

# A Finite Element approach for mesoscopic modelling of fracture

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1 Introduction

2 Damage model

3 Applications

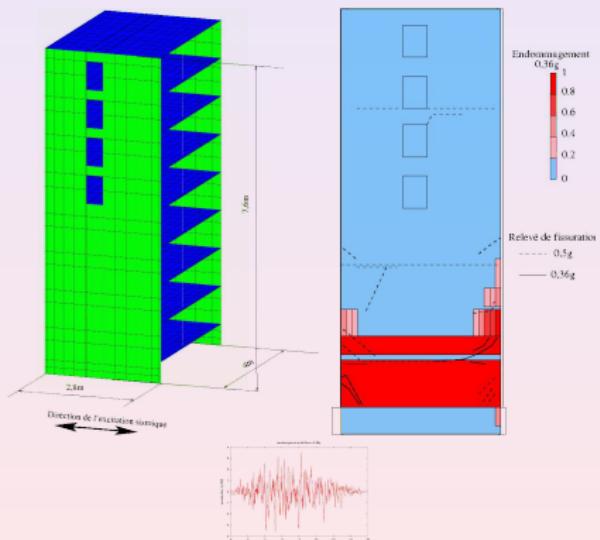
- Macroscopic behavior
- Young age concrete
- Scale effects

4 Conclusions

# Introduction : Scales of modelling

## Scales of Modelling

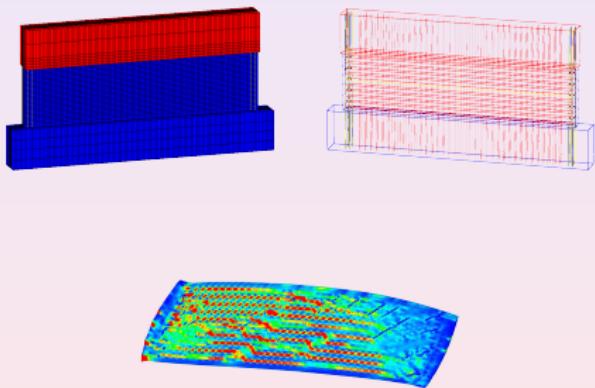
- Structure > 10m
- Macroscopic  $\approx$  1 – 10m
- Mesoscopic  $\approx$  1 – 10cm
- Microscopic < 1cm



# Introduction : Scales of modelling

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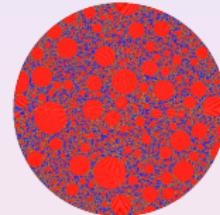


# Introduction : Scales of modelling

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Splitting test



D

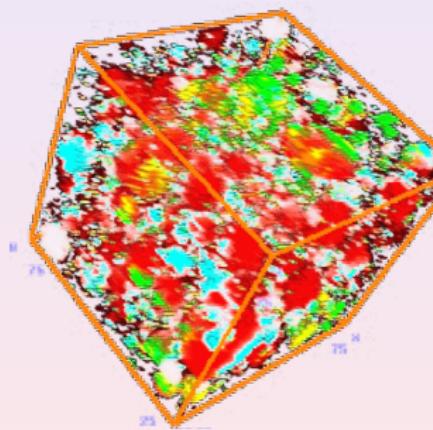
Crack width

# Introduction : Scales of modelling

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- Structure > 10m
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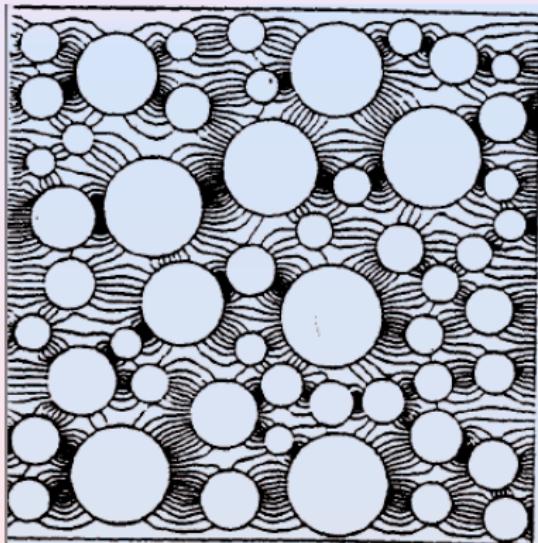
Cement paste  $E/C = 0.45$



from D.P. Bentz (NIST,  
CEMHYD3D)

