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MICROMECHANICAL ANALYSIS OF FATIGUE DAMAGE IN CONCRETE CAUSED BY MATRIX MICROCRACKS

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Abstract. A micromechanics based damage model is proposed to predict the behavior of plain concrete under fatigue loads. Microcracks present in concrete are responsible for its nonlinear behavior and damage under cyclic loads typically initiate from cracks pre-existing in the material. In the present study, the fatigue damage caused by microcracks present in mortar is investigated within the framework of continuum micromechanics. Concrete is modeled at the meso-scale as a composite material consisting of two phases- the coarse aggregates and the cement mortar. The mortar matrix, in turn, is assumed to be weakened by the presence of arbitrarily oriented sharp microcracks. The crack density parameter is used as the damage variable. The homogenized constitutive relations of the mortar-aggregate system is computed by employing the Mori-Tanaka homogenization scheme. The evolution of microcracks is predicted based on the principle of damage mechanics. The loading-unloading irreversibility principle is employed for considering the dissipation under fatigue loads. The model is used to predict the fatigue life of plain concrete beams. An attractive feature of the proposed model is its capability to take into account the physical mechanisms causing fatigue degradation in cement based materials. The influence of the different meso-scale properties of concrete are also predicted by the model, enabling a more fundamental understanding of fatigue damage in these materials.

1 INTRODUCTION

The deterioration in performance of a majority of engineering structures during their service life can be traced to the application of repeated loads, a phenomenon commonly known as fatigue. The material is significantly weakened under the action of the load cycles on the structure, eventually causing its failure. The widespread use of concrete as an infrastructure material makes it prone to fatigue damage. Offshore structures, bridges, airport runways, pavements are some examples of concrete structures which experience fluctuating loads. The sudden and catastrophic nature of fatigue failure merits a thorough understanding of the mechanisms responsible for fatigue damage to ensure safety and serviceability of structural components. While a lot of research has been devoted to understand the complex behavior of concrete under monotonic loads, literature pertaining to fatigue response analysis of concrete remains somewhat limited.

Damage under fatigue loads is triggered from imperfections or defects present in a material. Concrete, though treated as a homogeneous material for design purposes, has an intricate internal structure consisting of various phases and flaws at different length scales. Microcracks have been identified as the primary source of damage in cement based materials such as concrete. Under the influence of external load, the microcracks gradually grow, coalesce to form a macrocrack and final failure occurs by the propagation of the macrocrack. The degradation of the material under fatigue loads is exhibited by the gradual loss of load carrying capacity of the structure.

It has been experimentally observed that the mechanisms of failure under fatigue loads is dependent on the nature of loading applied to the structure. Low cycle fatigue, which is characterized by a fewer number of load cycles of high amplitude, causes plastic deformation of the material. In contrast, a large number of load cycles of low amplitude act on the structure in high cycle fatigue. The deformations remain elastic and damage occurs by the gradual propagation of microcracks [1]. In addition to loading conditions, the initiation and propagation of fatigue damage is also dependent on the microstructural features of the material. Therefore, the multiphase nature of the internal structure of concrete should be taken into account to accurately predict the fatigue response of a structural component.

Prediction of fatigue life of cementitious materials has been initially confined to the empirical S-N curves, in which the magnitude of the alternating stress is plotted against the number of cycles to failure for a given material. While simplistic in approach, the curves are unable to take into account the various factors that affect the fatigue life of the material [2]. A more mechanistic approach to analysis of fatigue performance of concrete is provided by the models based on fracture mechanics and continuum damage mechanics [3, 4]. However, numerical models to describe the fatigue behavior of concrete are mostly phenomenological in nature. These models involve a number of parameters which are fitted with experimental data. A more rational approach to describe the phenomenon of fatigue in cementitious materials can be adopted by developing micromechanical models. Continuum micromechanics is an effective framework to predict the overall response of heterogeneous materials by considering the properties of the microstructure and the different physical mechanisms occurring at the lower scales. Analysis is carried out at the micro or meso scale and the results are homogenized to finally obtain the macroscopic properties [5, 6]. Micromechanics based models for cementitious composites are relatively scarce and provide immense scope of research. In the present work, an attempt has been made to describe the response of plain concrete under fatigue loads with the aid of continuum micromechanics.

