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RELIABILITY ANALYSIS OF CONCRETE BEAMS REINFORCED WITH CARBON FIBER-REINFORCED POLYMER BARS

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Abstract: Reinforced concrete structures may have their service life considerably reduced due to steel corrosion. As an alternative to provide durability at low maintenance costs, carbon fiber-reinforced polymer (CFRP) bars are used instead of steel. The purpose of this paper is to calculate the CFRP reinforcement of nine residential building beams according to the provisions of ACI 440 1R-06 and subsequently, to perform a reliability analysis with reference to the ultimate and service limit states. The same elements are also designed considering steel reinforcement in order to compare the results for both types of materials. Once designed to fail due to concrete crushing, the reliability analysis of all beams is performed utilizing the program Strand© - Structural Risk and Analysis, which, through the First Order Reliability Model (FORM) and Monte Carlo Simulation, computes the reliability indexes, probabilities of failure and sensitivity factors. The material properties, applied loads and dimensions are treated as random variables with different statistical distributions provided by the literature, while the fracture modes are described by two limit state equations, accounting for bending and shear. Similarly, the noncompliance of serviceability requirements is modeled considering the direct method to compute deflections, the Frosch Equation for cracking, as well as the maximum crack width and deflections allowed by the ACI 440 1R-06 guideline. The results showed that the probabilities of failure due to bending and shear are on average higher for the CFRP reinforced beams. They are more likely to exhibit excessive deflections; however, crack widths hardly exceed the permissible limit of 0.7 mm. In general, the variables that most contributed to failure were the concrete compressive strength, CFRP Young Modulus, position of reinforcement and model uncertainty for cracking.

1 INTRODUCTION

Reinforcement corrosion is the most important factor contributing to service life reduction in reinforced concrete. The combination of chlorides, humidity and temperature reduces the concrete alkalinity, causing steel to corrode, which gradually compromises the structure capacity to resist loadings at service levels. Therefore, the use of FRP (Fiber-reinforced Polymer) bars in concrete has considerably increased [3].

Despite being non-corrosive, the FRP reinforcement behaves linearly until suddenly rupturing. For this reason, the main design guidelines recommend considering failure due to the crushing of concrete, which exhibits some plastic behavior prior to failure. Moreover, the Young Modulus of FRP reinforcement is generally lower than that of steel [17].

CFRP bars (Carbon Fiber-reinforced polymer) are the most resistant and stiffest among other types of FRP (glass, aramid and basalt), with an elasticity modulus ranging from 100 to 580 MPa [12]. High strength associated to low stiffness leads the design to either over-reinforced cross-sections or under-reinforced, preceded by excessive deformation and cracking. Therefore, in order to meet the serviceability requirements, concrete crushing often controls the design [13].

In addition to providing information with respect to physical and mechanical properties of FRP bars, the ACI (American Concrete Institute) committee 440 has developed guidelines for calculating flexural members. However, the procedures to determine the safety factors for loading and strength parameters have not always been based on a reliability analysis, but on the committee consensus [21].

The ACI committee of 2006 considered there had already been enough experimental data on FRP bars, enabling the calibration of resistance safety factors based on reliability analyses [21]. In sum, the main goal of analyses is to determine probabilities of failure regarding flexural members, due to uncertainties behind the design variables. It is necessary to establish LS (limit state) functions separating failure and reliability domains. They can be written as G = R – L, where *R* accounts for strength and *L*, loading effects. If G ≤ 0, failure occurs. Therefore, the probability of failure P_f is:

$$P_f = \int_{G \le 0} f(x_1, x_2, \dots, x_n)\mathrm{dV} \tag{1}$$

In the above equation, x_1 to x_n correspond to the random variables involved in the analysis. For FRP RC members, they are the concrete compressive strength, live and dead loads, FRP Young Modulus, reinforcement position, etc. Probability of failure P_f can be directly obtained from reliability index β, which is the minimum geometrical distance between the LS function and the point concentrating the means of all variables, as illustrated in Figure 1.

Generally, this function is non-linear and the variables have different statistical distributions. Thus, the First Order Reliability Model – FORM linearizes the function and converts the distribution of each variable to standard normal. By trial and error, vector **y***, for which g(**y**) = 0, is found. The unit vector in the direction of **y*** assembles sensitivity factors α_i regarding the random variables. Ranging from zero to one, each factor α_i accounts for the contribution of its variable to the probability of failure [7].

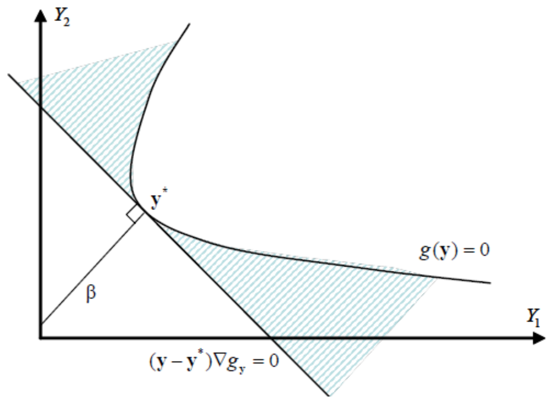

Figure 1: Graphical representation of FORM [7]

In spite of the good approximation provided by FORM, the nonlinearity degree of the LS function may lead to a substantial error. Since $P_f = \Phi(-\beta)$, the hatched area in Figure 1 illustrates the probability content neglected by FORM. Thus, the reliability area is greater and the actual probability of failure smaller [7].

Whereas FORM approximates the LS function, Monte Carlo Simulation can provide better accuracy, seeing that it computationally generates millions of samples. Those are tested in the LS function, which indicates failure or reliability. The probability of failure is then computed as the ratio between the number of samples indicating failure to the total [7].

