A Finite Element approach for mesoscopic modelling of fracture

C. La Borderie & collaborators

SIAME-EA4581
University of Pau, France

Berkeley, May 29, 2016
1. Introduction

2. Damage model

3. Applications
   - Macroscopic behavior
   - Young age concrete
   - Scale effects

4. Conclusions
Introduction: Scales of modelling

Scales of Modelling

- **Structure > 10m**
  - Macroscopic ≈ 1 – 10m
  - Mesoscopic ≈ 1 – 10cm
  - Microscopic < 1 cm
Introduction : Scales of modelling

Scales of Modelling

- Structure $> 10m$
- Macroscopic $\approx 1 - 10m$
- Mesoscopic $\approx 1 - 10cm$
- Microscopic $< 1cm$
Introduction: Scales of modelling

- **Structure**: $> 10 \text{ m}$
- **Macroscopic**: $\approx 1 - 10 \text{ m}$
- **Mesoscopic**: $\approx 1 - 10 \text{ cm}$
- **Microscopic**: $< 1 \text{ cm}$

**Splitting test**

$D$  
Crack width
Introduction: Scales of modelling

**Scales of Modelling**
- Structure $> 10m$
- Macroscopic $\approx 1 - 10m$
- Mesoscopic $\approx 1 - 10cm$
- Microscopic $< 1cm$

Cement paste $E/C = 0.45$

From D.P. Bentz (NIST, CEMHYD3D)
**Introduction**: mesoscopic models

- **Witmann 1988**: Application to drying of concrete
- **Mounajed, Menou & La Borderie ≈ 2002**: Symphonie, concrete at high temperatures
- **Implementation into Cast3M**: 2007
- **Smooth method**: 2010
Introduction: mesosopic models

- Witmann 1988: Application to drying of concrete
- Mounajed, Menou & La Borderie ≈ 2002: Symphonie, concrete at high temperatures
- Implementation into Cast3M: 2007
- Smooth method: 2010
Introduction: mesoscopic models

- Witmann 1988: Application to drying of concrete
- Mounajed, Menou & La Borderie ≈ 2002: Symphonie, concrete at high temperatures
- Implementation into Cast3M: 2007
- Smooth method: 2010
Introduction: mesoscopic models

- Witmann 1988: Application to drying of concrete
- Mounajed, Menou & La Borderie ≈ 2002: Symphonie, concrete at high temperatures
- Implementation into Cast3M: 2007
- Smooth method: 2010

Specifications:
- 2D or 3D random generation
- Granular compactness
- ITZ
- Shape of aggregates
Introduction: mesoscopic models

- Witmann 1988: Application to drying of concrete
- Mounajed, Menou & La Borderie ≈ 2002: Symphonie, concrete at high temperatures
- Implementation into Cast3M: 2007
- Smooth method: 2010

Specifications

- 2D or 3D random generation
- Granular compactness
- ITZ
- Shape of aggregates
The smooth meshing method

exact  discrete  smooth
Meshing methods

Exact method

- Each aggregate is meshed separately, then the matrix is meshed.
- Doesn’t work in case of small and large aggregates, particularly in 3D.
- ITZ can be modelled (but which parameter?)
Meshing methods

Discrete method

- Aggregates properties are projected on the mesh elements
- Costless method, irregularities at the boundary
- Total volume of aggregates is badly modeled
Meshing methods

Smooth method

- Aggregate properties are projected on the Gauss points
  - Each Gauss point owns the material properties of paste or aggregate
  - Costless method, unable to model ITZ
Meshing methods

Smooth method

- Aggregate properties are projected on the Gauss points
- Each Gauss point owns the material properties of paste or aggregate
- Costless method, unable to model ITZ
Aggregate drawing
Mesh samples

\[ \Phi_{\text{min}} = 1\,mm \]

\[ \Phi_{\text{min}} = 2.5\,mm \]

\[ \Phi_{\text{min}} = 2.5\,mm \]
Damage model

- Isotropic but unilateral (Fichant et al, 1999)
- Indirect effect of damage on compression
- Softening in tension
  - Mazars’ equivalent strain $\tilde{\varepsilon}$
  - mesh size $h$
  - parameters $f_t$ et $G_f$

\[
\bar{\sigma}_{ij} = \frac{E}{1+\nu} \varepsilon_{ij} + \frac{E\nu}{(1+\nu)(1-2\nu)} \varepsilon_{kk} \delta_{ij}
\]

\[
\sigma_{ij} = (1-d) \langle \bar{\sigma} \rangle_{ij}^{+} + (1-d)^{\alpha_1} \langle \bar{\sigma} \rangle_{ij}^{-}
\]

\[
d = 1 - \frac{f_t}{E\tilde{\varepsilon}} \exp \left( \frac{h f_t}{G_f} \left( \frac{f_t}{E} - \tilde{\varepsilon} \right) \right)
\]

