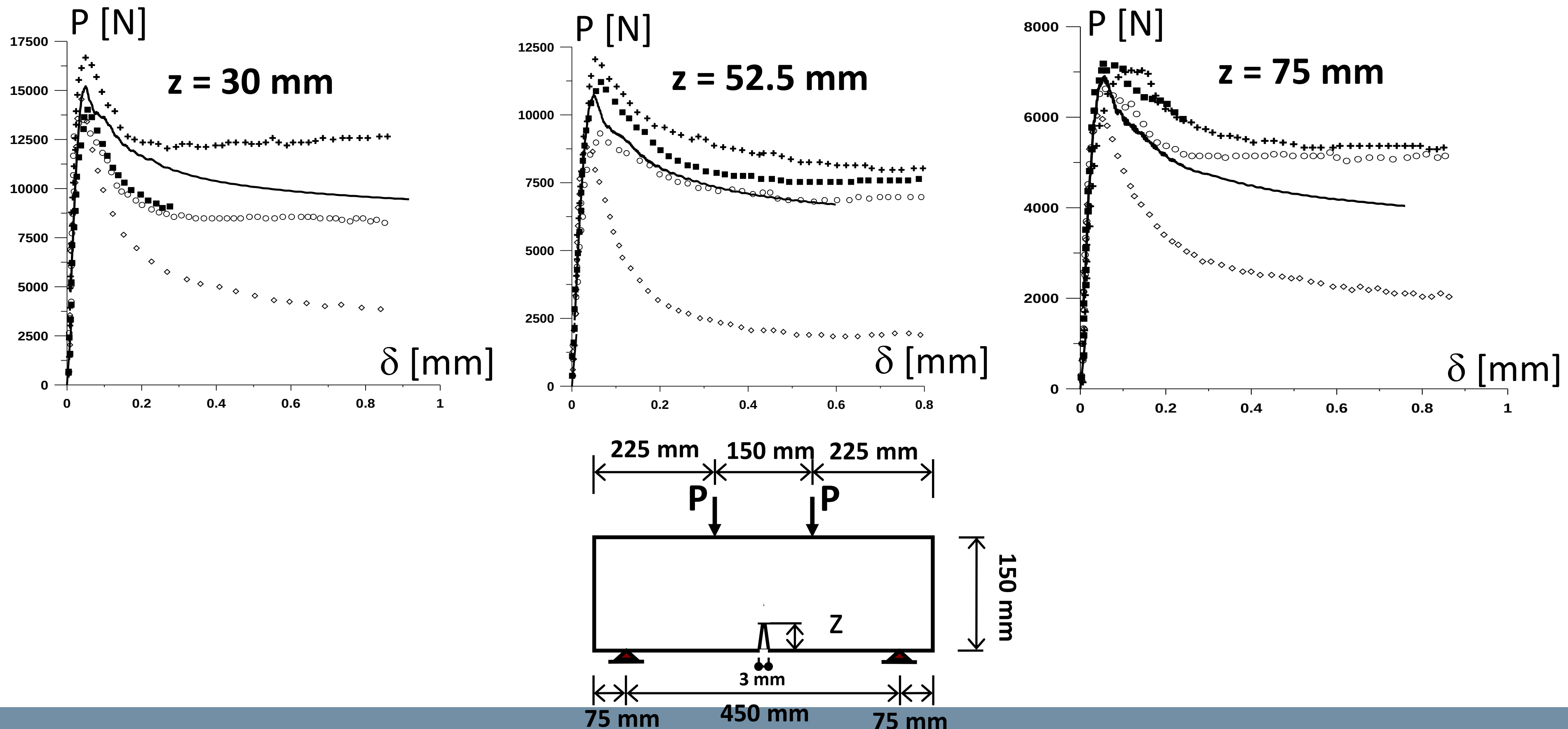




Microplane model for fiber reinforced concrete
(Di Luzio and Cedolin 2004)

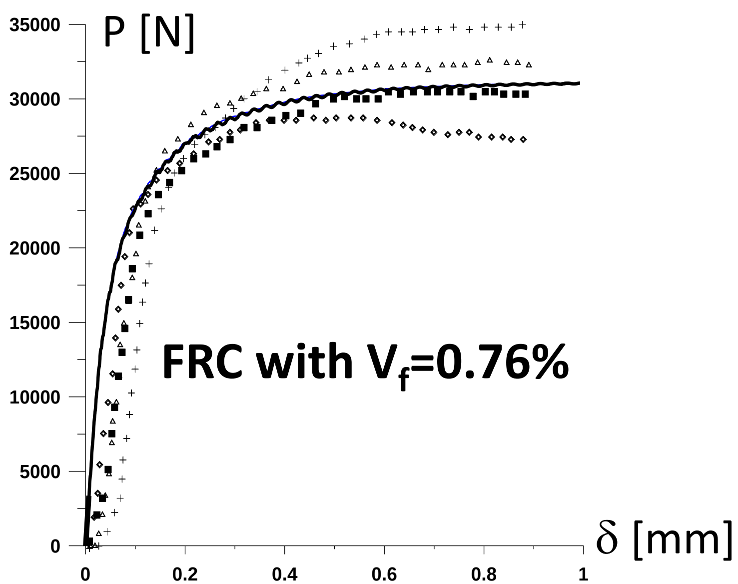
Simulations four-point-bending tests with sharp notch of different lengths.

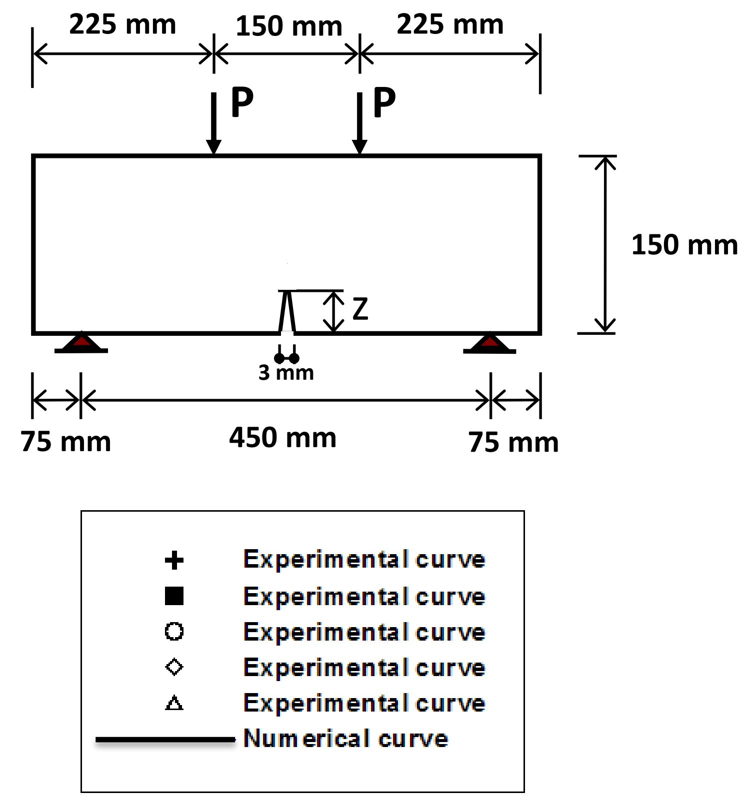
FRC with V_f=0.38%



Microplane model for fiber reinforced concrete
(Di Luzio and Cedolin 2004)

Simulations four-point-bending tests with sharp notch **z = 30 mm**





Numerical Simulation with Microplane Model
(Di Luzio, 2007; Di Luzio and Cedolin 2006; Di Luzio and Cedolin 2004)

Simulation of anchor failure by tension or shear load.