Since probabilities of failure related to structural engineering problems are usually of a 10^{-6} magnitude order, Monte Carlo simulation may require computationally costly numbers of samples to achieve accurate results. For this reason, the probability density functions must have their center transferred to the design point, for which G = 0 and the distance to all means is orthogonal. This technique is denominated Importance Sampling [7].

Because ACI 319-02 established that the ductility of a cross-section must rely on its curvature and not on the fracture mode, ACI 440 1R-06 decided to calibrate the safety factors such that reliability indexes β were 4.5 regardless of the type of failure governing the design [21].

This decision also led to different factors for distinct fracture modes. For FRP rupture-governed sections, a value of 0.55 was found. Beams failing due to concrete crushing led to safety factors of 0.65. Transitional cross sections, in turn, needed to have their factor interpolated between 0.55 and 0.65 according to the reinforcement ratio [21].

2 SCOPE

Even though FRP bars can be very attractive due to their high corrosion resistance, more research on the field of reliability is needed. As mentioned, the FRP reinforcement does not yield, which leads the flexural member to fail with no warning. It is thus necessary to investigate the safety levels resulting of usual design guidelines. The designer will therefore be able to judge whether replacing conventional steel with FRP bars yields good results, not only with regard to durability but also in terms of structural safety.

Given these circumstances, this paper presents a reliability analysis of nine residential building beams reinforced with CFRP – Carbon Fiber Reinforced Polymer Bars, designed according to the ACI 440 1R-06 document – Guide for the Design and Construction of Structural Concrete Reinforced with FRP bars. The main objective is to compare the results of this analysis with steel-reinforced beams.

3 DESIGN PROCEDURES

The studied beams had their CFRP reinforcement area calculated such that all elements fail due to concrete crushing in the ultimate limit state. As previously stated, this type of failure might come with some warning, which makes it preferable.

The structural schemes of the analyzed beams are illustrated in Figure 2. The flexural reinforcement is calculated considering the maximum bending moment in the spans and supports. The transversal reinforcement, in turn, has its area determined to resist the maximum shear force in the supports.

Since all beams must fail due to concrete crushing, the reinforcement ratios needed to be at least 1.4 greater than the balanced ratio. Thus, the flexural strength was reduced by a safety factor of 0.65, according to [3].

Balanced reinforcement ratio ρ is calculated through [3]:

$$\rho_b = 0.85\beta_1 \frac{f_c'}{f_{fu}} \frac{E_f \varepsilon_{cu}}{E_f \varepsilon_{cu} + f_{fu}} \tag{2}$$

In Equation (2), β_1 is the ratio between the depths of the ACI approximation stress block and the neutral axis. For concrete crushing governed sections, $\beta_1 = 0.8$. The terms f'_c, f_{fu}, E_f and ε_{cu} correspond to the concrete compressive strength, reinforcement tension strength, FRP Young Modulus and concrete ultimate strain, respectively.

