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THE HOMOGENIZATION OF A MASONRY UNIT CELL USING A LATTICE APPROACH: UNIAXIAL TENSION CASE

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Abstract: This study provides an explanation on how a lattice model is employed in homogenizing a heterogeneous anisotropic masonry unit cell made of brick, mortar and their interface using energy equivalence concepts. The direct tension test was only considered in this study. Other loading scenarios like shear and compression may also be included using the same approach. The purpose is to obtain a post-peak scalar damage parameter of a homogenized isotropic finite element from the fracture energy results of a lattice masonry unit cell. A 2-D plane strain lattice formulation was implemented to evaluate energy release rate values of the masonry unit cell. Different strength failure criteria were assigned to brick, mortar, and interface strength material properties that were mapped on top of the mechanical model of the lattice according to their geometric locations. An energy method was subsequently employed to obtain the energy release rate of the lattice mesh as the crack propagates which was obtained by the change in the global stiffness matrix of the lattice approach before and after of strut removal. It was assumed that the total strain energy released in the lattice masonry unit cell in direct tension as the crack propagates equals the total strain energy dissipated in the equivalent homogenized isotropic continuum finite element under the same loading. Since these dissipated energy values correspond to the crack propagation and the damage incurred in the masonry unit cell, a scalar damage parameter can be defined based on the dissipated strain energy and energy release rate values during the analysis. The homogenization technique may be regarded as a bridge between the micro-scale lattice analysis and macro-scale masonry wall.

1 INTRODUCTION

This study investigates how a lattice approach is used in homogenizing a masonry unit cell comprised of brick units, mortar joints and their interfaces according to an energy method. The masonry unit cell in direct tension loading scenario was considered to obtain a post-peak scalar damage parameter of a homogenized isotropic finite element from the fracture energy results of a lattice masonry unit cell. A 2D plane strain lattice approach was regarded to discretize the continuum domain of the unit cell based on Voronoi tessellation. The constructed Voronoi polygons or convex rigid particles have a point inside called nucleus or centroid which has a specific geometric definition [1,2]. These centroids constitute the computational points of the lattice with the defined degrees of freedom. For further details on the lattice approach, readers are encouraged to see Refs. [1–4].

Material structure overlay is one of the attractive features of lattice models where the material structure and the mechanics model are combined. Hence, different strength failure criteria were assigned to brick, mortar, and interface strength material properties that were mapped on top of the mechanical model of the lattice according to their geometric locations. **Figure 1** illustrates the material properties' assignment to the mechanical model of the masonry unit cell. 'Black', 'blue', and 'pink' struts in **Figure 1** represent elements on which the material properties of brick, mortar and interface are projected, respectively. The coordinates of each strut's nodes are firstly considered, having three phases of material properties. If both nodes of an element fall within a single phase, then that phase's material properties are assigned to that of the element. This is valid for the brick and mortar phases. However, if one node is located on one phase and the other node sits on another phase, then that element is considered as an interface receiving material properties of interface element. The connecting lines between the nodes in the lattice mesh represent the truss or frame elements which are the mechanical model of the numerical lattice simulation.

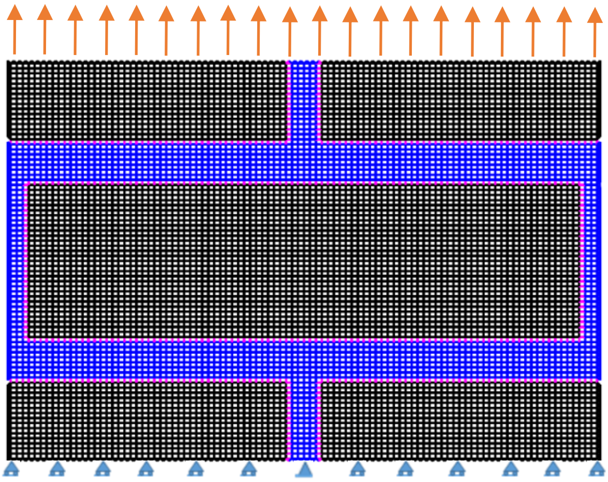

Figure 1 Overlaying material properties onto lattice structure. Black, blue and pink represent brick, mortar, and interface material properties, respectively.

