Simplified strategies based on damage mechanics for RC under dynamic loadings

Jacky Mazars*, S. Grange**
*Grenoble Institute of Technology
**Grenoble Alpes University - France

Question to solve:
For large size concrete structures such as building the use of FE techniques implies often to choose simplified modelling including multifiber beam or lattice descriptions

Is simplified modelling able to describe the response to low, medium and high velocity?

Content
- Concrete model
- MF beam and localization pbs
- Applications:
  - seismic response strain rate effects (spalling test)
  - impact on a RC beam
- Conclusions

Bazant Workshop - Berkeley 29 May 2016
From the coupling Elasticity – isotropic Damage, 

\[ \sigma = \Lambda_0 (1-d): \epsilon = (1-d) \left[ \lambda_0 \text{trace}(\epsilon) \mathbf{1} + 2\mu_0 \epsilon \right] \]

The objective is to describe the main non linear effects in concrete (cracking and damage, unilaterality, ….)

- **Main assumptions for the 3D version:**
  In order to stay as simple as possible:
  - only one damage variable \( d \) is used, which is the « activated» part of damage or « effective damage » (nil when cracks are closed)
  - no permanent strain

- **However for a 1D version** *(useful for simplified modelling)*
  Enhancements have been introduce such as :
  - hysteretic dissipation during cyclic loading
  - permanent strains
  - strain rate effects,

To distinguish « cracking » and « crushing », 2 equivalent strains are defined:

\[ \varepsilon_t = \frac{I_e}{2(1-2\nu)} + \frac{\sqrt{J_e}}{2(1+\nu)} \quad \text{and} \quad \varepsilon_c = \frac{I_e}{5(1-2\nu)} + \frac{6\sqrt{J_e}}{5(1+\nu)} \]

\[ I_e = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \]

\[ J_e = \frac{1}{2}[(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2] \]

From this, 2 thermodynamical variables are defined:

\[ Y_t = \max(\varepsilon_{0t}, \max(\varepsilon_t)) \quad \text{and} \quad Y_c = \max(\varepsilon_{0c}, \max(\varepsilon_c)) \]

2 loading surfaces are associated to these variables

\[ f_t = \varepsilon_t - Y_t \leq 0 \quad \text{and} \quad f_c = \varepsilon_c - Y_c \leq 0 \]

Evolution of the « effective damage »

\[ d = \text{fct.}(Y_0, Y, A, B) \quad \text{with} \quad Y = rY_t + (1-r)Y_c \quad \text{and} \quad Y_0 = r\varepsilon_{t0} + (1-r)\varepsilon_{c0} \]

\[ r \] is the triaxiality factor

\[ r = \frac{\sum_i \langle \tilde{\sigma}_i \rangle_+}{\sum_i |\tilde{\sigma}_i|} \quad \text{(Lee,Fenves 98)} \]

\[ \tilde{\sigma} = \Lambda_0 : \varepsilon \quad \text{(effective stress)} \]

A = f (At, Ac, r) \quad B = f (Bt, Bc, r) \quad : At, Bt, Ac, Bc material parameters (from tests on sample)

Failure surface: section $\sigma_3=0$

Uniaxial Compression

Biaxial Traction

Uniaxial Traction

Biaxial Compression

Pure Shear

Experiment (Kupfer et al 1973)
Principle:

- in each beam element the section is composed of // fibres (concrete – steel)

- the behaviour of each fiber is 1D (but the global behaviour is 3D)

- kinematics constraint at the connection between 2 elements:
  - plane section remains plane
Multifiber description and localization

On the same RC beam (1.5m long section 5x5 fiber) : 3 different types of loading are considered

<table>
<thead>
<tr>
<th>Type of loading</th>
<th>Global behaviour</th>
<th>Damage contour (at a given loading)</th>
<th>Type of response</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tensile loading</td>
<td><img src="image" alt="Graph" /></td>
<td><img src="image" alt="Damage contour" /></td>
<td>Strain is localized</td>
</tr>
<tr>
<td>(tie)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Four point bending</td>
<td><img src="image" alt="Graph" /></td>
<td><img src="image" alt="Damage contour" /></td>
<td>Strain is distributed</td>
</tr>
<tr>
<td>Three point bending</td>
<td><img src="image" alt="Graph" /></td>
<td><img src="image" alt="Damage contour" /></td>
<td>Strain is distributed</td>
</tr>
</tbody>
</table>

