10^{th} International Conference on Fracture Mechanics of Concrete and Concrete Structures
FraMCoS-X
G. Pijaudier-Cabot, P. Grassl and C. La Borderie (Eds)
https://doi.org/10.21012/FC10.235522

EFFECT OF INTERFACIAL TRANSITION ZONE ON FRACTURE ENERGY IN CONCRETE

DINESH K. SAMAL*, SONALISA RAY† AND T. HEMALATHA‡

*Indian Institute of Technology Roorkee
Roorkee, India
e-mail: dsamal@ce.iitr.ac.in

†Indian Institute of Technology Roorkee
Roorkee, India
e-mail: sonarfce@iitr.ac.in

‡CSIR-Structural Engineering Research Centre
Chennai, India
e-mail: hemalatha@serc.res.in

Key words: Fracture Energy, Interfacial Transition Zone, Dimensional Analysis, Theory of Intermediate Asymptotic

Abstract. Concrete is a three phase material comprising of aggregates, matrix and interface transition zone (ITZ) between the aggregate and matrix. In general, the microstructure of ITZ is different than that of bulk mortar and is considered to be the weakest link in concrete. It is important to understand the properties of ITZ and its effect on physical, mechanical and fracture properties of concrete. In this work, an attempt has been made to develop an analytical model to investigate the fracture properties in concrete namely, critical fracture energy while taking into account the effect of interfacial transition zone at mesoscopic level. The mathematical model has been developed from the fundamentals using the principle of dimensional analysis and theory of intermediate asymptotic. Various important parameters, which influence the fracture energy, are volume fraction and elastic modulus of constituents, maximum aggregate size, water to cement ratio and thickness and porosity of ITZ and have been considered in this study. The thickness and porosity of the transition zone has been analyzed as a function of degree of hydration and is seen to have significant effect on fracture energy. Results have shown that the fracture energy of concrete increase with the increase in aggregate size whereas decreases with the increase in transition zone porosity. The effect of water to cement ratio on fracture energy of concrete has also been studied and presented in the paper.

1 INTRODUCTION

It has been conclusively proven that the use of fracture mechanics can furnish much more efficient design of concrete structures than our conventional design practices against brittle cracking. The most vital feature of fracture mechanics is arguably the phenomenon known as ”The Size-Effect”. Size effect is defined as variation of nominal stress (σ_n) corresponding to ultimate load with structural sizes. Existence of considerable size of fracture process zone (FPZ) in concrete is perceived as the root cause of this phenomenon. Several attempts have been made to describe size efect behaviour in concrete. Few of the important fracture models explaining size effect phenomenon were given such as

Size effect model [1, 2], Two parameter model (TPM) [3] and Effective crack model(ECM) [4]. All these models are approximately equal and return similar results [2, 8, 9]. One of the parameters from the size effect model given by Bazant and Kazemi is size-independent fracture energy (G_f). Size independent fracture energy, as the name suggests is free from the structural shape and geometry.

Planas and Elices [10] studied the relation between fracture energy(G_F) corresponding to complete stress-separation curve of the cohesive crack model to the fracture energy (G_f) associated with area under the initial tangent of the stress separation curve in size effect model and were able to conclude that $G_F/G_f \approx 2.0 - 2.5$.

Due to heterogeneous nature of concrete, size independent fracture energy is dependent on various parameters of its different phases. Concrete is considered to be composed of three different phases viz. cement paste, aggregate and the interfacial transition zone (ITZ) surrounding the aggregate. Interfacial transition zone is a few micrometer thick (around 20µm-50µm) layer around the aggregate which has a cement paste with higher porosity than the bulk cement paste. This is due to a well known phenomenon known as ”The Wall Effect”. Cement particles which are in close vicinity to the aggregate are relatively small and thus the aggregate near the cement particles can be assumed to have an effect of wall. Thus the cement particles near the aggregate boundary arrange themselves to align with boundary itself resulting into a less compact and a more porous arrangement of cement particles around the aggregate. The several parameters dictating the size independent fracture energy in concrete are compressive strength, elastic modulus of composite as well as of its phases, tensile strength, water to cementitious materials ratio, max size of aggregate, volume fraction of the different phases, transition zone porosity and the unit weight of concrete [11–17].