Number of degrees of freedom at nodes or computational points determine the type of element in the simulation. In this study, 2-D 'frame' elements with three degrees of freedom at nodes were considered. With regard to constitutive behavior of struts in the lattice, a formulation introduced in Ref. [1] was taken into account which is helpful in determining the material properties of the interface struts.

Fracture analysis of brittle material has been the topic of many studies, e.g., [5,6] The simulation of fracture, in this study, was performed with a 'linear elastic' analysis of the lattice under loading and removing an element from the mesh which exceeds a certain fracture criterion, for instance a tensile or compressive stress based on the failure envelope. The 'gap' between the remaining elements is considered as a discontinuity or crack in the lattice mesh. After removing the element, the lattice mesh contains one less element. The simulation is continued by performing a linear elastic analysis of the new mesh, where the forces that were carried by the removed element are now redistributed over the neighboring elements. This procedure continues until the next element satisfies its 'fracture criterion', and so on [7,8]. Thus at each step, the external load on the lattice is increased and the critical element at the fracture threshold is removed. The erosion strategy leads to an 'instantaneous relaxation' of the load, carried by that removed part of the lattice. This was often observed during the lattice analyses of this study as a sudden drop in form of snap-backs in the load-displacement diagrams.

Fracture criterion for the failure of brick and mortar frame elements, i.e., "black" and "blue" struts in **Figure 1**, was defined as a function of normal force and bending moments at computational points of each frame member as

$$\sigma_{eff} = \frac{N}{A} \pm \alpha' \frac{(|M_i|,|M_j|)_{max}}{S} \geq f_t \ or \ f_c, \qquad (1)$$

where N is the normal force of the lattice element; A is its cross sectional area; M_i and M_j are the bending moments at the nodes i and j, respectively; S is the section modulus; f_t and f_c are the tensile and compressive strengths of the material, respectively; and $0 \leq \alpha' \leq 1$ is added to limit the effect of bending in the fracture law. The fracture criterion for the brick-mortar interface was determined based on a combination of experimental measurements and numerical parameter simulations. **Figure 2** shows the failure condition considered for the interface frame elements in the lattice model. This failure envelope has a compressive cap which was necessary for the simulation of triplet test under high normal confinements. It should be noted that the shear failure surfaces were neglected for brick and mortar elements since the main focus of this study was to evaluate the brick-mortar interface fracture properties, and with the simulations conducted, their shear failure envelopes were not activated. For details on how the material properties for brick units, mortar joints, and their interface were determined using Digital Image Correlation and image processing techniques see Refs. [2,3,9,10]. The same method was also used to evaluate the damage incurred in concrete structures [11,12]

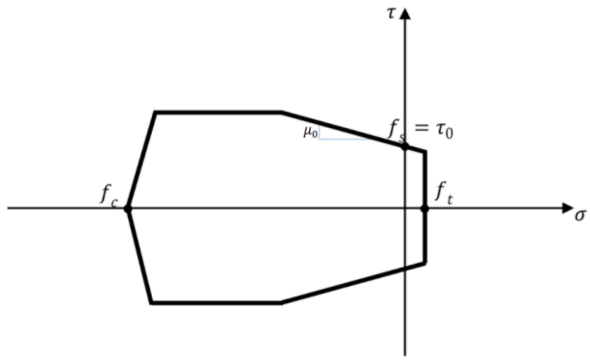

Figure 2 Failure surface for the brick-mortar interface employed in this study.

2 ENERGY APPROACH

The implemented 2D lattice model explained in Section 1 is capable of simulating crack path evolution in the form of strong discontinuities at a homogeneous or heterogeneous solid. Since the crack propagation is captured by the lattice during an analysis, it is postulated that the fracture mechanics quantities like the energy release rate or the stress intensity factors associated with the evolving crack may be determined by the lattice.

