Failure and Nonlocality

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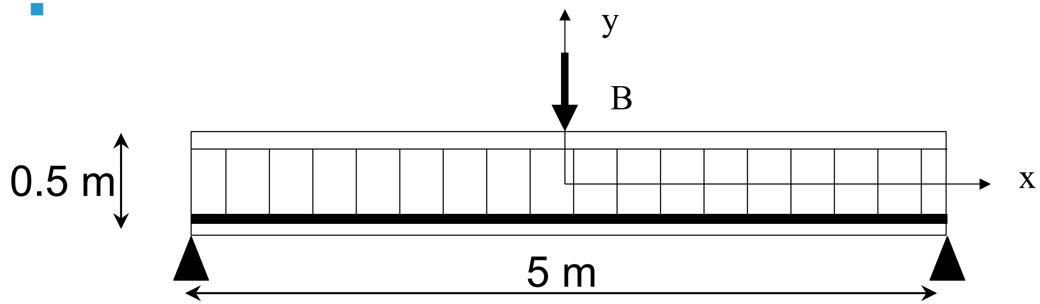
Structural computations

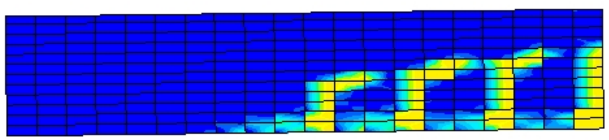


$$\varepsilon_{ij} = \frac{1+\nu}{E(1-D)}\sigma_{ij} - \frac{\nu}{E(1-D)}\left[\sigma_{kk}\delta_{ij}\right]$$

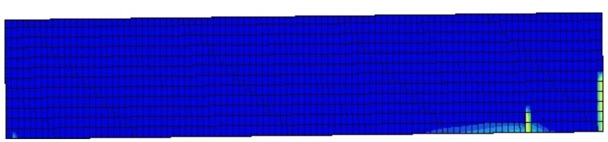
Local damage

Two meshes...

20 elements over the half span

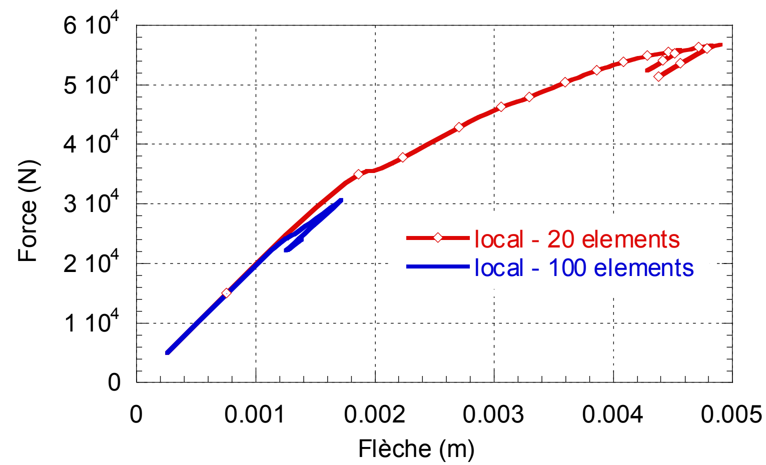


100 elements over the half span



Distribution of damage

… and two responses

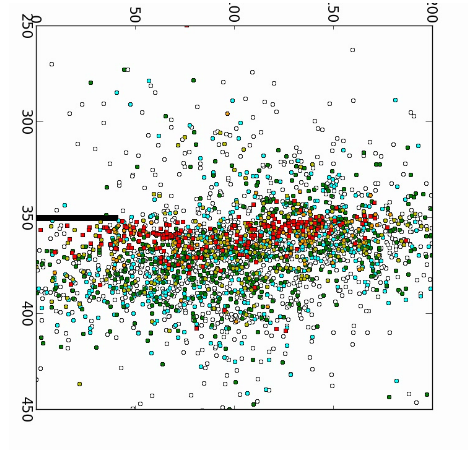


Stain and damage localisation

Mesh dependent solution

Fracture of quasi-brittle materials

Fracture test



Fracture approach:

- Linear elastic fracture mechanics, cohesive crack models
- XFEM implementation, phase fields and variational approach of fracture

Continuum-based approach:

- Continuum damage, enhanced continua (non local, gradient)
- Standard FE models, TLS approach

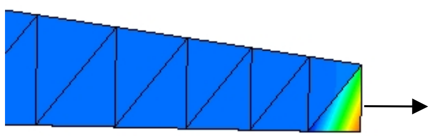
Lattice approach:

- Physical aspects of fracture (continuum = thermodynamic limit), scaled lattices

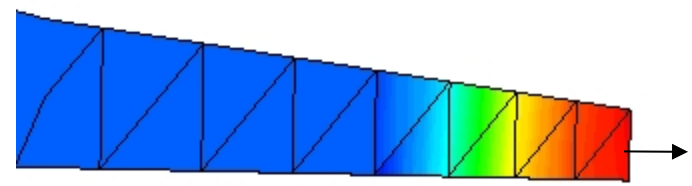
Comparison between local and non local computations

$$\varepsilon_{ij} = \frac{1+\nu}{E(1-D)}\sigma_{ij} - \frac{\nu}{E(1-D)}\left[\sigma_{kk}\delta_{ij}\right]$$

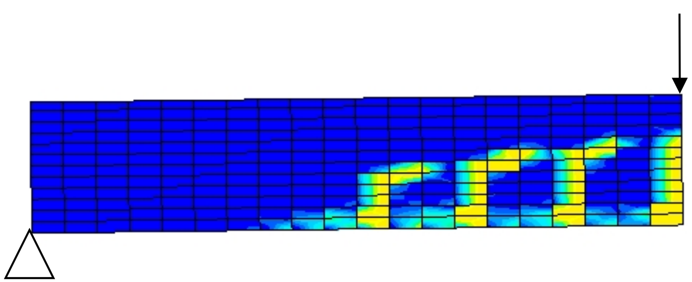
Tension Tension



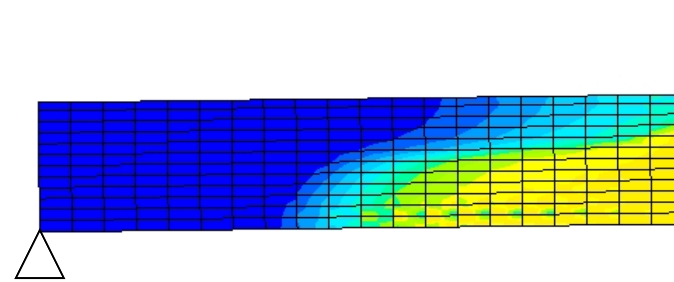
local / non local



Bending



local / non local



“Classical nonlocal models”

Limitations can be still observed in continuum-based approaches

- **incorrect crack initiation, ahead of the crack tip;**
- **propagating damage fronts after failure due to non-local averaging;**
- **incorrect shielding effect with non-zero non-local interactions across a crack surface;**
- deficiencies at capturing spalling properly in dynamics, with spalls of zero thickness when the expected spall size is below the internal length of the model

