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ERROR ESTIMATE IN THE PROBABILISTIC CHARACTERISATION OF CONCRETE FATIGUE

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Abstract. Experimental results of fatigue tests in concrete present a wide scatter, with a difference of even several orders of magnitude between the highest and lowest obtained fatigue lives. The behaviour of fatigue results can be described by a probability model like the Weibull distribution. However, it is not clear how many tests are necessary to properly estimate the distribution parameters for a given concrete and loading conditions. In this work, a method has been developed so that the number of tests can be selected on the basis of the admissible error for the characterisation of the probabilistic behaviour of fatigue. As support for the performed study, a fibre-reinforced concrete was produced and 105 cubic specimens were tested. Afterwards, the sampling method known as 'bootstrap' was applied, which consists in generating multiple random samples of a given number of fatigue life results taken from the experimental set. The difference between the distribution fitted to each sample and the reference distribution of the complete set of values is the error made by the sample to characterise the material. The distribution followed by the error itself after all the sample simulations allows to determine the expected maximum error for a given safety level in relation to the number of tests. Different sample sizes from 3 to 90 tests have been analysed. The results provided by the method also allow to define a fatigue design distribution for the studied concrete and loading conditions.

1 INTRODUCTION

Fatigue of materials, and specially fatigue of heterogeneous materials like concrete, is a phenomenon that presents a very high scatter of resisted loading cycles between different specimens made of the same material and under the same loading conditions. Due to this fact, it is common to find specimens that resist a low number of cycles together with others with a fatigue life several orders of magnitude higher or that have not even reached the failure point.

Therefore, fatigue presents a remarkable probabilistic behaviour or, in other words, reaching a determined number of cycles before failure has a higher or lower probability. In order to determine this probability, it is necessary to carry out a series of tests for the range of stresses and frequencies of interest. Afterwards, a probability distribution is fitted to the experimental results. As one of the extreme value distributions, the Weibull function represents very well the mechanical fatigue of concrete [1–3].

However, there are no general criteria for selecting the minimum number of tests that must be performed in order to properly reproduce the behaviour of concrete under fatigue and to know the upper and lower limits of the fatigue life range. That large scatter of results implies that an elevated number of tests could be necessary. Due to the cost in time and resources, fatigue characterisation usually keeps to a reduced number of tests or to the deterministic expressions given by design standards [4, 5], which provide a mean value of cycles for the specified stress level.

The probability distribution obtained from a small sample of specimens can greatly differ from the distribution to which the entire material tends. That difference is the error made in the characterisation, which could reach an inadmissible value without the awareness of being in that situation.

Some authors have proposed different methods to define the necessary sample size [6–9]. However, distributions other than Weibull are used in some cases and, in general, the methods are relatively complex and use coefficients given for particular probability levels.

In the present work [10], it is intended to show in a simpler manner the relationship between the number of performed tests and the maximum committed error. Results are given for different safety levels and along the range of failure probability. The study starts from a set of 100 fatigue tests carried out with specimens made with the same concrete and with the same loading configuration. By means of statistical sampling techniques, the distribution and range of the error has been determined for a wide variety of different sample sizes from 3 to 90 specimens.

The results are presented in graphs that can be used for design in cases with a similar order of magnitude of number of cycles and scatter or, in any case, as a reference of the extent of the possible error in characterising concrete fatigue. With these results, it is possible to define how many tests must be performed in order to have an error under an admissible value, both as a maximum error or at a given failure probability. Moreover, a fatigue design distribution can be defined so that the fatigue resistance is not overestimated for the specified safety level.

In the next section the material and the experimental programme are described; in Section 3, the followed method for the statistical analysis is explained; Section 4 shows the obtained results and, finally, the conclusions are presented in Section 5.

2 MATERIAL AND TESTS

A steel-fibre-reinforced self-compacting concrete was produced. The cement type was CEM I 52.5 SR and the aggregates were silicious with a maximum size of 8 mm. The fibres were straight of 13 mm in length and 0.20 mm in diameter and its proportion in the mix was 0.2% in volume.

The concrete was poured in prismatic moulds of $400 \times 100 \times 100\,\text{mm}^3$. The specimens for fatigue testing were cut from those prisms as cubes of 40 mm side.