2 MICROMECHANICAL MODEL

2.1 Damage due to matrix microcracks

Concrete is considered as a two phase particulate composite at the mesoscale: the stiffer aggregate particles being dispersed in the mortar matrix. The mortar phase and the aggregate phase are designated by ‘m’ and ‘a’ respectively. Each of the phases is considered to be elastic and isotropic. Microcracking is considered to be the primary source of damage. In the present analysis, randomly oriented microcracks are assumed to be present in the mortar matrix. The cracks are assumed to be planar. The aggregates are approximated to be circular in shape for simplifying the computation.

The stress and strain fields are computed at the mesoscale by employing the solution of an equivalent inclusion as given by Eshelby [7]. The stress field in an equivalent inclusion is effectively expressed in terms of the applied far field stress (or strain) by making use of Eshelbys tensor **L**. Eshelbys tensor, in turn, is a function of the shape of the inclusion and the elastic properties of the matrix. The Mori-Tanaka scheme of homogenization is implemented in order to obtain the effective macroscopic quantities. The macroscopic constitutive relation is given by:

$$\mathbf{\Sigma} = [\{f(\mathbf{C^a} \cdot (\mathbf{I} + \mathbf{L} \cdot \mathbf{T})) + (1-f)\mathbf{C^m}\} \cdot \{f(\mathbf{I} + \mathbf{L} \cdot \mathbf{T}) + (1-f)\mathbf{I}\}^{-1}] : \varepsilon \quad (1)$$

where $\mathbf{\Sigma}$ and ε are the macroscopic stress and strain respectively. The volume fraction of coarse aggregates is given by f. $\mathbf{C^a}$ and $\mathbf{C^m}$ are the stiffness tensors of the aggregate phase and the matrix phase. $\mathbf{I}$ represents the fourth order identity tensor. $\mathbf{T}$ is the stress concentration tensor which is given by:

$$\mathbf{T} = ((\mathbf{C^m} - \mathbf{C^a}) \cdot \mathbf{L} - \mathbf{C^m})^{-1} \cdot (\mathbf{C^a} - \mathbf{C^m}) \quad (2)$$

It relates the eigenstrain ϵ^* in the inclusion to the applied far field strain ϵ^∞ as:

$$\epsilon^* = \mathbf{T} : \epsilon^\infty \quad (3)$$

The homogenized stiffness tensor $\mathbf{C_1^{hom}}$ for the composite consisting of mortar matrix and coarse aggregates is thus given by:

$$\mathbf{C_1^{hom}} = \{f(\mathbf{C^a} \cdot (\mathbf{I} + \mathbf{L} \cdot \mathbf{T})) + (1-f)\mathbf{C^m}\} \cdot \{f(\mathbf{I} + \mathbf{L} \cdot \mathbf{T}) + (1-f)\mathbf{I}\}^{-1} \quad (4)$$

The inverse of the homogenized stiffness yields the homogenized compliance tensor $\mathbf{S_1^{hom}} = \mathbf{C_1^{hom}}^{-1}$. It can be seen from Equation 4 that the overall behavior of the composite, which is governed by the tensor $\mathbf{C_1^{hom}}$, is dependent on the elastic properties of the constituent phases and the volume fraction of the phases. The components of Eshelby's tensor, $\mathbf{L}$, for a circular inclusion under plain stress conditions are:

$$L_{ijkl} = \frac{3\nu^m - 1}{8}\delta_{ij}\delta_{kl} + \frac{3 - \nu^m}{8}(\delta_{il}\delta_{jk} + \delta_{jl}\delta_{ik}) \quad (5)$$

where ν^m is the Poisson's ratio for the matrix material and δ_{ij} is the second order identity tensor or the Kronecker delta.

$\mathbf{C_1^{hom}}$ is the undamaged stiffness tensor which does not consider any inelastic effects. The randomly distributed mortar microcracks are the source of damage which result in inelastic strains and alter the macroscopic stiffness of cementitious materials. The cracks are assumed to be line cracks or slit cracks in two dimensions with the length of the cracks being $2c$. For simplification of the present analysis, all the cracks are assumed to be of the same size. Further, since the response of the composite is considered under tensile loads, the cracks are assumed to be open at all stages of loading. The crack faces are assumed to be frictionless. Figure 1 shows a single microcrack inclined at an angle of θ with the x_1 axis. The local coordinate system of the crack is represented as x_1'-x_2'. The local coordinate system is chosen in a way such that the centre of the crack is at the origin of the coordinate system and the normal to the crack face coincides with the x_2' direction.