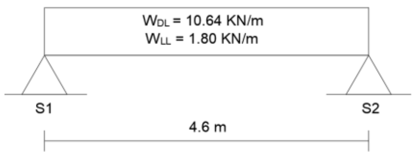

Figure 2.a: Beam 1

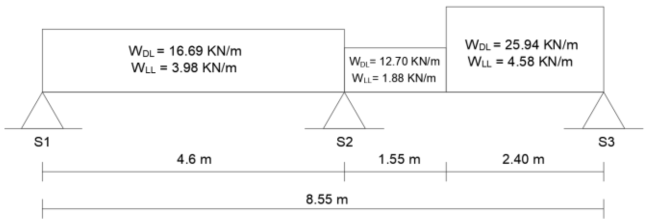

Figure 2.b: Beam 2

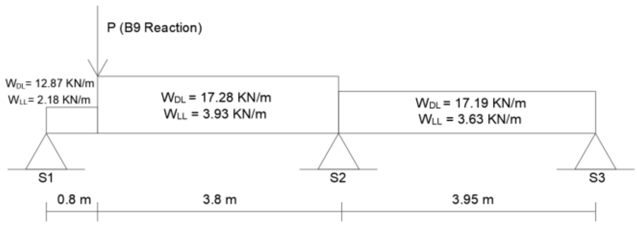

Figure 2.c: Beam 3

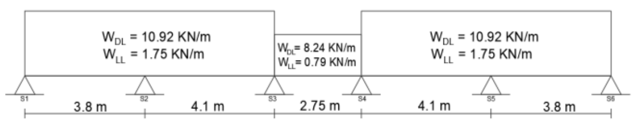

Figure 2.d: Beam 4

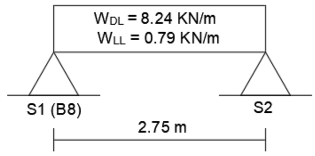

Figure 2.e: Beam 5

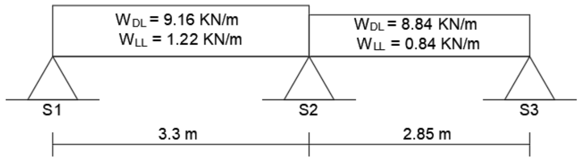

Figure 2.f: Beam 6

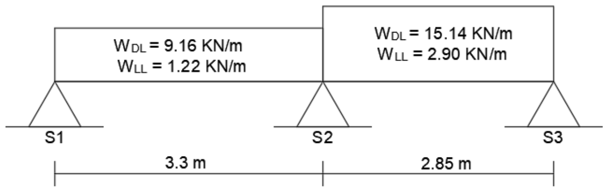

Figure 2.g: Beam 7

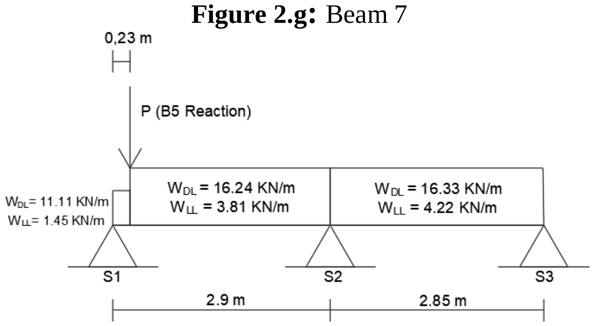

Figure 2.h: Beam 8

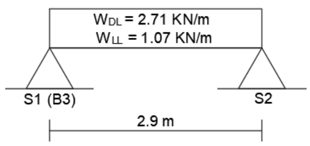

Figure 2.i: Beam 9

Figure 2: Structural Schemes of the residential building beams in study

Figure 3 shows the equilibrium and strain compatibility conditions for a generic section failing due to concrete crushing. In this case, $\alpha_1 = 0.85$, $\beta_1 = 0.8$ and $\varepsilon_{cu} = 0.003$ [3].

Regarding material parameters, all beams were designed considering Concrete C30. The tension strength is taken as $0.21f'_c{}^{2/3}$ according to [11]. The CFRP elasticity modulus and tension strength are 152 GPa and 2070 MPa, respectively [3].

The minimum reinforcement cover required by [2] is 3.81 cm, which is added to the stirrup diameter (ϕ_{str} = 6.4 mm) and half of the longitudinal bar ($\phi/2$) to calculate the flexural reinforcement center position d, with regard to the most compressed concrete fiber.

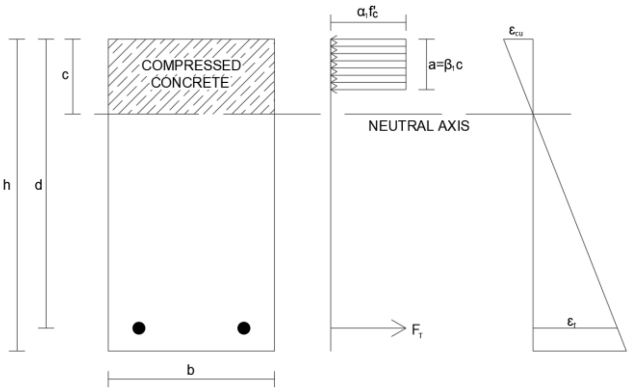

Figure 3: Stress and strain distributions for sections failing due to concrete crushing

In order to obtain the bending moment at the ultimate limit state, all dead and live loads were increased by safety factors of 1.2 and 1.6, respectively [2]. Considering the equilibrium and compatibility conditions shown in Figure 3, flexural strength M_r is:

$$M_r = 0.65 A_f E_f \varepsilon_f \left(d - 0.4 \frac{A_f E_f \varepsilon_f}{0.68 f_c' b d} \right) \quad (3)$$

Table 1 illustrates the design for all beams, where c corresponds to the neutral axis depth. SP and SU designate sections at spans and supports with maximum moment, respectively.

Table 1: Summary of design procedures – Moments M_u and M_r in KN.m, bar diameter ϕ in mm, d and c in cm.

Section		M_u	d	ϕ	ρ/ρ_b	c	M_r
1	**SP**	41.3	39.9	12.7	2.5	10.8	71.6
2	**SP**	51.8	40.1	9.5	1.4	8.5	57.6
	SU	73.2	39.9	12.7	2.5	10.8	71.6
3	**SP**	57.8	44.9	12.7	2.3	11.6	86.8
	SU	80.7	44.9	12.7	2.3	11.6	86.8
4	**SP**	16.5	25.1	9.5	2.3	6.5	27.1
	SU	27.7	24.9	12.7	4.1	8.2	33.0
5	**SP**	10.5	25.1	9.5	2.3	6.5	27.1
6	**SP**	10.8	25.1	9.5	2.3	6.5	27.1
	SU	15.2	25.1	9.5	2.3	6.5	27.1
7	**SP**	14.8	25.1	9.5	2.3	6.5	27.1
	SU	20.6	25.1	9.5	2.3	6.5	27.1
8	**SP**	16.8	25.1	9.5	2.3	6.5	27.1
	SU	28.1	24.9	12.7	4.1	8.2	33.0
9	**SP**	13.2	25.1	9.5	2.3	6.5	27.1

Beam 1 was initially designed with a 9.5 mm diameter. However, the deflection at midspan exceeded the maximum limit imposed by [2], which corresponds to 1/240 of span length. For this reason, a 12.7 mm diameter was adopted.

In order to calculate the effective moment of inertia I_{eff} for cracked sections, the modified Branson's Equation for FRP reinforced flexural members is utilized [3]:

$$I_{eff} = \left(\frac{M_{cr}}{M_a}\right)^3 \left(\beta_d I_g - I_{cr}\right) + I_{cr} \tag{3}$$

In Equation (3), M_{cr} and M_a correspond to the cracked and service applied moments, respectively; I_g and I_{cr} to the gross and cracked moments of inertia and β_d is a modification factor for FRP-reinforced members, equal to $0.2(\rho/\rho_b)$. Deflections are therefore calculated and compared to the permissible limit.