<table>
<thead>
<tr>
<th></th>
<th>$E$ (GPa)</th>
<th>$\nu$</th>
<th>$f_t$ (MPa)</th>
<th>$G_f$ (J/m$^2$)</th>
<th>$\alpha_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paste</td>
<td>15</td>
<td>0.2</td>
<td>3</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>Aggregates</td>
<td>60</td>
<td>0.2</td>
<td>6</td>
<td>60</td>
<td>30</td>
</tr>
</tbody>
</table>

Characteristics of components
Application to macroscopic behavior

2D

3D

C. La Borderie & collaborators  FE Meso Fracture
Uniaxial tension

Macroscopic behavior

$G_f = 54.2 \text{ J/m}^2$

$E = 36.4 \text{ GPa}$

$f_t = 3.31 \text{ MPa}$

Crack width

C. La Borderie & collaborators

FE Meso Fracture
Uniaxial tension

Damage field
at $E_{yy} = 1\%$

Crack width
$	ilde{A} \quad E_{yy} = 1\%$
Uniaxial compression

Comportement macroscopique

Ouverture de fissure

$E = 36.4 \text{ GPa}$

$f_c = 38.47 \text{ MPa}$
Uniaxial Compression

Damage field
\[ \tilde{A} \quad E_{yy} = -1.83\% \]

Crack width
\[ \tilde{A} \quad E_{yy} = -1.83\% \]
Effects of hydration on inner stresses

\[ \dot{\xi} = \tilde{A}(\xi) e^{-\frac{E_a}{RT}} \]

- \( \xi \) hydration degree
- \( \tilde{A}(\xi) \) Normalized affinity function
- \( E_a \) is the activation energy \((Jmol^{-1})\)
- \( R \) is the constant of perfect gas \((8.314 Jmol^{-1} K^{-1})\)
- \( T \) is the temperature in kelvin

Hydration
- Thermo-activation (Ulm)
- Hydration heat
- Endogenous shrinkage
Effects of hydration on inner stresses

Hydration
- Thermo-activation (Ulm)
- Hydration heat
- Endogenous shrinkage

\[ \tilde{A}(\xi) = \sum_{i=0}^{6} a_i x^i \]
Effects of hydration on inner stresses

\[ C \dot{T} = \nabla (K \nabla T) + L \dot{\xi} \]

- \( C \) specific heat capacity
- \( K \) thermal conductivity (\( Wm^{-1}K^{-1} \))
- \( L \) is the total activation energy (\( Jm^{-3} \))
Effects of hydration on inner stresses

\[ \dot{\varepsilon}_{au_{ij}} = -k \dot{\xi} \delta_{ij} \quad \text{for} \quad \xi > \xi_0 \]

- \( \varepsilon_{au} \) autogenous shrinkage
- \( k \) shrinkage coefficient
- \( \xi_0 \) is the setting value of \( \xi \) for which the paste becomes to be elastic

\[ \dot{\varepsilon}_{th_{ij}} = \alpha \dot{T} \delta_{ij} \]

- \( \varepsilon_{th} \) thermal strain
- \( \alpha \) coefficient of expansion

Hydration
- Thermo-activation (Ulm)
- Hydration heat
- Endogenous shrinkage
Evolution of the mechanical parameters (from de Schutter)

- **Effective coef of hydration**
  \[ \bar{\xi} = \langle \frac{\xi - \xi_0}{\xi_\infty - \xi_0} \rangle \]

- **Young’s modulus**
  \[ E(\xi) = E_\infty \bar{\xi}^\beta \]

- **Poisson’s ratio**
  \[ \nu = \nu_\infty \sin \frac{\pi \bar{\xi}}{2} + 0.5 e^{-10 \bar{\xi}} \]

- **Tension strenght**
  \[ f_t(\xi) = f_{t\infty} \bar{\xi}^\gamma \]

- **Fracture energy**
  \[ G_f = G_\infty \bar{\xi}^\gamma \]
Coupling

Evolution of the mechanical parameters (from de Schutter)

- Effective coef of hydration
  \[ \bar{\xi} = < \frac{\xi - \xi_0}{\xi_\infty - \xi_0} > + \]

- Young’s modulus
  \[ E(\xi) = E_\infty \bar{\xi}^\beta \]

- Poisson’s ratio
  \[ \nu = \nu_\infty \sin \frac{\pi \bar{\xi}}{2} + 0.5 e^{-10 \bar{\xi}} \]

- Tension strenght
  \[ f_t(\xi) = f_{t\infty} \bar{\xi}^\gamma \]

- Fracture energy
  \[ G_f = G_\infty \bar{\xi}^\gamma \]
Hydration of concrete

Temperature $T(K)$

degree of hydration $\xi$
Hydration of concrete

Damage

Hydrostatic stress $\sigma_{kk}$
Hydration of concrete

Along the middle line

messo-stress $\sigma_{xx}$
Hydration of concrete

Along the middle line

messo-stress $\sigma_{yy}$
Application to scale effects

**Dimensions**

<table>
<thead>
<tr>
<th>Thick (m)</th>
<th>$L_2$ (m)</th>
<th>$L$ (m)</th>
<th>$D$ (m)</th>
<th>Notch Height (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0,05</td>
<td>1,4</td>
<td>1</td>
<td>0,4</td>
<td>0,5D</td>
</tr>
</tbody>
</table>