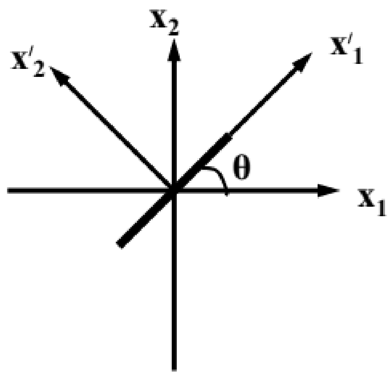

Figure 1: An arbitrarily oriented matrix microcrack

The crack opening displacements in the normal and tangential directions in terms of the local stresses under plane stress conditions are:

$$\|u_i\| = \frac{4}{E^m}\sqrt{c^2 - x^2}\sigma_{ii}' \quad (6)$$

where $i = 1, 2$ and $-c < x < c$ and E^m is the Young's modulus of the matrix.

The resultant inelastic strain $\epsilon^\mathbf{c}$ for a single microcrack is given by:

$$\epsilon_{ij}^c = \frac{1}{c^2}\int_{-c}^{c}(n_i\|u_j\| + n_j\|u_i\|)dx_1' \quad (7)$$

where n_i and n_j are the components of the unit normal on the crack face. The inelastic strain can be expressed as a function of the locally applied stress fields as:

$$\epsilon_{ij}^{c} = H_{ijkl}^{'}\sigma_{kl}^{'} \quad (8)$$

$H_{ijkl}^{'}$ is the additional compliance resulting due to the presence of microcracks in the matrix. The non-zero components of the tensor $\mathbf{H}'$ are:

$$H'_{2222} = \frac{2\pi}{E^m};$$
$$H'_{1212} = H'_{2121} = H'_{1221} = H'_{2112} = \frac{\pi}{2E^m} \quad (9)$$

When a number of microcracks are present in the matrix, each crack contributes to the inelastic strain as given by the previous equations. In order to obtain the total inelastic strain engendered by multiple microcracks, the crack density parameter d [8] is used. For two dimensional formulations, the crack density parameter is $d = Nc^2$, where N denotes the number of cracks of length $2c$ present per unit area of the RVE. d is the internal variable used in this micromechanical formulation to characterize the damage caused by microcracking.

In the present analysis, an isotropic distribution of microcracks is assumed. The additional compliance due to the presence of microcracks in all possible directions is obtained from the following equation:

$$\mathbf{H} = \frac{d}{2\pi}\int_0^{2\pi} \mathbf{Q^T H^{'} Q} d\theta \quad (10)$$

The equation yields the components of the additional compliance tensor $\mathbf{H}$ as:

$$H = \frac{\pi d}{E^m}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \quad (11)$$

The overall compliance tensor, $\mathbf{S^{hom}}$, of the composite includes the contributions of the coarse aggregate-mortar system and that of the randomly oriented microcracks and is given by:

$$\mathbf{S^{hom}} = \mathbf{S_1^{hom}} + \mathbf{H} \quad (12)$$

The existence and propagation of microcracks result in a reduction of the stiffness of the material. The homogenized stiffness tensor of concrete comprising of coarse aggregates, mortar matrix and matrix microcracks is:

$$\begin{aligned} \mathbf{C^{hom}} &= \mathbf{C_1^{hom}} - \mathbf{C_1^{hom}} : \mathbf{J} \\ \mathbf{J} &= \mathbf{H} : \mathbf{C^m} \end{aligned} \quad (13)$$

where $\mathbf{C^m}$ is the stiffness tensor of the mortar matrix.

In the initial stages of loading, the microcracks remain stationary. However, with the increase in the level of external load, the cracks begin to propagate. The propagation of microcracks is adequately described by using the evolution functions of a damage variable d. Energetic considerations based on damage mechanics, similar to some previous works [9, 10], are adopted in the present analysis. The strain energy release rate (SERR) is used as the criterion to determine the condition under which a microcrack with a particular orientation begins to propagate. The SERR is computed for each of the families of cracks at the mesoscale. When the SERR overcomes the resistance offered by the mortar matrix, the crack begins to grow which is given by the damage criterion:

$$f(F_i^d, d_i) = F_i^d - R(d_i) \leq 0 \quad (14)$$

$R(d_i)$ is the resistance offered by the matrix material to the growth of microcracks at the mesoscopic scale. In the present analysis, the crack resistance is assumed to depend linearly on the state of damage and is given by $R(d_i) = c_0 + c_1 d_i$. c_0 represents the initial damage threshold and can be interpreted as the fracture toughness of the matrix material. By imposing the normality rule, the damage evolution is finally obtained as:

$$\dot{d}_i = \dot{\lambda}_i^d \frac{\partial f_i}{\partial F_i^d} = \dot{\lambda}_i^d$$
$$\dot{d}_i = \begin{cases} 0 & \text{if } f_i \leq 0 \text{ and } \dot{f}_i < 0 \\ \dot{\lambda}_i^d & \text{if } f_i = 0 \text{ and } \dot{f}_i = 0 \end{cases} \quad (15)$$