Several attempts have been made to formulate a relation of fracture energy with the various dependent parameters [11, 18, 19]. These attempts were recognized as too rudimentary as they don't take into account all the parameters and are empirical in nature as well as the heterogeneity due to different concrete phases. Few of these attempts are described below.

Bažant and Oh [18] estimated mean fracture energy from direct tensile strength data (f'_t) for notched specimens which is described herein Equation 1. The setback for the formulation is that it was established on very little test data.

$$G_f = (2.72 + 0.0214 f'_t) f'^2_t \frac{d_a}{E} \quad (1)$$

Another mean estimate for fracture energy (G_F) was given by *comite$'$ Euro-International du Be$'$ton* in its CEB-FIP model code [19]:

$$G_F = (0.0469 d_a^2 - 0.5 d_a + 26)(\frac{f'_c}{10})^{0.7} \quad (2)$$

where f'_c and d_a are compressive strength and maximum size of aggregate respectively.

In an another attempt, Bažant and Giraudon [11] came up with their statistical prediction for fracture energy and are reported below.

$$G_f = \alpha_0 (f'_c/0.051)^{0.46} (1 + d_a/11.27)^{0.22} (w/c)^{0.30} \quad (3)$$

$$G_F = 2.5\alpha_0 (f'_c/0.051)^{0.46} (1 + d_a/11.27)^{0.22} (w/c)^{0.30} \quad (4)$$

Equations 3 and 4 have been reported with the coefficients of variation of 17.8% and 29.9% respectively, for the predictions from the formulae when compared to the test data. Although it was able to take into account the effect of compressive strength, water to cement ratio and the maximum size of aggregate, it hasn't taken the effect of transition zone which plays an important role in describing the crack path in fracture of concrete, and thus influences fracture energy too.

In this paper, an attempt has been made to derive an analytical expression for fracture energy taking into account the effect of ITZ and other important parameters such as compressive strength , water to cement ratio and volume fraction of different constituent phases, ITZ properties and their elastic moduli. Dimensional analysis and the theory of intermediate asymptotic have been used to derive closed form expression. Dependencies of involved parameters on fracture energy have been verified through existing experimental and analytical studies.

2 GOVERNING PARAMETERS

Concrete is considered to be a three phase material in which the overall material properties are governed by different phases present. Spatial arrangement and strength of phases encountered along the crack dictates its path and energy dissipated [20]. Therefore, it is reasonable to state that all the phases would have their influence on fracture energy which can be considered as a material property.

Various important parameters affecting fracture energy are compressive strength (f_c), water to cementitious materials ratio (w/cm), maximum size of aggreagte (d_a), volume fraction and elastic moduli values of different phases ($V_x,\ E_x,\ x = a,\ m,\ i,\ c;\ where\ a = aggregate,\ m = matrix,\ i = aggregate,\ c = concrete$). In addition, two other important parameters viz. thickness (t_i) and porosity (η_i) of ITZ are also expected to influence its own strength and consequently affecting the fracture energy. Experimental results from Tasdemir et al. [16] has shown that fracture energy increases with modulus of concrete. Several studies [30–33] have shown the influence of elastic modulus of transition zone on elastic modulus of concrete. The elastic modulus of interfacial transition zone can be estimated using inverse analysis as was demonstrated by Hashin and Monteiro [24]. Analytical expression provided by Lu and Torquato in their study [25] for polydispersed-sphere systems with any size distribution can be employed to compute volume fraction of ITZ in concrete. Figure 1 shows the schematic representation of quantities in polydispersed system. The volume fraction of matrix outside the spheres and shells of thickness r in polydispersed systems is represented as $ev(r)$ and has been measured by authors [25]. In case of concrete, the quantity $ev(r)$ would correspond to $ev(t_i)$ as shells are considered transition zone and can be computed as below.

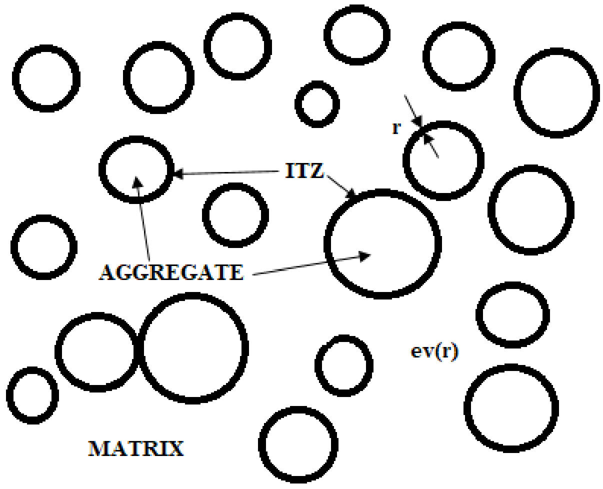

Figure 1: Schematic view of polydispersed systems.