The main quantity to be calculated is the energy release rate of the interface fracture. This value is determined by an energy approach using the total potential energy, Π, of the lattice solution. Assume that a lattice analysis has been performed on a given planar linear elastic body of '*unit thickness*' containing a crack. The total potential energy of the lattice model solution may be expressed as [13], [14]

$$\Pi = \frac{1}{2}\{u\}^T[K]\{u\} - \{u\}^T\{f\}, \qquad (2)$$

where $\{u\}$ is a vector of displacements associated with lattice computational or nodal points or nuclei, $[K]$ is the global stiffness matrix of the lattice mesh, and $\{f\}$ is the vector of prescribed nodal loads. The energy release rate, G, is obtained by differentiating Equation 2 with respect to crack length, a, as [15]

$$G = -\frac{\partial \Pi}{\partial a} = -\frac{1}{2}\{u\}^T \frac{\partial [K]}{\partial a}\{u\}. \qquad (3)$$

Equation 3 is the main ingredient to obtain the fracture properties of the interface using the numerical lattice model. In the numerical solution, $\partial[K]/\partial a$ may be approximated by the ratio $\Delta[K]/\Delta a$ in form of a simple forward finite difference scheme as

$$\frac{\partial[K]}{\partial a} \cong \frac{\Delta[K]}{\Delta a} = \frac{1}{\Delta a}[[K]_{a+\Delta a} - [K]_a], \quad (4)$$

where $[K]_{a+\Delta a}$ is the stiffness matrix after the crack growth Δa. Therefore, using Equation 3 the interfacial energy release rate may numerically be determined by the lattice analysis as the crack propagates through the interface. It should be mentioned that the value of G obtained from Equation 3 is for a specimen with unit thickness. If the thickness $t \neq 1$, then G should be divided by t.

3 HOMOGENIZATION

This section provides a brief explanation on how the lattice model might be employed in homogenizing a heterogeneous anisotropic masonry unit cell made of brick, mortar and their interface using energy equivalence concepts. The direct tension test was only considered here for the sake of simplicity. Other loading scenarios like shear and compression may also be included using the same approach. The purpose is to obtain a post-peak scalar damage parameter for a homogenized isotropic finite element from the fracture energy results of a lattice masonry unit cell. Elastic properties of the homogenized finite element, i.e., equivalent Young's modulus and Poisson's ratio, can easily be obtained from the linear elastic behavior of the lattice unit cell in $\sigma_y - \varepsilon_y$ and $\sigma_y - \varepsilon_x$ planes, respectively. **Figure 1** shows the lattice masonry unit cell under vertical direct tension, the dimensions of which are $8.5\ (in)$ long, $5\ (in)$ high, $4\ (in)$ wide, and with a mortar thickness of $0.5\ (in)$. Brick dimensions are $8\ (in) \times 2(in) \times 4\ (in)$.

It is assumed that the total strain energy released in the lattice masonry unit cell, ∂U_{cell}, in direct tension as the crack propagates equals

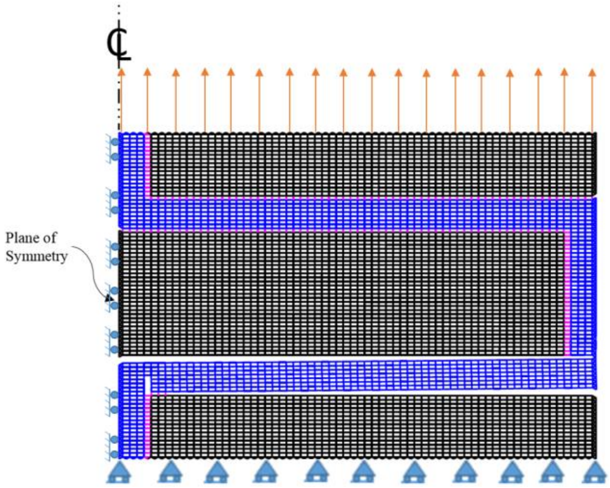

Figure 3 One half of the symmetric masonry unit cell mesh and its boundary conditions under direct tension in the last increment.