Experimental investigation (Cattaneo 2003; Cattaneo and Rosati 2004)

Mechanical properties of concrete

Young modulus	**42000 MPa**
Compressive strength	**96.16 MPa**
Tensile strength	**5.35 Mpa (estimated)**

Mechanical properties of adhesive (Adhesive Anchors)

Young modulus	**6000 MPa**
Poisson ratio	**0.33**
Shear strength	**19 Mpa**

Mechanical properties of steel (expansion and adhesive anchors)

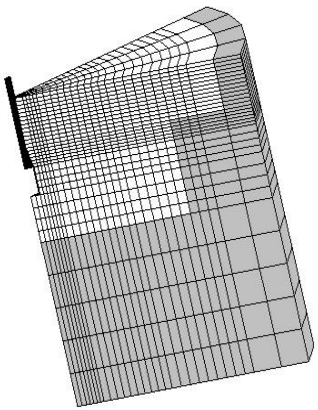
Young modulus	**210000 MPa**
Poisson ratio	**0.3**

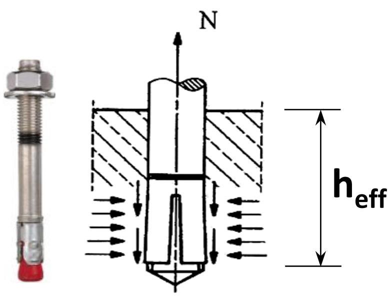
Elasto-plastic behavior with Huber-Henky-von Mises yield limit and isotropic hardening

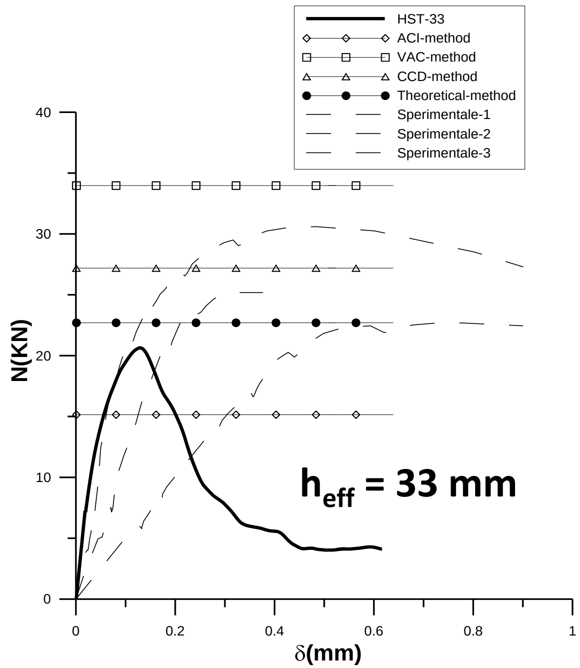
Numerical Simulation with Microplane Model
(Di Luzio, 2007; Di Luzio and Cedolin 2006; Di Luzio and Cedolin 2004)

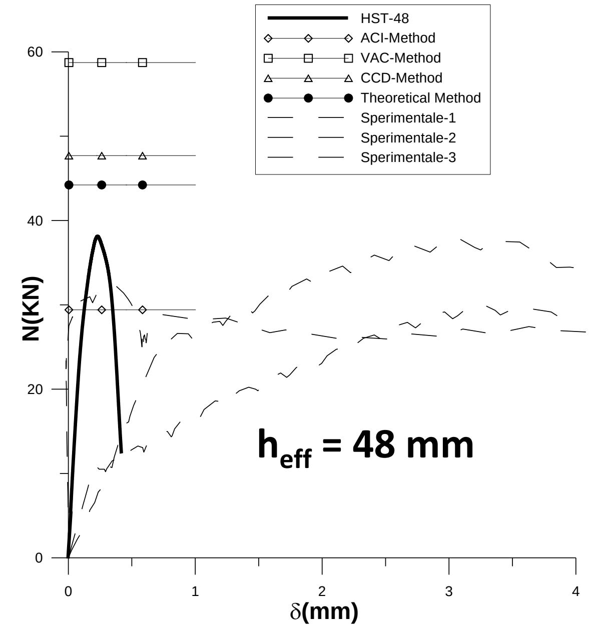
Simulation of expansion anchor failure by tension load

Original mesh



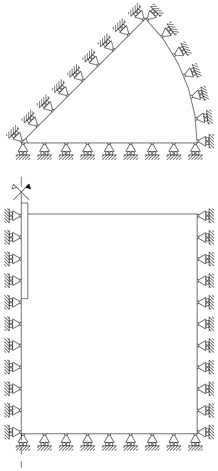


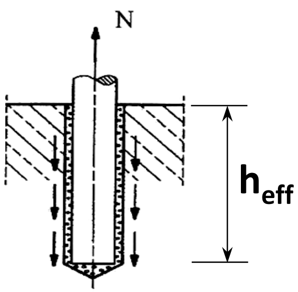


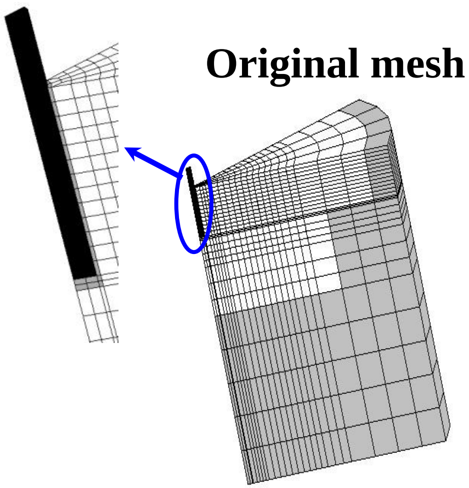


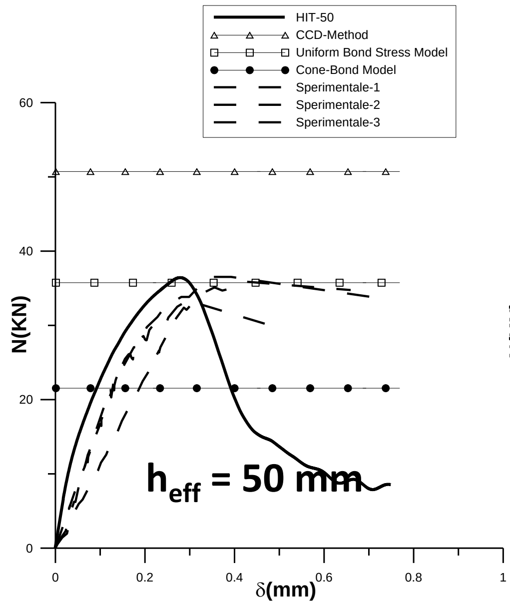
Numerical Simulation with Microplane Model
(Di Luzio, 2007; Di Luzio and Cedolin 2006; Di Luzio and Cedolin 2004)