Concerning the maximum crack widths, these cannot exceed 0.5 and 0.7 mm for internal and external environments, respectively [3]. For this study, in particular, the maximum allowed is 0.7 mm. To predict crack widths of FRP reinforced members, the Frosch Equation is used:

$$w = 2\frac{f_f}{E_f}\beta K_b \sqrt{d_c^2 + \left(\frac{s}{2}\right)^2} \tag{4}$$

In the above equation, w corresponds to the crack width in mm, f_f to the tension in reinforcement due to service loads, β to the strain gradient and k_b is a factor accounting for the bond between concrete and FRP bars. In turn, d_c is the distance between the reinforcement center and the most tensioned fiber, whereas s corresponds to the horizontal bar spacing.

Since service loads do not lead to fail, the ACI approximation block used for ultimate limit state design no longer applies. Therefore, different values of α_1 and β_1 were determined in consonance with the service applied moments.

Tables 2 and 3 summarize the checking for deflections and crack width, respectively.

Regarding the transversal reinforcement, ACI 440 1R-06 establishes a strength safety factor of 0.75, the same as steel. Since both concrete and transversal reinforcement resist shear forces, two strength capacities V_c and V_f are defined for concrete and CFRP, respectively:

$$V_c = 0.75\left(\frac{2}{5}\sqrt{f_c'}bc\right)$$

$$V_f = \frac{A_{fv} f_{fv} d}{s} \tag{5}$$

In Equation (5), f_{fv} is the stress at ultimate limit state, taken as the minimum value between $0.004E_f$ and f_{fb}, the tensile strength at bent portions of CFRP bars. A_{fv} is the transversal reinforcement area, which must be greater than the minimum established by the guideline [3].

Table 2: Checking for deflections v (mm), with M_a and M_{cr} in KN.m; I_{cr} and I_{eff} in cm^4.

B	M_a	M_{cr}	I_{cr}	I_{eff}	v_{max}	v_{lim}
1	32.9	13.7	15178	17985.9	13.1	19.2
2	31.9	13.7	9171.5	10831.9	16.3	18.9
3	51.6	13.7	19515	23270.3	9.6	19.2
4	39.1	16.9	3407.8	4473.6	10.9	15.8
5	52.8	16.9	3407.8	7399.3	3.0	11.5
6	13.2	6.1	3407.8	7130.3	3.6	13.6
7	22.1	6.1	3407.8	4938.3	5.4	12.1
8	8.5	6.1	3407.8	4473.6	6.4	12.1
9	8.7	6.1	3407.8	5465.9	5.7	12.0

Table 3: Checking for crack width w (mm), with f_f in MPa; d_c and s in cm.

Section		f_f	β	d_c	s	w
1	**SP**	353.9	1.16	5.09	2.56	0.40
2	**SP**	599.1	1.15	4.93	3.20	0.65
	SU	556.6	1.16	5.09	2.56	0.62
3	**SP**	372.1	1.14	5.09	2.56	0.41
	SU	503.2	1.14	5.09	2.56	0.55
4	**SP**	402.3	1.24	4.93	3.20	0.48
	SU	389.6	1.27	5.09	2.56	0.48
5	**SP**	258.6	1.24	4.93	3.20	0.31
6	**SP**	264.7	1.24	4.93	3.20	0.31
	SU	374.7	1.24	4.93	3.20	0.44
7	**SP**	356.4	1.24	4.93	3.20	0.42
	SU	500.4	1.24	4.93	3.20	0.59
8	**SP**	402.3	1.24	4.93	3.20	0.48
	SU	387.8	1.27	5.09	2.56	0.48
9	**SP**	322.7	1.24	4.93	3.20	0.38

For all the beams, stirrups have a 6.4 mm in diameter. The spacing between transversal bars is:

$$s = \frac{0.75 A_{fv} f_{fv} d}{V_u - 0.75 V_c} \tag{6}$$

The calculation of s and $V_r = V_c + V_f$ is summarized in Table 4. s_{adop} corresponds to the adopted spacing, which cannot exceed 0.5d or 60.96 cm [3].

Regarding the same beams reinforced with conventional steel, the design was based on the Brazilian code ABNT NBR 6118:2014 – *Projeto de Estruturas de Concreto*. Unlike ACI 440 1R-06, this code increases dead and live loads by the same safety factor, which is 1.4. The concrete compressive capacity is reduced by a factor of 1.4 and the yield strength of steel by 1.15. Moreover, the reinforcement cover is thinner, only 3 cm; and the maximum crack width allowed, 0.3 mm. The concrete utilized is the same (f'_c = 30 MPa), while the yield strength of steel f_y is 500 MPa.

All the reinforcement areas were computed such that the neutral axis was located between 0.259d and 0.45d. The objective was to conciliate serviceability and ductility requirements. As a result, some beams had their depth h reduced.

Table 4: Calculation of CFRP stirrup spacings and shear strengths. Forces are in KN and spacings in cm.

B	V_u	V_c	s_{calc}	s_{adop}	V_f	V_r
1	35.9	23.35	63.6	20	58.52	76.03
2	89.3	23.35	16.3	16	73.15	90.66
3	95.5	24.92	17.1	17	77.48	96.16
4	37.5	17.94	30.4	12.5	58.45	71.90
5	15.3	13.92	151.4	12.5	58.82	69.27
6	26	13.92	47.3	12.5	58.82	69.27
7	40.2	13.92	24.7	12.5	58.82	69.27
8	47.9	17.94	21.2	12.5	58.45	71.90
9	18.3	13.92	93.6	12.5	58.82	69.27

Table 5 and 6 summarize the design for flexure and shear of steel RC beams, respectively. The ratio between strength and ultimate shear forces are considerably high due to the minimum reinforcement area and stirrup spacing required by both codes.

Table 5: Design considering steel bars - Moments M_u and M_r in KN.m; h e d in cm and A_s in cm^2.