$$ev(r) = (1 - V_a)e^{-\pi\rho(cr+dr^2+gr^3)} \tag{5}$$

where V_a is the volume fraction of aggregates in the concrete, ρ is total number of particles per unit volume; c, d, g are defined in terms of number averages of the aggregate radius and aggregate radius squared and are defined below.

$$c = \frac{4 < R^2 >}{1 - V_a} \tag{6}$$

$$d = \frac{4 < R >}{1 - V_a} + \frac{12Z_2 < R^2 >}{(1 - V_a)^2} \tag{7}$$

$$g = \frac{4}{3(1-V_a)} + \frac{8Z_2{<}R{>}}{(1-V_a)^2} + \frac{16AZ_2^2{<}R^2{>}}{3(1-V_a)^3} \tag{8}$$

where, $Z_2 = 2\pi\rho < R^2 > /3, A = 0,\ 2,\ 3$ depending upon approximation chosen in theory [25] and $<>$ denotes the number average over any size distribution of aggregates. Volume fraction contribution from the term controlled by A is too small to make much difference [26]. Thus for concrete, considering A = 0 would be the appropriate choice.

Calculation of thickness of ITZ between aggregate and matrix can be done following the concept developed by Zheng et al. [21] for cement paste. Cement density function ($D_c(x)$) defined as the ratio of the sum of length of cement particles intersected at distance x to the length of square side a. Concept of cement density function can be utilized to compute porosity of the transition zone in the following manner. Integrating cement density function between the surface of aggregate and transition zone thickness can be represented as a limit of a sum as provided below.

$$\int_0^{t_i} D_c(x)dx = \Delta x \lim_{\Delta x \to \infty} [D_c(0) + D_c(\Delta x) + D_c(2\Delta x) + ... + D_c((n-1)\Delta x)] \quad (9)$$

where Δx is the thickness of assumed number of narrow strips within the transition zone for expressing the integral as a limit of sum as shown above.

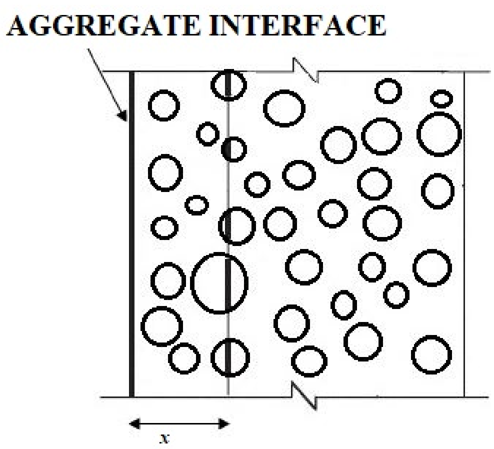

Figure 2: Cement density function ($D_c(x)$) [21].

If Δx is very small, total area of cement particles (A_c) in the ITZ can be written as:

$$A_c = [D_c(0) + D_c(\Delta x) + D_c(2\Delta x) + ... + D_c((n-1)\Delta x)].a.\Delta x \quad (10)$$

Comparing Equations 9 and 10 total area of cement particles is expressed as,

$$A_c = a. \int_0^{t_i} D_c(x)dx \quad (11)$$

Area porosity for transition zone can then be calculated as:

$$\text{Area porosity of transition zone} = \frac{a.t_i - A_c}{at_i} \quad (12)$$

Area porosity for two-dimensional plane-section through concrete can be rescaled to volume porosity for a three-dimensional media by using rescaling relations given by Virgin et al. [27]. Transition zone porosity can also be computed using the approach proposed by Zheng et al. [28]. It has been considered to be a function of water to cement ratio and degree of hydration.