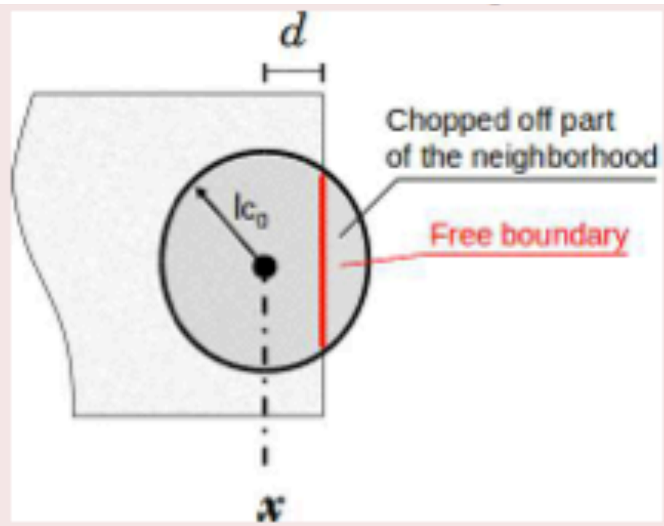
Classical nonlocality

Non-locality in classical non-local models (such as integral-type):

➔ The internal length is the parameter that encompasses the non-locality

This parameter should not be constant:

$$\begin{aligned}\bar{\varepsilon} &= \frac{1}{\Omega_r(x)}\int_{\Omega}\psi_0(x,\xi)\varepsilon(\xi)\mathrm{d}\xi\\ \psi_0(x,\xi) &= \exp\left(-\left(\frac{2||x-\xi||}{l_c}\right)^2\right)\\ \Omega_r(x) &= \int_{\Omega}\psi_0(x,\xi)\mathrm{d}\xi\end{aligned}$$



a) Boundary effects ($l_c \searrow$)

Classical nonlocality

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➔ The internal length is the parameter that encompasses the non-locality

$$\bar{\varepsilon} = \frac{1}{\Omega_r(x)} \int_\Omega \psi_0(x,\xi)\varepsilon(\xi)\mathrm{d}\xi$$

$$\psi_0(x,\xi) = \exp\left(-\left(\frac{2||x-\xi||}{l_c}\right)^2\right)$$

$$\Omega_r(x) = \int_\Omega \psi_0(x,\xi)\mathrm{d}\xi$$

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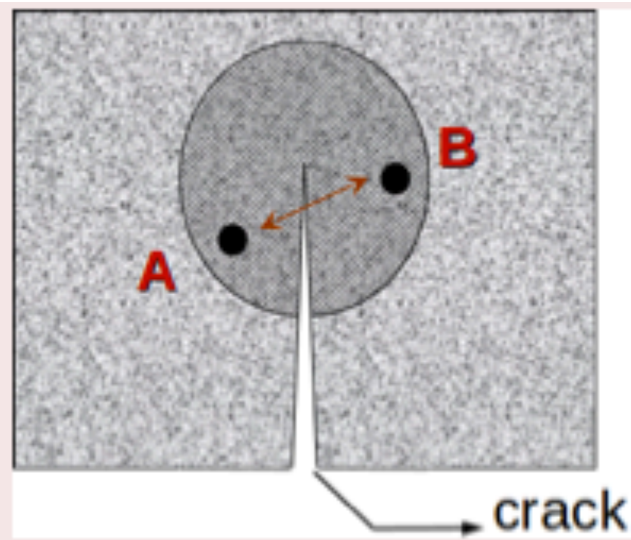


b) Shielding effect ($l_c \searrow$)

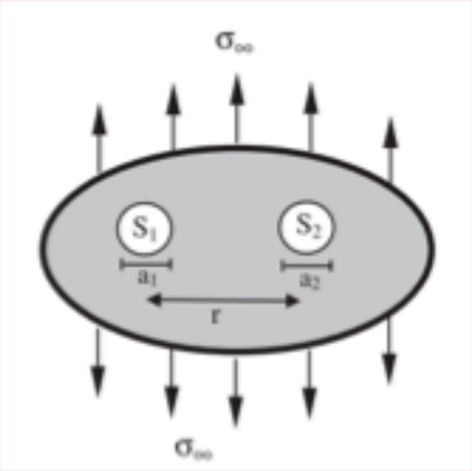
Classical nonlocality

Non-locality in classical non-local models (such as integral-type):

➔ The internal length is the parameter that encompasses the non-locality

$$
\begin{aligned}
\bar{\varepsilon} &= \frac{1}{\Omega_r(x)} \int_\Omega \psi_0(x,\xi)\varepsilon(\xi)\mathrm{d}\xi \\
\psi_0(x,\xi) &= \exp\left(-\left(\frac{2||x-\xi||}{l_c}\right)^2\right) \\
\Omega_r(x) &= \int_\Omega \psi_0(x,\xi)\mathrm{d}\xi
\end{aligned}
$$

This parameter should not be constant:



c) Voids growth effect ($l_c \nearrow$)

Modified nonlocality

Non-local models with varying internal length:

- lc = f(D,d)
  - D: damage ($0 \leq D \leq 1$)
  - d: boundary distance

  Krayani *et al.*, 2009
  Pijaudier-Cabot and Dufour, 2010
  Grégoire *et al.*, 2012
  Bažant *et al.*, 2010

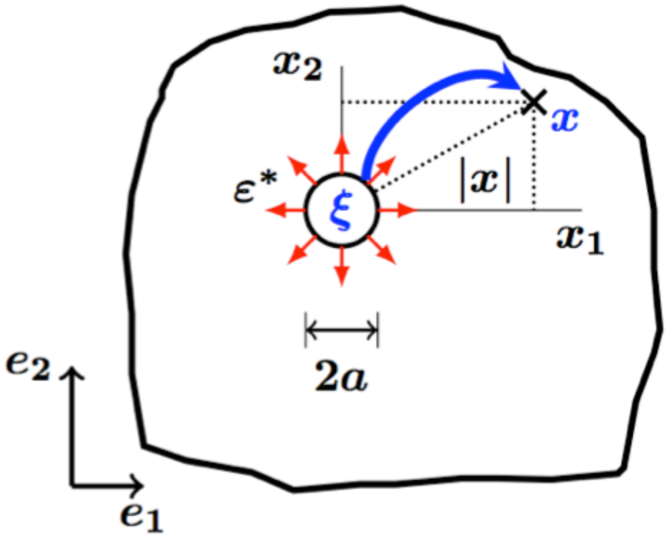
- Stress-based non local damage model

  Giry *et al.*, 2011

- …

➔ *Sometimes rather empirical*

interaction-based non-local model



Let us define an interaction as the effect on a given point x of the strain perturbation ε* located on a point ξ.

The strain induced may be estimated energetically by:

$$A(x,\xi,\varepsilon^*,a) = \sqrt{\sum_{i=1}^{3} |\varepsilon_i(x)|^2} \quad \textbf{(Euclidean norm)}$$

The interaction at x due to ξ may be given by:

$$A^*(x,\xi,a) = \frac{A(x,\xi,\varepsilon^*,a)}{||\varepsilon^*||}$$

Assuming that this interaction governs the non-locality averaging, we get:

$$\boxed{\bar{\varepsilon}_{\text{eq}}(x) = \frac{1}{\Omega_r}\int_\Omega \psi(x,\xi)\varepsilon_{\text{eq}}(\xi)d\xi \quad \text{with} \quad \psi(x,\xi) \equiv A^*(x,\xi,a) \quad \text{and} \quad \Omega_r = \int_\Omega A_0^*(x,\xi,a)d\xi}$$

interaction-based non-local model

Isotropic damage: $\sigma = (1 - D)\mathbb{C} : \varepsilon$

Equivalent strain: $\varepsilon_{eq} = \sqrt{\sum_{i \in [\![1,3]\!]} \langle \varepsilon_k \rangle_+^2}$ Mazars, 1986