Section		M_u	h	d	A_s	c/d	M_r
1	**SP**	46.1	35	29.6	3.95	0.28	45.1
2	**SP**	44.7	45	39.1	5.07	0.28	76.6
	SU	72.2	45	39.1	5.07	0.28	76.6
3	**SP**	54.7	40	35.1	4.68	0.28	63.2
	SU	73.9	40	34.7	5.39	0.33	70.5
4	**SP**	18.5	30	25.0	3.24	0.28	31.4
	SU	30.9	30	25.0	3.24	0.28	31.4
5	**SP**	11.9	30	25.0	3.24	0.28	31.4
6	**SP**	12.2	30	25.0	3.24	0.28	31.4
	SU	17.2	30	25.0	3.24	0.28	31.4
7	**SP**	16.4	30	25.0	3.24	0.28	31.4
	SU	23.0	30	25.0	3.24	0.28	31.4
8	**SP**	18.5	30	25.0	3.24	0.28	31.4
	SU	30.8	30	25.0	3.24	0.28	31.4
9	**SP**	14.8	30	25.0	3.24	0.28	31.4

Table 6: Calculations of steel stirrup spacings and shear strengths. Forces are in KN and spacings in cm.

Beam	V_u	V_c	s_{calc}	s_{adop}	V_s	V_r
1	40.0	36.0	39.6	18	41.4	77.4
2	82.2	47.6	28.4	23	42.8	90.3
3	87.5	42.2	19.3	19	46.0	88.2
4	41.9	30.5	39.6	15	42.0	72.5
5	17.4	30.5	39.6	15	42.0	72.5
6	29.1	30.5	39.6	15	42.0	72.5
7	44.5	30.5	39.6	15	42.0	72.5
8	52.6	30.5	28.4	15	42.0	72.5
9	20.7	30.5	39.6	15	42.0	72.5

4 RELIABILITY ANALYSIS

In order to perform the reliability analysis, the program StRAnd© - Structural Risk Analysis and Design – developed by [6] is utilized. The program employs the FORM and Monte Carlo methods to compute reliability indexes β, probabilities of failure P_f and sensitivity factors α. The results provided by both methods are compared; the error caused by the approximation of the LS functions by FORM can thus be evaluated.

As previously mentioned, failure occurs when the loading effects overcome the mechanisms of strength, which means the LS function is smaller than zero. Therefore, with respect to bending and shear, there are two LS equations.

Regarding the serviceability limit states, there are two modes of failure corresponding to deflections and crack widths being greater than the permissible limits.

The LS functions for bending G_B, shear G_S, deflections G_D and cracking G_C are as follows:

$$G_B = M_R - M_U$$

$$G_B = V_C + V_F - V_U$$

$$G_D = \frac{L}{240} - v_{max}$$

$$G_C = 0.7 - Cw_{max} \quad (7)$$

Loading and strength factors are not included in Equation 7. Otherwise, the results of the reliability analysis would not provide enough information about the safety levels these factors provide [5].

Coefficient *C* corresponds to the model uncertainty of the Frosch Method. Comparisons between experimental data and predicted values of crack widths are necessary to determine the statistical distribution as well as the mean and standard deviation of the variable.

Data on cracking of CFRP [20] and steel-reinforced beams [4] are compared to values predicted by Equation 4, which led to samples of *C*, as shown in Tables 7 and 8. Even though the data is not enough to define the probability density function of *C*, the uniform distribution is the one best fitting. Based on these samples, the mean μ_C, standard deviation σ_C and coefficient of variation CV_C for the beams with CFRP are 0.415, 0.232 and 0.56, respectively. For the ones with steel, they are 0.74, 0.19 and 0.254, respectively.

Since CFRP and steel RC members have been calculated according to different codes, average compressive strength $\mu f'_c$ as well as the elasticity modulus and ultimate strain of concrete have different values. For the CFRP RC beams, $\mu f'_c = f'_c + 1{,}34\sigma$ according to [2], whereas for the ones with steel, $\mu f'_c = f'_c + 1{,}65\sigma$ [1].

The elasticity modulus of concrete E_{ct} and its parameters are determined based on the compressive strength. The statistical distribution best fitting experimental results conducted by [9] is the normal distribution.

Variables such as beam width b, height h, and length L are treated as deterministic for this study. Concerning the other random variables, Tables 9 and 10 describe all their parameters.

Table 7: Determination of model uncertainty for FRP reinforced beams considering different applied loads P.

P (KN)	w_{exp}	w_{model}	C
20.68	0.04	0.37	0.09
23.50	0.07	0.52	0.14
27.26	0.18	0.70	0.25
31.68	0.34	0.90	0.38
35.73	0.58	1.09	0.53
39.49	0.85	1.30	0.65
43.25	1.03	1.55	0.66
48.89	1.24	2.05	0.61

Table 8: Determination of model uncertainty for steel reinforced beams considering different applied loads P.