3 MATHEMATICAL FORMULATION AND DISCUSSION

In order to develop a theoretical expression for the prediction of fracture energy, dimensional analysis method has been adopted [29].Critical fracture energy which is defined as the energy release rate for the crack growth is considered to be a material property. Concrete being a three phase material, critical fracture energy is a function of several aforementioned parameters and is represented as follows:

$$G_f = \phi(d_a, f_c, w/cm, t_i, \eta_i, V_x, E_x)....x = a, m, i, c \quad (13)$$

One could foresee from the first observation on the involved parameters, that there can be several non-dimensional quantities which could be formed using each of these parameters such as $G_f/f_c.d_a$, $G_f/f_c.t_i$,$G_f/E_x.d_a$, $G_f/E_x.t_i$, f_c/E_x, E_x/E_x, V_x/V_x $(x(Numerator) \neq x(denominator))$, w/cm, V_x, η_i, t_i/d_a. To take into account the effect of all the involved parameters into the formulation, such non-dimensional parameters are taken which could give a better insight to the problem in hand. Equation. 13 can be expressed in terms of different non-dimensional forms (π-terms) and is provided below.

$$\begin{aligned}
\pi &= \Phi(\pi_1, \pi_2, \pi_3, \pi_4) \\
\pi &= G_f/f_c.d_a \\
\pi_1 &= \sum_{x=a,i,m} (V_x.E_x)/E_c \\
\pi_2 &= t_i/d_a \\
\pi_3 &= w/cm \\
\pi_4 &= \eta_i
\end{aligned} \quad (14)$$

The first π-term (π_1) represents the relative volumetric weighted contribution from different phases in the elastic modulus of concrete. The Second pi-term (π_2) is aspect ratio of the projected area of the transition zone. This can be explained using a spherical aggregate with ITZ concentrically placed around it as shown in Figure 3. The value of parameter π_2 changes with size and the shape of the aggregate. The parameter π_2 can be related to the volume of the interfacial transition zone in concrete. This stands correct even when the aggregates are non-spherical.

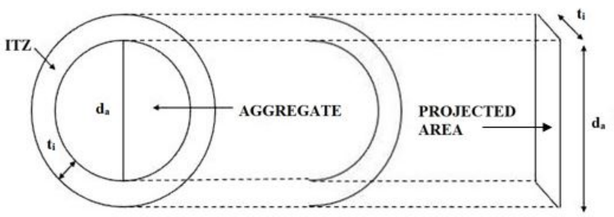

Figure 3: Description of π_2 using a spherical aggregate

In the present study, estimation of critical fracture energy has been done for the intermediate range of w/cm ratios keeping it independent from the initial and final boundary conditions.

In this context, the value of non-dimensional quantity π_2 is very small i.e. the ITZ thickness is much smaller than that of the maximum size of aggregate. The cement density function is used to calculate the value of t_i for π_2-term. Experimental results [12,22] has been used to calculate the values of non-dimensional parameter π_2. Further, the dependency of π on the quantity π_2 has been shown in Fig. 4. Incomplete similarity can be assumed for π_2 and thus the first equation of Equation 14 would reduce to

$$\pi = \pi_2^{\alpha}.\Phi_1(\pi_1, \pi_3, \pi_4) \tag{15}$$

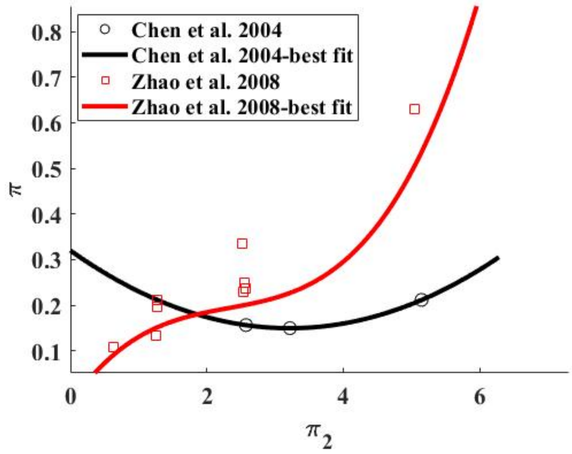

Figure 4: Variation of π with π_2.

According to the study by Bažant and Giraudon [11] fracture energy shows a power-law relationship with water to cementitious materials ratio which is π_3-term herein. Also experimental results from the literature [14,22,23] are depicted in Figure. 5 has shown the same kind of relationship.