2.2 Analysis of damage under fatigue loads

The damage model described in the previous section considers a two phase system in which the matrix is weakened by the presence of microcracks. The model is extended to evaluate the response of concrete under fatigue loads. Local damage variables are used to depict the state of damage in specific directions. For implementation in case of fatigue loads, the irreversibility condition for loading-unloading cycles, as proposed by Alliche [3], is used. According to this condition, the material undergoes damage when the local stress field is tensile in nature and the material is being loaded. When the material is unloaded in the loading cycle, no damage takes place. Due to the repeated loading and unloading cycles, energy is dissipated (as a result of growth of microcracks). The cumulative damage occurring in the material due to fatigue is satisfactorily defined from energetic considerations.

The model is used to analyze plain concrete notched beams under three point bending. A representative volume element (RVE) is considered in the beam. The propagation of crack under fatigue load occurs by subsequent failure of the RVE under consideration. The energy dissipation is computed at the mesoscale. The RVE fails when the value of the dissipated energy reaches the critical value. A nonlinear incremental analysis is carried out to compute the fatigue life, N_f , of plain concrete beams.

3 RESULTS AND DISCUSSIONS

3.1 Fatigue life of concrete beams

The micromechanics based damage model is utilized to estimate the fatigue life of plain concrete beams subjected to three point bending. The dissipative mechanisms responsible for causing damage are confined to a zone ahead of the notch. The evolution of damage due to propagation of the mortar cracks is responsible for causing fatigue failure in the structural element. The proposed model is validated by comparing the fatigue life, N_f, obtained from the model with the experimental results available in the literature. For this purpose, the experimental results of Toumi et. al. [11] are taken.

A series of experiments have been carried out by [11] on plain micro-concrete notched beams under constant amplitude fatigue loading. The span to depth ratio of the beams is 4.0 and the notch to depth ratio is 0.5. The specimens are subjected to three point bending. The minimum level of load, P_{min}, is maintained at $0.23P_u$, where P_u is the maximum load recorded under static loading. Three different values of the maximum load level, P_{max}, is considered ($0.87P_u$, $0.81P_u$, $0.76P_u$). The stress ratio R is given by $\frac{P_{min}}{P_{max}}$. Table 1 shows a comparison of the number of cycles to failure N_f as predicted by the model with the experimental ones. Given the wide variability commonly observed in fatigue test results, the model predictions are in good agreement with the experimental data.

Table 1: Comparison of predicted fatigue life with experimental results of Toumi et. al. [11]

Stress ratio	N_f (model)	N_f (experiment)	% error
0.26	39421	34314	14.8
0.28	77815	62255	25.0
0.30	78721	69784	12.8

3.2 Parametric study

The primary reason to adopt a micromechanical model is to specifically enunciate the dependence of fatigue response of the material on its microstructure. This enables one to gain a deeper insight about the material behavior and thus, design the material in an optimum way so as to enhance its performance under fatigue loading. Among the different parameters involved in the numerical model developed, the volume fraction of the coarse aggregates (given by the mix design) and the elastic properties of the mortar matrix are two vital parameters which play a decisive role on the response of cement based composites. In this section, the influence of these parameters on the fatigue life of plain concrete beams is illustrated with the help of S-N curves. Four different levels of maximum stress ($0.9P_u$, $0.8P_u$, $0.7P_u$, $0.6P_u$) are considered for each of the parameters under consideration. The number of cycles to failure for each of the amplitudes of load is estimated by the proposed model.

