

# ANALYTICAL MODEL FOR CONCRETE EDGE FAILURE OF MULTIPLE ROW ANCHORAGES WITH SUPPLEMENTARY REINFORCEMENT

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**Abstract:** In the accompanying paper, the details and results of the experimental campaign carried out on multiple row anchorages, without and with supplementary reinforcement, loaded in shear towards the edge were presented. It was shown that the models given in standards are insufficient to calculate the failure load for anchorages with supplementary reinforcement failing through concrete edge and reinforcement failure. This paper gives the details of a new analytical model developed to evaluate the failure load of anchorages with multiple anchor rows with supplementary reinforcement. The model is developed on the basis of the detailed evaluation of the results of an experimental campaign carried out on anchorages with up to four anchor rows. It has been shown that with the new model, the failure loads for the anchorages with supplementary reinforcement can be evaluated realistically considering different possible failure modes. In order to investigate the number of anchor rows participating to carry the shear loads, the experimental results are augmented through numerical simulations performed using software MASA at University of Stuttgart.

## 1 INTRODUCTION

The failure load for concrete edge failure of the anchorages with multiple anchor rows loaded in shear towards the edge can be significantly increased by using supplementary reinforcement in the form of stirrups and edge reinforcement. In case of anchor groups with supplementary reinforcement, once the concrete cracks, the stirrups get activated and provide resistance to the applied shear loads

until reinforcement yielding or bond failure occurs or the hook (node) of the supplementary reinforcement fails. Thus, the shear strength of the anchorage can be increased by increasing the amount of supplementary reinforcement. This increase in the shear capacity is limited by the anchor steel failure and/or the strut (compression) failure of concrete. In the current standards, such as EN1992-4 [1], the strut failure is neglected and a very conservative approach is

given to consider the steel failure of anchor, stirrup yielding and node failure.

In the accompanying paper, the details and results of the experiments performed on anchor groups with 2 to 8 headed studs cast in unreinforced and reinforced concrete, loaded in shear perpendicular to the edge were presented and discussed. The brief evaluation of the test results and comparison against the model given in EN1992-4 [1] clearly showed that the current models are over-conservative in estimating the failure loads for concrete edge failure for low to medium percentage of reinforcement, even if the failure crack is assumed to appear from the back anchor row. However, for high percentages of reinforcement, the current models are prone to over-predict the failure load as they do not recognize strut failure as a possible failure mode. Furthermore, it was shown that even when the failure crack appears from the back anchor row, almost all the anchors take up the shear loads (for anchor steel failure). Therefore, the current approach of assuming the shear load taken up only by last anchor row in case of the failure crack assumed from the back anchors is very conservative.

In this paper, the test results and observations are evaluated in detail and compared with the model proposed by Schmid [4] for anchorages with supplementary reinforcement loaded in shear towards the edge. Based on the detailed evaluation of the test results, a new model is proposed for predicting the concrete edge failure loads for anchorages with supplementary reinforcement by modifying the model proposed by Schmid [4]. It is shown that with the proposed model, the failure loads for the low to medium amount of reinforcement (where reinforcement failure dominates) can be predicted very well.

For high amounts of reinforcement, it is possible that the concrete strut failure limits the failure load prior to reinforcement yielding. To consider this, an approach to incorporate strut failure in case of anchorages with relatively high amount of reinforcement is also proposed analogous to the approach proposed by Berger [5] for anchorages with supplementary reinforcement subjected to

tension loads.

Although in the tests, steel failure of the anchors was avoided by design, the results clearly showed the conservatism in the current design approach for anchor steel failure. In order to investigate this aspect more in detail, numerical simulations were performed using the 3D FE software MASA at University of Stuttgart. At first, the numerical model was validated against the test results obtained for the 4x2 anchorage without supplementary reinforcement loaded in shear towards the edge. The numerical model was then used to investigate the participation of the anchors in taking up the shear loads.

## 2 MODEL PROPOSED BY SCHMID [4]

### 2.1 Description of the Schmid model

Based on a number of tests on single anchors and two anchors in a row (1x2 configuration) in concrete with supplementary reinforcement, Schmid [4] proposed the following formulation to evaluate the mean load carrying capacity of the anchorage. As per Schmid [4] model, the load carrying capacity of the anchor reinforcement can be divided into two parts: the contribution of the hook and the contribution of the bond.

The mean ultimate shear load corresponding to reinforcement failure is given as

$$V_{um,re} = \frac{N_{um,re}}{x} \geq V_{um,c} \quad (1)$$

Where,

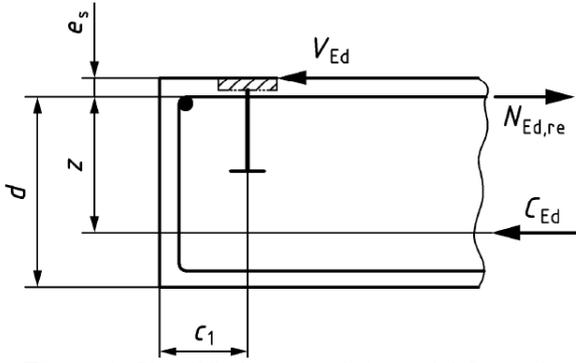
$V_{um,re}$  = mean shear capacity of an anchorage with anchor reinforcement

$N_{um,re}$  = total load carrying capacity of the anchor reinforcement

$V_{um,c}$  = shear capacity of anchorage without anchor reinforcement

$x$  is the factor to consider for the lever arm between the reinforcement and the applied shear load (refer Fig. 1) given as

$$x = \left( 1 + \frac{e_s}{z} \right)$$



**Figure 1:** Simplified strut-and-tie model for anchor reinforcement by EN1992-4 [1]

Thus, as per Eq. (1), the reinforcement failure load is considered as the failure load for the anchorage if it is more than the concrete edge failure load in unreinforced concrete. Else, only the concrete edge failure load in unreinforced concrete is considered as the failure load for the anchorage even with supplementary reinforcement. This is consistent with the model given in EN1992-4 [1].