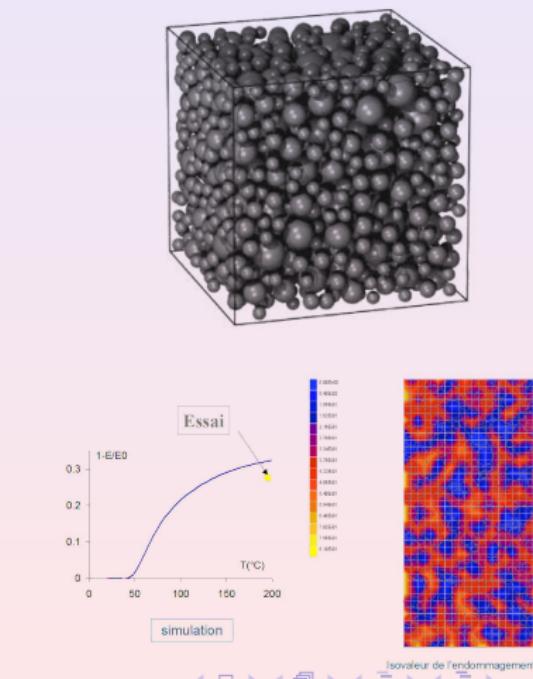
# Introduction : mesoscopic models

- Witmann 1988 : Application to drying of concrete
- Mounajed, Menou & La Borderie  $\approx$  2002 : Symphonie, concrete at high temperatures
- Implementation into Cast3M : 2007
- Smooth method : 2010



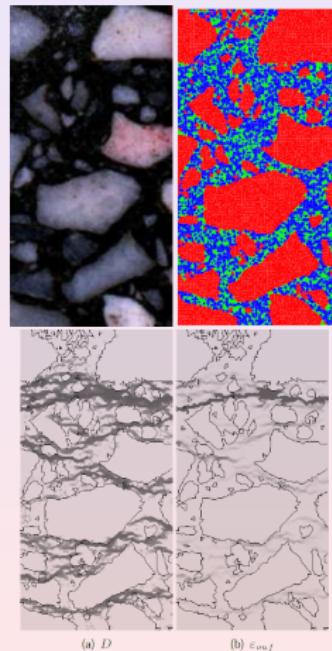
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## specifications

- 2D or 3D random generation
- granular compactness
- ITZ
- Shape of aggregates

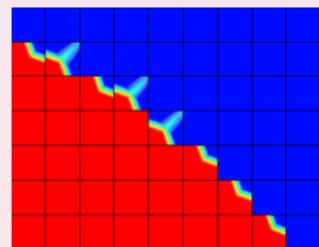
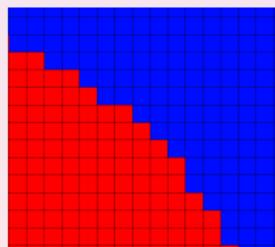
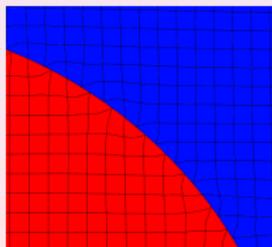
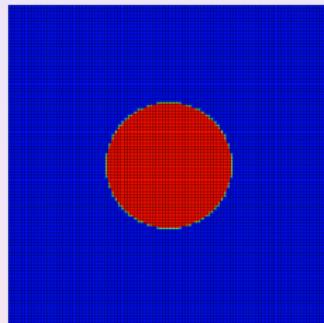
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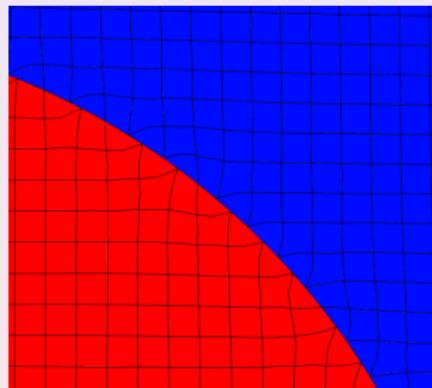
# The smooth meshing method



# Meshing methods

## Exact method

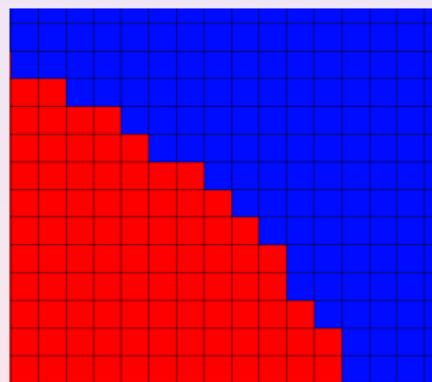
- Each aggregate is meshed separately, then the matrix is meshed
- Doesn't work in case of small and large aggregates, particularly in 3D
- ITZ can be modelled (but which parameter ?)



# Meshing methods

## Discrete method

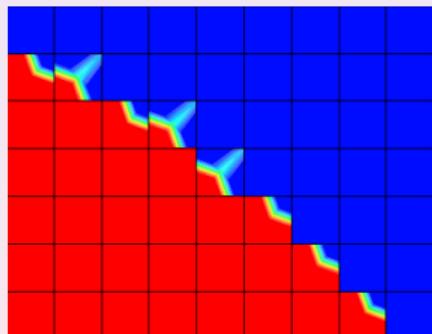
- Aggregates properties are projected on the mesh elements
- Costless method, irregularities at the boundary
- Total volume of aggregates is badly modeled