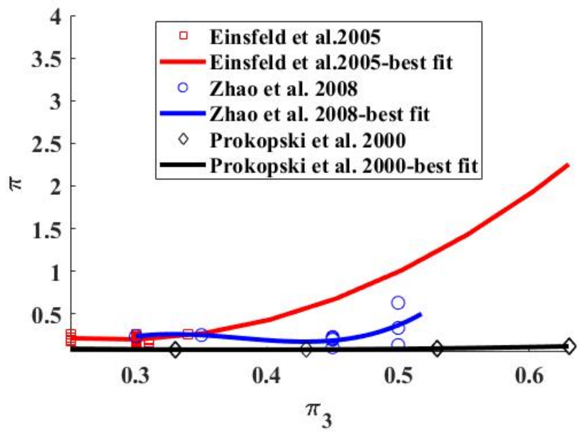

Figure 5: Variation of π with π_3.

Thus after assuming incomplete self-similarity for π_3, the expression 15 would lead to:

$$\pi = \pi_2^{\alpha}.\pi_3^{\beta}.\Phi_1(\pi_1, \pi_4) \tag{16}$$

On simplification eq. 16, reduces to the following form.

$$G_f = f_c.(d_a)^{1-\beta}.(w/cm)^{\alpha}.(t_i)^{\beta}\Phi_1(\pi_1, \pi_4) \tag{17}$$

where α and β are functions of similarity parameters π_1 and π_4.

From the literature [13, 14], it has been concluded that there is a direct relationship between fracture energy(G_f) and water to cementitious materials ratio (w/cm). The parameter α thus should be greater than 0 in order to establish the same kind of relation in formulation.

Furthermore, we know that fracture energy increases with increasing aggregate size [12,15, 17] and decreasing transition zone thickness as ITZ is considered as the weak link. Thus value of β should be negative in order for formulation to comply with experimental observations.

4 CONCLUSION

A mathematical framework has been proposed for size-independent fracture energy of concrete which takes into account various concrete parameters and the influence of properties of all the three phases viz. matrix, aggregate and interfacial transition zone using the principle of dimensional analysis and the theory of intermediate asymptotic. The initial and boundary conditions of a process were defined for applying the theory of intermediate asymptotic in the paper. Incomplete self-similarity has been assumed for the parameters π_2 and π_3 in order to obtain the present formulation. The exponents α and β in the derived formulation will be quantified by best-fitting the data acquired from experimental and analytical studies.

REFERENCES

[1] Bažant, Z.P., and Pfeiffer,P.A. 1987. Determination of fracture energy from size effect and brittleness number. *ACI Materials Journal.* **84(6)**:463–480.

[2] Bažant, Z.P., and Planas, J. 1998. *Fracture and Size Effect in Concrete and Other Quasibrittle Materials*, CRC Press, Boca Raton, FL.

[3] Jenq, Y.S., and Shah,S.P. 1985. Two parameter fracture model for concrete. *Journal of Engineering Mechanics.* **111(10)**:1227–1241.

[4] Karihaloo, B.L., and Nallathambi,P. 1989. An improved effective crack model for the determination of fracture toughness of concrete. *Cement and Concrete Research.* **19**:603–610.

[5] Karihaloo, B.L., and Nallathambi,P. 1989. Fracture toughness of plain concrete from three-point bend specimens. *Materials and Structures.* **22**:185–193.

[6] Karihaloo, B.L., and Nallathambi, p. 1991. *Notched beam test: Mode I fracture toughness*, Chapman & Hall, London (2010, 1-86 pp.).

[7] Nallathambi,P., and Karihaloo, B.L. 1986. Determination of specimen-size independent fracture toughness of plain concrete. *Magazine of Concrete Research.* **38(135)**:67–76.

[8] Bažant, Z.P. 1993. Discussion of 'Fracture mechanics and size effect of concrete in tension'. *Journal of Structural Engineering,American Society of Civil Engineers.* **119(8)**:2555–2558.

[9] Ouyang, C., and Tang, T. 1996. Relationship between fracture parameters from two parameter model and from size effect model. *Materials and Structures.* **29**:76–86.

[10] Planas, J., and Elices, M. 1990. Fracture Criteria for Concrete:Mathematical Approximation and Experimental Validation. *Engineering Fracture Mechanics.* **35**:87–94.

[11] Bažant, Z.P., and Becq-Giraudon, E. 2002. Statistical prediction of fracture parameters of concrete and implications for choice of testing standard. *Cement and Concrete Research.* **32(4)**:529–556.