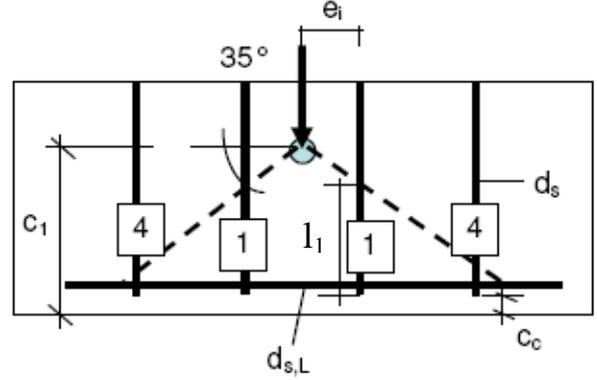
The anchorage capacity of one stirrup leg,  $N_{um,re}^0$  is given by summing the hook capacity,  $N_{um,hook}^0$  and the bond capacity,  $N_{um,bond}^0$  as

$$N_{um,re}^0 = N_{um,hook}^0 + N_{um,bond}^0 \leq A_{s,i} f_{ym} \quad (2)$$

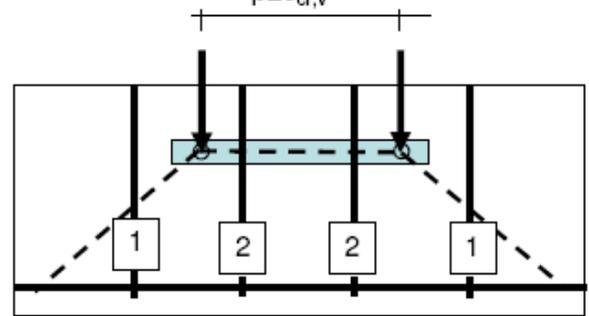
Where,  $A_{s,i}$  = area of one ( $i^{th}$ ) stirrup leg  
 $f_{ym}$  = mean yield strength of stirrup

The contribution of the hook of the stirrup,  $N_{um,hook}^0$ , is determined on the basis of the position of the stirrup relative to the theoretical crack. The stirrups that are first intercepted by the diagonal crack from a single anchor or from the outermost anchors in a group (stirrups marked 1 in Figure 2a and b) as well as the stirrups intercepted by the crack running parallel to the edge and in between the outermost anchors (stirrups marked 2 in Figure 2b) are considered as most effective. The other stirrups that are intercepted by the crack (stirrups marked 4 in Fig. 2a) are considered to be much less effective. Any stirrup that is not intercepted by the crack or whose anchorage length in the assumed breakout body is  $\leq 4d_s$  does not contribute towards the load carrying

capacity of the anchorage.



(a) Anchors first intercepted by the diagonal crack (marked 1)



(b) Anchors intercepted by crack parallel to the edge (marked 2) and anchors first intercepted by the diagonal crack

**Figure 2:** Effectiveness of stirrups - High effectiveness: stirrups 1 and 2; low effectiveness: stirrups 4 [4]

The ultimate mean value of the hook contribution for a particular stirrup leg is given as:

$$N_{um,hook,i}^0 = \psi_{1,i} \cdot \psi_2 \cdot \psi_3 \cdot A_s \cdot f_{ym} \cdot \left( \frac{f_{cm,cube}}{30} \right)^{0.1} \quad (3)$$

Where,

$f_{ym}$  is the mean yield strength of the reinforcement,

$f_{cm,cube}$  is the mean compressive strength of concrete obtained using 150mm cubes

The factor  $\psi_{1,i}$  considers the influence of the position of the stirrup. A value of  $\psi_{1,i} = 0.95$  is assumed for the most effective stirrups (marked 1 and 2 in Figure 2) and a value of  $\psi_{1,i} = 0.16$  for other stirrups (marked 4 in Figure 2).

The factor  $\psi_2$  considers the influence of the diameter of the edge reinforcement,  $d_{s,L}$

(Figure 2a) with respect to the diameter of the stirrup,  $d_s$  and is given as:

$$\psi_2 = \left( \frac{d_{s,L}}{d_s} \right)^{\frac{2}{3}} \leq 1.2 \quad (4)$$

The factor  $\psi_3$  considers the influence of the bond length,  $l_l$  (Fig. 5.1) and is given as:

$$\psi_3 = \left( \frac{l_{l,i}}{c} \right)^{0.4} \cdot \left( \frac{10}{d_s} \right)^{0.25} \quad (5)$$

Where,

$l_{l,i}$  is the bond length of the stirrup (Figure 2a)  
 $c$  is the edge distance for the anchors

The contribution of the bond of one stirrup leg is given as:

$$N_{um,bond,i}^0 = \pi \cdot d_s \cdot l_{l,i}' \cdot f_{bm} \quad (6)$$

With  $l_{l,i}' = l_{l,i} - 4d_s$

$f_{bm}$  = mean bond strength =  $1,33 \cdot 1,5f_{bd} = 2f_{bd}$   
 $f_{bd}$  = design bond strength given in EN1992-1-1 [6]