Tests were performed in a servo-hydraulic machine with a maximum capacity of 250 kN. The machine has two loading platens between which specimens are disposed and a ball-and-socket joint to align the load with the machine axis. This joint was fixed in each test once the platens were accommodated to the specimen faces at a pre-load of 4 kN. Besides, an additional self-produced ball-and-socket joint with a base of the same square dimensions that the specimen faces was placed on each one. With this device, the load is centred on the concrete cube, reducing the effect of possible eccentricities on the fatigue resistance [11].

In the first place, 15 quasi-static compression tests were performed at 0.3 MPa/s on the same type of specimens for fatigue tests. The compressive strength f_c was 58.9 MPa and the standard deviation was 3 MPa. For fatigue tests, the maximum and minimum stress levels were set as 82% and 36% of f_c, respectively. Tests were carried out at a load frequency of 10 Hz.

In total, 105 fatigue tests were performed, with 5 run-out specimens. The run-out limit was established in 1000000 cycles, around two or three orders of magnitude higher than the expected mean. Those 5 results were discarded for the analysis as they provided incomplete information. In this manner, the resulting probability distribution is on the safe side.

The probability distribution of the other 100 tests is taken as the real distribution of the material, taking into account that their number is high enough so as not to substantially vary with new tests. Such a distribution is described by means of the probabilistic Weibull model with two parameters, defined by Eq. 1.

$$F(x) = 1 - \exp[-(x/\lambda)^{\kappa}] \tag{1}$$

The variable x is the decimal logarithm of the number of cycles to failure ($\log N$), λ is the scale parameter and κ is the shape parameter.

This function is selected due to its suitability to describe phenomena with extreme values and, in the case of the Weibull distribution with two parameters, due to allow the possibility of very low values of N, as observed in real cases.

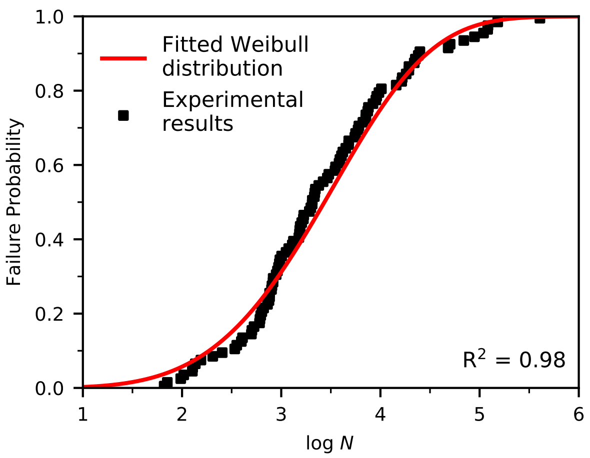

Figure 1: Cumulative Distribution Function of the complete set of tests.

Experimental results are presented in Fig. 1 in terms of $\log N$, represented as cumulative distribution. Likewise, the figure shows the Weibull function fitted to the data by the maximum likelihood method. The resulting distribution parameters are $\lambda = 3.72$ and $\beta = 4.56$. As it can be observed, the results cover from an order of 10^2 to 10^5 cycles.

3 METHOD

The applied method consists in a sampling technique ('bootstrap') where, for different sample sizes n, values are randomly extracted from the set of 100 results in multiple repetitions. For each sample of values, Eq. 1 is fitted and the difference with respect to reference distribution corresponding to the total of results is registered. That difference is the error made by that particular sample in characterising the fatigue behaviour of the material. The error is taken as the horizontal distance (in $\log N$) between both distributions along the cumulative probability axis. The expression that defines the observed error e_o in relative terms, at a certain failure probability p, is:

$$e_o(p) = \frac{\log N(p)_{\text{sample}} - \log N(p)_{\text{real}}}{\log N(p)_{\text{real}}} \tag{2}$$

The number of repetitions was determined by a convergence analysis of the difference between the cumulative mean value in $\log N$ of every extracted sample and the mean of the original 100 values. The convergence was analysed for a precision degree of 10^{-10} with several sample sizes. The most unfavourable result was adopted, which was 500000 repetitions.

A series of 23 different sample sizes was studied: 3, 5, 7, 9, 11, 13, 15, 17, 19, 22, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80 and 90.

After all the repetitions for each sample size, the distribution followed by the registered errors is obtained at each probability level, which follows a normal distribution. Defined this distribution, it is possible to obtain the value of the expected maximum error e_e at the desired safety level, that is to say, for the desired probability that the error does not exceed that value to the unsafe side, which is when the fatigue resistance is overestimated. This concept is sketched in Fig. 2. After selecting a determined safety level (SL), the evolution of the error for a given sample size can be represented by a curve along the cumulative probability of fatigue failure.