[12] Chen, B., and Liu, J. 2004. Effect of aggregate on the fracture behavior of high strength concrete. *Construction and Building Materials.* **18(8)**:585–590.

[13] Darwin, D., Barham, S., Kozul, R., and Luan, S. 2001. Fracture energy of high-strength concrete. *American Concrete Institute.*

[14] Prokopski, G., and Langier, B. 2000. Effect of water/cement ratio and silica fume addition on the fracture toughness and morphology of fractured surfaces of gravel concretes. *Cement and Concrete Research.* **30(9)**:1427–1433.

[15] Rao, G. A., and Prasad, B. R. 2002. Fracture energy and softening behavior of high-strength concrete. *Cement and Concrete Research.* **32(2)**:247–252.

[16] Tasdemir, M. A., and Karihaloo, B. L. 2001. *Effect of type and volume fraction of aggregate on the fracture properties of concrete: Fracture Mechanics of Concrete Structures*, Swets & Zeitlinger, Lisse, The Netherlands (90,825 pp.).

[17] Tasdemir, C., Tasdemir, M. A., Lydon, F. D., and Barr, B. I. 1996. Effects of silica fume and aggregate size on the brittleness of concrete. *Cement and Concrete Research.* **26(1)**:63–68.

[18] Bažant, Z. P., and Oh, B. H. 1983. Crack Propagation and The Fracture of Concrete. *Mate′riaux et construction* **16(3)**:155–177.

[19] *Comite′ Euro-International du Be′ton, CEB–FIP Model Code 1990*, Thomas Telford, London (1991).

[20] Yan, A., Wu, K. R., Zhang, D., and Yao, W. 2001. Effect of fracture path on the fracture energy of high-strength concrete. *Cement and Concrete Research.* **31(11)**:1601–1606.

[21] Zheng, J. J., Li, C. Q., and Zhou, X. Z. 2005. Effect of fracture path on the fracture energy of high-strength concrete. *Magazine of Concrete Research.* **57(7)**:397–406.

[22] Zhao, Z., Kwon, S. H., and Shah, S. P. 2008. Effect of specimen size on fracture energy and softening curve of concrete: Part I. Experiments and fracture energy. *Cement and Concrete Research.* **38(8-9)**:1049–1060.

[23] Einsfeld, R. A., and Velasco, M. S. 2006. Fracture parameters for high-performance concrete. *Cement and Concrete Research.* **36(3)**:576–583.

[24] Hashin, Z., and Monteiro, P. J. M. 2002. An inverse method to determine the elastic properties of the interphase between the aggregate and the cement paste. *Cement and Concrete Research.* **32(8)**:1291–1300.

[25] Lu, B., and Torquato, S. 2002. Nearest-surface distribution functions for polydispersed particle systems. *Physical Review A* **45(8)**:5530–5544.

[26] Garboczi, E. J., and Bentz, D. P. 1997. Analytical formulas for interfacial transition zone properties. *Advanced Cement Based Materials* **6(3-4)**:99–108.

[27] Virgin, B., Haslund, E., and Hilfer, R. 1996. Rescaling relations between two- and three-dimensional local porosity distributions for natural and artificial porous media. *Physica A: Statistical Mechanics and its Applications.* **232(1-2)**:1–20.

[28] Zheng, J., Zhou, X., and Jin, X. 2012. An n-layered spherical inclusion model for predicting the elastic moduli of concrete with inhomogeneous ITZ. *Cement and Concrete Composites.* **34(5)**:716–723.

[29] Ruzicka, M. C. 2008. On dimensionless numbers.*Chemical Engineering Research and Design.* **86(8)**:835–868.

[30] Zheng, J. J., Li, C. Q., and Zhou, X. Z. 2006. An analytical method for prediction of the elastic modulus of concrete.*Magazine of Concrete Research.* **58(10)**:665–673.

[31] Lee, K. M., and Park, J. H. 2008. A numerical model for elastic modulus of concrete considering interfacial transition zone.*Cement and Concrete Research.* **38(3)**:396–402.

[32] Ramesh, G., Sotelino, E. D., and Chen, W. F. 1996. Effect of transition zone on elastic moduli of concrete materials.*Cement and Concrete Research.* **26(4)**:611–622.

[33] Yang, C. C. (1998). Effect of the transition zone on the elastic moduli of mortar.*Cement and Concrete Research.* **28(5)**:727–736.