The total capacity of the anchor reinforcement is calculated by summing up the capacities of all effective stirrup legs:

$$N_{um,re} = \sum_n N_{um,re}^0 \quad (7)$$

With,

$n$  = number of effective stirrup legs of the anchorage. Effective are stirrups with an anchorage length,  $l_l \geq 4d_s$  in the theoretical breakout body

$N_{um,re}^0$  = capacity of one stirrup leg according to Eq. (2)

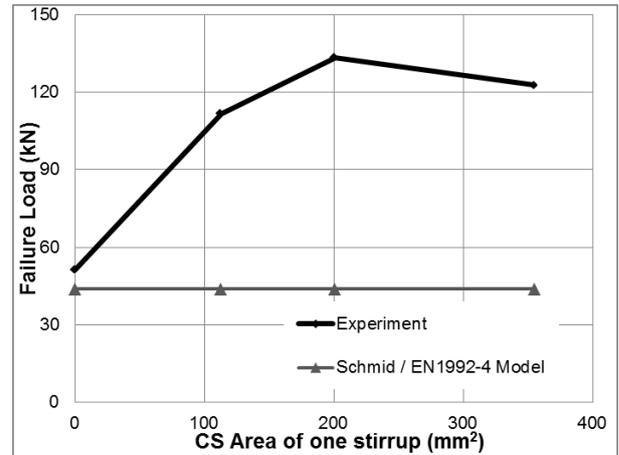
The resistance provided by the supplementary reinforcement against applied shear loads is then calculated using Eq. (1).

## 2.2 Comparison with experimental failure loads

The mean failure loads predicted by the Schmid model [4] model are compared with the experimental mean failure loads. The

analytical failure loads, evaluated using Schmid [4] model, are calculated considering crack once from front anchor row and once from back anchor row.

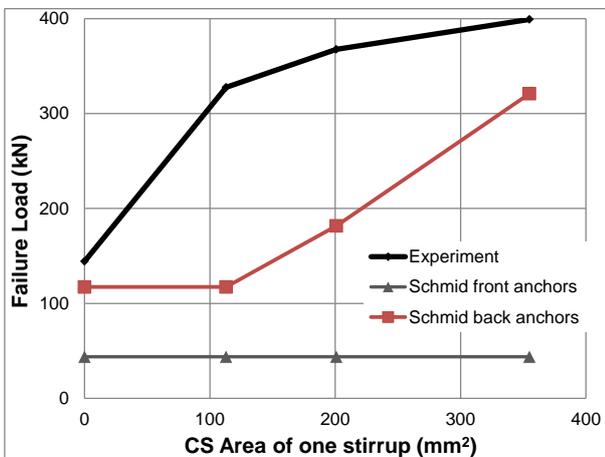
In case of groups 1x2 (Figure 3), the mean test failure load for the groups tested in unreinforced concrete matches reasonably well with the evaluated mean failure load value. However, due to a short anchorage length, in no case any contribution from stirrups is considered by the Schmid model. Therefore, for this case, the failure loads predicted by the Schmid model and the EN1992-4 [1] model are exactly the same and the analytically evaluated failure capacity for the anchor group 1x2 is independent of the diameter of stirrups and is equal to the capacity evaluated in unreinforced concrete. Although, the model seems to under-predict the measured failure loads of the group, given the unreliability of the contribution of the rope action of edge reinforcement for this anchor group, the approach given in the model seems reasonable.



**Figure 3:** Comparison of mean failure loads obtained from the tests [7] with the mean failure loads predicted by Schmid [4] and EN1992-4 [1] for groups 1x2

The comparison of experimentally obtained and analytically evaluated mean failure loads for group 2x2, as a function of cross-sectional area of one stirrup, as calculated by Schmid [4] model considering crack once from front and once from back anchors is given in Figure 4. If the crack is considered from the front anchors, no enhancement in the load carrying capacity is given by Schmid [4] model, since no stirrup is considered as effective. Further,

as per Schmid [4] model (same as per EN1992-4 model), the maximum of the failure loads corresponding to reinforcement failure and concrete failure is considered as failure load. When the crack is considered from the back anchors, for 12mm diameter stirrups, the failure load corresponding to reinforcement failure is less than that of concrete edge failure. Therefore, the failure load is equal to that of concrete failure in unreinforced concrete. However, for the other two cases (reinforced concrete with  $d_s16$  and  $d_s16+14$  stirrups), an enhancement of the failure loads is recognized by the Schmid model.

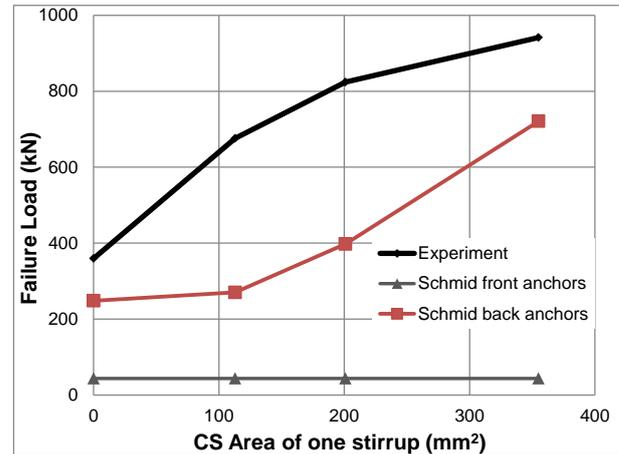


**Figure 4:** Comparison of mean failure loads obtained from the tests [7] with the mean failure loads predicted by Schmid [4] for groups 2x2

Nevertheless, the failure loads evaluated by the Schmid [4] model are rather conservative compared to the experimental failure loads. Another important aspect to note is that the trend of failure loads with respect to cross-sectional area of stirrups given by the Schmid [4] model (first constant and then increasing) is opposite to the real trend observed from the experiments (first increasing and then getting saturated). This is due to (i) considering only the contribution of concrete or reinforcement, whichever is greater, and (ii) no cap on failure load due to strut failure.