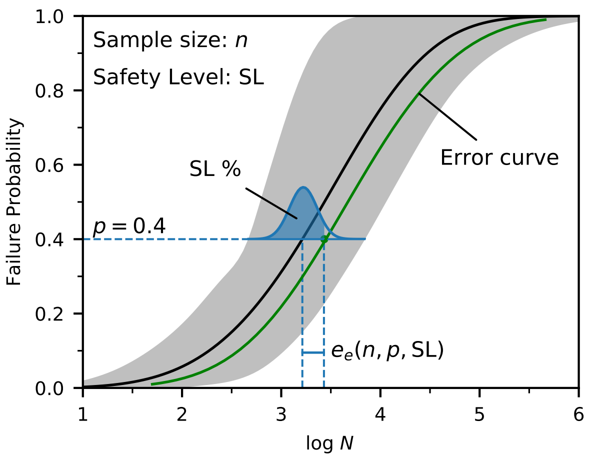

Figure 2: Error distribution, safety level and expected maximum error.

4 RESULTS

In the first place, the scatter of results obtained with samples of different sizes is compared by two examples (Fig. 3). The overlapped distributions of all the simulated samples for a given number of specimens form a band around the reference distribution of the total set of values. The width of each band shows the observed maximum scatter with respect to it. The distribution obtained by a random sample of that number of specimens will be traced within that band, which can differ more or less from the real distribution of the material.

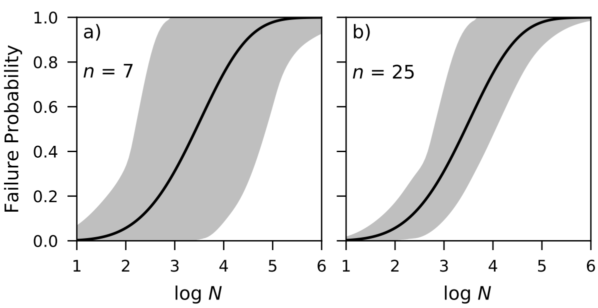

Figure 3: Total distribution and scatter bands for a) 7 tests and b) 25 tests.

As it can be appreciated, the scatter associated to a small number of tests can be very large, what is directly connected to the possible committed error. As the number of tests increases, the scatter and the maximum error diminish as each new specimen provides statistical information that improves the estimation of the general probability distribution.

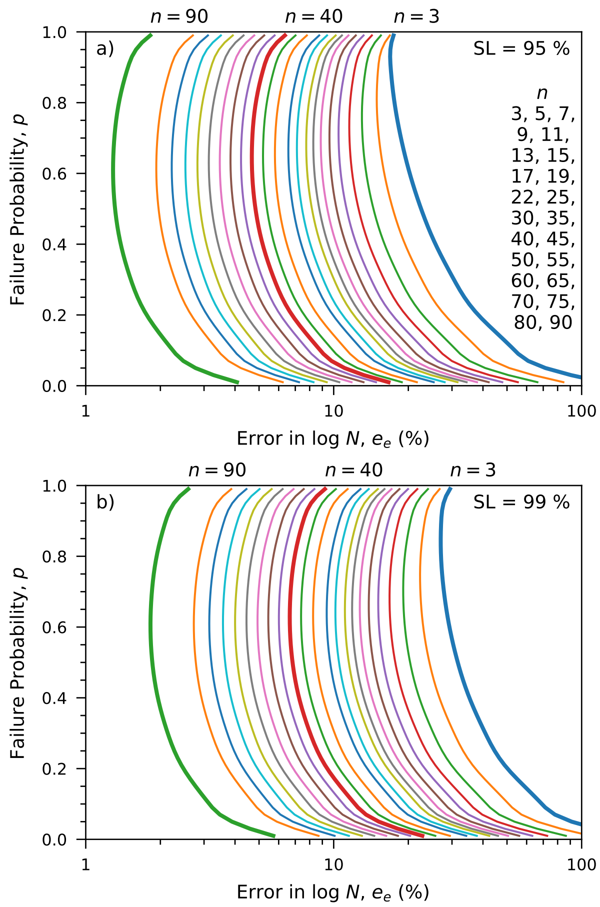

Figure 4: Error curves, e_e–p, for the safety levels of a) 95% and b) 99%.