# Meshing methods

## Smooth method

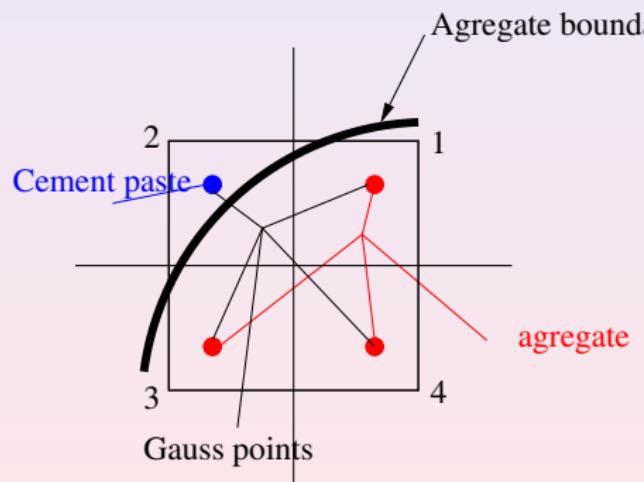
- Aggregate properties are projected on the Gauss points
- Each Gauss point owns the material properties of paste or aggregate
- Costless method, unable to model ITZ



# Meshing methods

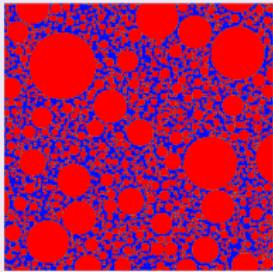
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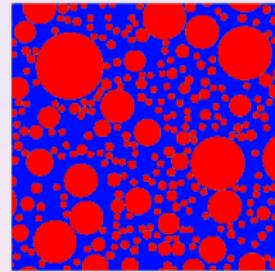


# Aggregate drawing

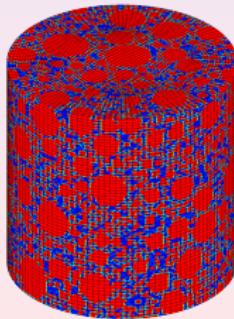
# Mesh samples



$\Phi_{min} = 1\text{mm}$



$\Phi_{min} = 2.5\text{mm}$



$\Phi_{min} = 2.5\text{mm}$

# Damage model

## Damage model

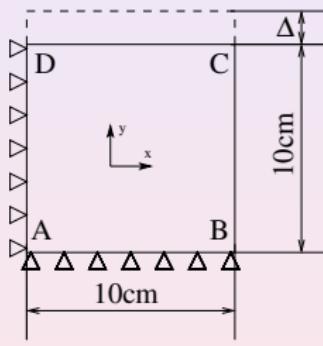
- Isotropic but unilateral (Fichant et al, 1999)
- Indirect effect of damage on compression
- Softening in tension
  - Mazars' equivalent strain  $\tilde{\varepsilon}$
  - mesh size  $h$
  - parameters  $f_t$  et  $G_f$

- $\bar{\sigma}_{ij} = \frac{E}{1+\nu} \varepsilon_{ij} + \frac{E\nu}{(1+\nu)(1-2\nu)} \varepsilon_{kk} \delta_{ij}$
- $\sigma_{ij} = (1 - d) \langle \bar{\sigma} \rangle_{ij}^+ + (1 - d)^{\alpha_1} \langle \bar{\sigma} \rangle_{ij}^-$
- $d = 1 - \frac{f_t}{E\tilde{\varepsilon}} \exp \left( \frac{hf_t}{G_f} \left( \frac{f_t}{E} - \tilde{\varepsilon} \right) \right)$

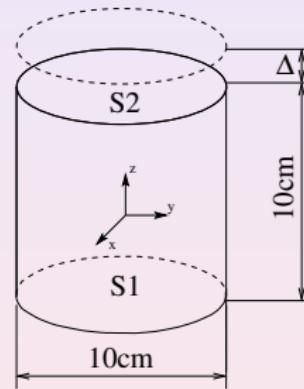
	$E(GPa)$	$\nu$	$f_t(MPa)$	$G_f(J/m^2)$	$\alpha_1$
Paste	15	0.2	3	20	10
Aggregates	60	0.2	6	60	30