P (KN)	w_{exp}	w_{model}	C
166.7	0.08	0.16	0.51
199.6	0.11	0.19	0.58
248.7	0.19	0.24	0.81
310.9	0.31	0.29	1.04
287.6	0.09	0.18	0.53
343.3	0.15	0.21	0.73
430.3	0.23	0.26	0.86
536.7	0.28	0.33	0.84

Table 9: Statistical Parameters of Random Variables for the CFRP RC beams

Variable	μ	CV	Dist.	Ref.
f_c (MPa)	34.6	0.10	Normal	[18]
ε_{cu} (‰)	3	0.15	Lognormal	[22]
E_{ct} (GPa)	32.96	0.05	Normal	[9]
E_f (GPa)	152	0.05	Lognormal	[22]
A_f (cm²)	A_{fn}	0.03	Normal	[21]
A_{fv} (cm²)	A_{fn}	0.03	Normal	[21]
d (cm)	$0.99d_n$	0.04	Normal	[21]
s_L (cm)	s_{Ln}	0.707/μ	Normal	[16]
s_T (cm)	s_{Tn}	*	Normal	[14]
C	0.415	0.56	Uniform	-
w_{DL} (KN/m)	w_{DLn}	0.10	Normal	[13]
w_{LL} (KN/m)	w_{DLn}	0.25	Gumbel Max	[13]

*The standard deviation of the transversal reinforcement spacing has been considered $\sigma_{st} = 0.006\mu + 0.4$ [14]

Table 10: Statistical parameters of Random Variables for the steel RC beams

Variable	μ	CV	Dist.	Ref.
f_c (MPa)	39.8	0.10	Normal	[18]
E_{ct} (GPa)	30.67	0.075	Normal	[9]
E_s (GPa)	210	0.10	Lognormal	[19]
A_s (cm²)	A_{sn}	0.03	Normal	[21]
A_{sv} (cm²)	A_{svn}	0.03	Normal	[21]
d (cm)	$0.99d_n$	0.04	Normal	[21]
s_L (cm)	s_{Ln}	0.707/μ	Normal	[16]
s_T (cm)	s_{Tn}	*	Normal	[14]
C	0.74	0.254	Uniform	-
w_{DL} (KN/m)	w_{DLn}	0.10	Normal	[13]
w_{LL} (KN/m)	w_{DLn}	0.25	Gumbel Max	[13]

*The standard deviation of the transversal reinforcement spacing has been considered σ_{st} = 0.006μ + 0.4 [14]

5 RESULTS

The results of the reliability analysis are displayed in seven tables.

Tables 11 and 12 describe the reliability indexes for CFRP and steel RC members. The failure modes corresponding to bending, shear, deflections and cracking are labeled from one to four, respectively. Tables 13, 14 and 15, in turn, describe the probabilities of failure calculated by FORM P_F and Monte Carlo P_M.

Table 11: Reliability indexes of CFRP β_C and steel β_S RC beams at the ultimate limit states - Strand©

Section		Mode 1		Mode 2	
		β_C	β_S	β_C	β_S
B1	SP	4.27	4.09	10.3	10.0
B2	SP	4.26	5.89	6.06	6.91
	SU	3.44	4.82		
B3	SP	4.97	4.94	6.36	6.50
	SU	4.26	4.49		
B4	SP	4.59	5.89	10.1	9.68
	SU	3.42	4.22		
B5	SP	5.59	5.98	18.7	12.4
B6	SP	5.55	5.98	14	11.3
	SU	4.79	5.90		
B7	SP	4.83	5.92	8.95	9.26
	SU	4.01	5.81		
B8	SP	4.59	5.90	5.85	6.64
	SU	3.52	4.59		
B9	SP	5.19	5.94	17.9	12

Table 12: Reliability indexes of CFRP β_C and steel β_S RC beams at the serviceability limit states - Strand©

Section		Mode 3		Mode 4	
		β_C	β_S	β_C	β_S
B1	SP	2.19	1.18	2.94	1.39
B2	SP	0.52	6.98	1.42	2.87
	SU			1.61	1.42
B3	SP	3.85	4.41	2.63	2.09
	SU			1.94	1.55
B4	SP	2.06	7.93	2.93	4.13
	SU			2.81	1.85
B5	SP	6.80	11.2	4.73	5.10
B6	SP	6.00	10.7	4.62	4.96
	SU			3.26	5.20
B7	SP	3.59	8.26	3.27	4.04
	SU			2.16	2.98
B8	SP	2.94	7.44	3.16	4.04
	SU			3.16	1.95
B9	SP	3.88	9.06	3.71	4.49

Table 13: Probabilities of failure for Mode 1 (10^{-6}) - Strand©

Section		CFRP		Steel	
		P_F	P_M	P_F	P_M
B1	SP	9.98	11.16	21.5	30.49
B2	SP	10.0	8.92	0.002	0.
	SU	287.8	318.9	0.71	1.07
B3	SP	0.338	0.289	0.386	0.380
	SU	8.27	10.17	3.51	5.08
B4	SP	2.24	2.71	0.0019	0.
	SU	311.4	367.9	11.92	15.96
B5	SP	0.0114	0.0139	0.00108	0.00104
B6	SP	0.0147	0.0154	0.00109	0.00114
	SU	0.84	1.29	0.00177	0.00175
B7	SP	0.69	0.78	0.00159	0.00157
	SU	30.1	41.2	0.00320	0.00330
B8	SP	2.18	0.98	0.00182	0.
	SU	214.1	262.9	2.26	3.67
B9	SP	0.10	0.12	0.00142	0.00146

There is no table comparing probabilities for Mode 2. Since they are too small, the Monte Carlo Simulation required a prohibitive number of samples. As a result, Strand© either showed the values as zero or did not compute them.

Tables 16 and 17 display the sensitivity factors α. They correspond only to sections with the highest probability of failure. For Mode 1, these sections are B4-SU (Beam 4 – Support) and B1-SP (Beam 1 – Span) related to CFRP and steel RC members, respectively.

Regarding Mode 2, they are B8 and B3 (section at these beams with greatest shear force), for Mode 3, B2 and B1, while for Mode 4, the elected sections are B2-SP and B1-SP.

Regarding the sections with high reliability, some other variables showed greater contribution. The rupture due to bending of steel RC section B6-SP, for instance, is attributed 99.6% to the concrete compressive strength. The steel contribution is neglected. Conversely, the position of reinforcement *d* contributes with 77.3% to failure due to shear of CFRP RC Beam 5. For steel, the concrete strength still plays the greatest contribution. In relation to serviceability requirements, contributions of variables did not change considerably as reliability increased.