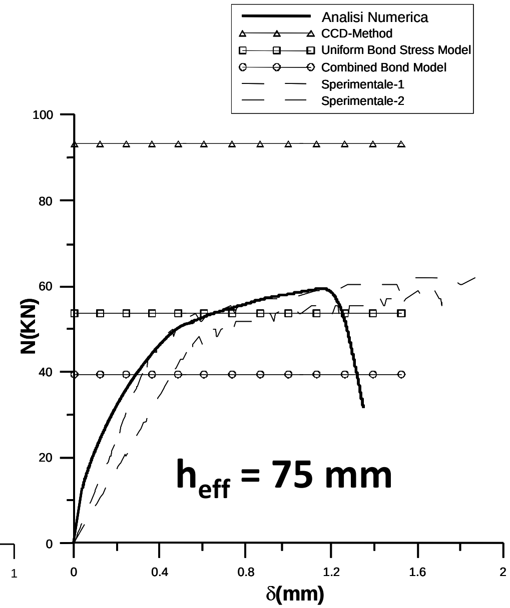
Simulation of adhesive anchor failure by tension load





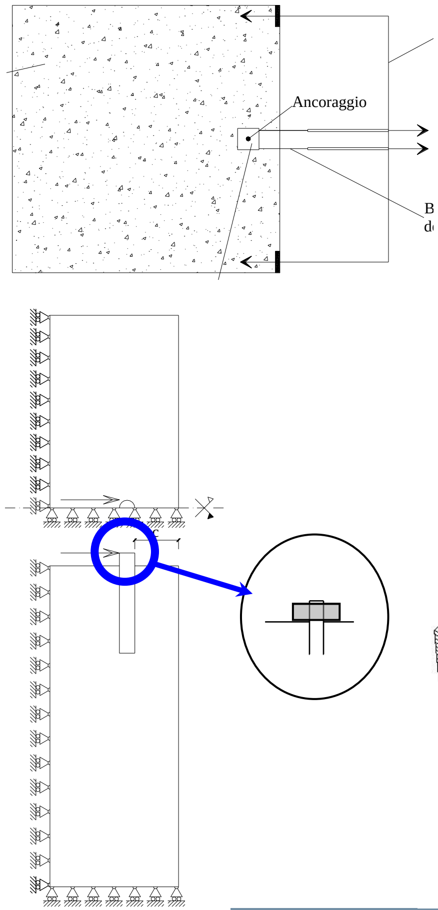


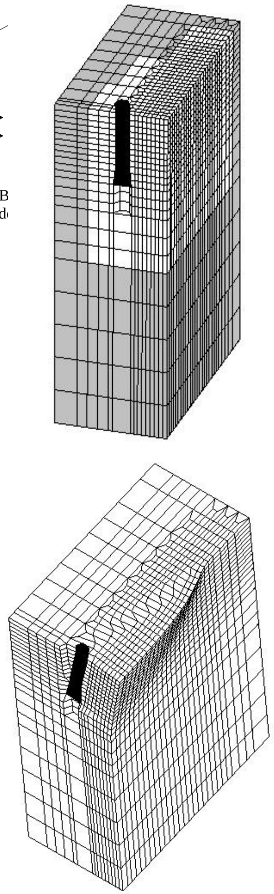


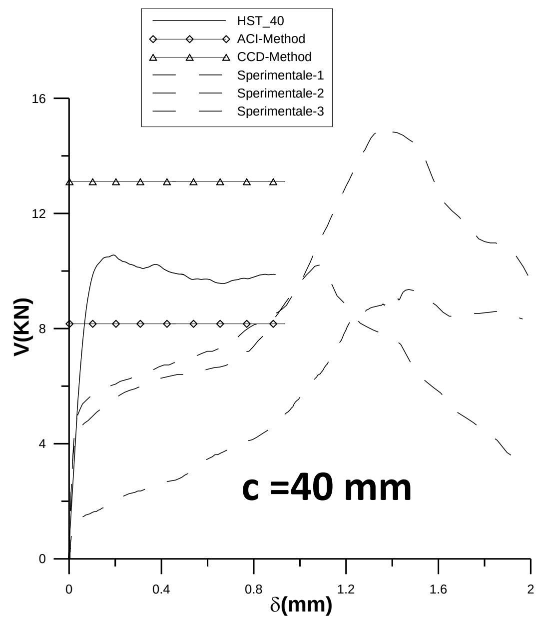


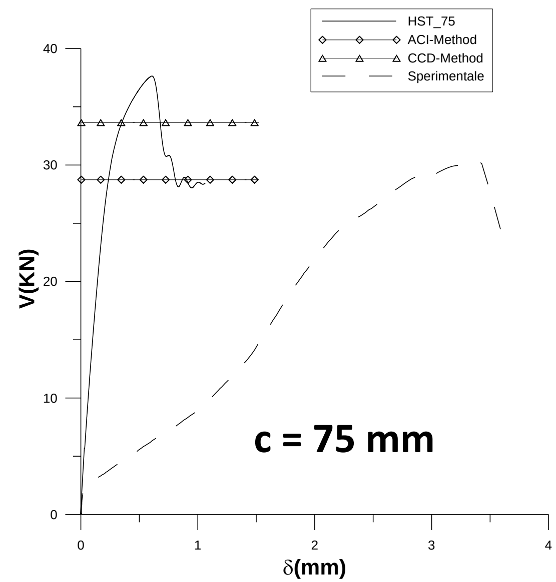
Numerical Simulation with Microplane Model
(Di Luzio, 2007; Di Luzio and Cedolin 2006; Di Luzio and Cedolin 2004)