Characteristics of components

# Application to macroscopic behavior



2D



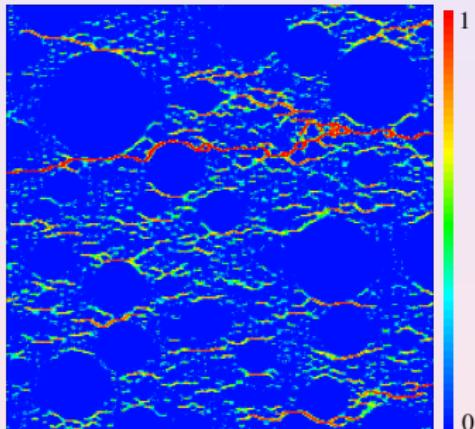
3D

# Uniaxial tension

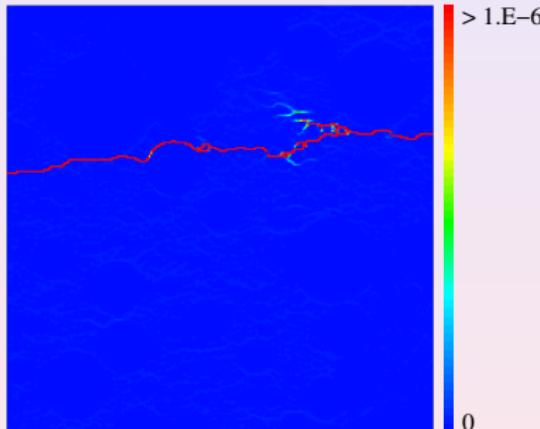
Macroscopic behavior

Crack width

# Uniaxial tension



Damage field  
at  $E_{yy} = 1\%$



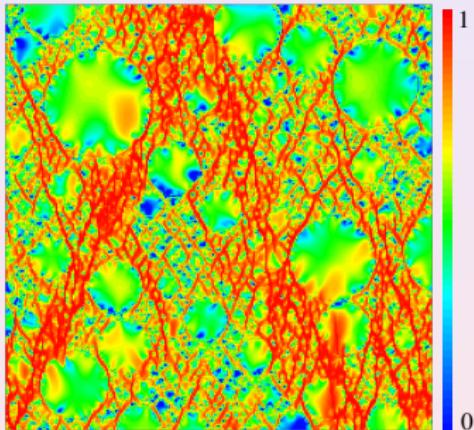
Crack width  
 $\tilde{A}$   $E_{yy} = 1\%$

# Uniaxial compression

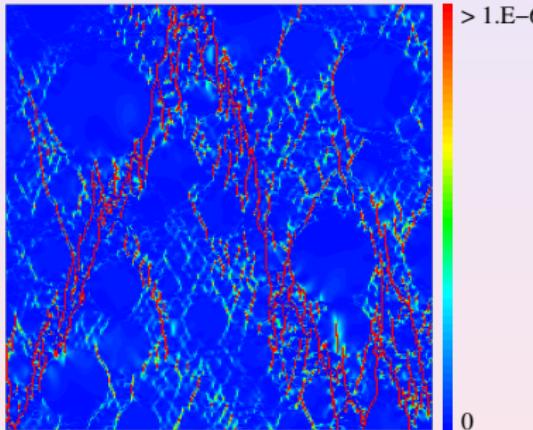
Comportement macroscopique

Ouverture de fissure

# Uniaxial Compression



Damage field  
 $\tilde{A} \quad E_{yy} = -1.83\%$



Crack width  
 $\tilde{A} \quad E_{yy} = -1.83\%$

# Effects of hydration on inner stresses

## Hydration

- Thermo-activation (Ulm )
- Hydration heat
- Endogenous shrinkage

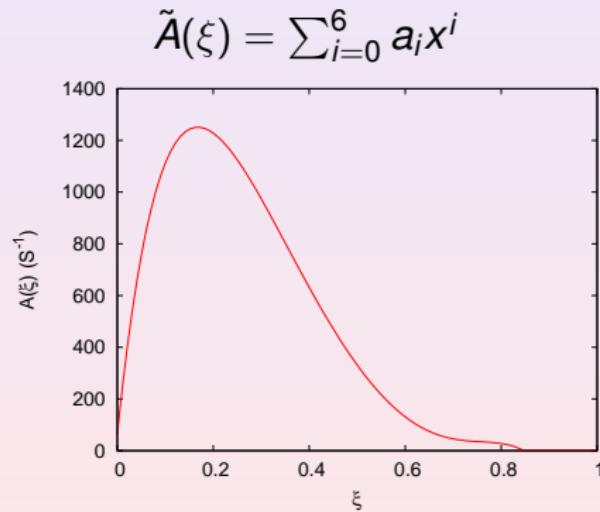
$$\dot{\xi} = \tilde{A}(\xi) e^{-\frac{E_a}{RT}}$$

- $\xi$  hydration degree
- $\tilde{A}(\xi)$  Normalized affinity function
- $E_a$  is the activation energy ( $Jmol^{-1}$ )
- $R$  is the constant of perfect gas ( $8.314 Jmol^{-1} K^{-1}$ )
- $T$  is the temperature in kelvin

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# Effects of hydration on inner stresses

## Hydration

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$$C\dot{T} = \nabla(K\nabla T) + L\dot{\xi}$$

- $C$  specific heat capacity
- $K$  thermal conductivity ( $Wm^{-1}K^{-1}$ )
- $L$  is the total activation energy ( $Jm^{-3}$ )

# Effects of hydration on inner stresses

## Hydration

- Thermo-activation (Ulm)
- Hydration heat
- Endogenous shrinkage

$$\dot{\varepsilon}_{au_{ij}} = -k\xi\dot{\delta}_{ij} \quad \text{for } \xi > \xi_0$$

- $\varepsilon_{au}$  autogenous shrinkage
- $k$  shrinkage coefficient
- $\xi_0$  is the setting value of  $\xi$  for which the paste becomes to be elastic

$$\dot{\varepsilon}_{th_{ij}} = \alpha \dot{T} \delta_{ij}$$

- $\varepsilon_{th}$  thermal strain
- $\alpha$  coefficient of expansion

# Coupling

## Evolution of the mechanical parameters (from de Schutter)

- Effective coef of hydration

$$\bar{\xi} = \left\langle \frac{\xi - \xi_0}{\xi_\infty - \xi_0} \right\rangle +$$