Nonlocal averaging: $\overline{\varepsilon}_{\text{eq}}(x) = \dfrac{1}{\Omega_r} \displaystyle\int_\Omega \psi(x,\xi)\varepsilon_{\text{eq}}(\xi)d\xi$

$$\psi(x,\xi) \equiv A^*(x,\xi,a) \quad \Omega_r = \int_\Omega A_0^*(x,\xi,a)d\xi$$

Damage evolution: $D(h,x) = \left[1 - (1 - A_t)\dfrac{\varepsilon_{D_0}}{h(x)} - A_t e^{(-B_t(h(x) - \varepsilon_{D_0}))}\right]$

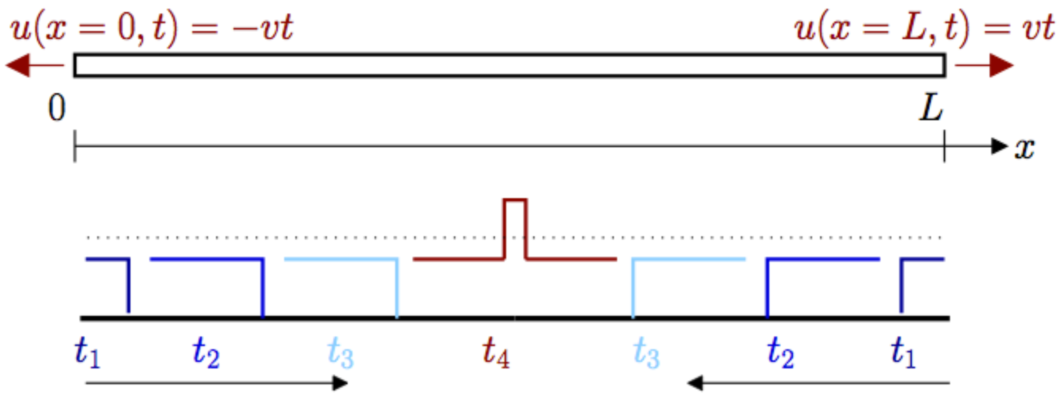
Mazars, 1986

Kuhn-Tucker condition:

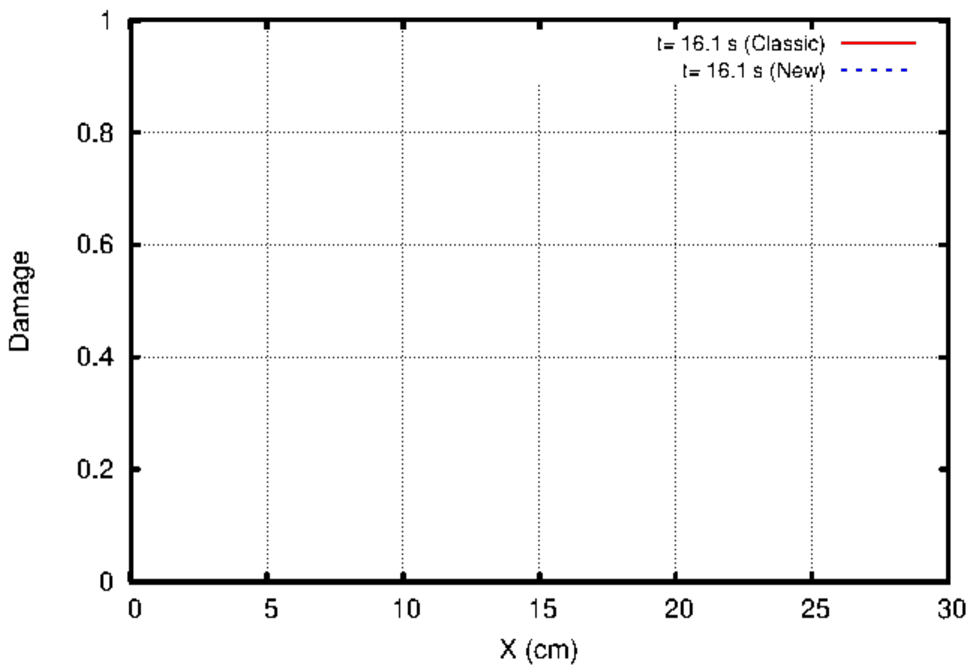
$$\Gamma(\varepsilon,h) = \overline{\varepsilon}_{eq}(\varepsilon) - h, \quad \Gamma(\varepsilon,h) \leq 0,$$
$$\dot{h} \geq 0, \quad \dot{h}\Gamma(\varepsilon,h) = 0$$
$$h = \max(\varepsilon_{D_0}, \max(\overline{\varepsilon}_{eq}))$$

Efficient shielding effect and assembly of small aggregates on the boundary: $a(x) \ = \ \min(a_0\sqrt{1 - D(x)}, d(x))$

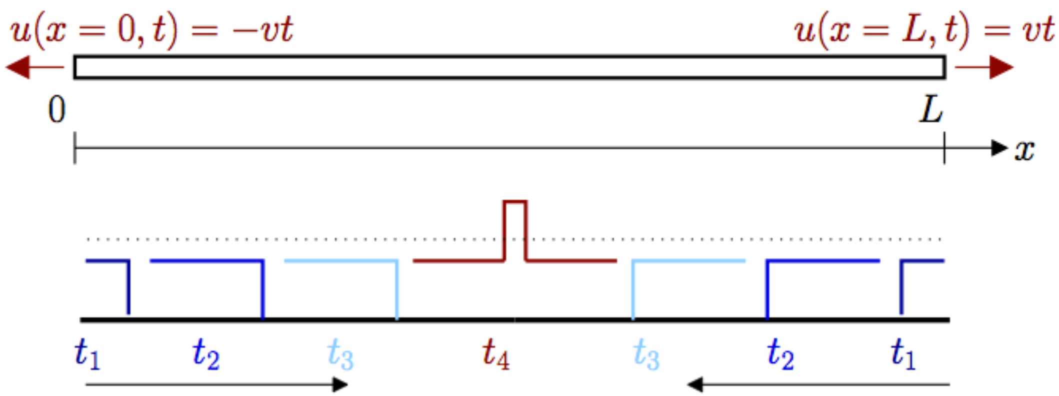
Dynamic failure of a bar



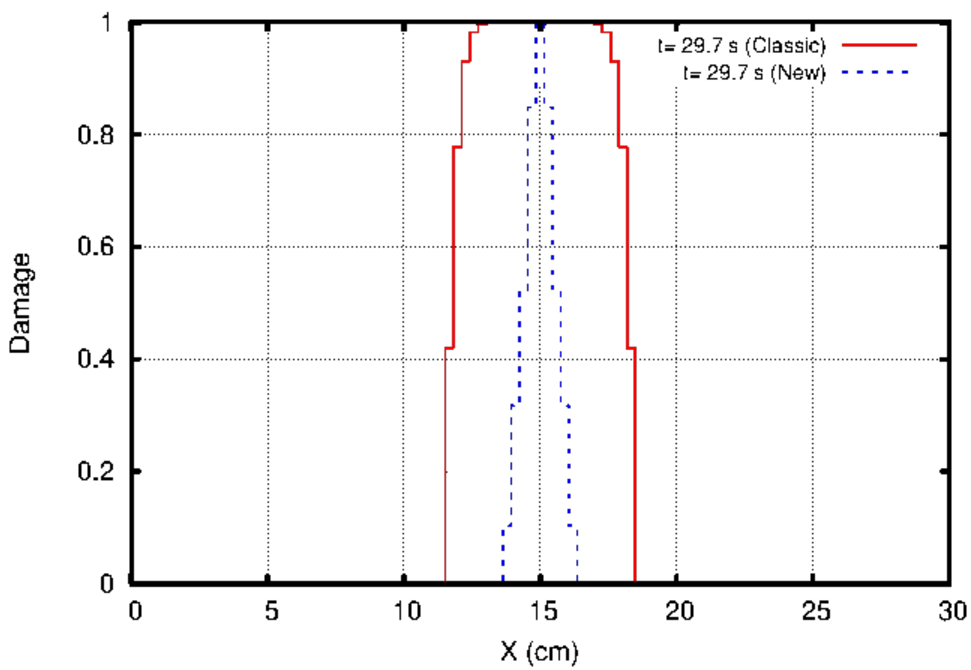
Damage:



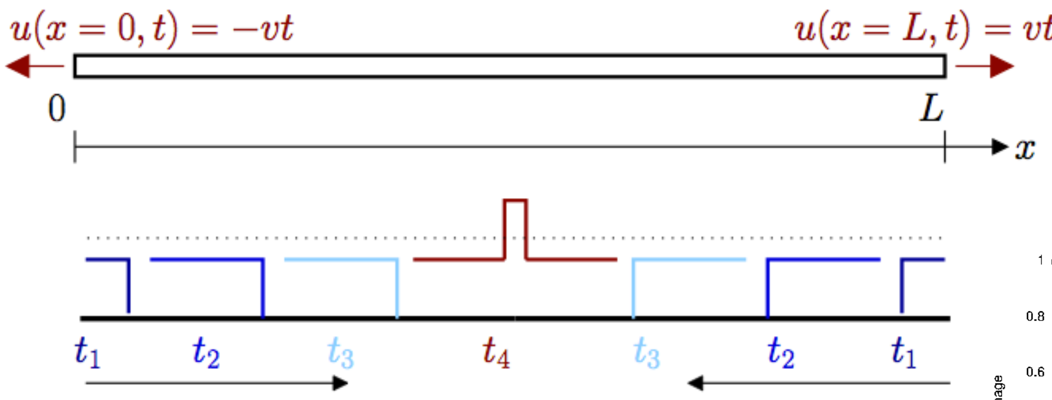
Dynamic failure of a bar



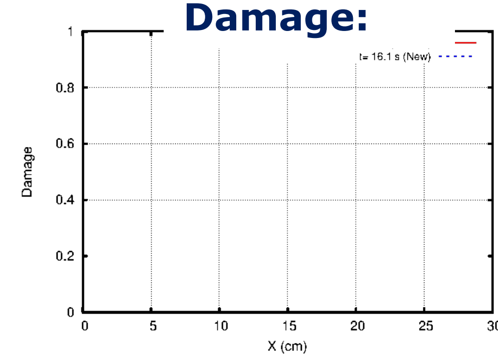
Damage:



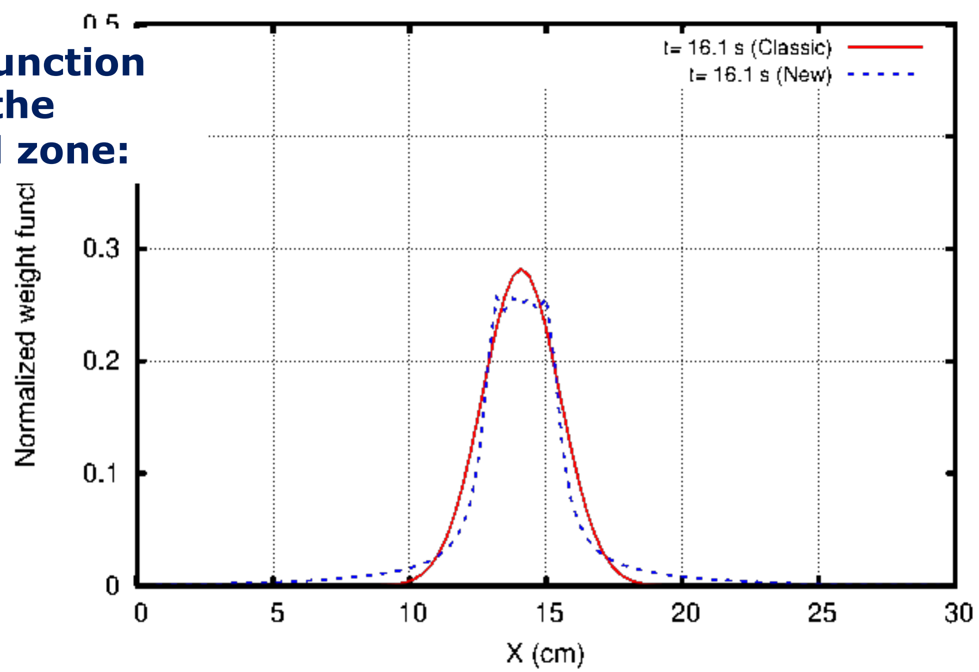
Dynamic failure of a bar



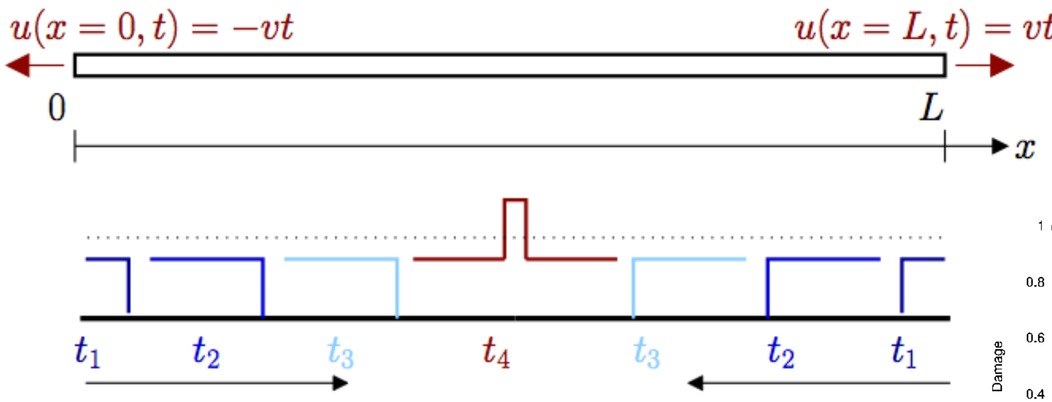
Damage:



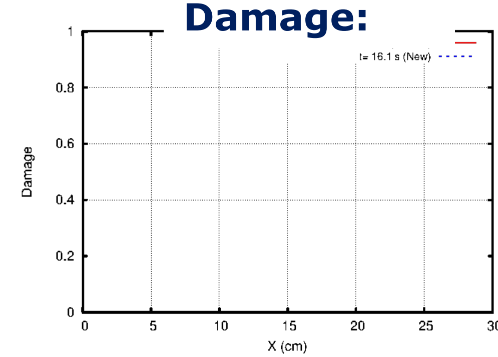
Weight function close to the damaged zone:



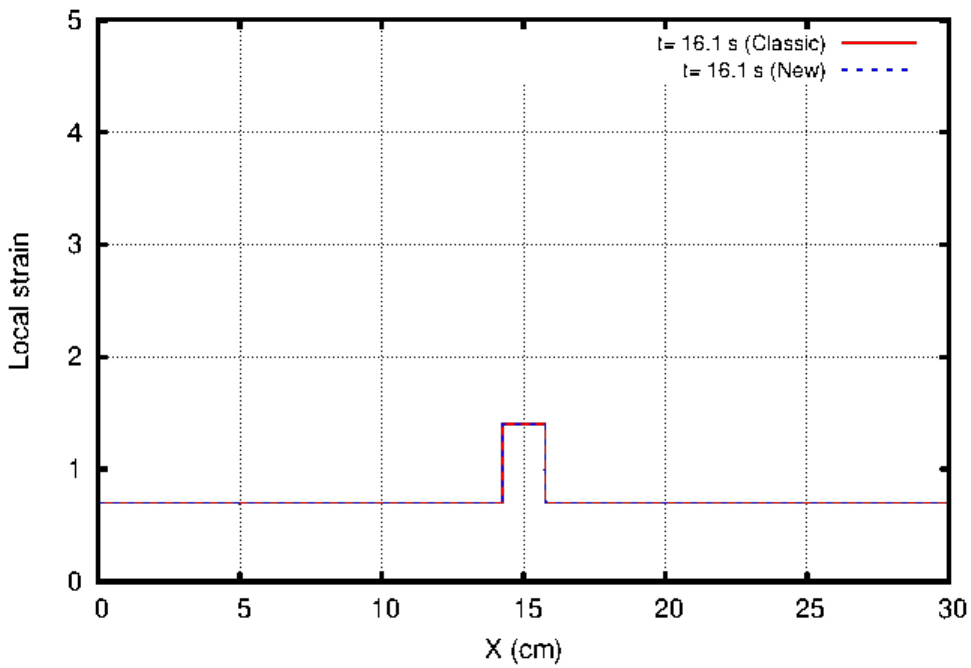
Dynamic failure of a bar



Damage:

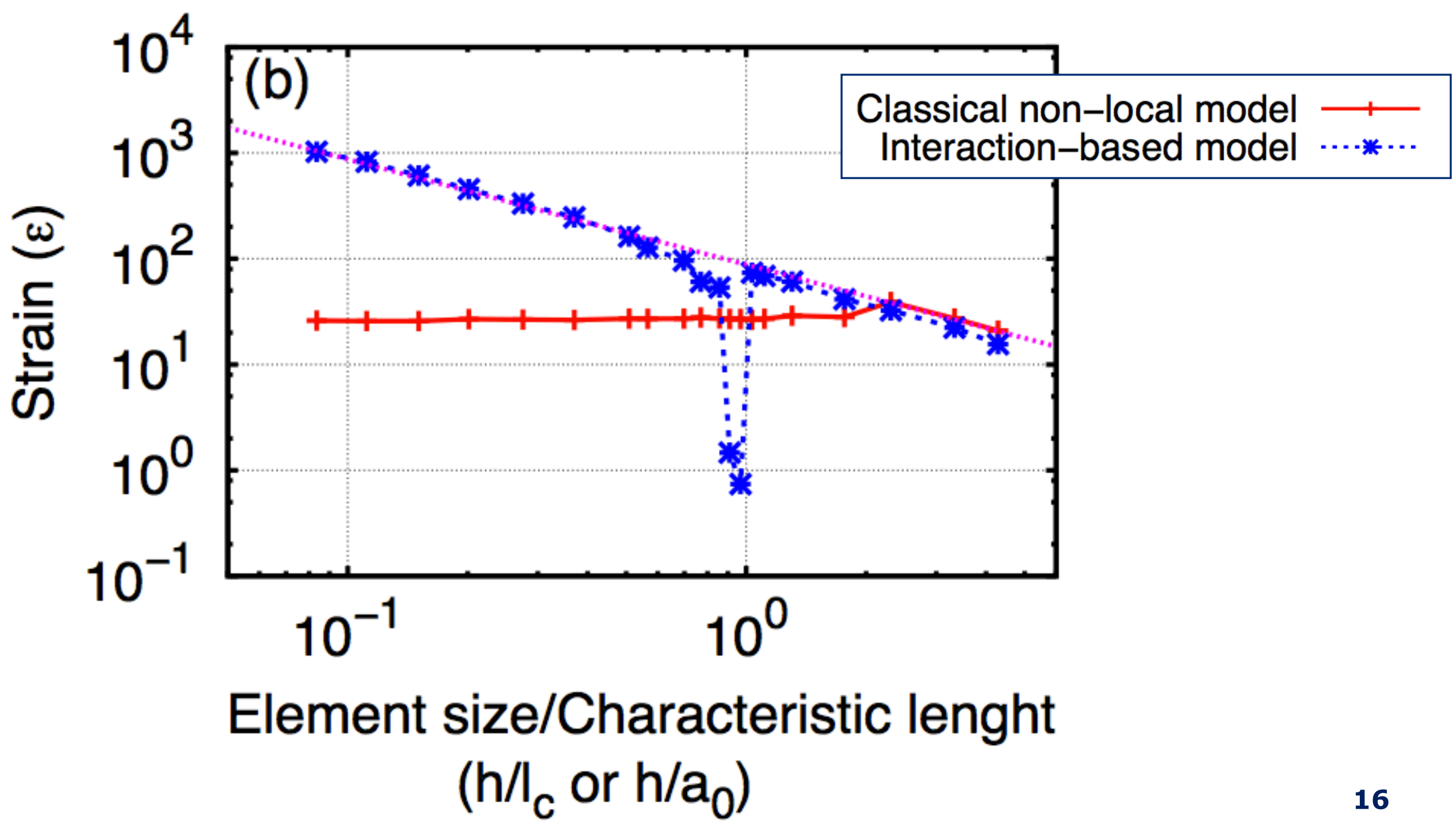


Local Strain:



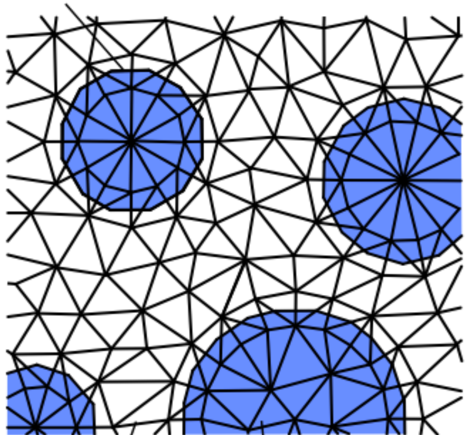
Dynamic failure of a bar

At complete failure, the crack opening should be independent of the element size. Assuming that the crack opening is smeared over the cracked finite element, we get: $[U](x) \approx \varepsilon h = \text{constant}$

