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ENERGY EQUIVALENCE APPROACH FOR ANALYSIS OF CONCRETE BEAMS UNDER MIXED MODE LOADING

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Abstract. Concrete being a composite, heterogeneous material, an accurate prediction of its behaviour under combined bending and shear is an open problem. In this work, a method is proposed to address this problem by bringing an equivalence between the normal stress (σ) and shear stress (τ). Finite element modelling is used as a tool for predicting the response of plain concrete beams under mode I and mixed mode I-II loading. Beams of three different sizes are modelled using finite element analysis and the results are validated through available experimental data. The proposed method considers a scaling factor, which is in proportion to the relative contribution of shear and normal stress. It is observed that the same amount of energy is associated with behaviour under a pure normal stress and a combined normal and shear stress field and consequently, a ratio is proposed to convert shear stress to an equivalent normal stress.

1 INTRODUCTION

Engineering structures such as bridge decks, offshore structures, pavements, wind towers etc. are subjected to a large number of load cycles during their service life. In general, these structures are subjected to a combination of stresses, due to asymmetries in geometry and complexities in loading. The failure of these structures could be due to fatigue under flexure, shear, torsion, or a combination of these. Concrete being a composite, non-homogeneous and non-isotropic material, an accurate determination of fatigue life under combined bending and shear remains an open problem.

For a quasi brittle material like concrete, the mechanism of fatigue is characterised by the formation of microcracks ahead of the crack tip known as a fracture process zone. These cracks eventually coalesce to form a propagating macro crack [1]. Although structural components in reality are frequently subjected to shear stresses, the behaviour of concrete under shear is not yet clearly understood. The shear behaviour of concrete beams depends on the development of two shear load transfer mechanisms namely, arch action and beam action. The extent of arch and beam action depends highly on the shear span to depth ratio of the beam [2]. Further studies are required to investigate the shear failure behaviour of concrete beams for various positions of loads that trigger shear cracks. This is critical for heavily reinforced beams and deep beams for which the predominant mode of failure is shear.

Many studies have been conducted to analyse plain and reinforced concrete beams under shear or a combination of shear and bending for both monotonic and fatigue loading. Ballatore et al. [3] studied two potential failure mechanisms, namely flexural failure at supports and mixed mode cracking, in concrete beams under four point bending by varying the distance between loading points. Mixed mode crack propagation was observed for smaller separation between load application points and larger depth of beam. Shear behaviour of reinforced concrete beams without stirrups was studied by Khuntia and Stojadinovic [4] and they proposed an approach that can be applied to members with any shear span to depth ratio, reinforcement ratio, concrete strength, presence of axial compression/tension, member size, loading type, and support conditions. Dong et al. [5] compared various stress based criteria for analysis of concrete specimens under mixed mode loading and concluded that mode II stress intensity factor has a significant impact on initial cracking load and crack propagation direction. Carmona et al. [6] conducted experimental studies on mixed mode crack propagation in reinforced concrete beams using asymmetrically notched specimens and observed that the final stretch of crack propagation induced a sudden drop in carrying capacity of the beam. Another model proposed by Gallego et al. [7] suggests that fatigue failure occurs when the propagation of diagonal crack has reduced the depth of compression zone so that it cannot resist the compressive stresses acting on it. But the methods available from literature are quite complex and give a large scatter in results because of numerous interacting parameters such as concrete strength, specimen size, shear span to depth ratio etc. In addition, it is found that the models available to predict crack propagation under flexure have a better correlation with experimental results. Converting the actual mixed mode loading to an equivalent mode I loading would simplify the analysis considerably. This could be done by considering a scaling factor between the contribution of normal stress and that of shear stresses. Anes et al. [8] proposed a stress scaling factor to convert the different applied stresses to a common stress space in case of metals. It was based on the assumption that different stress amplitudes in tensile and shear loading can lead to the same fatigue life. A stress scaling factor was defined considering axial stress amplitude and ratio of axial stress to shear stress as parameters. Another stress based criteria was proposed by Sajjadi et al. [9] for analysing a mixed mode problem by converting to an equivalent mode I problem. An equivalence of tangential stress was considered as the base for this approach.