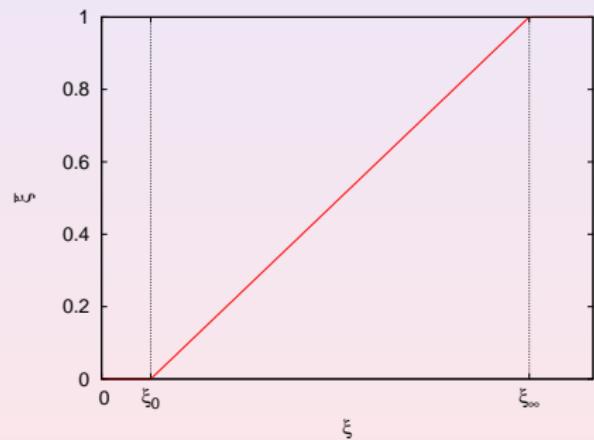
- Young's modulus  $E(\xi) = E_\infty \bar{\xi}^\beta$

- Poisson's ratio

$$\nu = \nu_\infty \sin \frac{\pi * \xi}{2} + 0.5 e^{-10 * \bar{\xi}}$$

- Tension strenght  $f_t(\xi) = f_{t\infty} \bar{\xi}^\gamma$

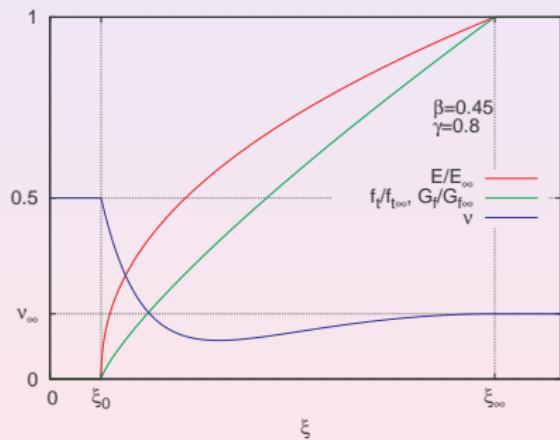
- Fracture energy  $G_f = G_\infty \bar{\xi}^\gamma$



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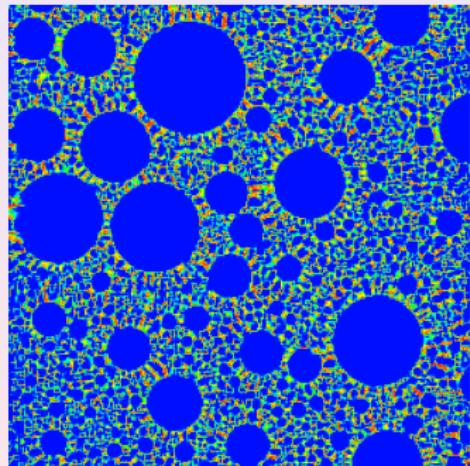