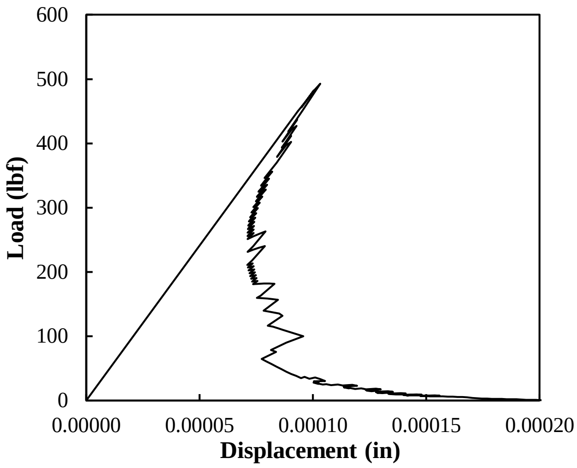

Figure 4 Load-displacement curve of the masonry unit cell in **Figure 3**.

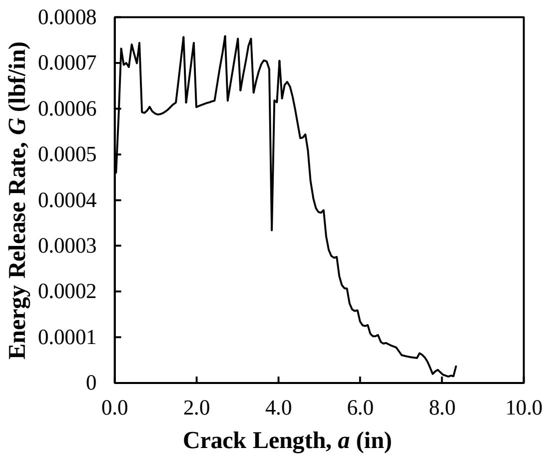

Figure 5 Variation of the energy release rate with respect to crack length for the masonry unit cell in **Figure 3**.

the total strain energy dissipated in the equivalent homogenized isotropic continuum finite element, ∂U_{cont}, under the same loading as

$$\partial U_{cell} = \partial U_{cont}. \quad (5)$$

According to Equation (3), the strain energy released or the change in the strain energy stored in a solid of *unit thickness* for a crack length growth of ∂a can be expressed as

$$\partial U = -\partial \Pi = G\partial a, \text{or } \partial U_{inc} \cong \Delta U_{inc} = G_{inc} \times \Delta a_{inc}, \quad (6)$$

where the subscript inc denotes the increment number in the lattice simulation, ΔU_{inc} is the strain energy dissipated for a crack length growth of Δa_{inc}, and G_{inc} is the energy release rate obtained from the lattice. For solids of thickness t, the value of ΔU_{inc} in Equation (6) must be multiplied by t. It should be noted that for increments where there is no increase in the crack length, i.e., $\Delta a_{inc} = 0$, there is then no strain energy dissipation and $\Delta U_{inc} = 0$.

Since these dissipated energy values correspond to the crack propagation and the damage incurred in the masonry unit cell, a scalar damage parameter can be defined based on the dissipated strain energy and energy release rate values during the analysis. Let D_{INC} and ΔU_{total} be the scalar damage parameter at increment INC and the total dissipated strain energy for the all increments, respectively. D_{INC} may be expressed as

$$D_{INC} = \frac{\sum_{inc=1}^{INC} \Delta U_{inc}}{\Delta U_{total}} = \frac{\sum_{inc=1}^{INC} \Delta U_{inc}}{\sum_{inc=1}^{INC\_ult} \Delta U_{inc}}; 0 \leq D_{INC} \leq 1, \quad (7)$$

where INC_ult is the ultimate increment number when the analysis is terminated. The numerator of Equation (7) is the accumulated strain energy released up to the increment INC.

Since the mesh and its boundary conditions in **Figure 1** are all symmetric, it is possible to analyze one half of the mesh to reduce computational cost. **Figure 3** illustrates the lattice mesh and boundary conditions of a one half symmetric masonry unit cell under direct tension. The mesh belongs to the last increment of the analysis when $D_{INC} = 1$. It is seen in **Figure 3** that the failure mostly occurred through the interface struts with penetrations into the mortar joints in the last increments.