Simulation of adhesive anchor failure by shear load

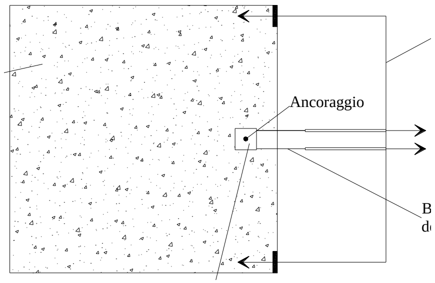


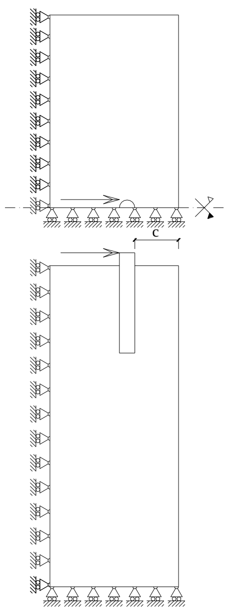


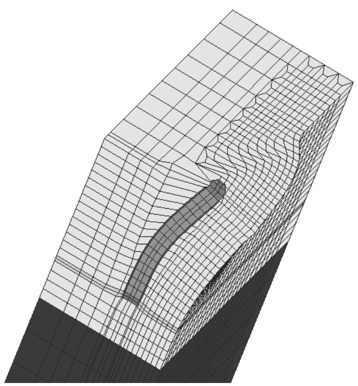


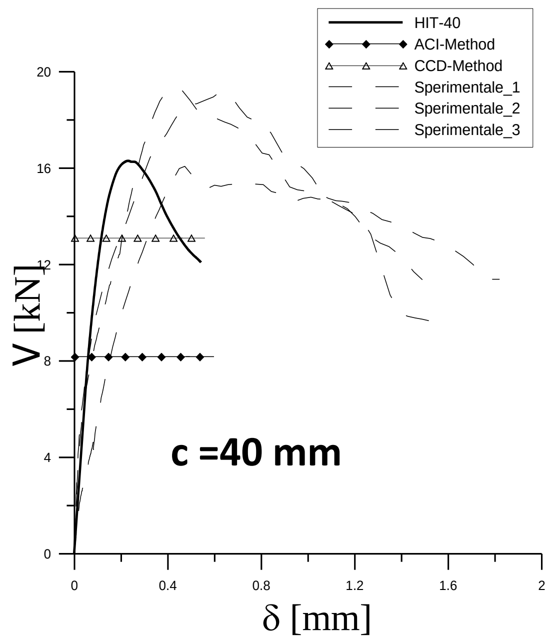


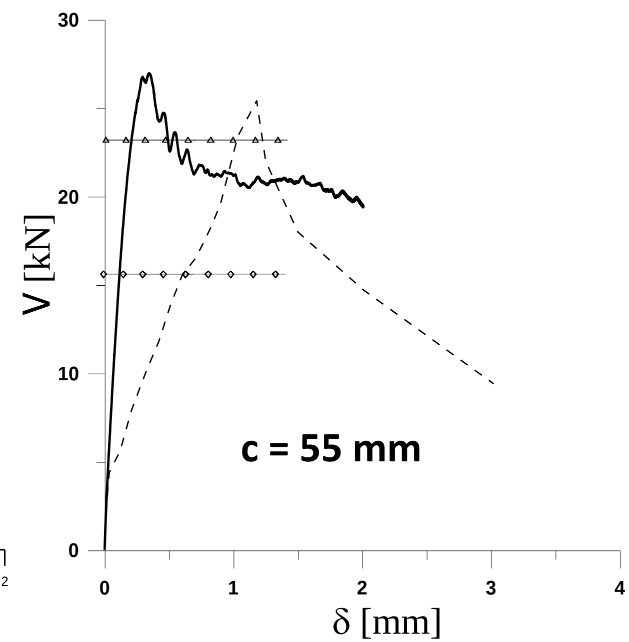
Simulation of adhesive anchor failure by shear load



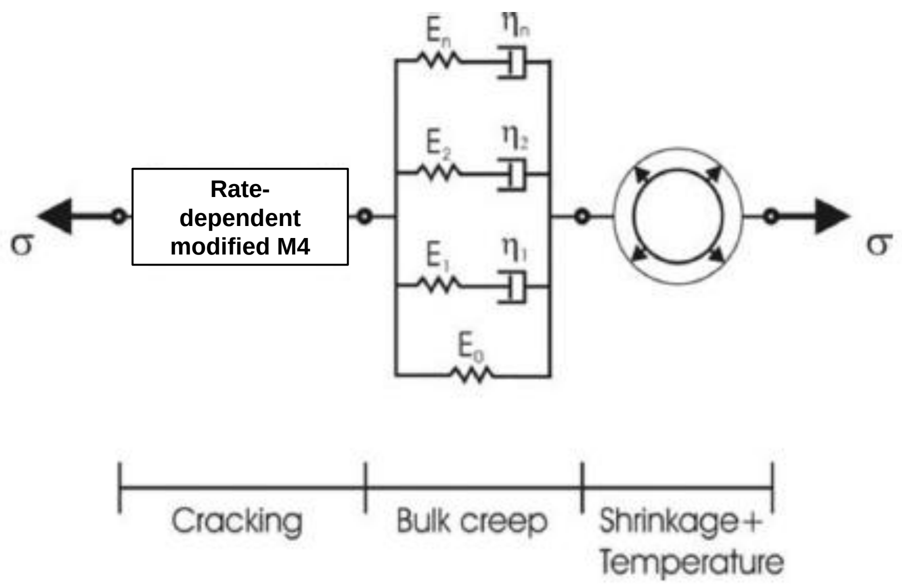








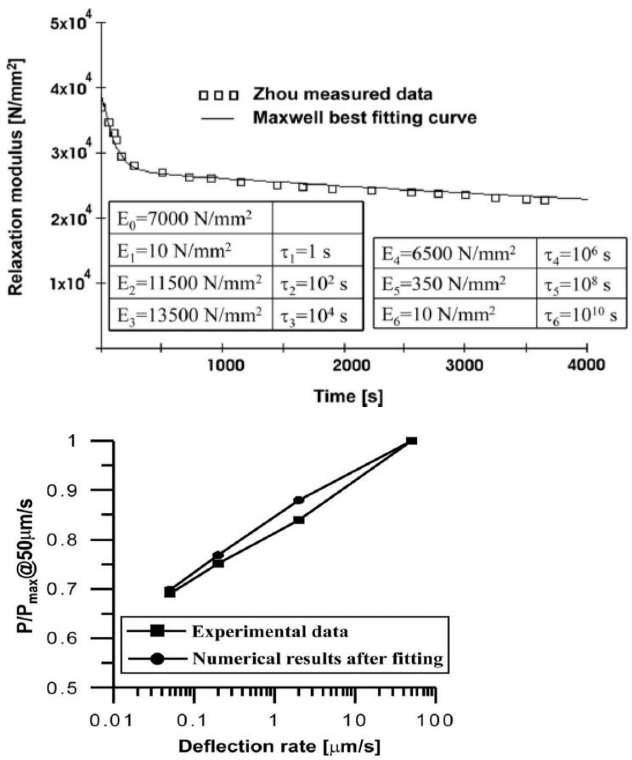
Microplane model for Time-Dependent Fracturing of Concrete
(Di Luzio 2009)



Microplane model for Time-Dependent Fracturing of Concrete
(Di Luzio 2009)

Three-point-bending tests under sustained loads of Zhou (1992)