The review of literature shows that the concept of scaling factor has been applied to metals and rocks by adopting a stress-based criterion. Such an attempt has not been reported on concrete. Considering the heterogeneity of concrete and possible combinations of loading, the complexity of the problem increases. Consequently, an energy based approach is suitable for modelling a quasi brittle material such as concrete as it avoids the need to consider singularity at crack tip.

In the present work, a method is proposed to bring in the normal and shear stresses to a common stress space using a scaling factor. A systematic procedure is developed in order to analyse plain concrete beams under mixed mode I-II loading conditions and define the scaling factor using the concept of energy equivalence.

2 PROPOSED METHOD

Plain concrete beams subjected to three point bending are considered in this work. Initially a finite element model for a beam subjected to monotonically increasing vertical displacement is set up in a commercial finite element code (ATENA) and the results are correlated with available experimental observations. The model is then calibrated to obtain a good agreement between the load-CMOD behaviour obtained from finite element analysis (FEA) and experimental results. Once the calibrated material properties are obtained, this model is further used to analyse the beam under several mode I and mixed mode I-II loading conditions. From the area under load-CMOD behaviour in each case and using the concept of energy equivalence, a method is proposed to separate the contribution of normal and shear stresses. The various steps involved are discussed in the subsequent sections.

3 CALIBRATION OF FE MODEL

Beams of three different sizes under three-point bending are modelled using the finite element code ATENA. The specimens subjected to monotonically increasing vertical displacement at midspan are considered for calibrating the model. Geometry, material properties and experimental results taken from Keerthana and Kishen [10] are used for this purpose. All specimens are geometrically similar with a notch to depth ratio of 0.2. Details of specimen geometry are tabulated in Table 1.

Table 1: Geometry of specimens

Specimen	Span S(mm)	Depth D(mm)	Thickness B(mm)
Large	1200	300	50
Medium	600	150	50
Small	300	75	50

The material model in ATENA is based on the fictitious crack model [11] and the propagation of crack is governed by exponential crack opening law. Stress strain behaviour is defined through equivalent uniaxial law which takes into account the non-linear behaviour of concrete in compression and tension as well as reduction in compressive strength after cracking. Beams are meshed using isoparametric quadrilateral elements and subjected to displacement-controlled load applied in steps of 0.01mm. Load versus CMOD behaviour obtained from FE analysis are compared with experimental curves and the material properties are calibrated so as to get a good match with experimental results as shown in Figure 1. It can be observed that the initial slope and maximum load values are in good agreement with experimental results. The final calibrated material properties in the FE analysis are tabulated in Table 2 and this calibrated model is now used for further analysis under mixed mode loading conditions.