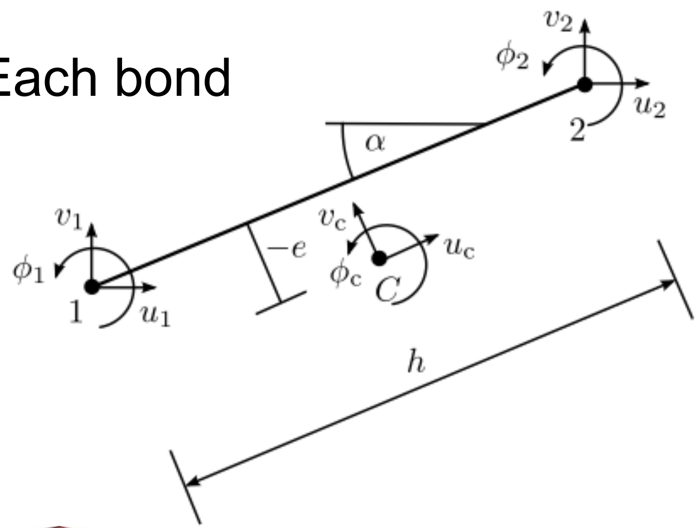


Meso-scale lattice model

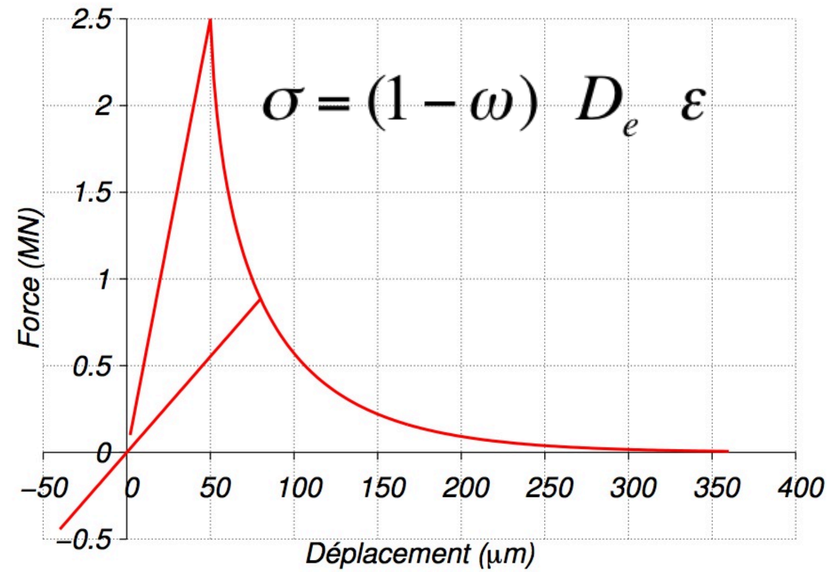
Meso-scale approach



Each bond



Grassl and Jirasek, Solids & Struct., 2010



Description of size effect

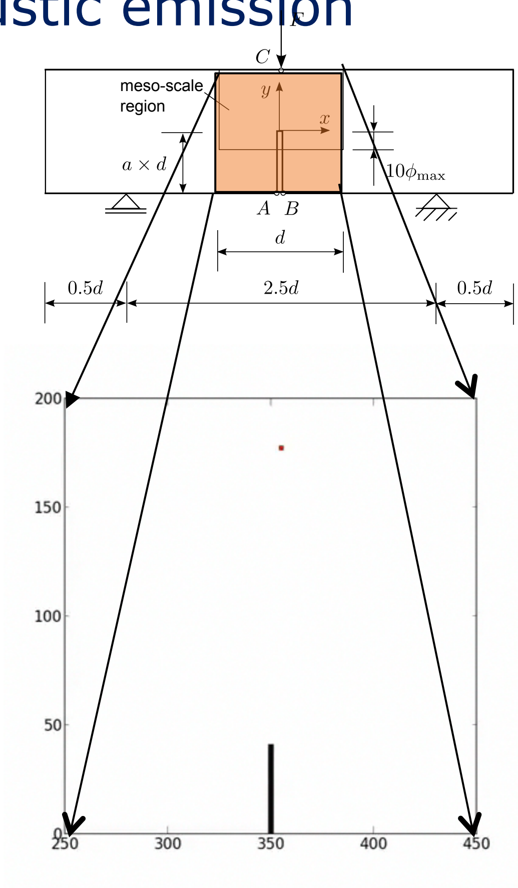
Grassl, Grégoire, Rojas-Solano and Pijaudier-Cabot, *Int. J. Solids Struct.*, 2012

Grégoire et al., *Int. J. Num. and Anal. Meths. Geomechanics*, 2013

Damage Process

Experiments

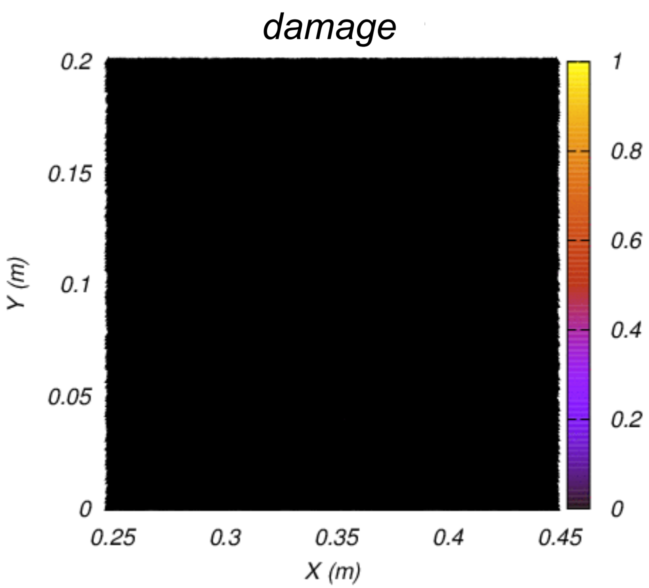
- Microcracks localization by acoustic emission



Grégoire et al., *Int. J. Num. and Anal. Meths. Geomechanics*, 2015

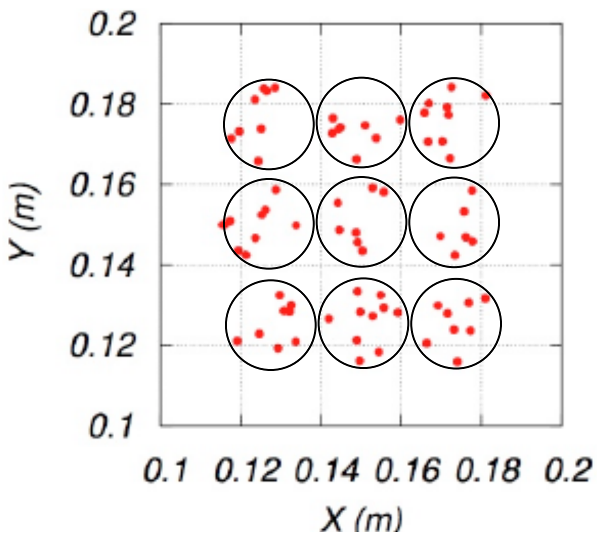
Numerical Model

- Identification of elements whose damage increase during the time step



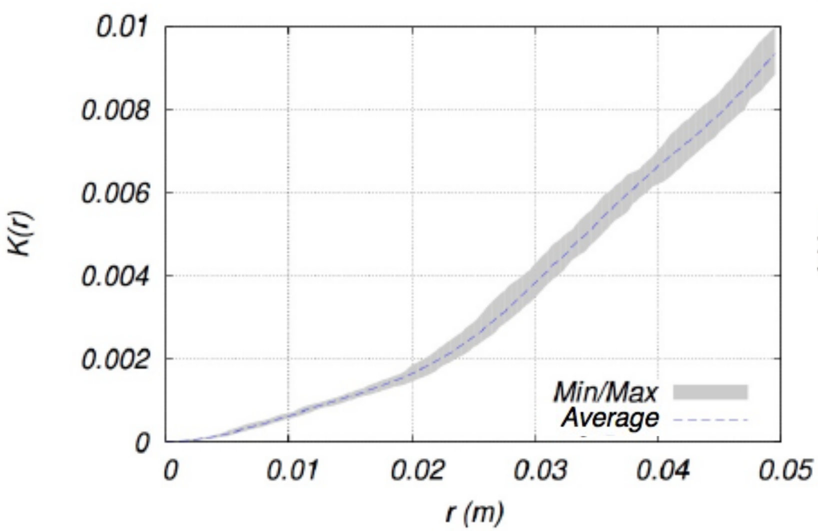
Ripley's functions

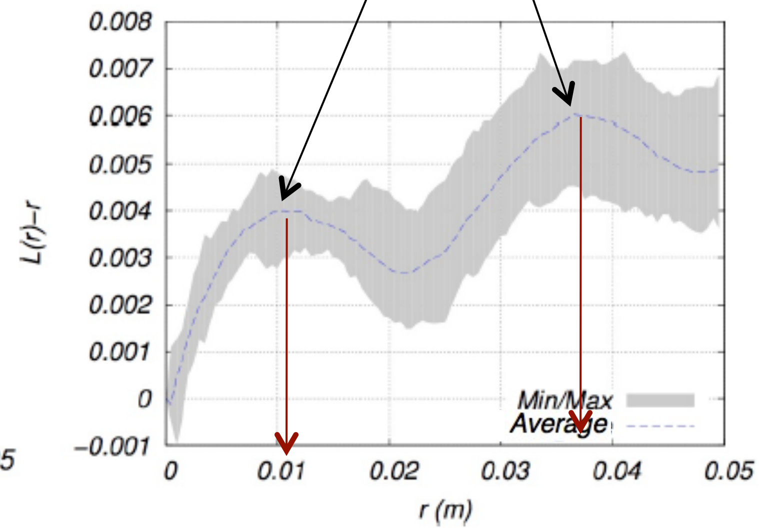
Points concentrated in 9 discs 1cm radius, spaced 2cm