# Hydration of concrete

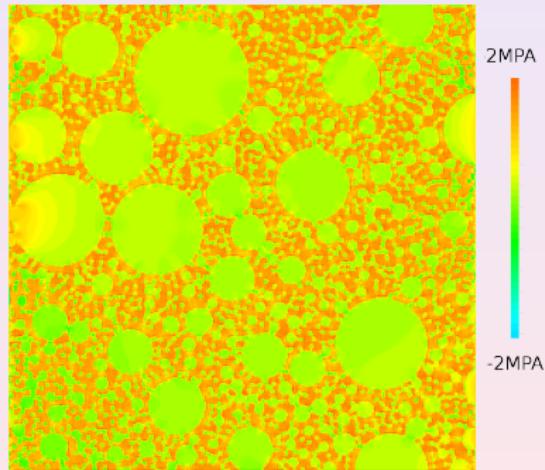
Temperature  $T(K)$

degree of hydration  $\xi$

# Hydration of concrete

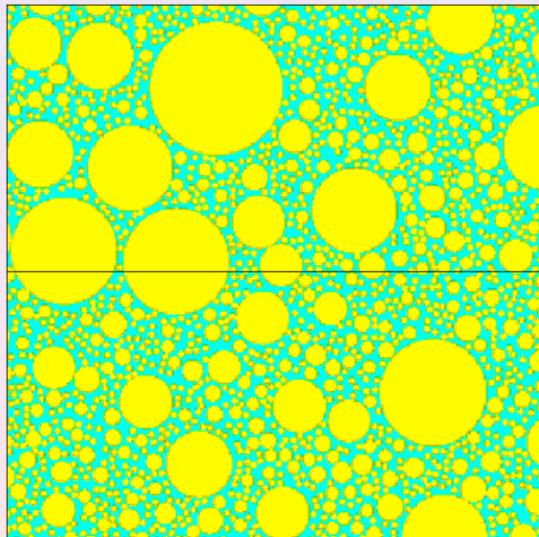


Damage

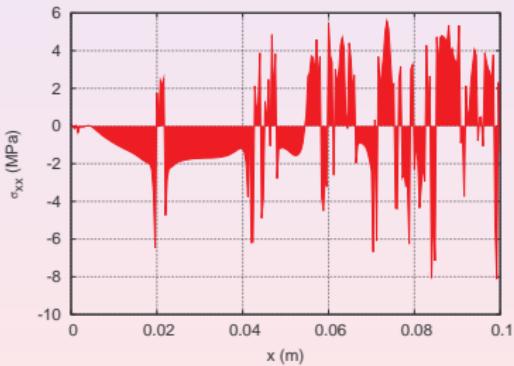


Hydrostatic stress  $\sigma_{kk}$

# Hydration of concrete

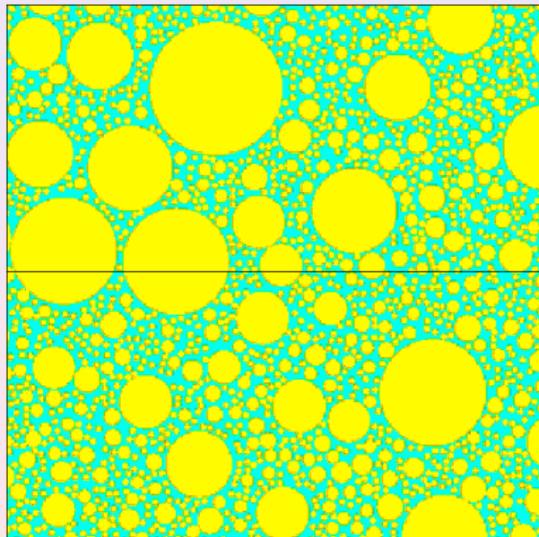


Along the middle line

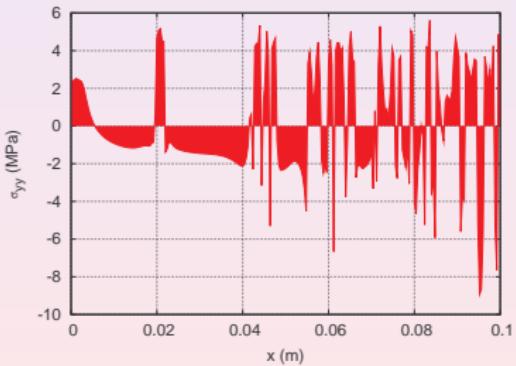


meso-stress  $\sigma_{xx}$

# Hydration of concrete



Along the middle line



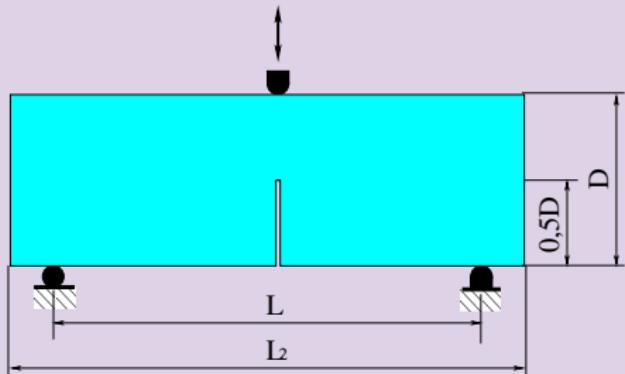
meso-stress  $\sigma_{yy}$

# Application to scale effects

## Dimensions

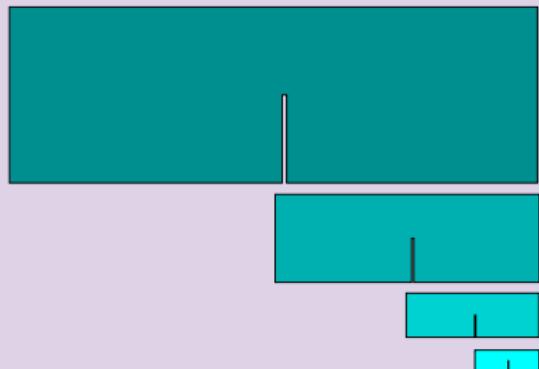
Thick (m)	$L_2$ (m)	$L$ (m)	$D$ (m)	Notch Height (m)
0,05	1,4	1	0,4	0,5D

Size of the beam



Experimental setup

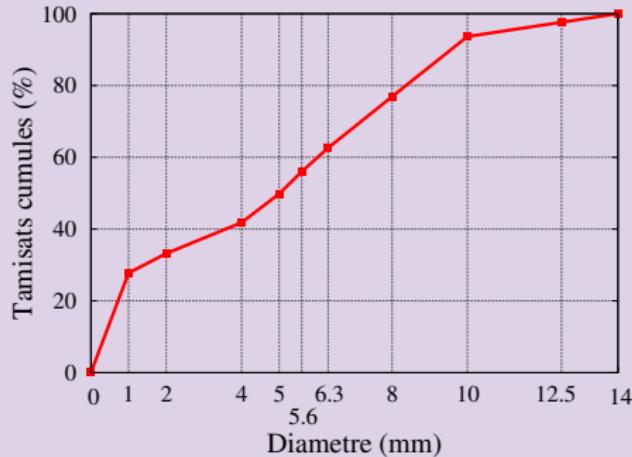
- Experiments from Rojas, Grégoire & Pijaudier-Cabot
- Homothety ratio :  $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}$