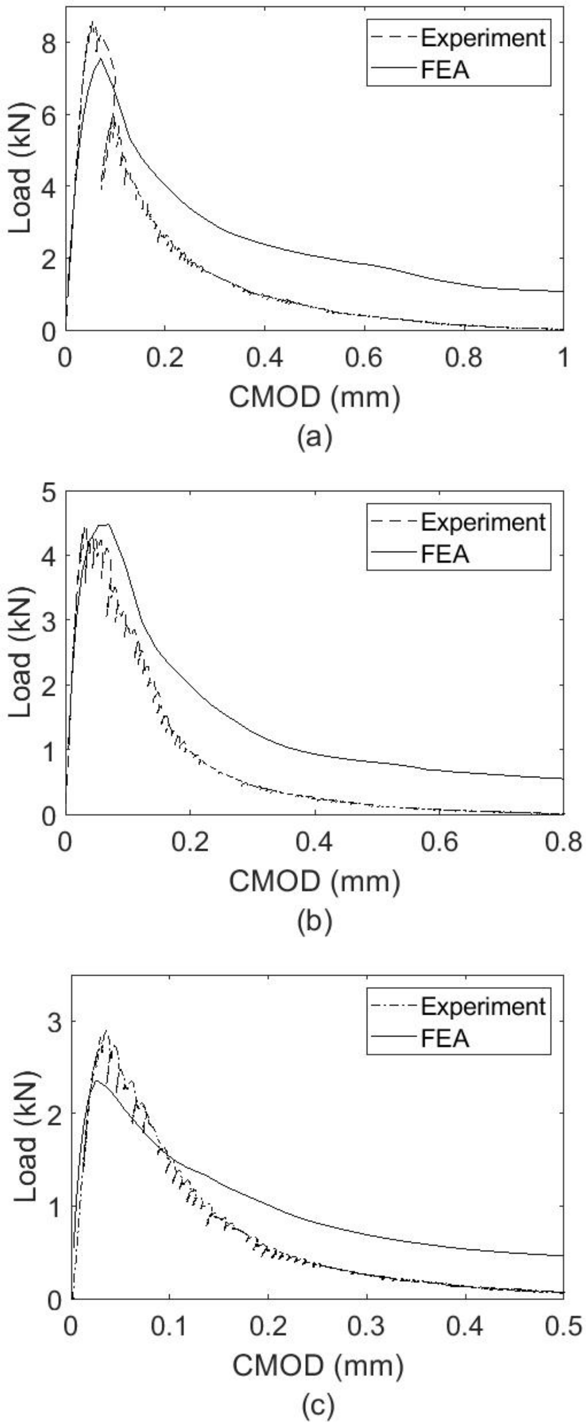

Figure 1: Load-CMOD curves for (a) large (b) medium and (c) small specimens

4 ANALYSIS FOR MIXED MODE LOADING CONDITIONS

Two sets of analysis, as represented in Figure 2, are carried out on plain concrete beams of three different sizes. In set 1, a notch is positioned at the same cross section as the load and the position of both together are varied across the span of the beam, i.e. $d_P = d_N$ and $0 < (d_P, d_N) < L$, where L is the span of the beam and d_P and d_N are the distances of point of load application and notch respectively, both measured from left support. In this case, crack which initiates from the critical section propagates vertically upwards towards the load application point under the influence of normal stresses alone. Thus, predominant mode I conditions exist at the crack tip for set 1.

Table 2: Calibrated material properties

Compressive strength, f_{ck}	56 MPa
Modulus of Elasticity, E	29400 MPa
Tensile strength f_t	3.6 MPa
Poisson's ratio, μ	0.12
Fracture energy, G_f	0.16 N/mm

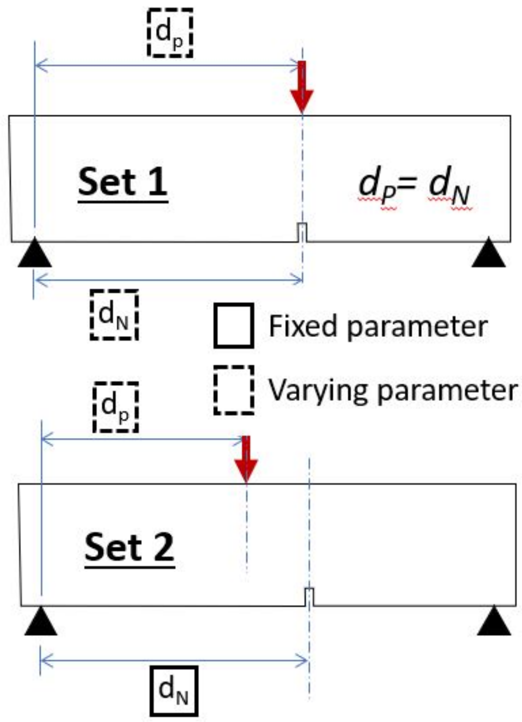

Figure 2: Schematic representation Set 1 and Set 2

In set 2, the position of notch (d_N) is kept fixed near midspan of the beam and the point of application of load (d_P) is varied across the span, $0 < d_P < L$. Here, the crack propagates in an inclined direction from the tip of the notch towards load point under the influence of both normal and shear stresses, making this a mixed mode case. Under each set 8 to 10 cases are considered by varying the values of d_P and/or d_N. The FE mesh and crack propagation under mode I and mixed mode loading are shown in Figure 3.