Calibration



Microplane model for Time-Dependent Fracturing of Concrete
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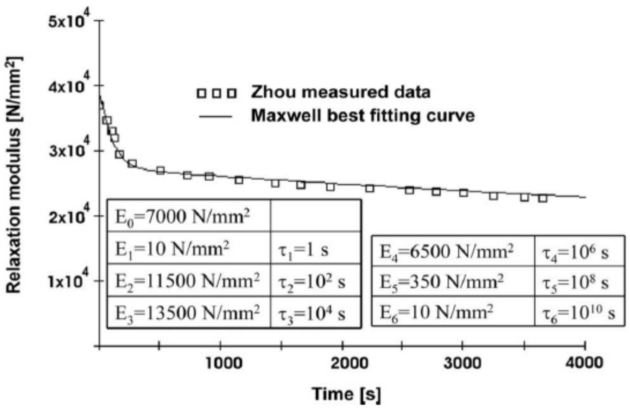
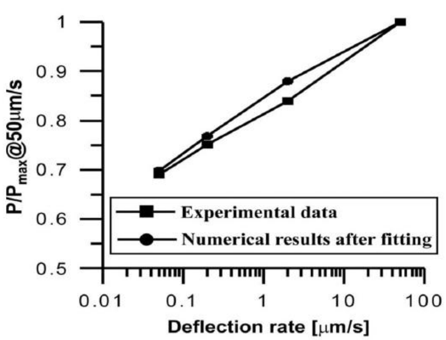
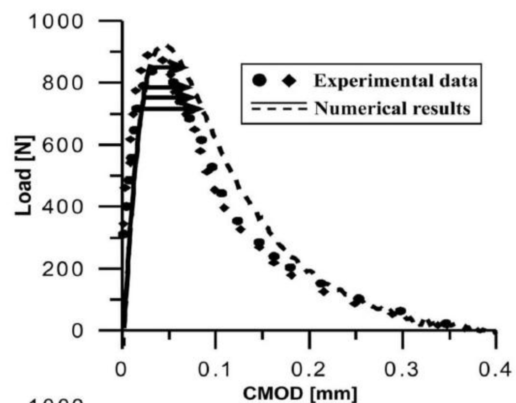
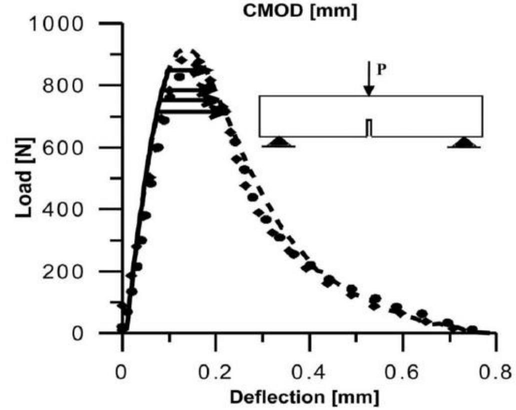
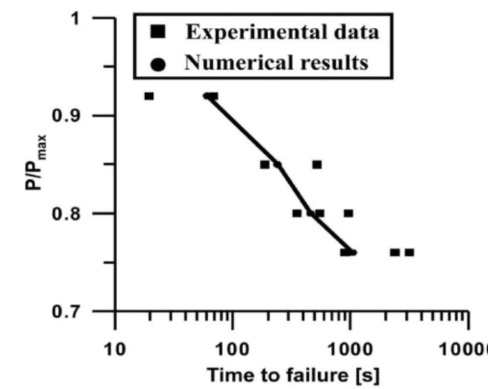
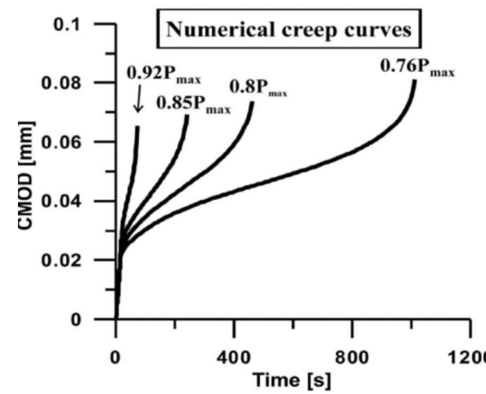




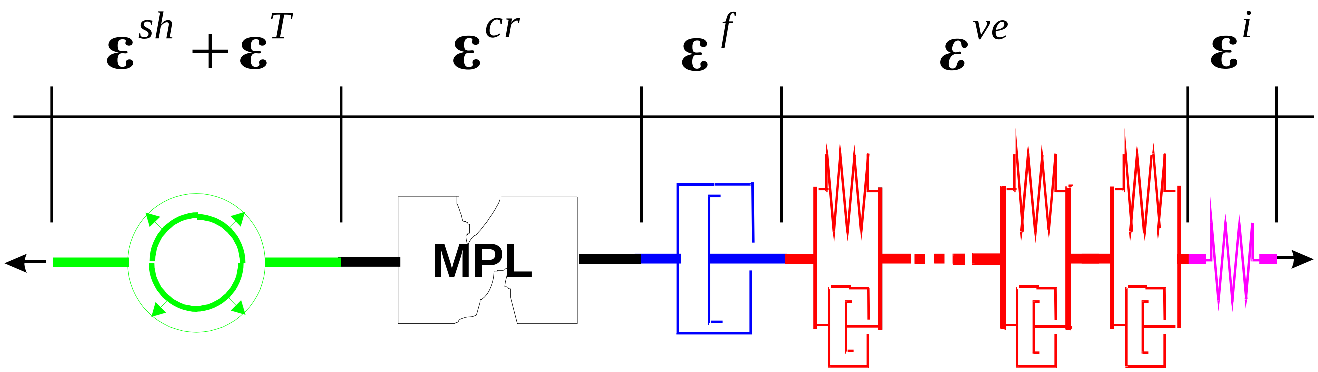








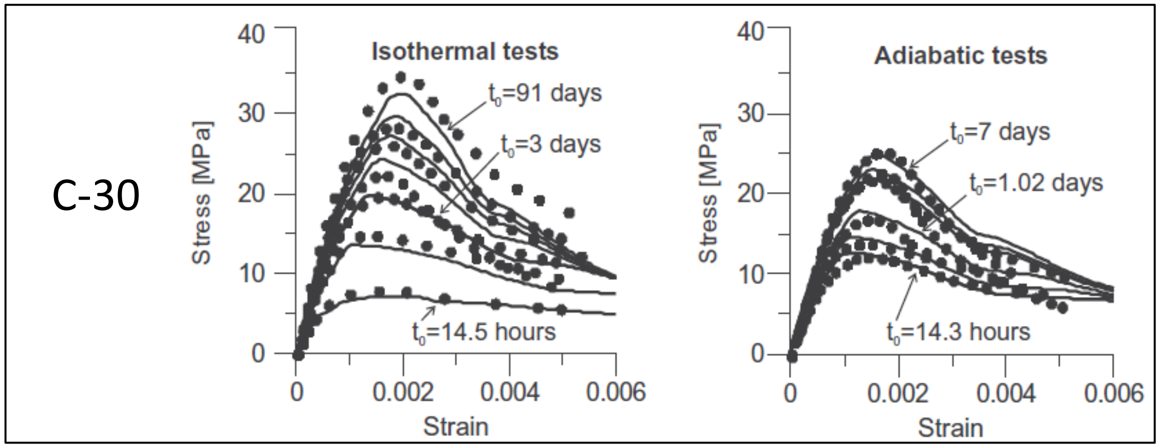
Solidification–Microprestress–Microplane model (SMM) theory
(Di Luzio and Cusatis 2013)

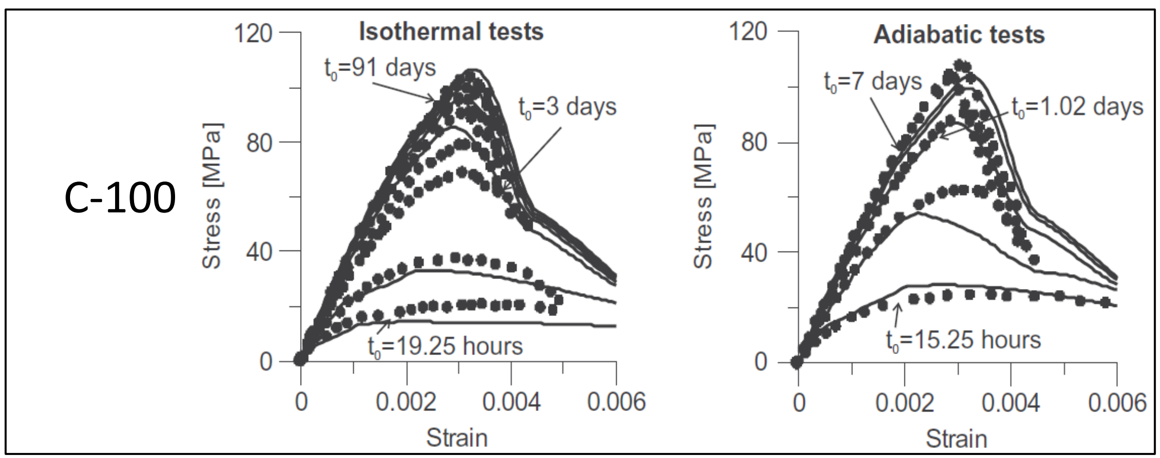


- Instantaneous strain is the strain appearing immediately after applying a uniaxial stress.
- Visco-elastic strain is originated in the solid gel of calcium silicate hydrates (explained by Solidification Theory)
- Purely viscous strain is the completely irrecoverable part of the creep strain (explained by Microprestress Theory)
- Inelastic (cracking) strain (age dependent modified Microplane model M4)
- Shrinkage and thermal strains