In the second place, Fig. 4 shows the evolution of the maximum error for each studied sample size and for the safety levels of 95% and 99%, along the cumulative failure probability. These curves evidence the large relative error that can be reached when using a reduced number of specimens for the fatigue characterisation of a concrete. With the increment of the number of tests, the maximum possible error decreases. The figure also shows that the error for lower failure probabilities is much larger than for higher probabilities in the cases of small samples, difference that reduces with the number of specimens.

With this figure, it is possible to determine what is the sample size necessary to keep to a given maximum admissible error o, inversely, to know the expected maximum error for the used number of tests. For the case of a given admissible error, the curves to the left of the vertical line traced at that value correspond to sample sizes that do not reach that error at any point of the failure probability range. Likewise, it is possible to find what curves have an admissible error only at a particular failure probability or starting at that point onwards.

Finally, a design curve for fatigue can be derived with the information of the error curves. After performing a given number of tests, the maximum error is now known for a certain safety level. However, this error still refers to the possibility that the distribution obtained from the experimental results is to the right of the real distribution of the material, which is the unsafe side. Therefore, the fatigue design curve for that material can be defined by subtracting the corresponding error at each probability p from the experimental distribution. The probability that this resulting curve is to the left, or safe side, of the real distribution is equal, by definition, to the safety level. This design curve can be expressed as in Eq. 3. An application example is shown in Fig. 5.

$$\log N(p)_{\text{design}} = \frac{\log N(p)_{\text{sample}}}{1 + e_e(n, p, \text{SL})} \qquad (3)$$

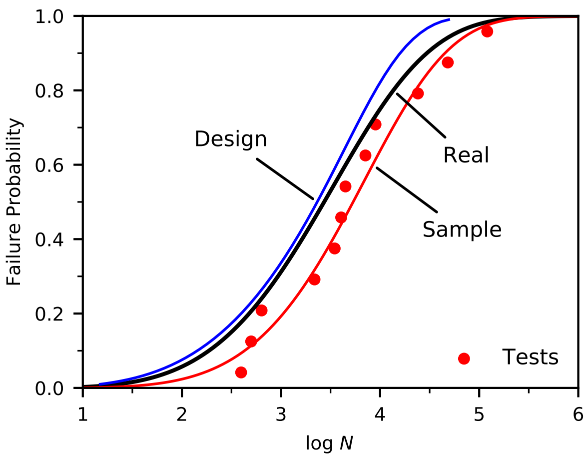

Figure 5: Example of fatigue design curve for a safety level of 95% with 12 tests.

5 CONCLUSIONS

This work highlights the large scatter of loading cycles to failure that can be found in the study of concrete fatigue. As a result, fatigue must be characterised by a probabilistic function, fitted to experimental results. The Weibull distribution is particularly appropriate for such a phenomenon. The number of tested specimens is key to the precision of the estimation of the probability distribution followed by the material. Due to that high scatter that affects this type of results, the distribution obtained from a random sample of specimens can be very different from the real distribution of the material if the number of tests is insufficient. This difference is the error made in the estimation of the real distribution by that sample.

The method developed in this work analyses the maximum committed error depending on the number of performed tests. After obtaining the distribution of the results of 100 tests, which is adopted as the real distribution of the studied concrete, a sampling technique has been applied that consists in randomly extracting multiple samples from that set of specimens. With each sample, the difference in $\log N$ with respect to the known real distribution is registered. After a total of 500000 repetitions, the scatter or error associated to a determined sample size is found to follow a normal distribution. From this error distribution, it is possible to know the possible maximum error for a given safety level.

On the one hand, two examples of scatter band obtained from different sample sizes are shown. For the smaller size, that band is much wider than in the other case, that is to say, there have been random samples that have yielded a distribution farther from the real distribution than when the number of used specimens is higher. On the other hand, the error curves of the 23 studied sample sizes, from 3 to 90 specimens, are presented for the safety levels of 95% and 99%. It is demonstrated how using a reduced number of specimens the committed error may be very high, which diminishes with each greater size. These results are useful for determining the necessary number of tests so that a given admissible error is not surpassed for a certain failure probability and safety level. Furthermore, a design fatigue distribution can be derived by subtracting from the experimental distribution the possible error corresponding to the required safety level.

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