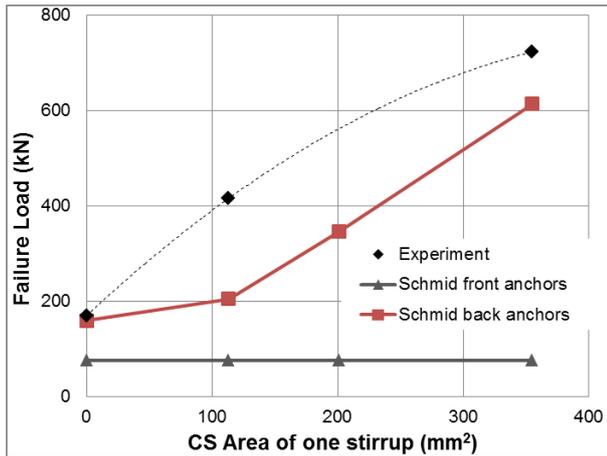
Figure 5 shows the comparison of the mean failure loads evaluated using Schmid [4] model and the mean failure loads obtained from the experiments for group 4x2. When the crack is considered from front anchors, again no stirrups are considered effective and hence

the failure load in reinforced concrete is the same as that in unreinforced concrete. Assuming the failure crack starts from the back anchors, the trend of the failure loads as predicted by the Schmid model is similar to that observed for the EN1992-4 model shown in the accompanying paper [7]. Although, as per Schmid model, more number of stirrups are activated on either side of the anchorage compared to the EN1992-4 model, the predicted loads are only marginally higher compared to the EN1992-4 model. This is because except for the first stirrups intercepted by the theoretical crack, all the other stirrups are assigned a value of  $\psi_1 = 0.16$  (see Eq. 3 above). Clearly, the predicted failure loads are always very conservative compared to experimental failure loads.



**Figure 5:** Comparison of mean failure loads obtained from the tests [7] with the mean failure loads predicted by Schmid [4] for groups 4x2

The comparison of mean experimental and analytical failure loads as per Schmid model for the 2x4 anchor group is given in Figure 6. It may be noted that in this case, the stirrups lying between the outermost anchors as well as the stirrups closest to outermost anchors are assigned a value of  $\psi_1 = 0.95$  as per Schmid model (see Eq. 3). Consequently, the failure loads for this group as per the Schmid model considering the failure crack from back anchors are very similar to the experimental failure loads compared to the prediction by the EN1992-4 model (shown in [7]).



**Figure 6:** Comparison of mean failure loads obtained from the tests [7] with the mean failure loads predicted by Schmid [1] for groups 2x4

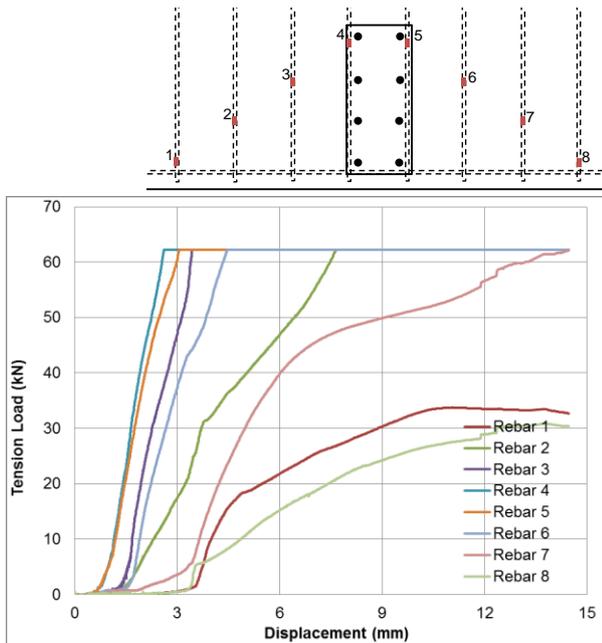
In summary, it was observed that if the failure crack is assumed from the front anchors, in none of the cases, any contribution from the stirrups could be considered because the anchorage length of the stirrups was smaller than the minimum required value. When the crack was considered from rear anchors, the stirrups contribution becomes significant. However, in the Schmid model, similar to the EN1992-4 model, either the concrete capacity or the reinforcement capacity, whichever is greater, is considered as the load carrying capacity of the anchor group. Therefore, in certain cases with low amount of reinforcement, the addition of reinforcement did not enhance the load carrying capacity of the anchor group loaded in shear perpendicular to the edge. Thus, the current models to evaluate failure loads for anchorages with more than one anchor row perpendicular to the edge in reinforced concrete loaded in shear perpendicular to the edge, are in general, over conservative even when the crack is assumed from the back anchors. Further, since no cap on the load carrying capacity for strut failure is assumed in both models, the analytical failure loads have a tendency to be unconservative for high amounts of shear reinforcement in the concrete slabs (higher than provided in the tests). Therefore, there is a need to develop a more rational and reliable method to analytically evaluate the shear failure loads of anchorages with more than one anchor row in reinforced concrete

#### 4 DETAILED EVALUATION OF TEST RESULTS

The Schmid model was developed based on the tests on single anchors and anchor groups with a single anchor row only. Further in these tests, the reinforcement did not yield. In order to understand the real behavior of the anchorages in reinforced concrete, the test results reported in [7] are evaluated in detail.