The specimens considered in this study are labelled in the following format with variables:

X_YZ_YZ

X- $\{L, M, S\}$ indicating size of the specimen as large, medium or small
YZ- indicates the position of notch/load. The first YZ corresponds to position of notch and the second YZ corresponds to position of load.
Y- $\{R, L\}$ indicating the position of notch/load from mid span. R indicates that notch/load is towards the right of mid span and L indicates that it is towards left of mid span.
Z- $\{0 - 50\}$ indicating the ratio of distance of notch/load from mid span to span of the beam multiplied by 100.

For example, M_R10_L25 corresponds to a medium beam with notch positioned at a distance of 0.10S towards right half and load at 0.25S towards left half of the beam (Figure 4) with S being the span.

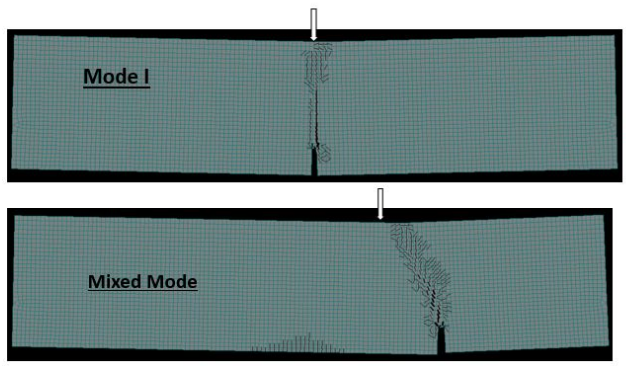

Figure 3: Crack propagation under mode I and mixed mode I-II loading

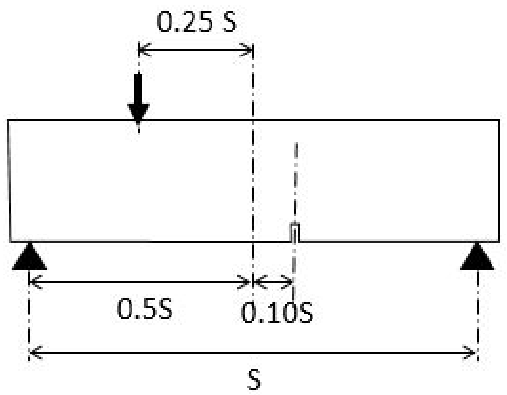

Figure 4: Geometry of M_R10_L25

5 RESULTS AND DISCUSSION

Load versus CMOD curves are obtained for both sets of analysis for all three sizes of beams. The typical behaviour of plain concrete beams under monotonic loading is observed in all cases. Load increases with increase in CMOD up to a maximum and then gradually reduces to zero. This post peak behaviour, referred to as softening, is a result of various toughening mechanisms in concrete such as crack deflection, micro crack shielding, crack tip blunting, grain bridging crack bridging etc. Load-CMOD plots from each set of analysis for large, medium and small beams are presented in Figure 5 and the area under each graph is computed to give energy at fracture. It is to be noted that only four cases from each set of analysis are presented in Figure 5 for clarity.

The results are explained based on the following suppositions:

1. Since bending moment as well as shear force at critical section are linearly proportional to the vertical load, stresses are computed considering the magnitude of load to be unity (1 kN).

2. The crack propagation occurs under a bending stress of M_x/z and shear stress of V_x/BD, where M_x and V_x are the bending moment and shear force at critical section and B and D are the width and depth of the specimen. Here, only the maximum normal and shear stress in the uncracked cross section is considered for analysis.

From Figure 5, it can be observed that as the bending moment (or normal stress) at the critical section decreases, area under the load-CMOD curve is larger indicating higher energy for fracture. This remains true for both mode I as well as mixed mode loading cases for large, medium and small beams. Energy at fracture computed in each case of set 1 and set 2 is now used to define the concept of scaling factor as explained in the next section.