Solidification–microprestress–microplane model (SMM) theory
(Di Luzio and Cusatis 2013)

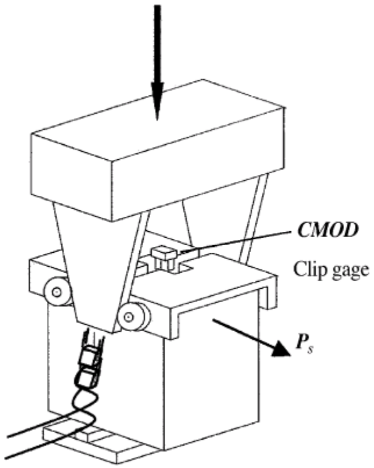
Uniaxial compression test under different conditions, isothermal and adiabatic, tests of Khan et al. (1995).

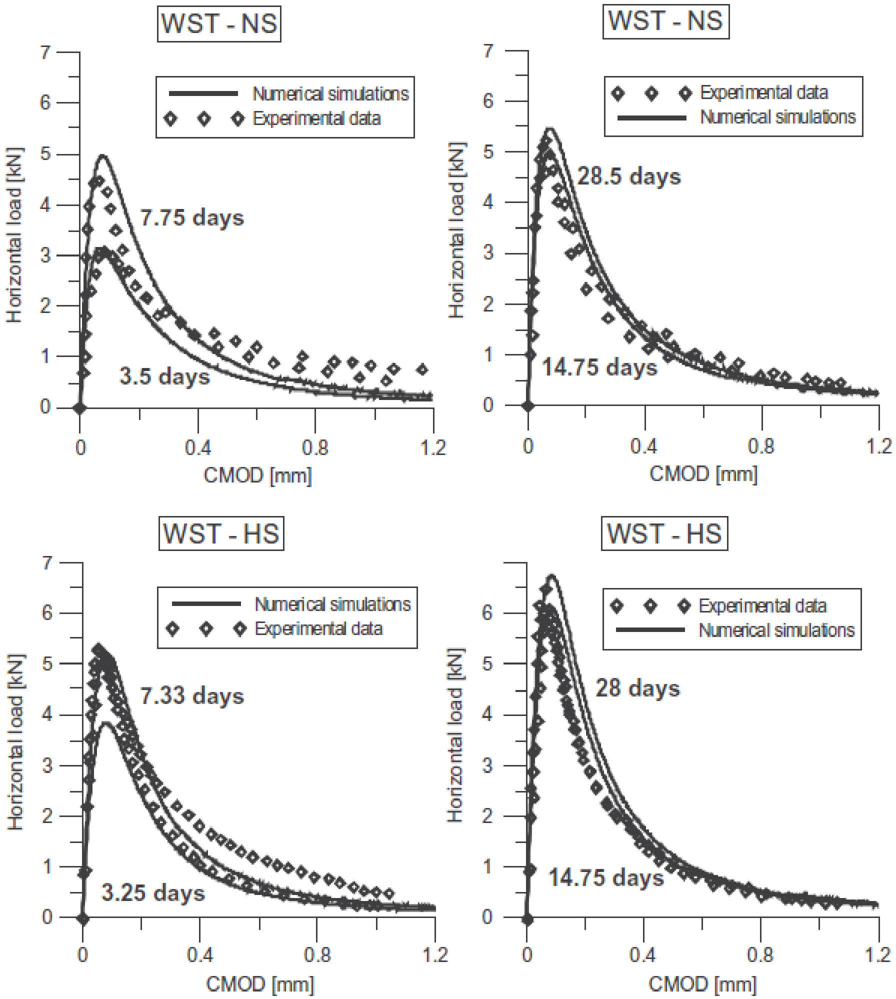




Solidification–microprestress–microplane model (SMM) theory
(Di Luzio and Cusatis 2013)

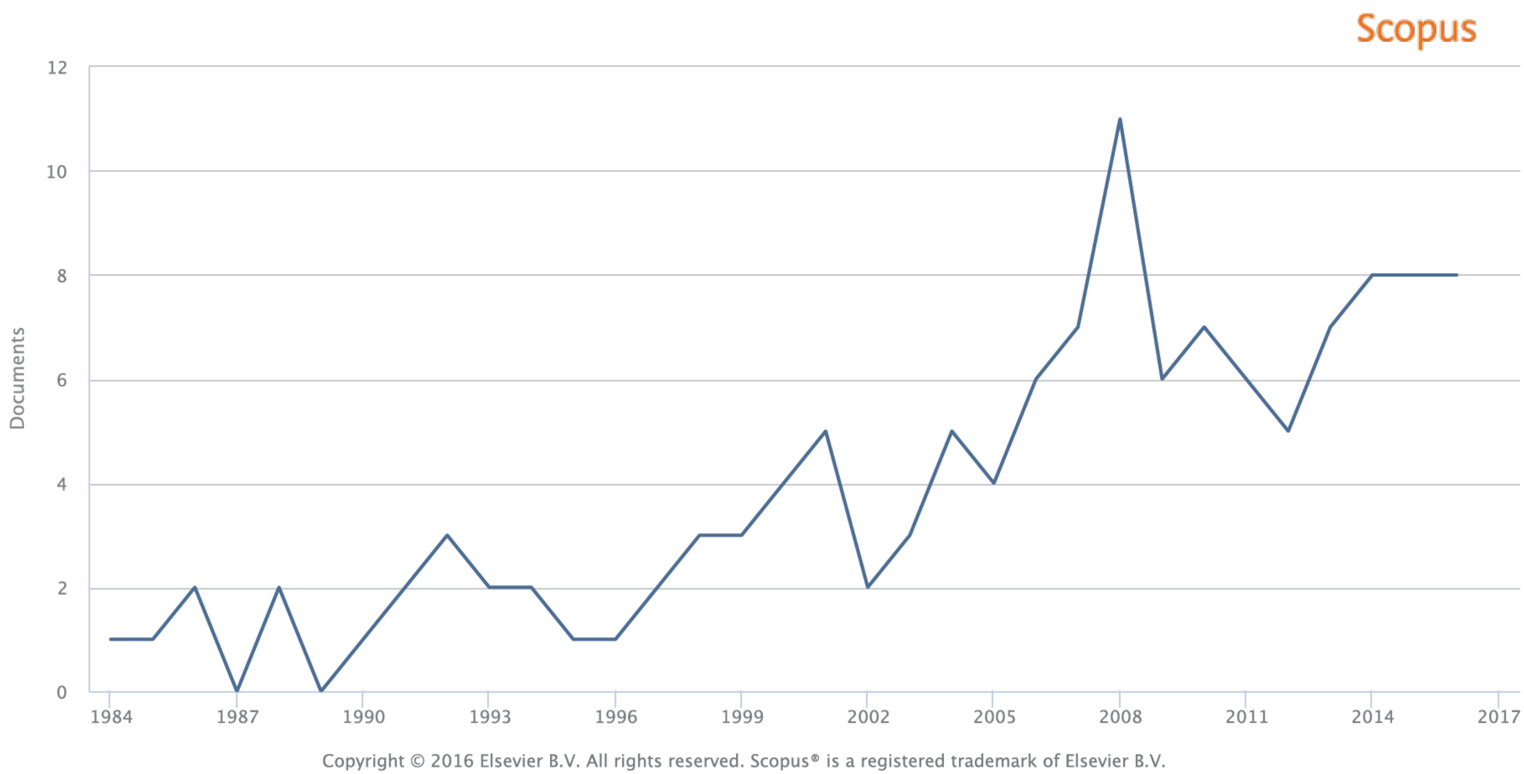
Short-term fracturing behavior experimental investigations of Kim et al. (2004).