The abscissa of each peak depends on the radius of the disks and their spacing

$$L(r) - r = \sqrt{\frac{K(r)}{\pi}} - r$$

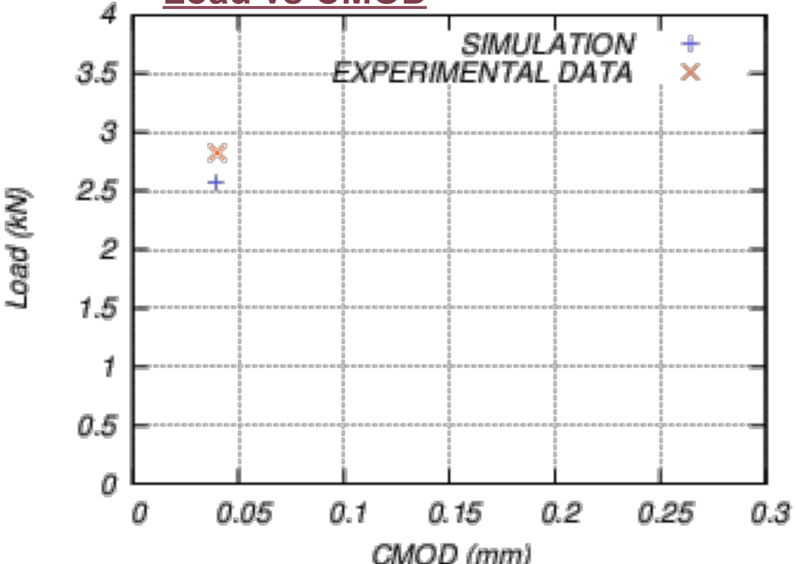




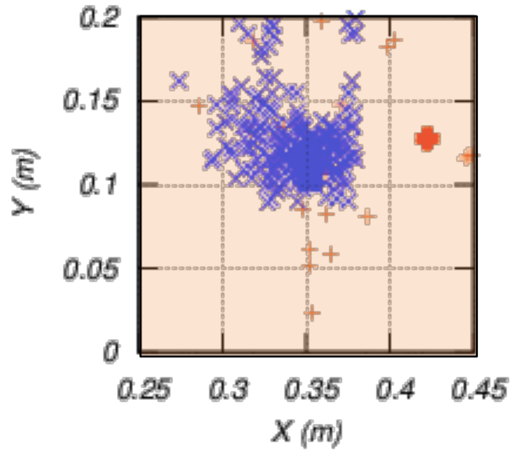
Half notched beam 700mm x 200mm

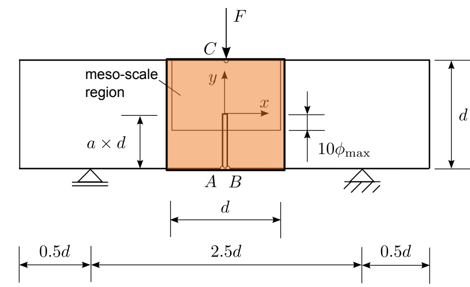
COMPARISON SIMULATION vs EXPERIMENTAL DATA

Load vs CMOD

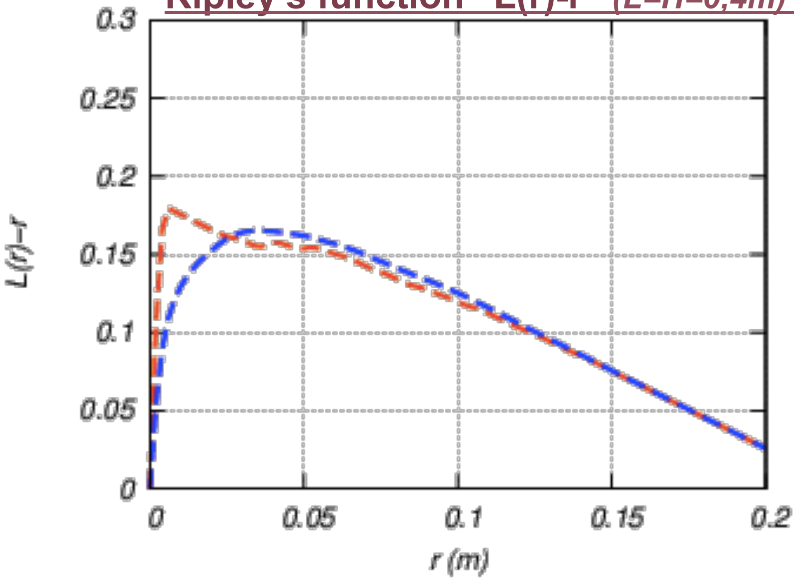


Position of events occurring during the time step

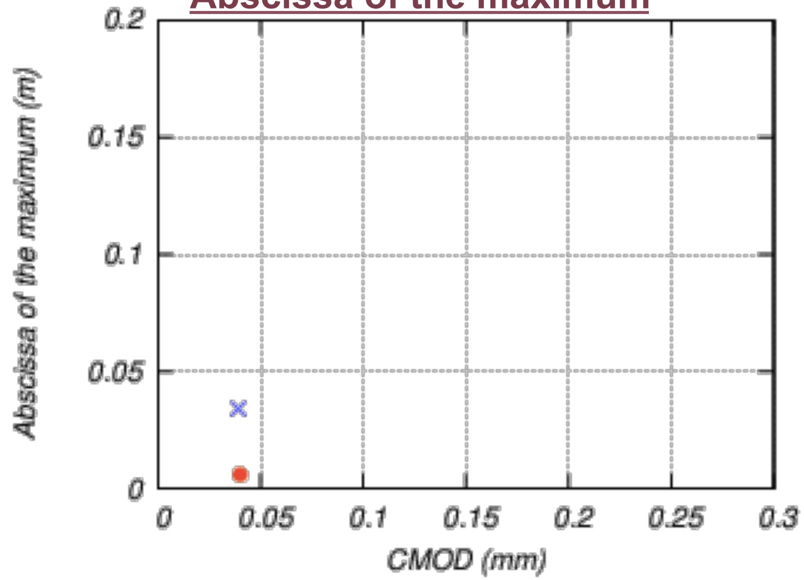




Ripley's function "L(r)-r" *(L=H=0,4m)*



Abscissa of the maximum



Concluding remarks

- **Need to introduce at least a length in order to bridge the gap between Continuum and fracture models**