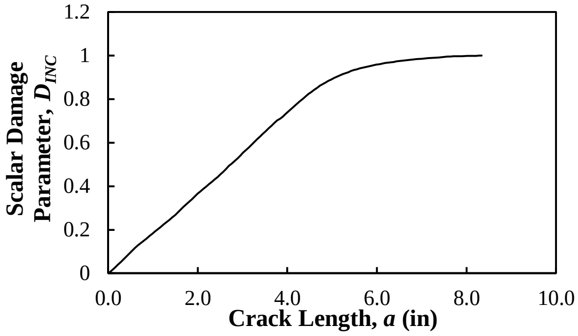

Figure 6 Variation of the scalar isotropic damage parameter with respect to crack length.

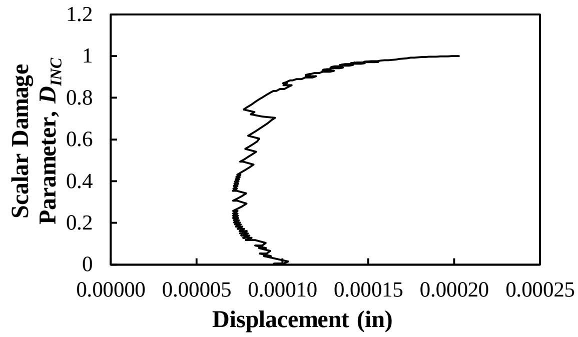

Figure 7 Variation of the scalar isotropic damage parameter with respect to the average displacement of the top nodes in **Figure 3** where the tension traction is applied.

Figure 4 and **Figure 5** depict the load-displacement curve and the variation of G against crack length for this simulation, respectively. Data obtained from **Figure 5** can be used to determine the scalar damage parameter, D_{INC}.

Using the fracture energy release rate values from **Figure 5** and Equation (7), one can determine the scalar damage parameter, D_{INC}, of the unit cell in **Figure 3**. **Figure 6** illustrates the variation of D_{INC} against the crack length propagation. D_{INC} first increases in a constant rate followed by a decreasing rate of change. The major linear part of the curve in **Figure 6** corresponds to the unzipping failure of the lower interface struts in **Figure 3**. In other words, the masonry unit cell experiences more degradation and damage due to the complete failure of the lower interface where $D_{INC} = 0.703$ and the load level drops to about $100\ (lbf)$. Furthermore, **Figure 7** shows the change of D_{INC} with respect to the average displacement of the top nodes in **Figure 3** where the tension traction is applied. Considering the assumption in Equation (5), it is possible to use the damage data in **Figure 7** to model the nonlinear behavior of the homogenized isotropic continuum finite element, equivalent to the anisotropic masonry unit cell in **Figure 1**, under direct tension based on damage formulations.

4 CONCLUSION

A 2D plane strain lattice approach was considered to extract the scalar damage parameter of a heterogeneous anisotropic masonry unit cell to be used in a homogenized isotropic masonry unit cell with elastic properties equivalent to those of the anisotropic one. The fracture energy outputs of the lattice was employed in homogenizing a heterogeneous anisotropic masonry unit cell made of brick, mortar and their interface using energy equivalence concepts. For simplicity, the direct tensile loading scenario was only regarded for this purpose. The post-peak scalar damage parameter of a homogenized isotropic finite element was determined from the fracture energy release rate values of a lattice masonry unit cell under tension. The scalar damage parameter at each increment was calculated from the accumulated dissipated strain energy values up to that increment divided by the total strain energy dissipated throughout the analysis. These damage data in terms of displacements could be used to model the nonlinear behavior of a homogenized isotropic continuum finite element which is equivalent to the anisotropic masonry unit cell under direct tension based on damage formulations. This homogenization technique may be regarded as a bridge between the micro-scale lattice analysis and macro-scale masonry wall. It should be noted that for other masonry unit cell configurations and loadings the lattice analysis with the same approach could be conducted to obtain the damage curve similar to **Figure 7**. However, the general approach is the same and the damage parameter could be extracted from the lattice based on the energy equivalence concept. Other loading scenarios like biaxial tension or tension-compression and pure shear may be the subject of future research.

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