Microplane models in the scientific literature

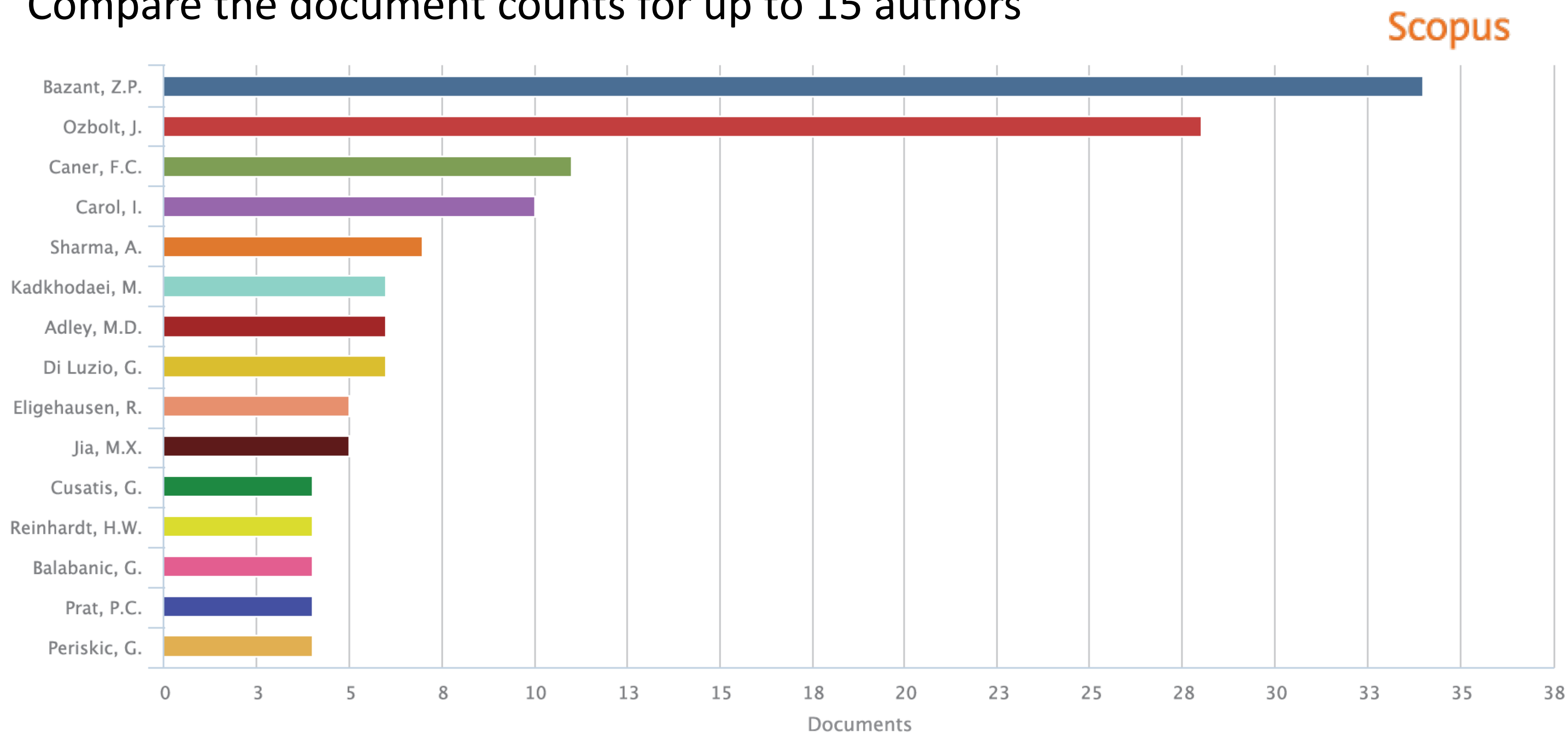
Journal paper on Microplane model from 1984 to 2016: 128 articles (SCOPUS)



Microplane models in the scientific literature

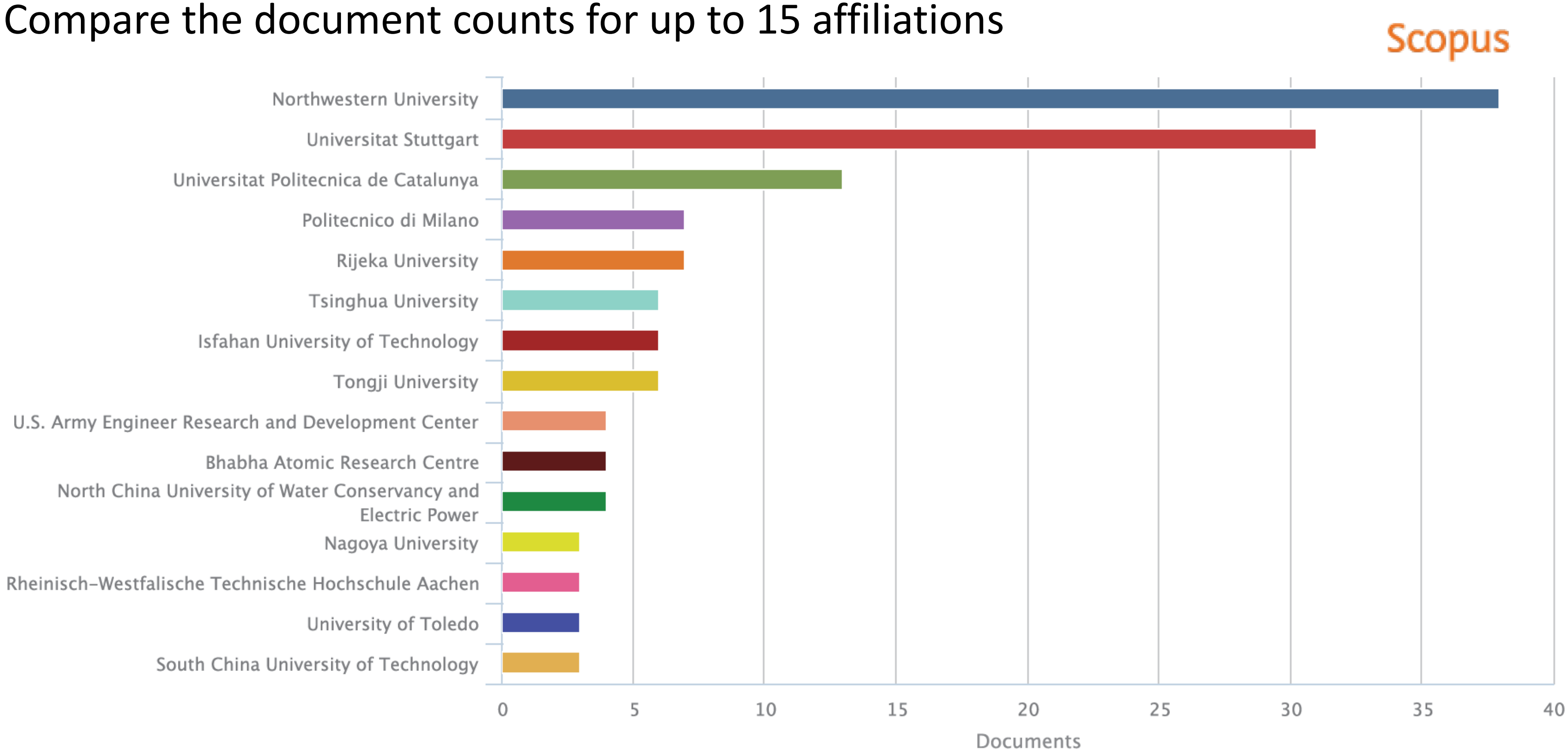
Documents on Microplane Models by author