As mentioned in [7], in two of the three slabs reinforced with 12mm diameter stirrups, strain gauges were provided at specified locations where the . The readings from the strain gauges were evaluated in order to estimate the amount and extent of forces carried by reinforcing bars. The strain gauges were provided on the reinforcing bars at the location where the anticipated crack would intersect the reinforcing bars. The strains recorded by the strain gauges were converted into stress in the reinforcing bar assuming an elastic-perfectly plastic stress-strain curve with a yield stress of 550 MPa, which was justified by the actually measured stress-strain curves for the rebar used. The stress in the reinforcing bar was multiplied by the cross-sectional area to obtain the tensile force carried by the rebar. The tensile forces carried by all the reinforcing bars intercepted by the crack were added up to obtain the total tension force carried by the activated stirrups. This total tensile force was converted by using Eq. (1) into the contribution of reinforcing bars towards resisting the applied shear loads. The difference of the total applied shear load and the shear force resisted by stirrups gave the contribution of concrete in taking up the shear forces.

Figure 7 displays the tension force carried by individual reinforcing bars for the tests performed on 4x2 group. For this group, four stirrups on either side of headed studs were provided with strain gauges. It can be observed that load is first carried by stirrups marked 4 and 5 that are closest to the headed studs. When these stirrups are stressed close to yield, the next two stirrups marked 3 and 6 become more effective.



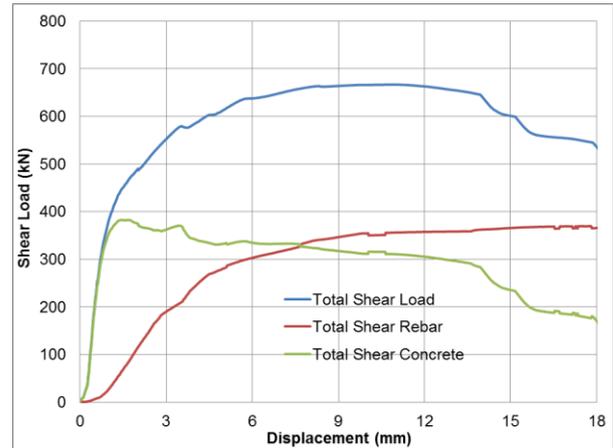
**Figure 7:** Force carried by individual stirrup as a function of displacement for the group 4x2

Similarly, once these stirrups reach yield, the next stirrups marked 2 and 7 become effective and finally the stirrups 1 and 8 that are farthest from the headed studs become active. When the peak load is reached, six out of eight stirrups have reached the yield strength (62.2 kN). This suggests that the effectiveness of the stirrups in carrying load depends on yielding or non-yielding of the stirrup closer to the anchorage intersected by the crack than the stirrup in consideration. This aspect could not be captured in the tests performed by Schmid [4] because in those tests, none of the stirrups yielded.

The tensile forces carried by the individual rebars were added and converted to the shear contribution of the rebars taking into account the lever arm between the line of action of applied force and position of stirrups (compare Eq. 1). This shear contribution of the rebars is plotted as a function of displacement in Figure 8. The total contribution of reinforcing bars towards carrying the shear forces was deducted from the total applied shear force to obtain the shear force carried by concrete.

Initially, the plot of total load coincides with the plot of concrete contribution. After reaching a shear load of approximately 350kN, which is close to the failure load valid for concrete edge failure in unreinforced concrete,

the concrete contribution saturates and the reinforcement starts to take up load.



**Figure 8:** Separate contribution of concrete and reinforcement towards resisting the applied shear load in case of tests performed on 4 x 2 group

Even at relatively small amount of reinforcement, the contribution of the reinforcement in taking up shear loads is significant. It is interesting to note that once the applied load reaches the load corresponding to concrete edge failure, the concrete does not drop the load suddenly but continues to carry this load even at very large displacements. However, this could be due to the fact that the uplift of the base plate was prevented by the test setup. Similar behavior was observed in all the tests. The peak load corresponds to the point when reinforcement reaches its yield strength and concrete retains the load corresponding to concrete edge failure.

Based on these observations, a new model is proposed to evaluate the failure loads for anchorages with supplementary reinforcement loaded in shear perpendicular to the edge.

## 5 NEW MODEL

### 5.1 Description of the proposed model

Based on the information gathered from the tests, a new model is proposed that is, in principle, a modification of the existing Schmid [4] model. The following major modifications are proposed in the existing Schmid [4] model:

1. The failure crack is always assumed from the back row of anchors.
2. The value of the effectiveness factor,  $\psi_{1,i}$  for the  $i^{\text{th}}$  stirrup, used in the Schmid model (Eq. 3) is dependent on yielding/non-yielding of the  $(i-1)^{\text{th}}$  stirrup previously intercepted by the crack.
3. The total failure load for an anchorage is given by adding the contribution of concrete to the contribution of reinforcement

In the proposed model, the following step-by-step procedure is followed to assign the value to the effectiveness factor,  $\psi_{1,i}$ .

Step 1: Assign the effectiveness factor,  $\psi_{1,1} = 0,95$  to the stirrups lying between the outermost anchors as well as to the stirrups that lie outside the anchorage but would be first intercepted by the theoretical crack (same as in original Schmid model).

Step 2: Evaluate the hook resistance (Eq. 3) and bond resistance (Eq. 6) for the first intercepted stirrups and obtain total stirrup resistance (Eq. 2)

Step 3: If yielding of the stirrups first intercepted by the crack takes place, the next stirrup is assigned a value of effectiveness factor,  $\psi_{1,2} = 0,95$  else,  $\psi_{1,2} = 0,16$

Step 4: Repeat steps 2 and 3 for all the stirrups intercepted by the theoretical crack

Through this procedure, the individual contribution of the activated stirrups is considered one after the other.