Conclusion : Localization appears only on specific cases (RC tie, plain concrete beam,...) In these cases, solutions to solve mesh dependency are the same as 2D-3D FE
RC beam under tensile loading : tie
(Mivelaz – EPFL 1996)

For both calculations:
- Same model ($\mu$ model)
- Same size of elements
- Same materials parameters
1. In a 2D-FE beam: localization generates section warping

In the framework of the crack-band theory, cracking is localized in a band of elements (size $h$) the behaviour of which is calibrated from the Hillerborg method:

$$G_f/h = \int \sigma \cdot d\varepsilon$$

2. In a MF beam: sections remain plane which thwart localization

Then the damage-cracking processes for one crack is distributed on both side of the crack over a volume defined by the distance $s_c$ ($s_c$ is the crack spacing)

$$G_f/s_c = \int \sigma \cdot d\varepsilon$$

**Conclusion**: there is no localization problem,

**But**: material parameters for MF beam description are different from the ones used for FE-2D calculations

**Finding**: this way leads to a mesh independency
1D concrete behaviour: introduction of hysteretic loops and permanent strain

- **Permanent strain** *(Pontiroli 1995)*
  \[(\sigma - \sigma_{ft}) = E (1-d_i) (\varepsilon - \varepsilon_{ft}) \quad d_i = d_t \text{ or } d_c \]
  \[\varepsilon_{ft} = \frac{\varepsilon_{ft0}}{(1-d_c)}(\varepsilon_{ft} - \varepsilon_{fc}) - \varepsilon_{fc} \cdot d / (1-d_c)\]
  \[\sigma_{ft} = E (1-d_c) (\varepsilon_{ft} - \varepsilon_{fc}) + E \cdot \varepsilon_{fc}\]
  \[\varepsilon_{ft0} \text{ and } \varepsilon_{fc} \text{ are 2 materials parameters}\]

- **Hysteretic loop**
  \[\sigma_t = \sigma + \sigma_d, \quad \sigma = E(1-d)\varepsilon, \]
  \[\sigma_d = (\beta_1 + \beta_2 d_i) \cdot E(1-d_i)\varepsilon \cdot f(\varepsilon) \cdot \text{sign} \]
  \[\beta_1 \text{ and } \beta_2 \text{ are 2 parameters}\]
  \[f(\varepsilon) \text{ gives the form}\]
  \[\text{and the size of the loop}\]
  \[\text{sign is – for unloading}\]
  \[\text{and + for reloading}\]
What about cyclic loading response for large deformation?

In the real life: when crack opens and Rebar yield, a debonding zone appears in the vicinity of the crack → relative slide between steel and concrete:

**Proposition**: introduce this sliding in the behaviour of the steel ($\varepsilon_s = \kappa \varepsilon_p$)

\[
\varepsilon = \varepsilon_e + \varepsilon_p + \varepsilon_s
\]
Cyclic loading: Simulations (MF)/Experiments

Loading path

Fiber beam model:
- 30 beam elements
- Section, 5x5 fibers

Experiments from LMT Cachan: F. Ragueneau et al - 2010
1. Mock-up representative of a building of a power plant
   - 1/4 scale - weight = 45t
   - asymmetrical structure
   - bidirectional loading

2. Loading program:
   Phase 1: artificial signal in respect of the design spectra
   Phase 2: natural earthquake (Northridge)
   Phase 3: Northridge replica

Schéma de la maquette

T. Chaudat, P-E Charbonnel – SMART 2013 Experimental campaign, 2014
Shear wall simplified modelling: lattice « equivalent reinforced concrete»