5.1 Concept of Scale factor

When a crack propagates under mode I, the total energy at fracture is a contribution of normal stress alone. But in case of mixed mode crack propagation, the total energy is associated with a normal as well as a shear stresses. Figure 6 shows a typical plot of maximum stress at the critical section versus energy at fracture for both set 1 and set 2 analysis. Under set 1, maximum stress at the critical section has only a normal component. By varying the value of d_N and d_P together, we obtain mode I loading with varying normal stress at critical section and each loading case will have a corresponding energy at fracture. The curve denoted as *σ(Mode I)* in Figure 6 is obtained by plotting the maximum normal stress in each case against corresponding energy at fracture. Similarly, for set 2, maximum stress at critical section will have both shear and normal components. When the value of d_P is varied, the resulting cases correspond to mixed mode loading conditions, each one with different value of normal and shear stress. The curve denoted as *σ(Mode I-II)* and *τ(Mode I-II)* are obtained by plotting the max normal stress and shear stress at critical section respectively against the magnitude of energy at fracture.

In Figure 6, AC is the magnitude of normal stress under mode I loading (σ_C) and AD and AB are the magnitudes of normal (σ_D) and shear stresses (τ_B) respectively, under mixed mode loading, all of which corresponds to a

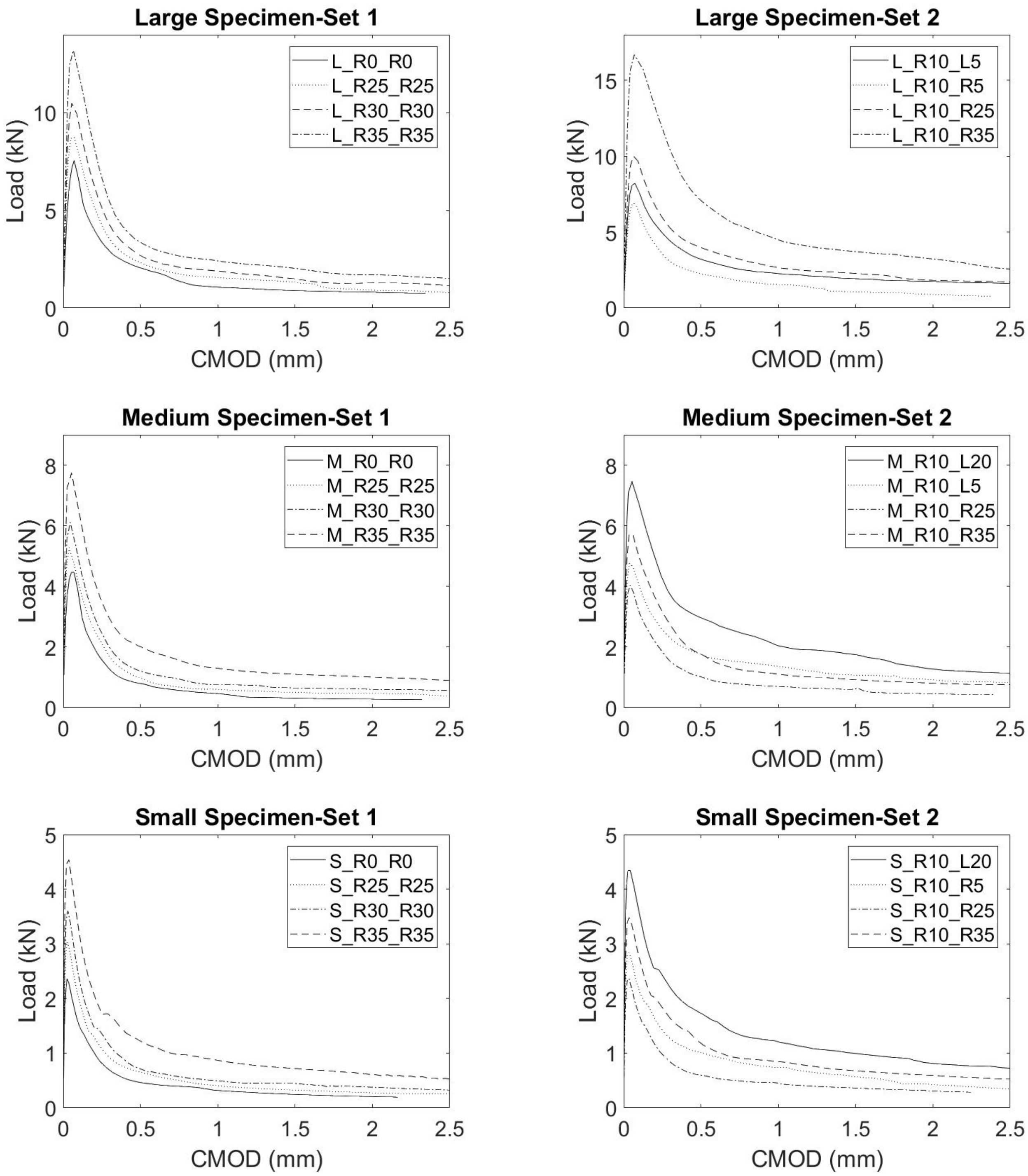

Figure 5: Load versus CMOD for set 1 and set 2

given magnitude of energy for fracture, denoted by point A. Hence, the energy at fracture for a pure mode I case subjected to normal stresses at the critical section is same as that for a mixed mode I-II case subjected to normal and shear stresses. In other words, $E_{\sigma_C} = E_{(\sigma_D+\tau_B)}$. As observed earlier, from the load-CMOD behaviour, the energy at fracture associated with a higher normal stress is lower. Consequently, $E_{\sigma_C} > E_{\sigma_D}$. This difference in energy corresponds to the difference in normal stresses in mode I and mixed mode loading ($\sigma_C - \sigma_D = \sigma_{CD}$) and is compensated by the shear stress in mixed mode loading (τ_B). Hence, the contribution to energy by normal stress σ_{CD} can be considered equivalent to that of shear stress τ_B. Using this concept of energy equivalence, a scaling factor can be defined based on the ratio σ_{CD}/τ_B for converting a given mixed mode problem to an equivalent mode I problem.

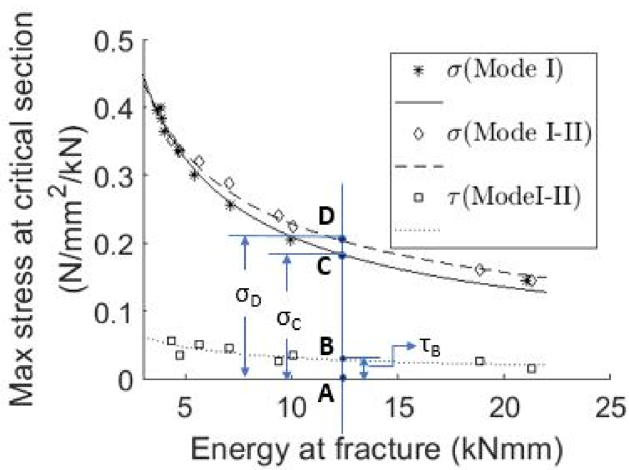

Figure 6: Illustrative case for definition of scaling factor

Figure 7 shows the energy at fracture associated with maximum stress at critical section for large, medium and small beams. In all sizes of beam, the same trend as explained above is observed.