Further, in this model it is proposed that the peak failure load is given as the failure load corresponding to concrete edge failure in unreinforced concrete plus the load corresponding to reinforcement failure calculated in accordance with the new model. This is in contrast to the existing Schmid [4] model where the maximum of the failure loads corresponding to concrete failure in unreinforced concrete and reinforcement failure is considered as the failure load for the anchorage. Thus, as per new model, the mean shear resistance for an anchorage is given by

$$V_{Rm} = V_{Rm,c} + V_{Rm,re} \quad (8)$$

## 5.2 Comparison with experimental failure loads

Following the step-by-step procedure, the failure loads were calculated for all anchor groups in reinforced concrete based on the new model. It may be noted that in the new model, the contribution of only edge reinforcement is unaccounted for due to its unreliable nature. Therefore, no difference in the failure loads in case of group 1x2 is observed between original Schmid [4] model and the new proposed model. This is therefore, not repeated here.

In the following comparison, steel shear failure load of headed studs are considered as well based on the mean ultimate strength of the headed studs (518.3 MPa). The mean value of strength in shear to strength in tension is considered as 0.75. Therefore, the mean shear strength of a single headed stud is obtained as 147.8 kN.

Figure 9 displays the comparison of experimentally obtained and theoretically predicted mean failure loads for the group 2x2 as a function of the area of reinforcing bars used as stirrups.

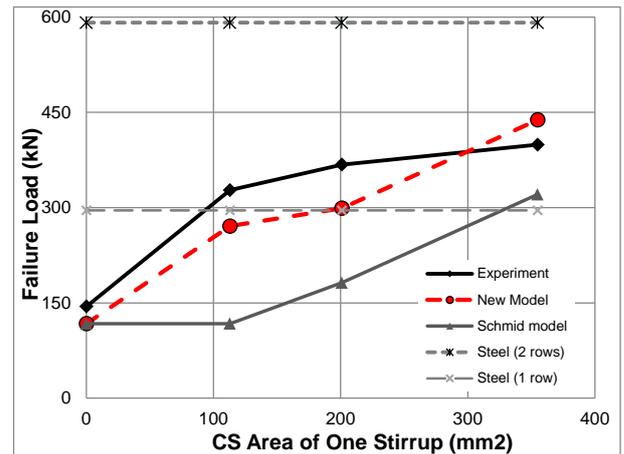


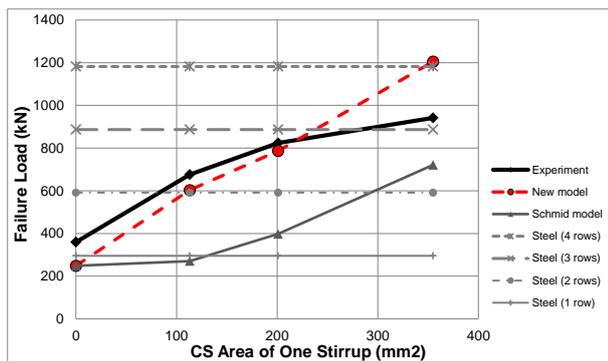
Figure 9: Comparison of mean failure loads obtained from the tests [7] with the mean failure loads calculated using the new model for the groups 2x2

It is clearly observed that the failure loads predicted by the new model are much closer to the actual failure loads obtained from the

experiments, compared to the loads predicted by the Schmid [4] model. Thus, the new model provides a significant improvement to the original Schmid model. It should further be observed that the anchor steel failure load, if only one anchor row (i.e. two studs) are participating is exceeded by all the tests performed in reinforced concrete, clarifying that more than one anchor row is participating towards resistance to anchor steel failure.

For the case of tests performed in unreinforced concrete, and tests performed in concrete slabs reinforced with stirrups of 12mm and 16mm diameter, the calculated failure loads as per new model are not only close to the experimental failure loads but also follow the same trend. This is an important aspect, which is not captured by the Schmid model or the EN1992-4 model. For the case of tests in concrete reinforced with ds16+14 bundled stirrup, the calculated failure load is slightly over-predicting the measured failure load. This may be attributed to the fact that in the tests, strut failure may have been started. This suggests that it is important to provide an upper limit, given by strut failure, to the enhancement of the failure load for anchor groups due to presence of reinforcement.

For group 4x2 with four anchor rows perpendicular to the edge, the comparison of experimentally determined mean failure loads and those calculated using the new proposed model is given in Figure 10.



**Figure 10:** Comparison of mean failure loads obtained from the tests [7] with the mean failure loads calculated using the new model for the groups 4x2

Again, the calculated loads for the group as per new model show a significant

improvement over the loads calculated by Schmid model, when compared to the experimental mean failure loads.

The failure loads predicted by the new model for group 4x2 in concrete reinforced with 12mm and 16mm stirrups are very close to the experimental results. The experimental failure mode obtained for these cases was reinforcement failure. Thus, it can be said that the new model is able to predict the failure loads corresponding to reinforcement failure quite realistically. The steel failure load, if only one anchor row is assumed to participate, is exceeded even by the tests performed in unreinforced concrete. This proves that the current assumption of only one anchor row contributing towards steel resistance is very conservative. For the case of tests in concrete reinforced with ds16+14 bundled stirrups, the calculated failure load is significantly higher than the measured value. However, the test results show that the failure load for this case was limited by strut failure.