- **Nonlocality is a complex issue !**

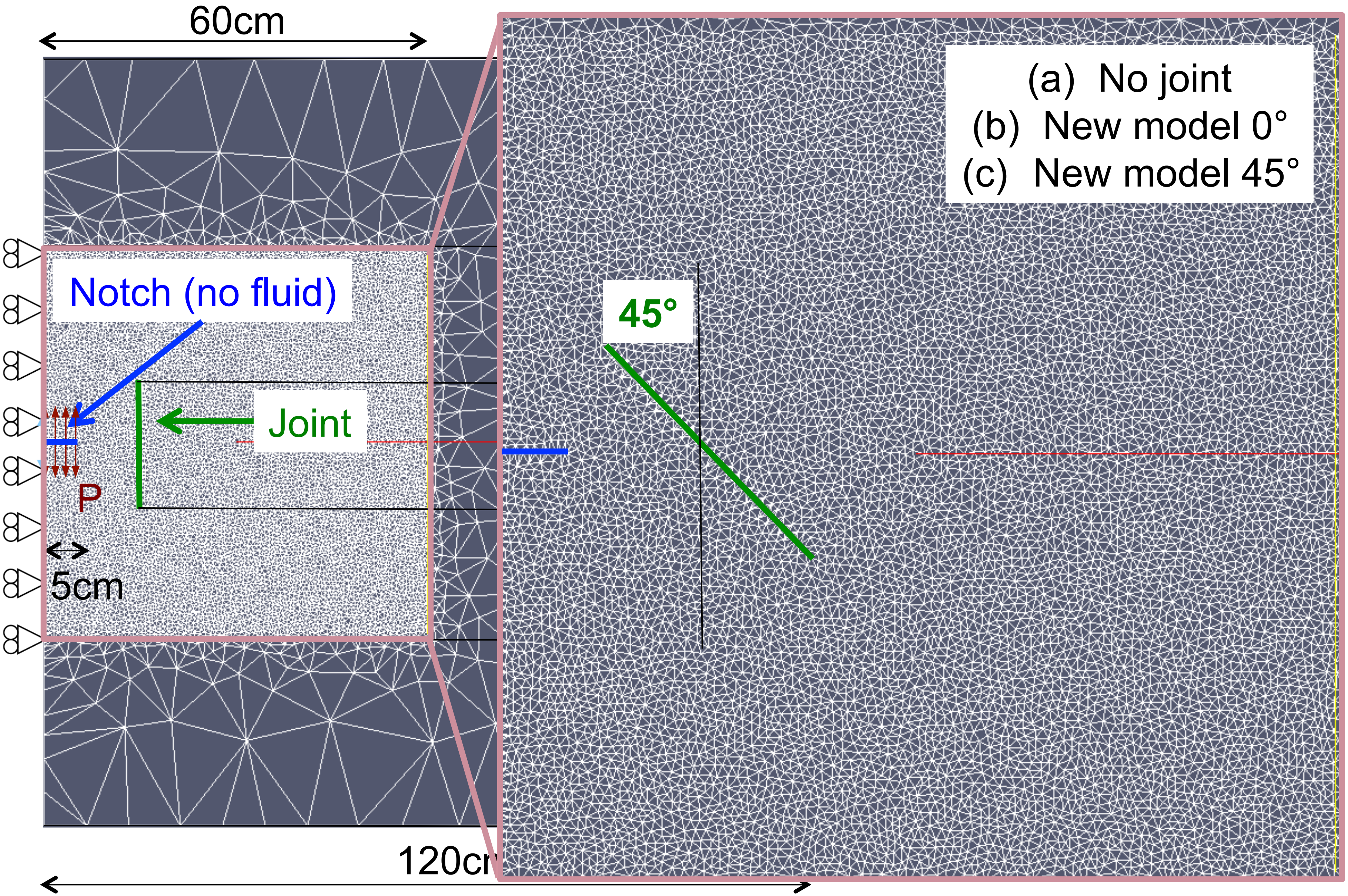
- **Interaction-based model provide some background**

- **The lattice meso-scale approach provides a consistent description of fracture**

- **Ripley’s functions provide the evolution of a characteristic length upon localization of damage during the fracture process**

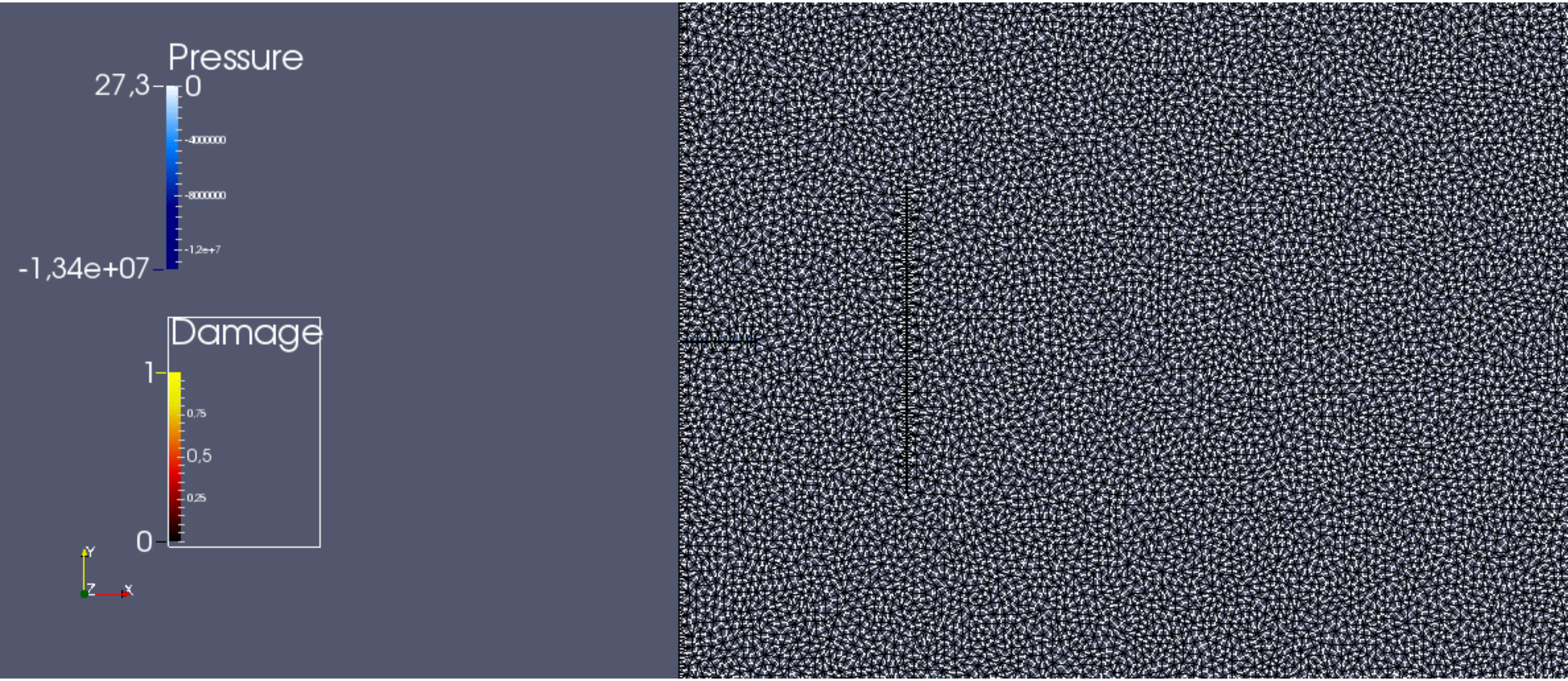
→ Lattice approach may be implemented in more complex configurations

Hydraulic fracturing



Hydraulic fracturing

Joint at 0°



Aknowledgements

Failflow Advanced Grant (27769)

Thank you for your attention

Thank you so much Zdenek…