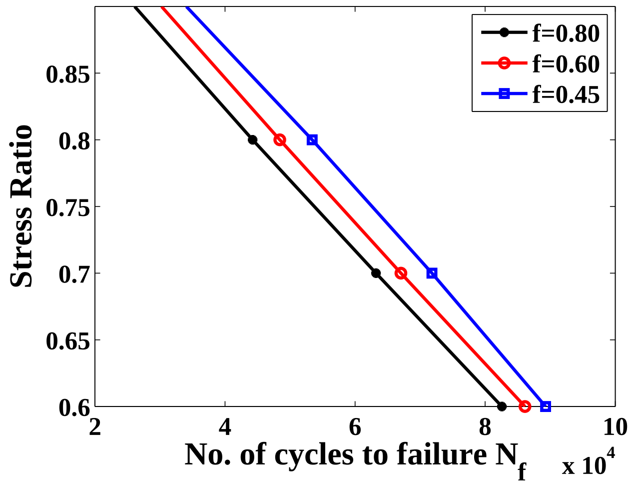

Figure 2: Effect of aggregate volume fraction on fatigue life of plain concrete

3.2.1 Volume fraction of aggregates

Coarse aggregates constitute about 30% to 80% of the total volume of plain concrete and may affect the fatigue response to a great extent. The importance of the role of coarse aggregates on the mechanical response of concrete under monotonic loads has been studied by several researchers. However, very little information regarding the influence of this parameter on fatigue life of concrete is available in current literature. The fatigue life is computed for different load ratios by considering three different values of aggregate volume fraction f. As illustrated in Figure 2, the fatigue life is seen to increase with a decrease in the volume fraction of coarse aggregates. The behavior can be explained based on micromechanical arguments. It has been shown that a higher aggregate content lowers the tensile strength of concrete. Also, the aggregate particles act as stress concentrators, increasing the local stress in the matrix. Therefore the number of load cycles required for the damage to initiate, i.e., propagation of the matrix cracks, decreases. This results in decreasing the number of cycles required for the failure of the material.

3.2.2 Elastic modulus of mortar

The macroscopic constitutive relation is a function of the elastic properties of both the coarse aggregates and the mortar. Variation of the elastic properties of the phases influences the fatigue life of the composite. The effect of the elastic modulus of the phases on the macroscopic behavior is studied by varying the elastic modulus of the mortar matrix. Figure 3 shows the dependence of the fatigue life on the elastic modulus of mortar matrix, E^m. Three different values of E^m is considered and the corresponding number of cycles to failure of the beam are calculated. From the S-N curve it is observed that an increase in the elastic modulus of mortar has a beneficial effect on the fatigue life of concrete. Since the aggregate is usually stiffer in comparison to the surrounded matrix, solely increasing E^m (keeping E^a constant) reduces the stress concentration and the perturbation stress arising from the differences in the elastic properties of the constituents. Therefore, the the load level at which growth of microcracks begins, is lowered. Once the microcracks begin to propagate, the stiffer matrix provides a greater resistance. These result in increasing the number of cycles to failure of the material under fatigue load.

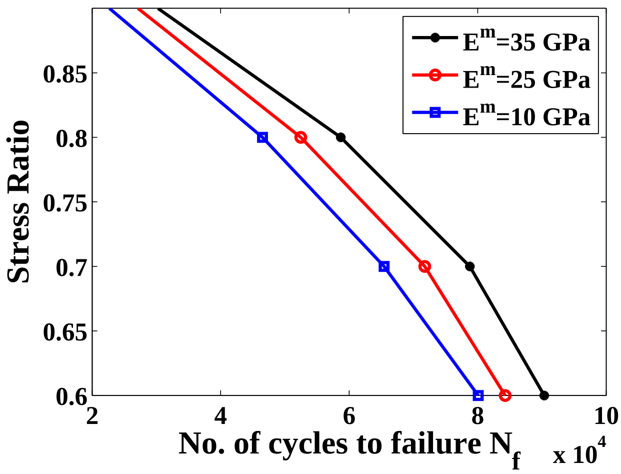

Figure 3: Effect of elastic modulus of mortar on fatigue life of plain concrete

4 CONCLUSIONS

In the present work a micromechanics based damage model has been proposed to analyze the response of cementitious composites under fatigue load. The model is based on the premise of continuum micromechanics and explicitly accounts for the mechanism responsible for fatigue damage. Pre-existing matrix microcracks are the source of damage. Degradation under repeated load cycles is brought about by the growth of these cracks. The different properties of the constituents are included in the constitutive relations of the composite material. The model is capable of predicting the fatigue life of plain concrete beams satisfactorily. The dependence of fatigue life on microstructural features such as aggregate content and elastic property of the mortar matrix are investigated through a parametric study. It is observed that a higher content of coarse aggregates lowers the fatigue life while the fatigue performance is improved when the stiffness of mortar is increased. The present study thus provides an understanding regarding the role of the microstructural mechanisms on fatigue life of plain concrete. The results presently are qualitative in nature and warrants extensive study coupled with experimental validation to quantitatively relate the fatigue life to the microstructural properties of plain concrete.

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