Table 14: Probabilities of failure for Mode 3 (10^{-2}) - Strand©

Section	CFRP		Steel	
	P_F	P_M	P_F	P_M
B1	1.44	1.44	11.8	11.3
B2	30.1	31.0	$1.5{\bullet}10^{-10}$	$1.3{\bullet}10^{-10}$
B3	0.0059	0.0089	0.00051	0.00047
B4	1.99	1.93	$1.1{\bullet}10^{-13}$	$6.2{\bullet}10^{-14}$
B5	$5{\bullet}10^{-10}$	$6{\bullet}10^{-10}$	0.	0.
B6	$9{\bullet}10^{-10}$	$6{\bullet}10^{-10}$	0.	0.
B7	0.0163	0.0030	$5.5{\bullet}10^{-15}$	$5.0{\bullet}10^{-15}$
B8	1.65	1.62	$4.8{\bullet}10^{-12}$	$3.1{\bullet}10^{-12}$
B9	0.0051	0.0056	0.	0.

Table 15: Probabilities of failure for Mode 4 (10^{-2}) - Strand©

Section		CFRP		Steel	
		P_F	P_M	P_F	P_M
B1	**SP**	0.17	0.10	8.28	6.55
B2	**SP**	7.79	5.62	0.20	0.034
	SU	5.36	3.72	7.77	6.25
B3	**SP**	0.43	0.21	1.81	0.78
	SU	2.61	1.68	6.07	4.36
B4	**SP**	0.17	0.085	0.0018	0.
	SU	0.25	0.164	3.22	2.54
B5	**SP**	0.00011	$6.3{\bullet}10^{-5}$	$1.7{\bullet}10^{-5}$	$3.0{\bullet}10^{-6}$
B6	**SP**	$1.9{\bullet}10^{-4}$	$9{\bullet}10^{-5}$	$3.5{\bullet}10^{-5}$	$3.0{\bullet}10^{-5}$
	SU	0.0550	0.0388	$9.7{\bullet}10^{-6}$	$1.6{\bullet}10^{-6}$
B7	**SP**	0.052	0.0312	0.0028	0.0012
	SU	1.53	0.931	0.142	0.115
B8	**SP**	0.08	0.12	0.0027	0.00032
	SU	0.08	0.06	2.58	1.77
B9	**SP**	0.01	0.007	0.0004	0.0003

Table 16: Sensitivity factors in % of CFRP α_C and steel α_S RC beams – Modes 1 and 2 - Strand©

Variable	Mode 1		Mode 2	
	α_C	α_S	α_C	α_S
f_c	58.68	0.33	0.45	32.75
ε_{cu}	18.9	-	-	-
E_{ct}	-	-	0.079	-
$E_{f\|s}$	1.9	-	7.02	-
f_y	-	68.32	-	17.11
$A_{f\|s}$	0.66	1.78	0.003	-
$A_{fv\|sv}$	-	-	2.14	0.54
d	12.98	6.76	5.38	10.85
s_L	-	-	-	-
s_T	-	-	3,00	0.65
C	-	-	-	-
w_{DL}	6.00	18.33	18.48	11.60
w_{LL}	0.88	4.48	63.45	26.50

Table 17: Sensitivity factors in % for CFRP α_C and steel α_S RC beams – Modes 3 and 4 - Strand©

Variable	Mode 3		Mode 4	
	α_C	α_S	α_C	α_S
f_c	1.85	3.22	0.03	0.33
ε_{cu}	-	-	-	-
E_{ct}	0.42	2.29	-	-
$E_{f\|s}$	3.27	18.62	1.43	9.64
f_y	-	-	-	-
$A_{f\|s}$	1.18	1.08	0.52	0.56
$A_{fv\|sv}$	-	-	-	-
d	11.30	25.61	41.40	42.73
s_L	-	-	0,08	0.
s_T	-	-	-	-
C	-	-	47.86	37.04
w_{DL}	63.35	42.35	6.71	8.48
w_{LL}	18.63	6.83	1.97	1.22

6 DISCUSSION

Regarding failure due to bending (Mode 1), the reliability indexes for the CFRP RC beams ranged from 3.42 to 5.59, while for the steel RC, this interval is narrower, from 4.09 to 5.98.

One major factor contributing to lower reliability indexes of CFRP reinforced beams is the concrete compressive strength average $\mu f'_c$ considered in the reliability analysis. Although the specified strength is the same (30 MPa), the mean values differ because ACI 318-05 and ABNT NBR 6118:2014 specifies different equations to calculate the nominal strength from experimental data. Since no experiments were performed, the mean values are determined from the specified strength, which led to 34.6 e 39.8 MPa for CFRP and steel RC members, respectively. Thus, the probability that the actual strength is smaller than 30 MPa is higher for the concrete of CFRP RC members. Therefore, reliability indexes are lower and probabilities of failure, higher.

Moreover, the computed sensitivity factors shown in Table 16 reveal that differently from the steel RC beams, the concrete properties significantly contribute to the probability of failure due to the bending of CFRP RC members. It may be a concern for beams with reinforcement ratio close to the balanced ratio. If the actual compressive strength is much higher, the section can fracture due to FRP rupture instead of concrete crushing. Thus, the design considerations would no longer be valid.

Although the steel RC members were designed to fail due to the crushing of concrete, the reinforcement properties were the ones that most contributed to bending rupture, as shown in Table 16. Higher yield strengths, for example, could compromise the ductility of sections, leading to brittle failure, which would require extra reliability [8]. The same may occur to the beams with CFRP if the actual strength and stiffness of the reinforcement is much higher than the specified project values. Although both concrete and CFRP are fragile, the curvature of sections would be lower, not providing enough warning as regards the structure conditions.