## 6 APPROACH FOR STRUT FAILURE

Through the experimental results, it is clear that although the capacity of the anchorage under shear loading can be increased by providing anchor reinforcement, this increase is not unlimited. Beyond a certain amount of supplementary reinforcement, the failure load for the anchorage is limited by the capacity of the concrete struts provided that anchor steel failure does not occur. This is the absolute upper limit for the concrete edge resistance of an anchorage group. However, so far, there is no information on this limit due to strut failure and consequently En1992-4 [1] does not give any guidance to consider it. The original Schmid model [4] tries to eliminate strut failure indirectly, by limiting the applicability of the model to stirrups with  $d_s \leq 20$  mm. However, this approach may not be objective. Therefore, it is required to include parameters in the model to deal with strut failure in a direct way.

An approach to consider strut failure for anchorages with headed studs in reinforced concrete subjected to tension loads is proposed

by Berger [5]. The strut formation for headed studs with supplementary reinforcement loaded in shear perpendicular to the edge can be considered analogous to the strut formation for studs loaded in tension and enclosed by supplementary reinforcement. Considering an analogous approach to the one proposed by Berger [5] for tension, the maximum failure load for the anchorage with supplementary reinforcement would be given as

$$V_{Rm,max} = \psi_{strut,V} \cdot V_{Rm,c} \quad (9)$$

Thus, the factor  $\psi_{strut,V}$  is the ratio of maximum achievable strength corresponding to shear failure of an anchorage with supplementary reinforcement to that of the same anchorage in unreinforced concrete. As per Berger [5], for tension loads, this ratio is equal to 2.5.

From the tests performed on anchor groups with 2 anchors in a row, i.e. groups 1x2, 2x2 and 4x2, the ratio of mean peak loads in reinforced concrete to mean peak loads in unreinforced concrete are tabulated in Table 1.

**Table 1:** Ratio of mean peak loads for anchorages with supplementary reinforcement to that of anchorages in unreinforced concrete

Stirrups	Group 1x2	Group 2x2	Group 4x2
None	1.00	1.00	1.00
ds12	2.18	2.27	1.88
ds16	2.61	2.55	2.29
ds16+14	2.40	2.77	2.62

From Table 1, the average of the highest value of the ratio of shear strength of the anchorage with supplementary reinforcement to that of the anchorage in unreinforced concrete is obtained as 2.67. Thus, the coefficient for strut failure of anchorages loaded in shear perpendicular to the edge is given as

$$\psi_{strut,V} = 2,67 - 1,11 \frac{x}{c_1} \geq 1,0 \quad (10)$$

Where,

$x$  = distance between the nearest stirrup and outer anchor, and

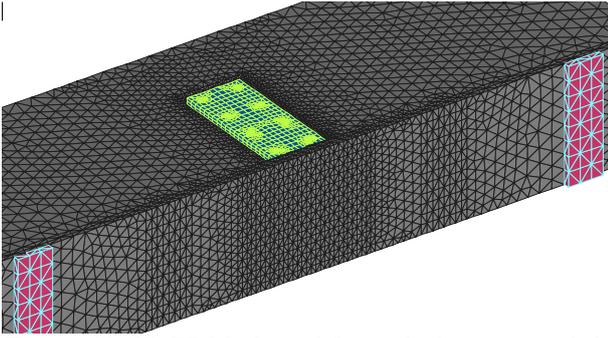
$c_1$  = edge distance

For the group 2x4, with four anchors in a row, the ratio of maximum capacity in reinforced concrete to the capacity in unreinforced concrete as observed from the tests was 4.27. The coefficient to calculate strut failure load using Eq. (10) comes out to be 2.32, which would be over-conservative for this case. This is because, in case of anchorages with 4 anchors in a row, more struts than in case of anchorages with 2 anchors in a row can form. Due to large number of possible struts, a stronger resistance against strut failure is obtained. Further research is needed to consider this aspect towards strut failure capacity of the anchorages subjected to shear forces.

## 7 ANCHOR STEEL FAILURE

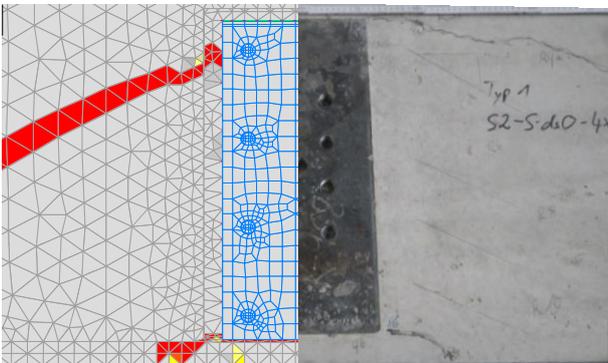
As mentioned in the accompanying paper [7], if the failure crack for concrete edge failure is assumed from the back anchor row, then only the anchors in the last anchor rows are considered to contribute towards anchor steel failure [2-3]. It was observed from the tests that the failure crack always appears from the back anchor row. From the evaluation presented in [7], it is clear that the assumption of only one anchor row participating in steel failure is very conservative and the test results indicate that probably all the anchors take up shear loads. However, anchor steel failure did not occur in any of the tests.

In order to investigate the participation of anchors in taking up shear loads, numerical simulations within the framework of fracture mechanics approach were performed for 4x2 anchor group in unreinforced concrete using 3D FE software MASA at University of Stuttgart. Figure 11 shows the FE model utilized for the calculations. The nonlinear concrete behavior was modelled using Microplane model with relaxed kinematic constraint [8], while the von-Mises yield criterion was considered for steel anchors. Trilinear stress-strain curve was used as the constitutive law for steel.