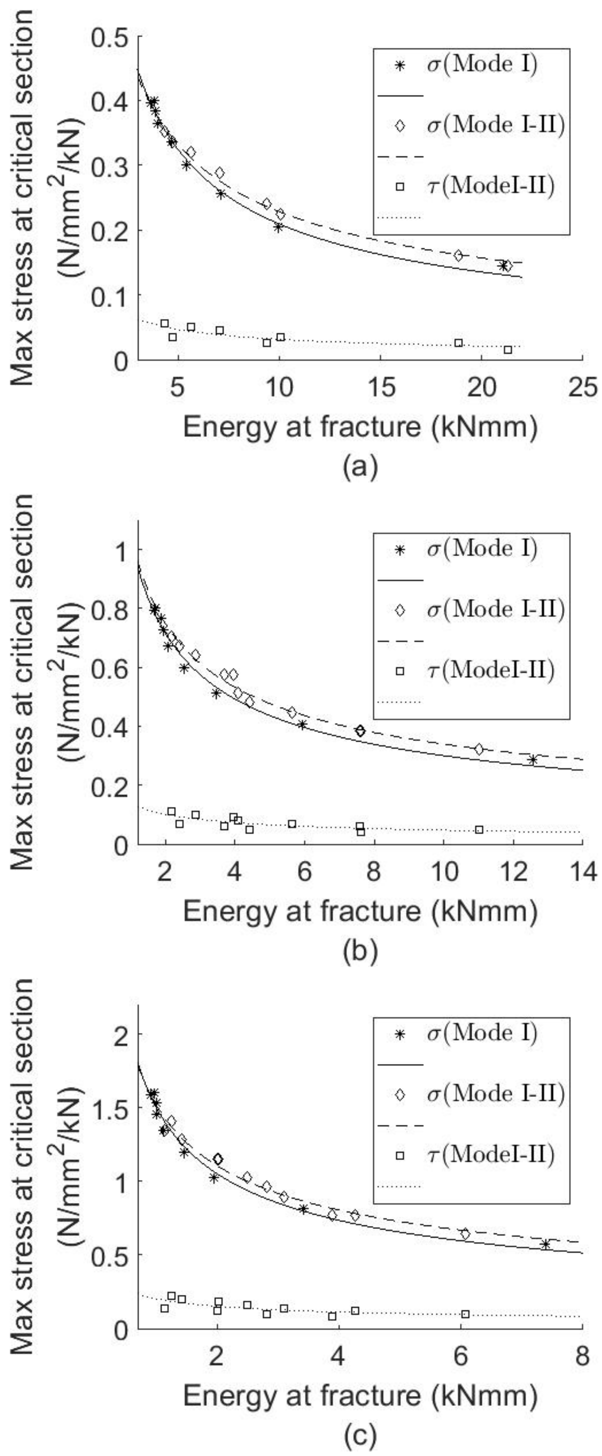

Figure 7: Variation of energy at fracture with maximum stress at critical section for (a) large (b) medium and (c) small beams

5.2 Size Effect

Three different sizes of specimens are considered in this analysis. Based on the load-CMOD curves from set 1 analysis for large, medium and small beams, it can be observed that, for the same position of notch/load, as the size of specimen increases, the total area under load-CMOD curve increases and maximum stress corresponding to unit load at critical section decreases. The variation of logarithm of total energy for fracture with logarithm of normal stress at the critical section for the three beam sizes for different mode I cases are plotted in Figure 8. It is observed that for the same magnitude of normal stress, energy for fracture is highest for small specimen. Also, it can be noted that the linear trend lines for all three sizes are nearly parallel indicating a similarity in behaviour under monotonic loading. This could imply that the scaling factor, when expressed quantitatively, would exhibit a linear variation with size of specimen.

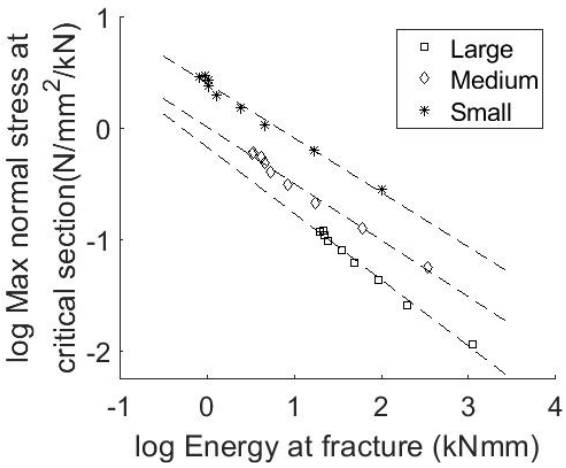

Figure 8: Illustration of size effect

6 CONCLUSION

In this work, an attempt has been made to propose a method to bring normal and shear stresses in plain concrete to the same stress platform. A scaling factor is defined based on the concept that the same amount of energy at fracture is associated with a mode I and a mixed mode I-II loading conditions. This factor is found to be dependent on the magnitudes of maximum shear and normal stress at the critical section as well as the size of the specimen. Once the scaling factor is quantified in terms of stresses at critical section, conversion of a mixed mode problem to an equivalent mode I problem can easily be achieved. Also, since large, medium and small sized specimens exhibit a similar behaviour under mode I loading conditions, the scaling factor is also expected to vary linearly with size. Research work is in progress to obtain the numerical form of the scale factor.

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