Regarding failure due to shear (Mode 2), the reliability indexes varied from 5.85 to 18.7 for CFRP reinforced beams and from 6.64 to 12.4 for the ones with steel. The results were quite similar for most beams; however, the concrete compressive strength almost did not contribute to the failure of CFRP RC beams. For the ones with steel, this contribution is considerably greater. The reinforcement, in turn, has a small contribution for both types of materials.

Sensitivity factors α referring to shear failure show that material properties do not have great influence on failure for the two types of beams with the lowest reliability indexes. The greatest contribution comes from the dead and live loads. For the beams with high reliability, it was found that the position of reinforcement d is the one most contributing to failure of CFRP RC members, while for S RC ones, the concrete compressive $f'c$ and steel yield strength f_y have the greatest contribution.

Regarding the acceptability of the reliability indexes and probabilities of failure, the JCSS (Joint Committee on Structural Safety) establishes that the minimum reliability index for typical design situations including residential buildings is 4.2 for the ultimate limit states. This is the target index for structures whose consequences of failure include risk to life and considerable economic damages [15].

Table 1 allows observing that four CFRP RC sections have reliability indexes below the target value, while just one steel RC does. In these particular cases, more safety can be provided by carefully adding extra reinforcement area. However, the designer should pay close attention to the curvatures of sections, not allowing the strain in the reinforcement to fall below the limit of 0.005 established by [2]. Otherwise, the rupture is no longer ductile, and a higher reliability index is required if considering that sudden failures result in higher costs [8].

It is also possible to achieve higher reliability indexes for these sections by improving material quality control, which will result in specimens with lower variations in strength and, consequently, lower probabilities of failure.

The EN 1990 Eurocode also defines a target reliability index with regard to the ultimate limit states. For the case in study, the target is 4.7 [10]. Seven CFRP RC sections did not meet this standard with regard to bending failure, while four indexes concerning steel RC sections are below the target. Adding more reinforcement area may not be a good alternative. Perhaps, calibrating the resistance safety factors to achieve the target reliability indexes of EN 1990 constitutes an alternative.

Concerning the serviceability limit states, the CFRP RC beams reliability indexes ranged from 0.52 to 6.80 for deflections and from 1.42 to 4.63 for cracking. For the steel RC beams, this interval was 1.18 to 9.06 for deflections and 1.39 to 5.20 for cracking. The dead and live loads were the ones that most contributed to the noncompliance of the serviceability requirements related to the maximum allowed deflection for both CFRP and steel reinforcement. The material properties and position of reinforcement *d*, in turn, have lower contribution. Yet, large variations in these parameters may decrease the effective moment of inertia, causing large deflections.

In relation to the noncompliance of the maximum crack width imposed by the design codes, the major variables contributing to failure was model uncertainty *C* related to the Frosch Equation and the position of reinforcement *d* for both types of materials. Position *d* affects the tension stress in the reinforcement as well as its cover. Therefore, if imprecisions during construction makes *d* higher than the nominal value, wider cracks will appear.

Concerning the acceptability of the reliability indexes related to the serviceability limit states, the JCSS establishes a target value of 1.7 for a reference period of one year, considering that the limit states are irreversible and the cost to improve safety is normal [15].

Table 12 shows that just one CFRP RC and one Steel RC beam did not meet this standard for Mode 3; nevertheless two CFRP RC and four steel RC members had reliability indexes below the target for Mode 4. This result was already expected since the maximum crack width for steel RC members is considerably smaller than the one corresponding to CFRP.

The EN 1990 Eurocode is more conservative for the serviceability limit states. The target reliability index is 2.9 under the same conditions [10]. Three CFRP and one Steel RC beam did not meet this standard for Mode 3, while six CFRP and seven Steel RC sections had indexes below the target for Mode 4.

Regarding the methods of analysis, considerable discrepancies occurred for Mode 2 (shear failure). The high non-linearity of the LS functions associated to very small probabilities of failure (smaller than 10^{-10}) causes FORM to converge to a value $G(\mathbf{X}) > 0$, which leads the probability of failure to be smaller than its actual value. Moreover, when using Monte Carlo Simulation, the program Strand© did not compute most of the probabilities related to Mode 2. Some of which were displayed as zero and others not even calculated. Nevertheless, the reliability indexes provided by FORM are satisfactory for the purposes of this paper, since the values to which $G(\mathbf{X})$ converged is too small in comparison to the approximated mean $G(\boldsymbol{\mu}_X)$.

7 CONCLUSIONS

In face of the differences between the two types of reinforcement, a reliability analysis accounting for the variability of design parameters was crucial to identify the safety levels of concrete members reinforced with CFRP bars.

The results show that the probabilities of failure related to the ultimate limit states are, on average, higher for CFRP RC members. The variables with the greatest contributions were compressive strength and ultimate strain of concrete for bending, and the CFRP Young Modulus for shear. Steel yield strength is the variable with greatest responsibility for failure due to the bending of steel RC beams. Regarding shear, the concrete compressive strength has the greatest importance. Moreover, CFRP RC members are more likely to exhibit excessive deflections, while the probability to present crack widths above the permissible limit is smaller.

As the reliability indexes of the CFRP RC beams are lower, the costs to provide great safety levels may be considerable. However, if compared to the costs of maintenance and repair inherent of conventional steel RC members, replacing one type of reinforcement with another can be compensating. Therefore, more research on the use of CFRP bars as flexural and shear reinforcement is necessary for reducing the uncertainties concerning design models and variables.

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