beam size

# Modeled problem

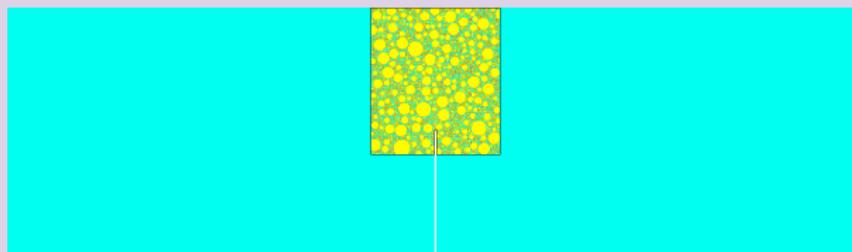
## Grading curves



# Model

## Meshing strategy

- Notched beams of different homothety ratios  $\frac{1}{2}; \frac{1}{4}; \frac{1}{8}$ .
- Mix meso-macro model



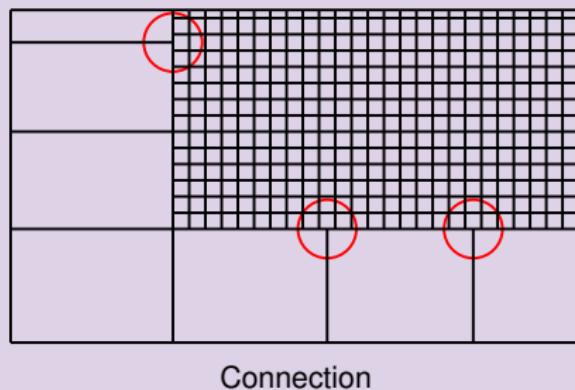
Mesh and aggregate drawing for the homothety ratio of  $\frac{1}{2}$

- 3 different drawings are used for each beam.

# FE Modeling

## Meshing strategy

- Nodes do not match.
- Cinematic coupling.

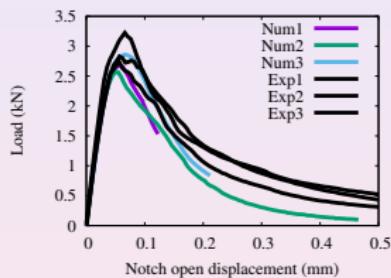


# FE Modeling

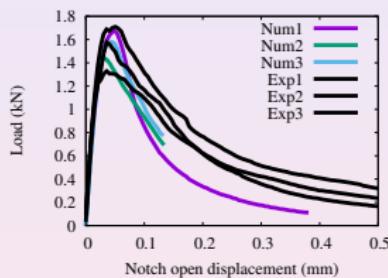
## Mechanic parameters

	Young's modulus $E$ (GPa)	Poisson's Ratio $\nu$	Tension strenght $f_t$ (MPa)	Fracture energy $G_f$ (J/m <sup>2</sup> )
Paste	25	0,2	3	20
Aggregates	55	0,2	6	60
Homogenized	39, 61	0,2	-	-