Compare the document counts for up to 15 authors



Microplane models in the scientific literature

Documents on Microplane Models by affiliation
Compare the document counts for up to 15 affiliations



Microplane models in the scientific literature

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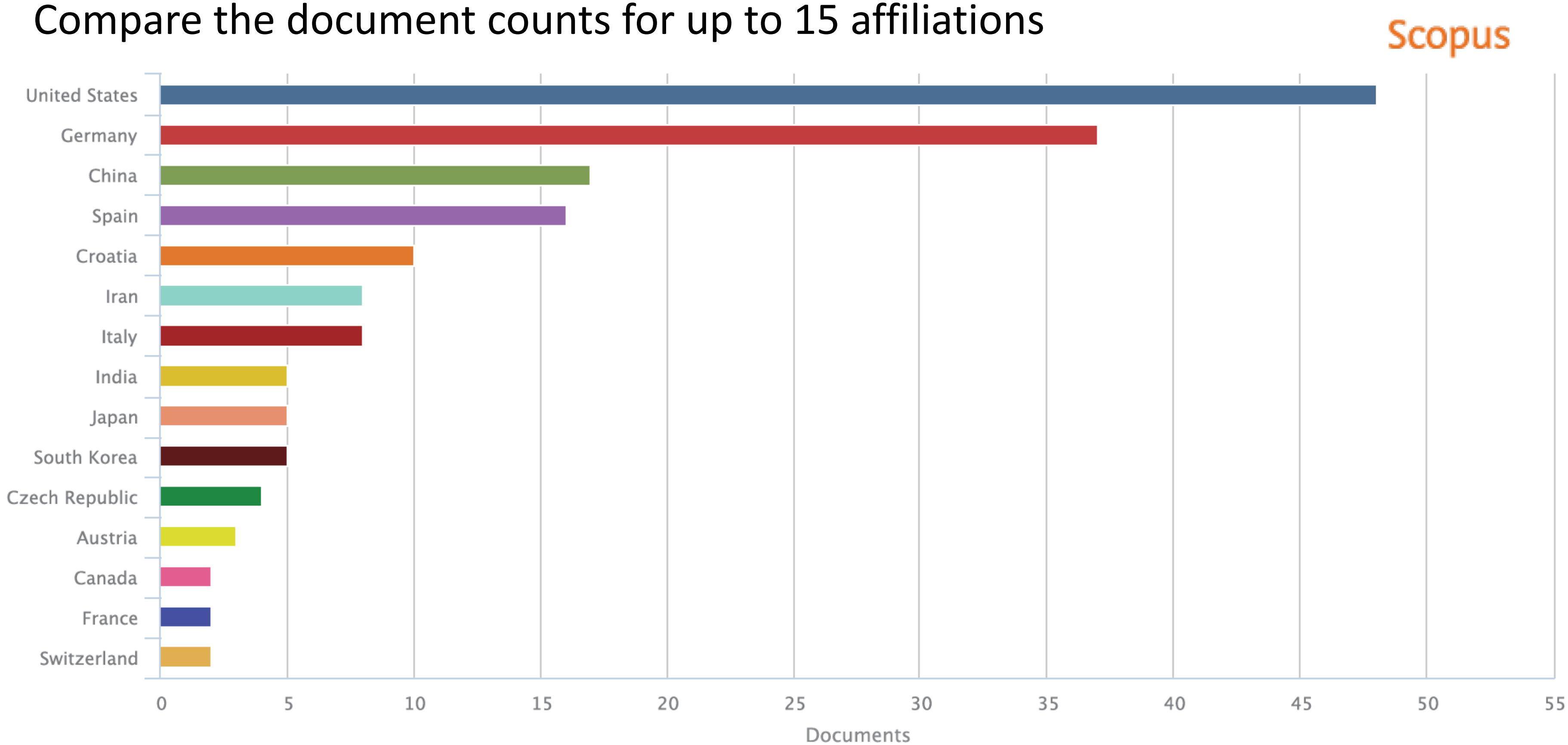

Conclusions

Albert Einstein:

“Everything should be made as simple as possible, but not simpler”

Conclusions

Thank to prof. Bažant for your unique contribution!

Microplane formulations

Advantages of microplane model (Bažant et al 2000)

1. The constitutive law is written in terms of vectors rather than tensors.
2. The inelastic physical phenomena associated with surfaces, such as slip, friction, tensile cracking, or lateral confinement on a given plane or its spreading, can be characterized directly in terms of the stress and strain on the surface on which they take place.
3. In computational practice, macroscopic tensorial plastic-damage models with only one or two loading surfaces are generally used. For such models, most materials appear to exhibit large apparent deviations from the normality rule for the plastic strain increments.
4. Unlike the classical plasticity models used in computational practice, the microplane model captures the so-called vertex effect (Caner et al 2002).
5. Extension to strain softening requires making the yield surfaces dependent on the strains.
6. The interaction of microplanes due to the kinematic constraint suffices to provide all the main cross effects on the macrolevel, such as the pressure sensitivity of inelastic shear strain and the dilatancy.

Microplane formulations

7. A vast number of combinations of loading or unloading on various microplanes is possible, and some microplanes unload even for monotonic loading on the macroscale.
8. The microplane model can naturally model fatigue, as well as hysteresis during cyclic loading.
9. With the microplane approach, anisotropic materials are not appreciably more difficult to model than isotropic materials.
10. The philosophies of the microplane approach and finite elements blend well.