**Concrete bars of the truss**

\[
\begin{align*}
A_h &= \frac{3}{8} (3-k^2) \alpha t \\
A_v &= \frac{3}{8} \frac{k^2 - 1}{k} \alpha t \\
A_d &= \frac{3}{16} \frac{(1+k^2)^{3/2}}{k} \alpha t
\end{align*}
\]

from Hrennikoff (1941)

**Steel model**

**Concrete damage \(\mu\) model**
vertical additional bars are added \((section\;area = 0)\)

\textit{bending and torsion inertia} identified from a FE modal analysis

Modal analysis of the « ERC » wall

Modal analysis

FE shell elements
SMART 2013 (EDF-CEA) : Benchmark results
(B. Richard et al., benchmark report SMART 2013)

Point C, level 3 – direction X - design earthquake (0.22g)

3D: solid elements (71600) (Labib et al. 2014)

1D (present modeling): MF beam elements(69) & lattice elements (10800 bars) (de Biaso, Grange, 2014)

2D: beam elements(230) & shell elements (8600) (Crespo et al., 2014)
Performance of the « 1D » calculation
SMART 2013 (M. di Biaso, S. Grange, Benchark report SMART 2013)

Beams and columns = NL multifiber beams; Walls = NL Equivalent Reinforced Concrete truss; Slabs = linear shell

Exp: 6.47Hz
Calcul: 6.28Hz

Exp: 9.13Hz
Calcul: 7.86Hz

Exp: 17.85Hz
Calcul: 16.5Hz

Mode 1

Mode 2

Mode 3

Design earthquake (0.22g)

Northridge earthquake (1.1g)
High velocity loading
Spalling test

(Forquin et al. 2011)
High velocity loading
Spalling test

Experimental observations:
1. Tensile strength increases with strain rate
2. Strain rate changes fracture processes
   - Induces multi-fracturing
   - Increases fracture energy

Mesoscopic model: effect of strain rate on the tensile behaviour
(Gatuingt et al. 2013)
Tensile behavior evolutions with strain rate

Strain rate effects taken into account using the **retarded damage concept**

\[ R_t = \frac{\varepsilon'^d}{\varepsilon'^s} \]  

**Two domains:**

- **Low and medium velocity** ($\dot{\varepsilon} < 10/s$)
  \[ R_t = 1.0 + a_t \dot{\varepsilon}^b \]

- **High velocity** ($\dot{\varepsilon} > 10/s$)
  \[ R_t = \min [c, \dot{\varepsilon}^d, M_t] \]

Tests from Forquin et al. 2011

Spalling test: Simulation (explicit 1D)

(Forquin et al. 2011)

Imput loading

"Imput" Velocity

"Output" Velocity

Strain at mid specimen

Impact on a RC beam: Medium velocity loading
Ågårdh L, Magnusson J, Hansson H, 1999

Experiment from Swedish Defence Research Agency (FOI)

Mass: 718kg
Height: 2.68m
Impact velocity: 6.7m/s
Experiment/MF calculation
(30 beam elts, 10x2 fibers)

Impact section
velocity
strain on rebar
displacement
acceleration
Tension damage contour

\[ t = 2 \times 10^{-3} \text{ s} \]

Compression damage contour

\[ t = 45 \times 10^{-3} \text{ s} \]
Comparison of various calculations

1. Present simplified modelling (1D)

2. Tuan Ngo and Priyan Mendis (RC impulsive model – 2D- 2005)

3. Present simplified modelling (1D)

4. Pontiroli et al (2D) PRM model 2004

5. M. Unosson - K&C concrete model (2D) 2001
Conclusions

Central question: *is simplified modelling (MF beams, lattice elements) able to describe the response to low, medium and high velocity?*

*Yes, but some requirement are needed*

1. **Have relevant models:**
   - Damage model for concrete including major behaviour effects (crack-closure, permanent strains, hysteretic loop, ...)
   - Including the steel – concrete debounding effects

2. **Take care to localization problems**

3. **Introduce strain rate effects**
   - Retarded damage concept + Fracture energy evolution (2 domains):
     - low and medium velocity (<10/s)
     - high velocity (>10/s)
   - Adapted dynamic solver

*...... major advantage: robustness and very low computer cost*