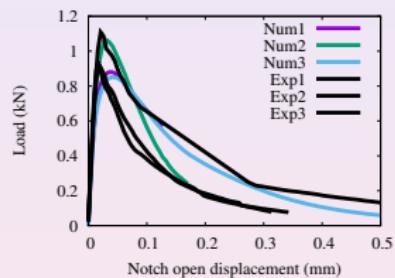
# FE Results



Homothety ratio 1/2



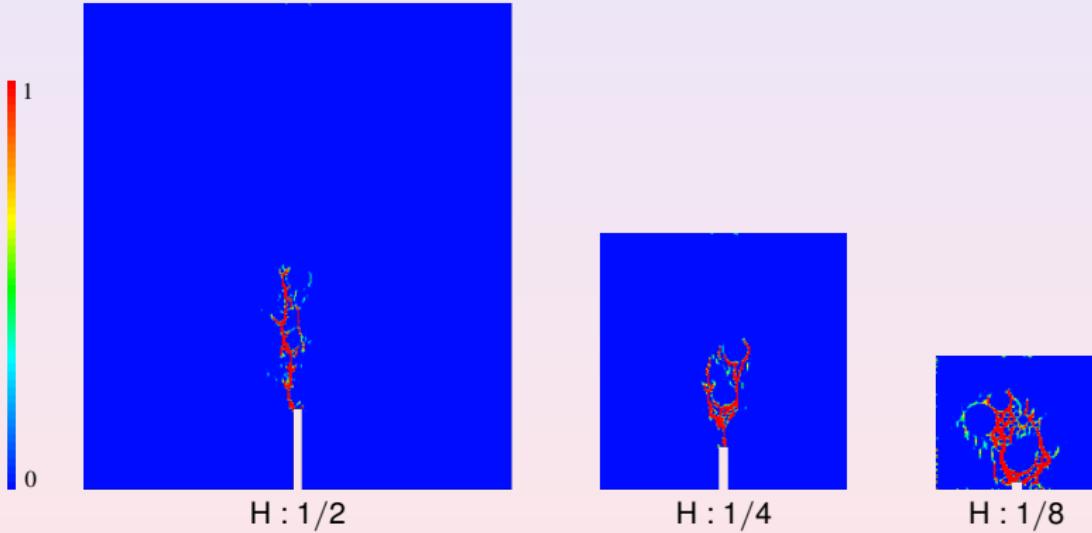
Homothety ratio 1/4



Homothety ratio 1/8

CMOD / load from experiments and simulations

# Results



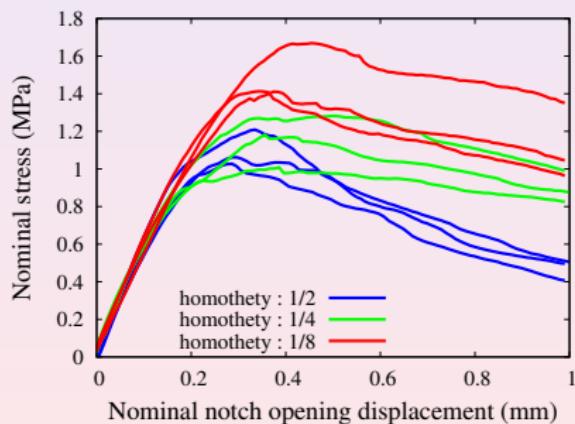
Damage at the peak load for different homothety ratio

# Scale effect law (Bažant, 1976)

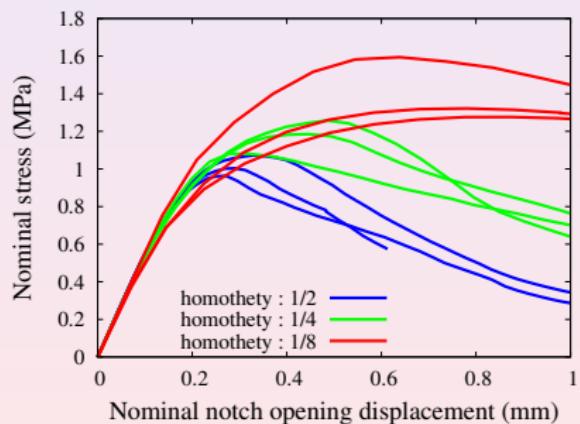
Nominal stress :

$$\sigma_n = \frac{3PL}{2eD^2}$$

Nominal notch opening :  $U_n = \frac{U}{D}$



$\sigma_n(U_n)$  curves



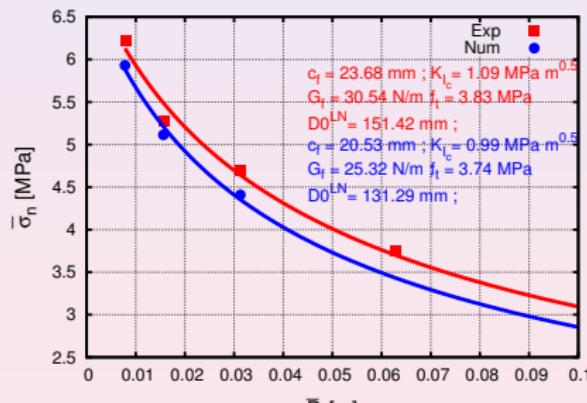
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Intrinsic size :

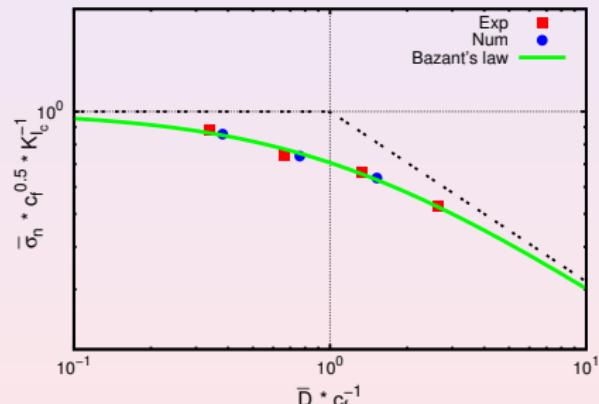
$$\bar{D} = 0.15637481182436 * D$$

Intrinsic nominal stress

$$\bar{\sigma}_{nu} = 4.35395101484703 * \sigma_{nu}$$



$\bar{\sigma}_{nu}(\bar{D})$  curve



Scale effect law from Bažant

# Conclusions

## Smooth FE method for mesoscopic scale

- Can be used in 2D or 3D even if 3D needs long computation time
- Take into account the granular compacity
- Useful for couplings
- Easy to use with any model as long as it is the same for aggregates and paste
- Does not account for the ITZ

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Thanks !!!

- Ghassan Mounajed, Abdellah Menou, Hocine Boussa
- Claire Lawrence, Olivier Maurel, Atef Daoud
- Farid Benboudjema, Matthieu Briffault
- The Dung Nguyen, Wen Chen, Mohammed Matallah
- Gilles Pijaudier-Cabot, David Grégoire
- Stéphane Morel, Alexandre Gangnant, Hatem Kallel
- Olivier Nouailletas, Laurie Buffo-Lacarrière