**Figure 11:** 3D FE Mesh used for analysis on group 4x2

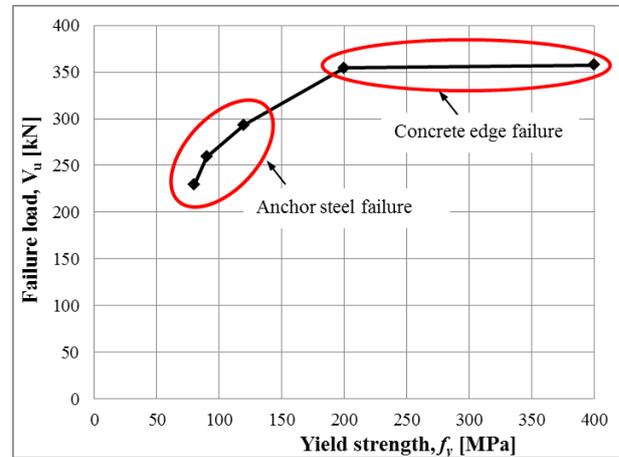
First the numerical model was validated against the experimental results. For this analysis, based on the test results, the yield strength of the steel was taken as 401 MPa and the ultimate strength as 517 MPa. Figure 12 shows the comparison of experimental and numerical crack patterns. Both the cracks appearing from the front anchor row as well as from the back anchor row could be captured using the numerical model. The mean experimental failure load was obtained as 359.9kN, while the numerically obtained failure load was 357.8kN.



**Figure 12:** Comparison of experimental and numerical crack patterns obtained for the group 4x2

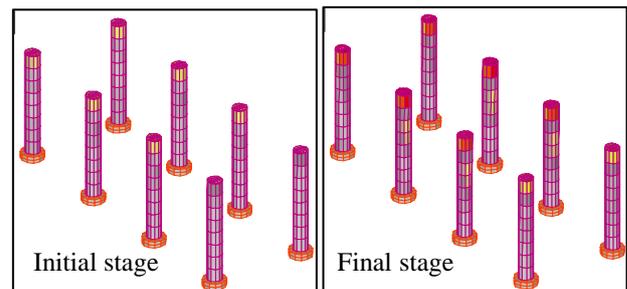
The validated numerical model was then used to carry out further studies by keeping every other parameter the same except the steel strength. Four additional cases were analyzed considering the steel yield strength as 200MPa, 120MPa, 90MPa and 80MPa. The ultimate strength was taken such that the ratio of ultimate to the yield strength remained same as in case of the steel used in the tests. Figure 13 shows the influence of anchor steel strength on the failure load obtained for this group. As

expected, for high steel strength, a clear concrete edge failure is obtained and there is almost no influence of steel strength. For smaller values of steel strength, anchor steel failure occurs and the failure load increases with the steel strength.



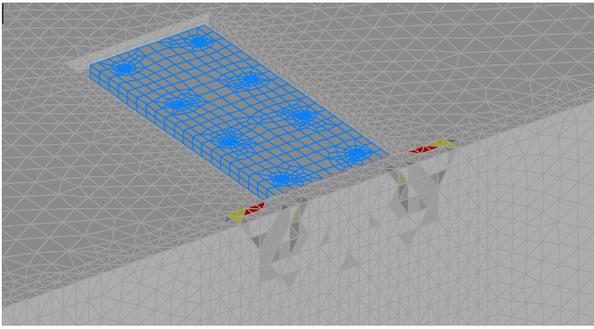
**Figure 13:** Influence of anchor steel strength on group failure load obtained for the group 4x2

The stresses in the headed studs were evaluated from the numerical analysis. Figure 14 shows the stress distribution in the anchors obtained in case of concrete edge failure. It can be noticed that initially, all the anchors take up the load equally. After the formation of the front crack, the front anchor row releases a part of the load, while the other three anchor rows are fully active.



**Figure 14:** Stress distribution in anchors for group 4x2

In case of groups with low steel strength as well (90 MPa), the failure crack from the front anchors appear (see Figure 15). The evaluation of the stresses in the headed studs shows a similar pattern as shown in Figure 14. Therefore, the numerical results indicate that initially all the anchors take up the shear loads until the crack from the front anchors form.



**Figure 15:** Cracks appearing from front anchor row even in case of anchor steel failure for the group 4x2

After this, the front anchors release a part of the load (to approx. 50%), while the other anchors fully participate in taking up shear forces. These calculations must be confirmed through experiments, where the steel failure may be achieved easily by using smaller sized anchors instead of using low steel strength.

## 6 CONCLUSIONS

In this paper, a new model is proposed to calculate failure loads for anchor groups in with supplementary reinforcement loaded in shear perpendicular to the edge. The model is based on the detailed evaluation of the test results reported in the accompanying paper [7] on anchor groups with up to four anchor rows perpendicular to the edge tested in concrete with four different levels of shear reinforcement.

The new model is based on realistic assumptions for participation of activated stirrups as well as combination of concrete and reinforcement contribution towards the shear resistance of the groups. The model is able to predict the failure loads of anchorages in reinforced concrete quite well, if the failure mode is governed by reinforcement failure. In order to consider the upper limit of beneficial effect of the supplementary reinforcement, an approach to consider strut failure is included in the model.

Further, it has been shown with the help of numerical analysis that even when the failure crack appears from the back anchor row, all the anchors take up the shear loads, however, after the appearance of the crack from front anchor row, the participation of the front

anchors is reduced. These results need to be confirmed by experiments.

## 7 ACKNOWLEDGEMENTS

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