- Experiments from Rojas, Grégoire & Pijaudier-Cabot
- Homothety ratio: $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}$

**Experimental setup**

**Beam size**
Modeled problem

Grading curves

![Grading curves graph](image-url)
Meshing strategy

- Notched beams of different homothety ratios $\frac{1}{2}; \frac{1}{4}; \frac{1}{8}$.
- Mix meso-macro model

Mesh and aggregate drawing for the homothety ratio of $\frac{1}{2}$

3 different drawings are used for each beam.
Meshing strategy

- Nodes do not match.
- Cinematic coupling.

Connection
Mechanic parameters

<table>
<thead>
<tr>
<th></th>
<th>Young’s modulus $E$ (GPa)</th>
<th>Poisson’s Ratio $\nu$</th>
<th>Tension strength $f_t$ (MPa)</th>
<th>Fracture energy $G_f$ (J/m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paste</td>
<td>25</td>
<td>0, 2</td>
<td>3</td>
<td>20</td>
</tr>
<tr>
<td>Aggregates</td>
<td>55</td>
<td>0, 2</td>
<td>6</td>
<td>60</td>
</tr>
<tr>
<td>Homogenized</td>
<td>39, 61</td>
<td>0, 2</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

C. La Borderie & collaborators
FE Results

Homothety ratio 1/2
CMOD / load from experiments and simulations

Homothety ratio 1/4

Homothety ratio 1/8
Results

Damage at the peak load for different homothety ratio

H : 1/2
H : 1/4
H : 1/8

C. La Borderie & collaborators  FE Meso Fracture
Scale effect law (Bažant, 1976)

Nominal stress:
\[ \sigma_n = \frac{3PL}{2eD^2} \]

Nominal notch opening:
\[ U_n = \frac{U}{D} \]

Experiments and Numerical \( \sigma_n(U_n) \) curves

C. La Borderie & collaborators
FE Meso Fracture
Scale effect law (Bažant, 1976)

Intrinsic size: \( \bar{D} = 0.15637481182436 \times D \)
Intrinsic nominal stress: \( \bar{\sigma}_{nu} = 4.35395101484703 \times \sigma_{nu} \)

\( \sigma_n(\bar{D}) \) curve

Scale effect law from Bažant

\( c_f = 23.68 \) mm; \( K_I = 1.09 \) MPa m\(^{0.5} \)
\( G_f = 30.54 \) N/m; \( f_t = 3.83 \) MPa
\( D_{0LN} = 151.42 \) mm

\( c_f = 20.53 \) mm; \( K_I = 0.99 \) MPa m\(^{0.5} \)
\( G_f = 25.32 \) N/m; \( f_t = 3.74 \) MPa
\( D_{0LN} = 131.29 \) mm
Smooth FE method for mesoscopic scale

- Can be used in 2D or 3D even if 3D needs long computation time
- Take into account the granular compacity
- Useful for couplings
- Easy to use with any model as long as it is the same for aggregates and paste
- Does not account for the ITZ
Smooth FE method for mesoscopic scale

- Can be used in 2D or 3D even if 3D needs long computation time
- Take into account the granular compacity
  - Useful for couplings
  - Easy to use with any model as long as it is the same for aggregates and paste
- Does not account for the ITZ
Conclusions

Smooth FE method for mesoscopic scale

- Can be used in 2D or 3D even if 3D needs long computation time
- Take into account the granular compacity
- Useful for couplings
- Easy to use with any model as long as it is the same for aggregates and paste
- Does not account for the ITZ
Conclusions

Smooth FE method for mesoscopic scale

- Can be used in 2D or 3D even if 3D needs long computation time
- Take into account the granular compacity
- Useful for couplings
- Easy to use with any model as long as it is the same for aggregates and paste
- Does not account for the ITZ
Conclusions

Smooth FE method for mesoscopic scale

- Can be used in 2D or 3D even if 3D needs long computation time
- Take into account the granular compacity
- Useful for couplings
- Easy to use with any model as long as it is the same for aggregates and paste
- Does not account for the ITZ
Thanks!!!

- Ghassan Mounajed, Abdellah Menou, Hocine Boussa
- Claire Lawrence, Olivier Maurel, Atef Daoud
- Farid Benboudjema, Matthieu Briffault
- The Dung Nguyen, Wen Chen, Mohammed Matallah
- Gilles Piaudier-Cabot, David Grégoire
- Stéphane Morel, Alexandre Gangnant, Hatem Kallel
- Olivier Nouailletas, Laurie Buffo-Lacarrière